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Quark number density and susceptibility calculation under one-loop correction in the mean-field potential

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Abstract. We calculate quark number density and susceptibility under one-loop correction in the mean-field potential. The calculation shows continuous increase in the number density and susceptibility up to the temperature $T = 0.4$ GeV. Then the values of number density and susceptibility approach the very weakly result with higher values of temperature. The result indicates that the calculated values fit well with increase in temperature to match the lattice QCD simulations of the same quantities.

Keywords. Quark-susceptibility; quark-gluon plasma; QCD.

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1. Introduction

One of the predictions of quantum chromodynamics (QCD) is that the quark-hadron phase transition happens under extreme nuclear density and high temperature. In the transition [1,2] phenomena there exists a system called deconfined phase of free quarks and gluons, which is also known as quark-gluon plasma (QGP). It is obtained at very large temperature and at large nuclear density. The system has a short lifetime and subsequently changes from the plasma state to a confined phase of hadrons of bound quarks at lower temperature. The system is considered to be a complicated phenomenon and is supposed to have been the first constituent at the beginning of early Universe. To investigate the nature of the Universe, a number of experimental facilities such as relativistic heavy-ion collision (RHIC) at BNL, large hadron collider (LHC) at CERN etc. are set up around the globe. Besides these, there are other experimental facilities like FAIR at Darmstadt and NICA at Dubna, where the study has focussed on dense baryonic matter and the baryonic matter at Nuclotron (BM@N) experiments with ion beams extracted from modernized Nuclotron. All these facilities are trying to detect the existence of this early phase transition, formation of QGP and QCD phase structure [3–6]. So, the study of QGP in ultrarelativistic heavy-ion collisions has become an exciting field in heavy-ion collider physics [7,8]. In this brief article, we focus on the calculation of quark number density and susceptibility through the free energy evolution of QGP with one-loop correction in the mean-field potential. The calculations of the number density and susceptibility can provide information of QCD phase structure. To evaluate the quark number density and susceptibility, we need to understand the free energy of the system so that we can correlate the number density and susceptibility through the free energy. The free energy is obtained through the modification in the density of state by introducing one-loop correction factor in the mean-field potential. Due to the correction factor in the mean-field potential through coupling value [9–13], there are changes in the free energy expansion of QGP fireball, and it also impacts in the stability of droplet with the variation of dynamical quark and gluon flow parameters. So the flow parameters take the role of stability in forming droplet with the differing temperatures.

In brief, we review free energy evolution through the density of state which is modified through one-loop correction. The loop correction is introduced in the coupling parameter and hence modified the density of state constructed through the coupling value. Then, we calculate quark number density and susceptibility with the relevant flow parameters of stable droplets and the corresponding speed of sound through thermodynamic entities like entropy, specific heat etc. In conclusion, we give details of the evolution of QGP fireball with different flow parametrization values and show the results of number density, susceptibility and speed of sound.

2. The free energy evolution

The free energies of quarks and gluons can be obtained through the thermodynamic canonical ensemble of the system. The partition function of the system is given by [14] by

$$Z(T, \mu, V) = \mathrm{Tr}\{\exp[-\beta(\hat{H} - \mu\hat{N})]\}, \tag{1}$$

where μ is the chemical potential of the system, $\hat{N}$ is the quark number and $\beta = 1/T$. Using this partition function, we can correlate the free energy through a simple model modifying the density of state with the inclusion of one-loop correction factor, and it is calculated by [15–18]

$$F_i = T \ln Z(T, \mu, V) \tag{2}$$

$$F_i = -\eta T g_i \int \rho_{q,g}(p) \times \ln\left[1 + \eta \mathrm{e}^{-(\sqrt{m_i^2+p^2}-\mu)/T}\right]\mathrm{d}p, \tag{3}$$

where $\eta = -1$ for the bosonic particle and $\eta = +1$ for the fermionic particles. The integral is evaluated with minimum momentum cut-off as

$$p_{\min} = (\gamma_{g,q} N^{1/3} T^2 \Lambda^4/2)^{1/4}, \tag{4}$$

where $N = (4/3)[12\pi/(33-2n_f)]$ and the corresponding masses of quarks are considered as zero except the strange mass. The minimum cut-off controls the integral to a finite value by avoiding the infrared divergence produced by considering the magnitude of Λ and T as taken in the same order of lattice QCD. g_i is the colour and particle–antiparticle degeneracy for quarks and gluons.

$\rho_{q,g}$ is the density of states in phase space by including one-loop correction in the interacting potential. It is reviewed through our earlier paper [19]:

$$\rho_{q,g}(p) = \frac{\nu}{\pi^2}\left[\frac{\gamma_{q,g}^3 T^2}{2}\right]^3 g^6(p) A, \tag{5}$$

where

$$A = \left\{1 + \frac{\alpha_s(p)a_1}{\pi}\right\}^2 \left[\frac{(1 + \alpha_s(p)a_1/\pi)}{p^4} + \frac{2(1 + 2\alpha_s(p)a_1/\pi)}{p^2(p^2 + \Lambda^2)\ln(1 + (p^2/\Lambda^2))}\right]. \tag{6}$$

Here, $\gamma_{q,g}$ is the quark and gluon parametrization factors taken as $\gamma_q = 1/8$ and $\gamma_g = (8-10)\gamma_q$. These factors determine the dynamics of QGP flow and enhance the transformation to hadrons. It also explains the stability of droplet formation. ν is the volume occupied by the QGP. $g^2(p) = 4\pi\alpha_s(p)$, where

$$\alpha_s(p) = \frac{4\pi}{(33 - 2n_f)\ln(1 + (p^2/\Lambda^2))}, \tag{7}$$

in which parameter Λ is considered in the scale of QCD as 0.15 GeV. The coefficient a_1 used in the above expression is due to the one-loop correction in their interactions and it is given as [20–22]

$$a_1 = 2.5833 - 0.2778 n_l, \tag{8}$$

where n_l is the number of light quark elements [23–26]. In addition to these free energies, we consider an interfacial energy taken from simple statistical model of free energy evolution [10,27]. The energy takes care of the hydrodynamic effects in the system and replaces the Bag energy constant. It is:

$$F_{\text{interface}} = \frac{\gamma T R^2}{4}\int p^2 \delta(p - T)\mathrm{d}p, \tag{9}$$

in which γ is the root mean square value of quark γ_q and gluon flow parameter γ_g and R is the size of QGP droplet. The value calculated through this interfacial energy takes the role of Bag energy in the system. The value reduces the drawback to a maximum in comparison to MIT model of calculation. The hadronic contributions of free energies with the corresponding degeneracy factor of g is [28]

$$F_h = (gT/2\pi^2)\nu \int_0^\infty p^2 \ln\left[1 - \mathrm{e}^{-\left(\sqrt{m_h^2+p^2}-\mu/T\right)}\right]\mathrm{d}p. \tag{10}$$

In the calculation of hadronic energy, m_h is considered to be the mass of hadronic particles. We can thus compute the total modified free energy F_{total} as

$$F_{\text{total}} = \sum_i F_i + F_{\text{interface}} + F_h, \tag{11}$$

where i stands for u, d and s quark and gluon.

3. Quark number density, susceptibility and speed of sound

The quark number density, susceptibility and speed of sound are calculated from the total free energy. The number density and susceptibility are calculated by the following relations [29–32]:

$$n = \left(\frac{\partial F}{\partial \mu_i}\right)_{\mu_i=0}, \tag{12}$$

$$\chi = \left(\frac{\partial^2 F}{\partial \mu_i^2}\right)_{\mu_i=0}. \tag{13}$$

The number density and susceptibility value can be analysed from the figure by varying temperatures. In the case of speed of sound we take the entropy and specific heat of the one-loop correction which can be obtained from the free energy. So the speed of sound is obtained as the ratio of the thermodynamic property entropy to specific heat calculated through the free energy, and it is given as [33,34]

$$C_s^2 = \frac{S/T^3}{C_v/T^3}. \tag{14}$$

The value of speed of sound is also shown in the following figure with the corresponding temperature.

4. Results

Analytical calculation of free energy of QGP-hadron fireball evolution with one-loop correction factor in the interacting mean-field potential is made by computing quark number density, susceptibility and speed of sound. The free energy evolution of the fireball is also described in this work. The stable droplets are found when the quark and gluon parametrization factors $\gamma_q = 1/8$ and $10\gamma_q \geq \gamma_g \geq 8\gamma_q$. At these particular ranges, we numerically calculate the free energy, quark number density, quark susceptibility and speed of sound for one particular flow parameter out of the

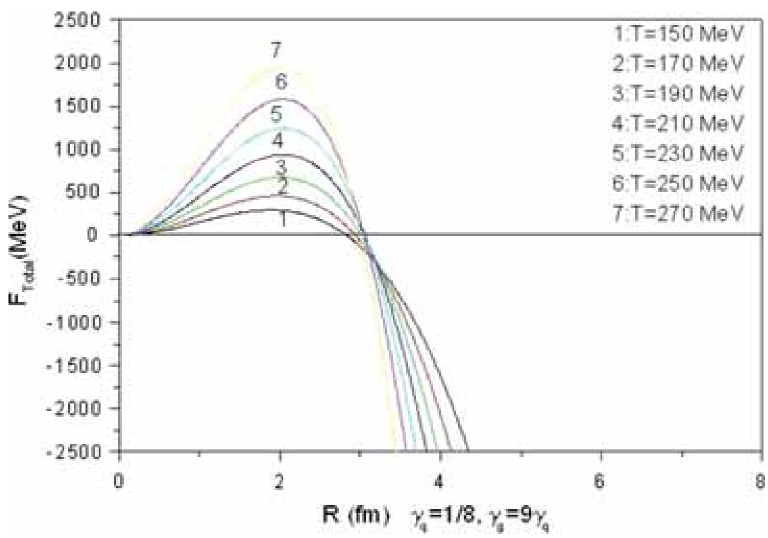

Figure 1. Free energy vs. R at $\gamma_q = 1/8$, $\gamma_g = 9\gamma_q$.

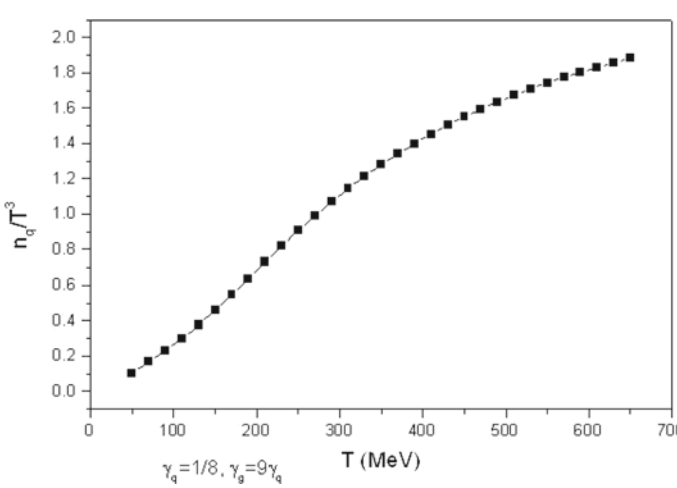

Figure 2. Quark number density vs. T at $\gamma_q = 1/8$, $\gamma_g = 9\gamma_q$.

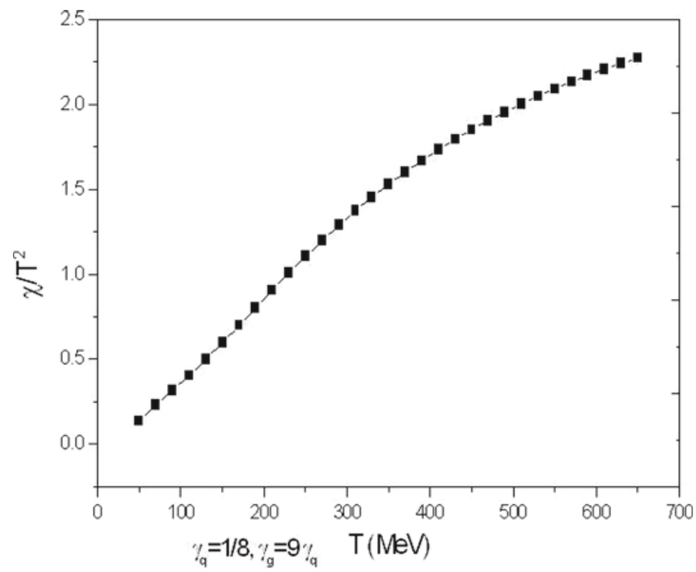

Figure 3. The quark susceptibility vs. T at $\gamma_q = 1/8$, $\gamma_g = 9\gamma_q$.

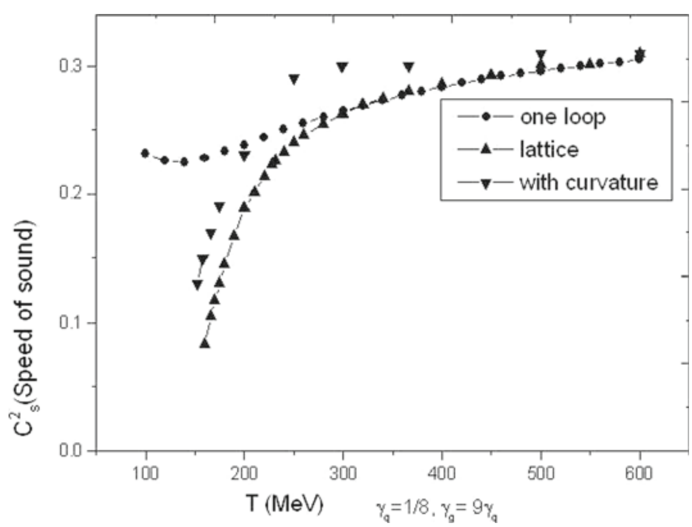

Figure 4. Speed of sound vs. T at $\gamma_q = 1/8$, $\gamma_g = 9\gamma_q$.

above ranges. The choice of this particular flow value is because of highly stable in the QGP droplet. In figure 1, we show the free energy evolution of the system with the change in droplet size. The free energy spectrum of the droplet is found to be almost stable when the radius $R = 2.8$ fm at $\gamma_q = 1/8$ and $\gamma_g = 9\gamma_q$. In figure 2, we plot the quark number density vs. temperature. The figure shows that the system with the modification of one-loop correction in the mean-field potential agrees with many other works [33–36]. It indicates that the choice of parameter during QGP formation plays a very important and significant role. Its value is probably a kind of Reynold number in the expansion of this dense nuclear fluid. The flow parameter can give the most stable droplet size of QGP. So we look quark susceptibility and the speed of sound at this particular flow parameter with this one-loop correction. The susceptibility and speed of sound are represented by figures 3 and 4. The susceptibility is following similar outputs with other theoretical works [31,36]. From figure 4, the speed of sound is almost matched with the recent data of speed of sound at higher range of temperature above the critical temperature whereas at temperatures below the critical temperature our result shows larger value of speed of sound because in our model, we exclude larger components of hadronic matter, only a few light hadronic particles are considered as we have already calculated the creation of QGP fireball with stability droplets. Overall, the result is in agreement with other theoretical works.

5. Conclusion

We can conclude from these results that due to the presence of loop correction in the mean-field potential, the stability of droplets increases while size decreases in comparison with the result of uncorrected potential. So, we study quark number density, quark susceptibility and velocity of sound on the basis of these smaller droplets. The results are very consistent with the current results of lattice data as evidenced by figure 4.

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