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Efficient schemes for deterministic joint remote preparation of an arbitrary four-qubit W-type entangled state

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Abstract. We present three schemes for the joint remote state preparation (JRSP) of an arbitrary four-qubit W-type entangled state with complex coefficients via four and two three-qubit GHZ states as the quantum channel. In these schemes, two senders (or N senders) share the original state which they wish to help the receiver to remotely prepare. To complete the JRSP schemes, some novel sets of mutually orthogonal basis vectors are introduced. It is shown that, only if two senders (or N senders) collaborate with each other, and perform projective measurements under suitable measuring basis on their own qubits, the receiver can reconstruct the original state by means of some appropriate unitary operations. It is shown that, in all our schemes, the total success probability of the JRSP can reach 1. Specially, compared with the first scheme in our paper, the entanglement resource in the second scheme can be reduced. This means that the scheme is more efficient and economical.

Keywords. Joint remote state preparation; arbitrary four-qubit W-type entangled state; four- and two-qubit projective measurement.

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1. Introduction

Quantum entanglement is one of the most striking features of quantum mechanics. Entangled systems are thus important resources for many quantum information processing schemes including quantum computation and quantum communication [1]. One of the remarkable exhibitions of entanglement is quantum teleportation [2], first proposed by Bennett *et al*, in which an unknown quantum state can be transmitted from a sender (Alice) to a receiver (Bob) with the assistance of a prior shared Einstein–Podolsky–Rosen (EPR) pair and two bits of classical information. In the last decade, another significant application of quantum entanglement, namely remote state preparation (RSP) [3–5], has been presented. Since then, RSP has attracted much attention, various theoretical schemes for generalization of RSP have been proposed and experimental implementation of RSP have been presented [6–27]. Especially, some researchers [22,28–30] have discussed the RSP of four-qubit W-type entangled state.

Recently, a novel aspect of RSP, called the joint RSP (JRSP), has been proposed [31–48]. In JRSP, two senders (or N senders) partly know the original state they wish to remotely prepare. If and only if all the senders agree to collaborate, the receiver can reconstruct the original quantum state. However, there is no scheme for JRSP of an arbitrary four-qubit W-type entangled state (FWES) till now. In this paper, we proposed three schemes for JRSP of an arbitrary FWES with four and two three-qubit GHZ states as quantum channel, respectively. To complete the JRSP schemes, several novel sets of four-qubit measuring basis are introduced. In these schemes, two senders (or N senders) share the original state, but each sender only partly knows the state. It is shown that, if and only if two senders (or N senders) agree to collaborate, the receiver can reconstruct the original quantum state. Moreover, it is shown that, in the three schemes presented here, the total successful probability of JRSP can reach 1. This paper is organized as follows. In §2, the joint remote preparation of a four-qubit W-type state with four three-qubit GHZ states as quantum channel is presented. Section 3 presents the scheme for JRSP with two three-qubit GHZ states as the quantum channel. In §4, the JRSP with N senders is discussed. Conclusions are given in §5.

2. JRSP of an arbitrary FWES with four three-qubit GHZ states as quantum channel

Suppose that two senders Alice and Bob wish to help the receiver Charlie remotely to prepare the state

$$|\psi\rangle = x_0\,|0\,0\,0\,1\rangle + x_1 e^{i\delta_1}\,|0\,0\,1\,0\rangle + x_2 e^{i\delta_2}\,|0\,1\,0\,0\rangle \\ + x_3 e^{i\delta_3}\,|1\,0\,0\,0\rangle, \tag{1}$$

where x_j and δ_j $(j = 0, 1, 2, 3)$ are real, $\delta_0 = 0$ and $\sum_{j=0}^{3} x_j^2 = 1$. Assume that Alice and Bob share the state $|\psi\rangle$ and they know the state partly, that is Alice knows x_j $(j = 0, 1, 2, 3)$ and Bob knows δ_j $(j = 0, 1, 2, 3)$, but Charlie does not know them at all. We also suppose that the states shared by Alice, Bob, and Charlie as quantum channel are four GHZ states

$$|\varphi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\,0\,0\rangle + |1\,1\,1\rangle\right)_{A_1B_1C_1},$$

$$|\varphi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\,0\,0\rangle + |1\,1\,1\rangle\right)_{A_2B_2C_2},$$

$$|\varphi_3\rangle = \frac{1}{\sqrt{2}}\left(|0\,0\,0\rangle + |1\,1\,1\rangle\right)_{A_3B_3C_3},$$

$$|\varphi_4\rangle = \frac{1}{\sqrt{2}}\left(|0\,0\,0\rangle + |1\,1\,1\rangle\right)_{A_4B_4C_4}, \tag{2}$$

where the qubits A_i $(i = 1, 2, 3, 4)$ belong to Alice, qubits B_i $(i = 1, 2, 3, 4)$ to Bob, and qubits C_i $(i = 1, 2, 3, 4)$ to Charlie, respectively.

In order to complete the JRSP, Alice and Bob should construct their measuring bases. The first measuring bases chosen by Alice are two sets of mutually othogonal basis vectors (MOBVs) $\{|\lambda_k^i\rangle\}$ $(k = 0, 1, \ldots, 7;\ i = 1, 2)$, which are given by

$$\left(|\lambda_0^{(1)}\rangle, |\lambda_1^{(1)}\rangle, |\lambda_2^{(1)}\rangle, |\lambda_3^{(1)}\rangle, |\lambda_4^{(1)}\rangle, |\lambda_5^{(1)}\rangle, |\lambda_6^{(1)}\rangle, |\lambda_7^{(1)}\rangle\right)^T \\ = F\left(|\xi_0^{(1)}\rangle, |\xi_1^{(1)}\rangle, |\xi_2^{(1)}\rangle, |\xi_3^{(1)}\rangle, |\xi_4^{(1)}\rangle, |\xi_5^{(1)}\rangle, |\xi_6^{(1)}\rangle, |\xi_7^{(1)}\rangle\right)^T, \\ \left(|\lambda_0^{(2)}\rangle, |\lambda_1^{(2)}\rangle, |\lambda_2^{(2)}\rangle, |\lambda_3^{(2)}\rangle, |\lambda_4^{(2)}\rangle, |\lambda_5^{(2)}\rangle, |\lambda_6^{(2)}\rangle, |\lambda_7^{(2)}\rangle\right)^T \\ = F\left(|\xi_0^{(2)}\rangle, |\xi_1^{(2)}\rangle, |\xi_2^{(2)}\rangle, |\xi_3^{(2)}\rangle, |\xi_4^{(2)}\rangle, |\xi_5^{(2)}\rangle, |\xi_6^{(2)}\rangle, |\xi_7^{(2)}\rangle\right)^T, \tag{3}$$

where

$$F = \begin{pmatrix} x_0 & x_1 & x_2 & 0 & x_3 & 0 & 0 & 0 \\ x_1 & -x_0 & 0 & -x_2 & 0 & -x_3 & 0 & 0 \\ x_2 & 0 & -x_0 & x_1 & 0 & 0 & x_3 & 0 \\ 0 & x_2 & -x_1 & -x_0 & 0 & 0 & 0 & -x_3 \\ x_3 & 0 & 0 & 0 & -x_0 & x_1 & -x_2 & 0 \\ 0 & x_3 & 0 & 0 & -x_1 & -x_0 & 0 & x_2 \\ 0 & 0 & -x_3 & 0 & x_2 & 0 & -x_0 & x_1 \\ 0 & 0 & 0 & x_3 & 0 & -x_2 & -x_1 & -x_0 \end{pmatrix} \tag{4}$$

and

$$|\xi_0^{(1)}\rangle = |0\,0\,0\,0\rangle, \quad |\xi_1^{(1)}\rangle = |0\,0\,1\,1\rangle, \\ \xi_2^{(1)}\rangle = |0\,1\,0\,1\rangle, \quad |\xi_3^{(1)}\rangle = |0\,1\,1\,0\rangle, \\ |\xi_4^{(1)}\rangle = |1\,0\,0\,1\rangle, \quad |\xi_5^{(1)}\rangle = |1\,0\,1\,0\rangle, \\ |\xi_6^{(1)}\rangle = |1\,1\,0\,0\rangle, \quad |\xi_7^{(1)}\rangle = |1\,1\,1\,1\rangle, \\ |\xi_0^{(2)}\rangle = |0\,0\,0\,1\rangle, \quad |\xi_1^{(2)}\rangle = |0\,0\,1\,0\rangle, \\ |\xi_2^{(2)}\rangle = |0\,1\,0\,0\rangle, \quad |\xi_3^{(2)}\rangle = |0\,1\,1\,1\rangle, \\ |\xi_4^{(2)}\rangle = |1\,0\,0\,0\rangle, \quad |\xi_5^{(2)}\rangle = |1\,0\,1\,1\rangle, \\ |\xi_6^{(2)}\rangle = |1\,1\,0\,1\rangle, \quad |\xi_7^{(2)}\rangle = |1\,1\,1\,0\rangle. \tag{5}$$

The second measuring bases chosen by Bob are two sets of MOBVs $\{|\eta_{j(i)}\rangle\}$ $(j = 0, 1, \ldots, 7;\ i = 1, 2)$, which are given by

$$\left(|\eta_{0(1)}\rangle, |\eta_{1(1)}\rangle, |\eta_{2(1)}\rangle, |\eta_{3(1)}\rangle, |\eta_{4(1)}\rangle, |\eta_{5(1)}\rangle, |\eta_{6(1)}\rangle, |\eta_{7(1)}\rangle\right)^T \\ = H\left(|\xi_0^{(1)}\rangle, |\xi_1^{(1)}\rangle, |\xi_2^{(1)}\rangle, |\xi_3^{(1)}\rangle, |\xi_4^{(1)}\rangle, |\xi_5^{(1)}\rangle, |\xi_6^{(1)}\rangle, |\xi_7^{(1)}\rangle\right)^T, \\ \left(|\eta_{0(2)}\rangle, |\eta_{1(2)}\rangle, |\eta_{2(2)}\rangle, |\eta_{3(2)}\rangle, |\eta_{4(2)}\rangle, |\eta_{5(2)}\rangle, |\eta_{6(2)}\rangle, |\eta_{7(2)}\rangle\right)^T \\ = H\left(|\xi_0^{(2)}\rangle, |\xi_1^{(2)}\rangle, |\xi_2^{(2)}\rangle, |\xi_3^{(2)}\rangle, |\xi_4^{(2)}\rangle, |\xi_5^{(2)}\rangle, |\xi_6^{(2)}\rangle, |\xi_7^{(2)}\rangle\right)^T, \tag{6}$$

where

$$H = \begin{pmatrix} 1 & r_1 & r_2 & 0 & r_3 & 0 & 0 & 0 \\ 1 & -r_1 & r_2 & 0 & r_3 & 0 & 0 & 0 \\ 1 & -r_1 & -r_2 & 0 & -r_3 & 0 & 0 & 0 \\ 1 & r_1 & -r_2 & 0 & r_3 & 0 & 0 & 0 \\ 1 & -r_1 & r_2 & 0 & -r_3 & 0 & 0 & 0 \\ 1 & r_1 & -r_2 & 0 & -r_3 & 0 & 0 & 0 \\ 1 & -r_1 & -r_2 & 0 & r_3 & 0 & 0 & 0 \\ 1 & r_1 & r_2 & 0 & -r_3 & 0 & 0 & 0 \end{pmatrix}, \tag{7}$$

where $r_m = \mathrm{e}^{-i\delta_m}$ $(m = 1, 2, 3)$.

Now let Alice first perform four-qubit projective measurement (FPM) on the qubits A_1, A_2, A_3 and A_4 under the basis $\{|\lambda_k^{(i)}\rangle\}$ $(k = 0, 1, \ldots, 7; i = 1, 2)$ and announce her measurement outcome. Next, in accordance with Alice's result, Bob should employ suitable unitary operations on his qubits B_1, B_2, B_3, B_4 and choose one of the measuring bases $\{|\eta_{j(i)}\rangle\}$ $(j = 0, 1, \ldots, 7; i = 1, 2)$ to measure these qubits. After the measurement, Bob informs Charlie of his result by the classical channel. According to Alice and Bob's results, Charlie can reconstruct the original state $|\psi\rangle$ by a suitable unitary operation. For example, without loss of generality, assume Alice's measurement outcome is $|\lambda_5^{(2)}\rangle_{A_1A_2A_3A_4}$, the qubits B_1, B_2, B_3, B_4, C_1, C_2 , C_3, C_4 will be collapsed into the state

$$|p\rangle = \frac{1}{4}(x_3\,|0\,0\,0\,0\,1\,1\,0\,0\rangle - x_1\,|1\,1\,0\,0\,0\,0\,0\,0\rangle \\ - x_0\,|1\,1\,0\,0\,1\,1\,1\,1\rangle \\ + x_2\,|1\,1\,1\,1\,1\,1\,0\,0\rangle)_{B_1C_1B_2C_2B_3C_3B_4C_4}. \tag{8}$$

Then Bob can make the unitary operations $(\sigma_x)_{B_1}$, $(I)_{B_2}$, $(\sigma_x)_{B_3}$, $(I)_{B_4}$ on his qubits B_1, B_2, B_3, B_4, respectively. After that, the state $|p\rangle$ will be transformed into the state

$$|p'\rangle = \frac{1}{4}(-x_0\,|0\,1\,0\,0\,0\,1\,1\,1\rangle + x_1\,|0\,1\,0\,0\,1\,0\,0\,0\rangle \\ - x_2\,|0\,1\,1\,1\,0\,1\,0\,0\rangle \\ + x_3\,|1\,0\,0\,0\,0\,1\,0\,0\rangle)_{B_1C_1B_2C_2B_3C_3B_4C_4}. \tag{9}$$

Next, Bob employs the FPM on the qubits B_1, B_2, B_3, B_4 under the bases $\{|\eta_{j(2)}\rangle\}$ $(j = 0, 1, \ldots, 7)$, and informs Charlie of his result by the classical channel. Assume that the result of Bob's second measurement is $|\eta_{6(2)}\rangle_{B_1B_2B_3B_4}$, the qubits C_1, C_2, C_3, C_4 will be collapsed into the state

$$|p''\rangle = \frac{1}{4}(-x_0\,|1\,0\,1\,1\rangle - x_1\mathrm{e}^{i\delta_1}\,|1\,0\,0\,0\rangle \\ + x_2\mathrm{e}^{i\delta_2}\,|1\,1\,1\,0\rangle \\ + x_3\mathrm{e}^{i\delta_3}\,|0\,0\,1\,0\rangle)_{C_1C_2C_3C_4}. \tag{10}$$

According to Alice and Bob's public announcements, Charlie can perform the local unitary operations $(-i\sigma_y)_{C_1}(I)_{C_2}(\sigma_x)_{C_3}(I)_{C_4}$ on the qubits C_1, C_2, C_3 and C_4, the desired state $|\psi\rangle$ can be reconstructed. If Alice's first results are the other 15 cases in the basis $\{|\lambda_k^{(i)}\rangle_{A_1A_2A_3A_4}\}$, Bob should choose appropriate unitary operations on his qubits B_1, B_2, B_3 and B_4, and then measure these qubits under the appropriate measuring bases $\{|\eta_{j(i)}\rangle\}$. The corresponding relation of Alice's first measurement outcomes $\{|\lambda_k^{(i)}\rangle_{A_1A_2A_3A_4}\}$ and Bob's second measuring bases $\{|\eta_{j(i)}\rangle\}$ $(j = 0, 1, \ldots, 7; i = 1, 2)$ can be described as

$$|\lambda_k^{(1)}\rangle\ (k = 0, 1, \ldots, 7) \to |\eta_{j(1)}\rangle\ (j = 0, 1, \ldots, 7), \\ |\lambda_k^{(2)}\rangle\ (k = 0, 1, \ldots, 7) \to |\eta_{j(2)}\rangle\ (j = 0, 1, \ldots, 7). \tag{11}$$

It is easily found that, for all the 128 measurement results of Alice and Bob, the receiver Charlie can reconstruct the original state $|\psi\rangle$ by appropriate unitary operations and the success probability of the JRSP process is 1. The required classical communication cost is eight bits in the scheme.

3. JRSP of an arbitrary FWES with two three-qubit GHZ states as the quantum channel

Now let us further propose a more economic scheme for JRSP of an arbitrary FWES with two three-qubit GHZ states as the quantum channel. Suppose that the state Alice and Bob wish to help Charlie remotely prepare is still in state $|\psi\rangle$ (see eq. (1)) and the states shared by Alice, Bob and Charlie as the quantum channel are two three-qubit GHZ states, which are given by

$$|\varphi_1\rangle = \frac{1}{\sqrt{2}}(|0\,0\,0\rangle + |1\,1\,1\rangle)_{A_1B_1C_1}, \\ |\varphi_2\rangle = \frac{1}{\sqrt{2}}(|0\,0\,0\rangle + |1\,1\,1\rangle)_{A_2B_2C_2}, \tag{12}$$

where the qubits A_1, A_2 belong to Alice, B_1, B_2 to Bob, C_1, C_2 to Charlie, respectively. The first measuring basis chosen by Alice is a set of MOBVs $\{|\nu_k\rangle\}$ $(k = 1, 2, 3, 4)$, which is given by

$$\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \end{pmatrix} = G \begin{pmatrix} |0\,0\rangle \\ |0\,1\rangle \\ |1\,0\rangle \\ |1\,1\rangle \end{pmatrix}, \tag{13}$$

where

$$G = \begin{pmatrix} x_0 & x_1 & x_2 & x_3 \\ x_1 & -x_0 & x_3 & -x_2 \\ x_2 & -x_3 & -x_0 & x_1 \\ x_3 & x_2 & -x_1 & -x_0 \end{pmatrix}. \tag{14}$$

The second measuring basis chosen by Bob is a set of MOBVs $\{|\varepsilon_j\rangle\}\,(j = 1, 2, 3, 4)$, which is given by

$$\begin{pmatrix} |\varepsilon_1\rangle \\ |\varepsilon_2\rangle \\ |\varepsilon_3\rangle \\ |\varepsilon_4\rangle \end{pmatrix} = R \begin{pmatrix} |0\,0\rangle \\ |0\,1\rangle \\ |1\,0\rangle \\ |1\,1\rangle \end{pmatrix}, \tag{15}$$

where

$$R = \begin{pmatrix} 1 & \xi_1 & \xi_2 & \xi_3 \\ 1 & -\xi_1 & \xi_2 & -\xi_3 \\ 1 & -\xi_1 & -\xi_2 & \xi_3 \\ 1 & \xi_1 & -\xi_2 & -\xi_3 \end{pmatrix}, \tag{16}$$

and $\xi_m = \mathrm{e}^{-i\delta_m}$ $(m = 1, 2, 3)$.

Now let Alice first perform the two-qubit projective measurement on her qubits A_1 and A_3 under the basis $\{|\nu_k\rangle\}$ $(k = 1, 2, 3, 4)$ and publicly announce her result. Next, in accordance with Alice's outcome of measurement, Bob should employ suitable unitary operations on his qubits. Without loss of generality, assume that Alice's result is $|\nu_3\rangle_{A_1A_3}$, the qubit B_1, B_2, C_1, C_2 will be collapsed into the state

$$|q\rangle = \frac{1}{2}(x_2|0\,0\,0\,0\rangle - x_3|0\,0\,1\,1\rangle - x_0|1\,1\,0\,0\rangle + x_1|1\,1\,1\,1\rangle)_{B_1C_1B_2C_2}. \tag{17}$$

Bob should make a unitary operation $(-i\sigma_y)$ on his qubit B_1, and the state $|q\rangle$ will be transformed into the state

$$|q'\rangle = \frac{1}{2}(x_2|1\,0\,0\,0\rangle - x_3|1\,0\,1\,1\rangle + x_0|0\,1\,0\,0\rangle - x_1|0\,1\,1\,1\rangle)_{B_1C_1B_2C_2}. \tag{18}$$

Then, Bob measures his qubits B_1 and B_2 under the basis $\{|\varepsilon_j\rangle\}$ $(j = 1, 2, 3, 4)$ and informs Charlie of his result. Assume that Bob's result is $|\varepsilon_2\rangle_{B_1B_2}$, the qubits C_1 and C_2 will be collapsed into the state

$$|q''\rangle = \frac{1}{2}(x_0|1\,0\rangle - x_1\mathrm{e}^{i\delta_1}|1\,1\rangle + x_2\mathrm{e}^{i\delta_2}|0\,0\rangle - x_3\mathrm{e}^{i\delta_3}|0\,1\rangle)_{C_1C_2}. \tag{19}$$

According to Alice and Bob's public announcements, Chalie can carry out unitary operations $(\sigma_x)_{C_1}$ and $(\sigma_z)_{C_2}$ on his qubits C_1 and C_2, and the state $|q''\rangle$ can be transformed into the state

$$|q'''\rangle = \frac{1}{2}(x_0|0\,0\rangle + x_1\mathrm{e}^{i\delta_1}|0\,1\rangle + x_2\mathrm{e}^{i\delta_2}|1\,0\rangle + x_3\mathrm{e}^{i\delta_3}|1\,1\rangle). \tag{20}$$

One can see that the state $|q'''\rangle$ (see eq. (20)) contains full information of the original state $|\psi\rangle$. In order to complete the JRSP, Charlie introduces two auxiliary qubits C_3 and C_4 with the initial states $|0\rangle_{C_3}$ and $|1\rangle_{C_4}$, and the state $|q'''\rangle$ can be described as

$$|\tilde{q}\rangle = \frac{1}{2}(x_0|0\,0\rangle + x_1\mathrm{e}^{i\delta_1}|0\,1\rangle + x_2\mathrm{e}^{i\delta_2}|1\,0\rangle + x_3\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2} \otimes |0\rangle_{C_3} \otimes |1\rangle_{C_4}. \tag{21}$$

Then Charlie in turn employs eight C-NOT gates $C_{C_2-C_3}$, $C_{C_3-C_2}$, $C_{C_3-C_4}$, $C_{C_1-C_2}$, $C_{C_4-C_1}$, $C_{C_1-C_2}$, $C_{C_1-C_3}$ and $C_{C_2-C_4}$ on the qubits C_1, C_2, C_3 and C_4, where C_{i-j} denotes i as the control qubit and j as the target one. After that, the state (21) can be transformed into the original state $|\psi\rangle$ and the JRSP succeeds in this situation. If Alice's results are the other 3 cases, Bob should choose suitable unitary operations on the qubits B_1 and B_2, and then measure them under the basis $\{|\varepsilon_j\rangle\}$. After that, in accordance with Alice and Bob's public announcements, Charlie can employ appropriate unitary operations on his qubits C_1 and C_2, and the state $|q'''\rangle$ (see (20)) can be obtained. Next, Charlie should introduce two auxiliary qubits C_3 and C_4 with the initial states $|0\rangle_{C_3}$ and $|1\rangle_{C_4}$, and then perform eight C-NOT gates on the qubits C_1, C_2, C_3 and C_4, and the desired state $|\psi\rangle$ can be reconstructed. It is easily found that, for all the 16 measurement outcomes of Alice and Bob, the receiver Charlie can reconstruct the original state $|\psi\rangle$ with unit successful probability, and the required classical communication cost is four bits in this scheme.

4. JRSP of an arbitrary FWES with *N* senders

The scheme in §3 can be generalized to the case of N senders. Suppose that Alice and Bob$_1$, Bob$_2$, ..., Bob$_{N-1}$ wish to help the receiver Charlie to remotely prepare an arbitrary FWES

$$|\Phi\rangle = x_0\,|0\,0\,0\,1\rangle + x_1\mathrm{e}^{i\mu_1}\,|0\,0\,1\,0\rangle + x_2\mathrm{e}^{i\mu_2}\,|0\,1\,0\,0\rangle + x_3\mathrm{e}^{i\mu_3}|1\,0\,0\,0\rangle, \tag{22}$$

where $x_l\,(l = 0, 1, 2, 3)$ and $\mu_m\,(m = 1, 2, 3)$ are real, and $\sum_{l=0} x_l^2 = 1\,(l = 0, 1, 2, 3)$, $\mu_m = \mu_m^{(1)} + \mu_m^{(2)} + \cdots + \mu_m^{(N-1)}$ $(m = 1, 2, 3)$. Assume that the N senders know the state $|\Phi\rangle$ partly, i.e., Alice knows x_i $(i = 0, 1, 2, 3)$, Bob$_1$ knows $\mu_m^{(1)}$, Bob$_2$ knows $\mu_m^{(2)}$,

$\ldots$, Bob$_{(N-1)}$ knows $\mu_m^{(N-1)}$, respectively, but Charlie does not know them at all. We also suppose that the N senders and receiver Charlie share two $(N+1)$-qubit GHZ states as quantum channel, which are given by

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes(N+1)} + |1\rangle^{\otimes(N+1)})_{A_1 B_1^{(1)} B_1^{(2)} \cdots B_1^{(N-1)} C_1},$$
$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes(N+1)} + |1\rangle^{\otimes(N+1)})_{A_2 B_2^{(1)} B_2^{(2)} \cdots B_2^{(N-1)} C_2}, \quad (23)$$

where the qubits A_1, A_2 belong to Alice, qubits $B_1^{(1)}$, $B_2^{(1)}$ to Bob$_1$, $\ldots$, qubits $B_1^{(N-1)}$, $B_2^{(N-1)}$ to Bob$_{(N-1)}$, and C_1, C_2 to Charlie, respectively.

As in the above scheme, the N senders must construct their own measurement basis. The first measuring basis chosen by Alice is still in eqs (13) and (14), and the measuring bases chosen by Bob$_1$, Bob$_2$, $\ldots$, Bob$_{N-1}$ are $4(N-1)$ sets of MOBVs $\{|\tau_{js}^{(k)}\rangle\}$ $(k, j = 1, 2, 3, 4;\ s = 1, 2, \ldots, N-1)$, which are given by

$$\begin{pmatrix} |\tau_{1s}^{(k)}\rangle \\ |\tau_{2s}^{(k)}\rangle \\ |\tau_{3s}^{(k)}\rangle \\ |\tau_{4s}^{(k)}\rangle \end{pmatrix} = G_s^{(k)} \begin{pmatrix} |0\,0\rangle \\ |0\,1\rangle \\ |1\,0\rangle \\ |1\,1\rangle \end{pmatrix}, \quad (24)$$

where

$$G_s^{(1)} = \begin{pmatrix} 1 & \sigma_{1s} & \sigma_{2s} & \sigma_{3s} \\ 1 & -\sigma_{1s} & \sigma_{2s} & -\sigma_{3s} \\ 1 & -\sigma_{1s} & -\sigma_{2s} & \sigma_{3s} \\ 1 & \sigma_{1s} & -\sigma_{2s} & -\sigma_{3s} \end{pmatrix},$$
$$G_s^{(2)} = \begin{pmatrix} \sigma_{1s} & 1 & \sigma_{3s} & \sigma_{2s} \\ \sigma_{1s} & -1 & \sigma_{3s} & -\sigma_{2s} \\ \sigma_{1s} & -1 & -\sigma_{3s} & \sigma_{2s} \\ \sigma_{1s} & 1 & -\sigma_{3s} & -\sigma_{2s} \end{pmatrix},$$
$$G_s^{(3)} = \begin{pmatrix} \sigma_{2s} & \sigma_{3s} & 1 & \sigma_{1s} \\ \sigma_{2s} & -\sigma_{3s} & 1 & -\sigma_{1s} \\ \sigma_{2s} & -\sigma_{3s} & -1 & \sigma_{1s} \\ \sigma_{2s} & \sigma_{3s} & -1 & -\sigma_{1s} \end{pmatrix},$$
$$G_s^{(4)} = \begin{pmatrix} \sigma_{3s} & \sigma_{2s} & \sigma_{1s} & 1 \\ \sigma_{3s} & -\sigma_{2s} & \sigma_{1s} & -1 \\ \sigma_{3s} & -\sigma_{2s} & -\sigma_{1s} & 1 \\ \sigma_{3s} & \sigma_{2s} & -\sigma_{1s} & -1 \end{pmatrix}, \quad (25)$$

and $\sigma_{ms} = \mathrm{e}^{-i\mu_m^{(s)}}$ $(m = 1, 2, 3;\ s = 1, 2, \ldots, N-1)$.

Now, let Alice first make the two-qubit projective measurement on her qubits A_1 and A_3 under the basis $\{|\nu_k\rangle\}$ $(k = 1, 2, 3, 4)$ (see eqs (13) and (14)) and publicly announces her result. In accordance with the Alice's result, Bob$_1$, Bob$_2$, $\ldots$, Bob$_{N-1}$ should choose suitable measuring basis in the MOBVs $\{|\tau_{js}^{(k)}\rangle\}$ to measure their own qubits $B_1^{(s)}$ and $B_2^{(s)}$ $(s = 1, 2, \ldots, N-1)$, and then inform Charlie of their results. According to the outcomes of N senders, the receiver Charlie can obtain the state $|q'''\rangle$ (see eq. (20)) by using appropriate unitary operations on his qubits C_1 and C_2. For instance, without loss of generality, suppose that Alice's result is $|\nu_1\rangle_{A_1 A_2}$, then Bob$_1$, Bob$_2$, $\ldots$, Bob$_{N-1}$ should choose suitable measuring bases $\{|\tau_{j1}^{(1)}\rangle\}, \{|\tau_{j2}^{(1)}\rangle\}, \ldots, \{|\tau_{j(N-1)}^{(1)}\rangle\}$ $(j = 1, 2, 3, 4)$ to measure their own qubits $(B_1^{(1)}, B_2^{(1)})$, $(B_1^{(2)}, B_2^{(2)})$, $\ldots$, $(B_1^{(N-1)}, B_2^{(N-1)})$, respectively. Assume that Bob's result is only $|\tau_{21}^{(1)}\rangle_{B_1^{(1)} B_2^{(1)}}$ while all other senders' outcomes are $|\tau_{1m}^{(1)}\rangle_{B_1^{(m)} B_2^{(m)}}$ $(m = 2, 3, \ldots, N-1)$, and the qubits C_1 and C_2 will be collapsed into the state

$$|Q\rangle = \frac{1}{2}(x_0|0\,0\rangle - x_1\mathrm{e}^{i\mu_1}|0\,1\rangle + x_2\mathrm{e}^{i\mu_2}|1\,0\rangle - x_3\mathrm{e}^{i\mu_3}|1\,1\rangle)_{C_1 C_2}. \quad (26)$$

In accordance with the results of N senders, Charlie should perform the unitary operation $(I)_{C_1} \otimes (\sigma_z)_{C_2}$ on his qubits C_1 and C_2, then the state $|q'''\rangle$ can be obtained. As described in §3, Charlie can introduce two auxiliary qubits C_3 and C_4 with the initial state $|0\rangle_{C_3}$ and $|1\rangle_{C_4}$, then in turn employ three C-NOT gates $C_{C_2-C_3}$, $C_{C_3-C_4}$ and $C_{C_1-C_4}$ on the qubits C_1, C_2, C_3 and C_4, and the original state $|\Phi\rangle$ can be recovered. If N senders obtain other results, similar to the above method, the receiver Charlie can reconstruct the original state $|\Phi\rangle$ by appropriate unitary operations, and the total success probability of the JRSP is still 1. Here, we no longer depict them one by one. The required CCC is $2N$ bits.

5. Conclusion

In conclusion, we have presented three new schemes for the joint remote preparation of an arbitrary four-qubit W-type entangled states. In these schemes, the coefficients of the original states to be co-prepared are all complex. In the first scheme, two senders share an arbitrary four-qubit W-type state, but each sender only partly knows the state, and four three-qubit GHZ states are exploited as the quantum channel. In order to help the receiver to remotely prepare the original state, in accordance with the knowledge of the original state which she/he has known, each sender must construct her/his own four-qubit measuring basis. First, a sender performs a four-qubit projective measurement on her qubits, then, another sender should employ, according to the result of the first sender, suitable unitary operations on his qubits and then measure these qubits, and inform the receiver of his outcomes of measurement. After these projective measurements, the receiver can reconstruct the original state by means of appropriate unitary operations. Specially, in the second scheme, according to the senders' public announcements, the receiver should introduce two auxiliary qubits and employ appropriate C-NOT gates on his qubits, and the original state can be reconstructed. Compared with the first scheme, the advantage of the second scheme is that the entanglement resource can be reduced. Hence, we can believe that our schemes are more efficient and economical. Finally, we generalize the second scheme to the case of N senders. It is shown that, for all the presented schemes, the total successful probability of JRSP can reach 1. Moreover, it should be pointed out that, the presented schemes are similar to the quantum state sharing [49–51] (QSTS), where the secret is an arbitrary unknown quantum state. Different from the QSTS, in the presented schemes the split information is an arbitrary known quantum state. Especially, in secret communications, the scheme in §4 is more secure with the number of senders (information sharer) increasing. In this sense, we hope that our schemes will be helpful for the deeper understanding of the process of JRSP, and may be useful in further studies on quantum cryptography.

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