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Nonplanar electrostatic shock waves in an opposite polarity dust plasma with nonextensive electrons and ions

M AMINA[1], S A EMA[2,*] and A A MAMUN[1]

[1]Department of Physics, Jahangirnagar University, Savar, Dhaka 1342, Bangladesh
[2]Department of Electrical and Electronic Engineering, Sonargaon University, Dhaka 1215, Bangladesh
*Corresponding author. E-mail: ema.plasma@gmail.com

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Abstract. A rigorous theoretical investigation has been carried out on the propagation of nonplanar (cylindrical and spherical) dust-acoustic shock waves (DASHWs) in a collisionless four-component unmagnetized dusty plasma system containing massive, micron-sized, positively and negatively charged inertial dust grains along with q (nonextensive) distributed electrons and ions. The well-known reductive perturbation technique has been used to derive the modified Burgers equation (which describes the shock wave properties) and its numerical solution. It has been observed that the effects of charged dust grains of opposite polarity, nonextensivity of electrons and ions, and different dusty plasma parameters have significantly modified the fundamental properties (viz., polarity, amplitude, width, etc.) of the shock waves. The properties of DASHWs in nonplanar geometry are found to be significantly different from those in one-dimensional planar geometry. The findings of our results from this theoretical investigation may be useful in understanding the nonlinear features of localized electrostatic disturbances in both space and laboratory dusty plasmas.

Keywords. Dust-acoustic waves; shock waves; nonextensive plasmas; dust of opposite polarity; nonplanar geometry.

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1. Introduction

A plasma system that contains extremely massive (in comparison to electrons and ions) charged dust grains is called dusty plasma. Because of the existence of dust particles in most space and astrophysical plasma systems such as different parts of our solar system (e.g., planetary atmosphere, asteroid zones, Saturn ring, Earth's ionosphere, cometary tails, etc.), interstellar clouds, interstellar media, and nebula [1–3], dusty plasma has become a very interesting field of study. Several processes are responsible for charging of dust grains [4,5]. The charge of the dust grains highly depends on the size of the dust surface; larger dust grains are found to be negatively charged and smaller ones are positively charged [3,6,7]. Due to the collisions with mobile electrons, typical micron-sized dust grains in space or laboratory plasmas are most often negatively charged, but there are environment such as cometary tails [3,8], upper mesosphere [9], Jupiter's magnetosphere [6], etc., where positively charged dust grains have been found. Mainly, three processes are responsible for a dust grain to be positively charged. These are the photoemission in the presence of ultraviolet (UV) photons flux, the thermionic emission induced by the radiative heating, and the secondary emission (electron emission) from the surface of the dust grains [10]. The coexistence of positively and negatively charged dust grains has been observed in the Earth's mesosphere [11,12] as well as in cometary tails and comae [6].

The propagation of various types of nonlinear waves in dusty plasmas, viz., dust-acoustic (DA) waves [13], dust-ion-acoustic (DIA) waves [14,15] and dust-lattice (DL) waves [16,17] have been studied by many researchers. Over the last few decades, the study of nonlinear propagation of DA waves has received a great deal of attention in the field of dusty plasma physics. At first, theoretical prediction of low-frequency DA waves in an unmagnetized dusty plasma was given by Rao *et al* [13], where inertia was provided by the mass of the dust particle and restoring force was given by the thermal pressure of Boltzmann-distributed electrons and ions. The phase speed of such DA waves is very low in comparison to electrons and ions and lies between the dust thermal speed and ion thermal speed [7]. In 1996, the existence of DA waves was experimentally verified by Barkan *et al* [18]. Mamun *et al* [19] have investigated the propagation of nonlinear DA waves in a two-component unmagnetized dusty plasma system consisting of a negatively charged cold dust fluid and Maxwellian ions. Again, the characteristics of nonlinear DA waves have been investigated by many researchers [7,20–25] by considering different dusty plasma situations. Many researchers [1,26–28] have also studied the roles of opposite polarity dust grains in different dusty plasma situations. It has been observed that the presence of additional positive dust component not only modifies the basic properties of solitary/shock structures but also allows the coexistence of positive and negative potentials solitary/shock structures. A large number of space and laboratory plasma situations have been observed where particles are not exactly Maxwellian and particles follow distributions that are deviated from the well-known Maxwell–Boltzmann distribution (viz., nonthermal, superthermal, nonextensive, and kappa distributions [29–32]). A new type of distribution, namely nonextensive distribution, which was proposed by Tsallis [30] has become the focus of interest among the plasma physics researchers because of its vast number of applications in many astrophysical and cosmological objects like stellar polytropes [33], hadronic matter and quark-gluon plasma [34], proton–neutron stars [35], dark-matter halos [36], Earth's bow-shock [37], upper ionosphere of Mars, vicinity of the Moon, magnetospheres of Jupiter and Saturn [38], etc., as well as in many laboratory environments such as nanomaterials, microdevices, and microstructures [39]. By assuming nonextensive distributed electrons or ions or both, many researchers have studied different types of waves, viz., ion-acoustic (IA) waves, electron-acoustic (EA) waves, DIA waves, and DA waves [31,40–46].

Many researchers have investigated theoretically the nonplanar propagation DIA and the DA solitary and shock waves in nonplanar geometry [47–50] and the observations show that the properties of solitary/shock waves in a bounded nonplanar (cylindrical and spherical) geometry are totally different from those in an unbounded planar geometry. In 2007, Mushtaq [51] investigated the basic features of cylindrical and spherical dust-acoustic solitary waves (DASWs) by considering a quantum dusty plasma system. Mamun and Shukla [52] have studied the fundamental features of nonplanar DIA solitary waves for an unmagnetized dusty plasma system containing nonthermal electrons, inertial ions, and stationary dust of opposite polarity. Again, the propagation of nonplanar double layers in dusty plasmas with opposite polarity dust grains was theoretically investigated by Mamun and Mannan [28]. Sahu and Tribeche [17] considered a plasma system containing nonextensive ion and inertial dust grains and studied the properties of DASWs and dust-acoustic shock waves (DASHWs) in a nonplanar geometry. In 2011, Eslami *et al* [53] have studied the characteristics of nonplanar DASWs in dusty plasmas after considering nonextensive distributed electrons and ions. Ghosh *et al* [54] investigated a plasma system containing nonextensive electrons and arbitarily-charged dust and observed the basic features of the nonplanar Gardner solitons. Many have studied DASHWs by considering nonextensive dusty plasmas (electron or ion or both) with dust of opposite polarity [43,44] but most of their investigations are limited to planar geometry.

As a new physical approach, in our present work, we have studied the fundamental properties of DASHWs in nonplanar (cylindrical and spherical) geometry by considering q-nonextensive distributed electrons and ions and dust of opposite polarity.

2. Governing equations

We consider the nonplanar (cylindrical and spherical) geometry of the DASHWs in a collisionless four-component unmagnetized dusty plasma system which contains massive, micron-sized, positively and negatively charged inertial dust grains and nonextensive distributed electrons and ions. Thus, at equilibrium, we have, $n_{i0} + Z_p n_{p0} - n_{e0} - Z_n n_{n0} = 0$, where n_{s0} represents the equilibrium number densities of the species s (here $s = e,\ i,\ p,\ n$ for electron, ion, positively charged dust, and negatively charged dust, respectively) and Z_p (Z_n) represents the number of the positive (negative) charges on the dust grain surface. The number densities of q-nonextensive distributed electrons [44] and ions [17] are

$$n_e = n_{e0}\left[(1 + (q-1)\frac{e\psi}{k_{\rm B}T_e}\right]^{(1+q)/2(q-1)},$$

$$n_i = n_{i0}\left[(1 - (q-1)\frac{e\psi}{k_{\rm B}T_i}\right]^{(1+q)/2(q-1)},$$

where q is the nonextensive parameter which describes the degree of nonextensivity, i.e., $q \rightarrow 1$ corresponds to Maxwellian distribution and $q < 1\ (> 1)$ denotes the nonextensive q-distribution.

The nonlinear dynamics of low phase speed (in comparison to the thermal speed of electrons and ions) DA waves is governed by the normalized equations of the form

$$\frac{\partial n_s}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_s u_s) = 0, \tag{1}$$

$$\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial r} = -\alpha\frac{\partial \psi}{\partial r} + \eta_p\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial u_p}{\partial r}\right), \tag{2}$$

$$\frac{\partial u_n}{\partial t} + u_n\frac{\partial u_n}{\partial r} = \frac{\partial \psi}{\partial r} + \eta_n\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial u_n}{\partial r}\right), \tag{3}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial \psi}{\partial r}\right) = \mu_1[1+(q-1)\sigma\psi]^{(q+1)/2(q-1)} - \mu_2[1-(q-1)\psi]^{(q+1)/2(q-1)} - \beta n_p + n_n, \tag{4}$$

where $\nu = 0$ represents one-dimensional planar geometry and $\nu = 1$ (2) corresponds to nonplanar cylindrical (spherical) geometry. The parameter n_s denotes the number density of the dust species s ($s = p$ for positively and $s = n$ for negatively charged dust grains) normalized by the equilibrium values of dust number densities n_{s0}, u_s is the dust fluid speed normalized by $C_d = (Z_n k_B T_i/m_n)^{1/2}$, ψ is the electrostatic wave potential normalized by $k_B T_i/e$, η_s is the viscosity coefficient normalized by $m_n n_{n0}\omega_p \lambda_D^2$, where m_p (m_n) is the mass of positive (negative) dust grain, k_B is the Boltzmann constant, T_i (T_e) is the ion (electron) temperature, and e is the magnitude of the electron charge. The time variable t is normalized by $\omega_p^{-1} = (m_n/4\pi e^2 Z_n^2 n_{n0})^{1/2}$ and the radial space variable r is normalized by $\lambda_{Di} = (k_B T_i/4\pi Z_n^2 e^2 n_{n0})^{1/2}$. We have defined another parameter arised in eq. (2) as $\alpha = Z_p m_n/Z_n m_p$ (negative-to-positive dust mass ratio). The parameters arised in eq. (4) are defined as $\mu_1 = n_{e0}/n_{n0}Z_n$ (electron-to-negative dust number density ratio), $\mu_2 = n_{i0}/n_{n0}Z_n$ (ion-to-negative dust number density ratio), $\beta = Z_p n_{p0}/Z_n n_{n0}$ (positive-to-negative dust number density ratio), and $\sigma = T_i/T_e$ (ion-to-electron temperature ratio).

3. Derivation of the modified Burgers equation

To derive a nonlinear dynamical equation for DASHWs from our basic eqs (1)–(4), we employ reductive perturbation method. We first introduce the stretched coordinate [46] as

$$\zeta = -\epsilon(r + V_p t), \qquad \tau = \epsilon^2 t, \tag{5}$$

where ϵ is an expansion parameter ($0 < \epsilon < 1$) and V_p is the phase speed of the DA waves normalized by C_d. We note that ζ is normalized by λ_{Di}. We can expand the perturbed quantities n_s, u_p, u_n, and ψ about the equilibrium values in power series of ϵ as

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \cdots, \tag{6}$$

$$u_p = 0 + \epsilon u_p^{(1)} + \epsilon^2 u_p^{(2)} + \cdots, \tag{7}$$

$$u_n = 0 + \epsilon u_n^{(1)} + \epsilon^2 u_n^{(2)} + \cdots, \tag{8}$$

$$\psi = 0 + \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \cdots. \tag{9}$$

Now substituting eqs (5)–(9) into eqs (1)–(4), different sets of equations in various powers of ϵ can be developed. To the lowest order in ϵ, eqs (1)–(4) give

$$u_p^{(1)} = \alpha\frac{\psi^{(1)}}{V_p}, \quad n_p^{(1)} = \alpha\frac{\psi^{(1)}}{V_p^2}, \tag{10}$$

$$u_n^{(1)} = -\frac{\psi^{(1)}}{V_p}, \quad n_n^{(1)} = -\frac{\psi^{(1)}}{V_p^2}, \tag{11}$$

$$V_p = \sqrt{\frac{2(1+\alpha\beta)}{(q+1)(\mu_1\sigma_1+\mu_2)}}. \tag{12}$$

The expression for V_p in eq. (12) represents the linear dispersion relation of the DA waves propagating in the dusty plasma system under consideration. It is observed in eq. (12) that the phase speed of DA waves is significantly modified by the nonextensivity of electrons and ions. To the next higher order of ϵ, i.e., taking the coefficients of ϵ^3 from eqs (1)–(3), and ϵ^2 from eq. (4), we obtain another set of simultaneous equations for $\psi^{(1)} = \psi$, $\psi^{(2)}$, $n_p^{(2)}$, $n_n^{(2)}$, $u_p^{(2)}$, and $u_n^{(2)}$. After some algebraic calculation (omitted here), we obtain the nonplanar Burgers equation as

$$\frac{\partial \psi}{\partial \tau} + \frac{\nu\psi}{2\tau} + A\psi\frac{\partial \psi}{\partial \zeta} = B\frac{\partial^2 \psi}{\partial \zeta^2}, \tag{13}$$

where the nonlinear coefficient A and the dissipative coefficient B are given by

$$A = \frac{V_p^3}{2(1+\alpha\beta)}\left[\frac{(q+1)(q-3)}{4}(\mu_1\sigma^2 - \mu_2) - \frac{3}{v_p^4} + \frac{3\alpha^2\beta}{v_p^4}\right], \tag{14}$$

$$B = \frac{\alpha\beta\eta_p + \eta_n}{2(1+\alpha\beta)}. \tag{15}$$

Equation (13) is the Burgers equation which is modified by an extra term $\nu\psi/2\tau$ arising due to the effect of nonplanar geometry ($\nu = 1$ for cylindrical and $\nu = 2$ for spherical). So, eq. (13) can also be termed as modified Burgers (mB) equation. The mB equation includes the effect of nonextensivity of electron and ion and kinematic viscosity. Our aim is now to analyse the Burgers equation numerically. From eq. (13) it is seen that the nonplanar geometrical effect is negligible for large values of $|\tau|$ and significant for small values of τ. The initial condition we have used in our numerical analysis is the stationary shock wave solution of eq. (13) without the term $\nu\psi/2\tau \to 0$, i.e.

$$\psi = \psi_m[1 - \tanh(\xi/\Delta)], \tag{16}$$

which is obtained by transforming the independent variables ζ and τ to

$$\xi = \zeta - U_0\tau', \qquad \tau' = \tau, \tag{17}$$

where U_0 is normalized by C_d and ξ is normalized by λ_{Di}, and imposing the appropriate boundary conditions $\psi \to 0$ and $\mathrm{d}\psi/\mathrm{d}\xi \to 0$ at $\psi \to -\infty$. The shock wave amplitude $\psi_m = U_0/A$ and the width $\Delta = 2B/U_0$. From eq. (16), it is observed that for $A > 0$ ($A < 0$), the plasma system under consideration supports compressive (rarefactive) DASHWs which are associated with a positive (negative) potential, and when the nonlinear coefficient A is close to zero then no shock wave exists. We see that the nonlinearity coefficient A is a function of $\mu_1, \mu_2, \sigma, \alpha, \beta$, and q for the considered plasma model. Thus $A = 0$ introduces the critical value of nonextensive parameter q. Therefore, $A(q = q_c) = 0$ leads q_c in the form $q_c = -3(\mu_2 - \mu_2^2 + 2\mu_2\alpha\beta + \mu_2^2\alpha^2\beta + \mu_2\alpha^2\beta^2 - 2\mu_1\mu_2\sigma + 2\mu_1\mu_2\alpha^2\beta\sigma - \mu_1\sigma^2 - \mu_1^2\sigma^2 - 2\mu_1\alpha\beta\sigma^2 + \mu_1^2\alpha^2\beta\sigma^2 - \mu_1\alpha^2\beta^2\sigma^2)/(-\mu_2 - 3\mu_2^2 - 2\mu_2\alpha\beta + 3\mu_2^2\alpha^2\beta - \mu_2\alpha^2\beta^2 - 6\mu_1\mu_2\sigma + 6\mu_1\mu_2\alpha^2\beta\sigma + \mu_1\sigma^2 - 3\mu_1^2\sigma^2 + 2\mu_1\alpha\beta\sigma^2 + 3\mu_1^2\alpha^2\beta\sigma^2 + \mu_1\alpha^2\beta^2\sigma^2)$ which shows that shock waves exhibit a compressive character below q_c and rarefactive character above q_c. The critical value of q for which $A = 0$ is given by $q = q_c = 0.73$ [obtained from $A(q = q_c) = 0$ for a set of plasma parameters, viz., $\sigma = 0.01$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\alpha = 0.2$, and $\beta = 0.3$].

Figure 1 shows the variation of q_c with σ and μ_1. The shock profile associated with $q < 0.73$ or $q > 0.73$ is shown graphically in figures 2, 3, 4, 5, 6 and 7, representing the effect of dust's kinematic viscosity in both planar and nonplanar (cylindrical and spherical) geometries. Figures 2, 4, and 6 show the variation of positive potential shock profile with τ for planar ($\nu = 0$), cylindrical ($\nu = 1$), and spherical ($\nu = 2$) geometry respectively, for $\sigma = 0.01$, $\alpha = 0.2$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$. Figures 3, 5, and 7 show the variation of negative potential shock profile with τ for planar ($\nu = 0$), cylindrical ($\nu = 1$), and spherical ($\nu = 2$) geometry respectively, for $\sigma = 0.01$, $\alpha = 0.2$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$. The variation of amplitude ψ_m with α for $q < q_c$ ($q > q_c$) is shown in figure 8 (9) for $\sigma = 0.01$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$. Figure 10 shows the variation of the width of the shock waves Δ with speed of the shock wave U_0 for different kinematic viscosity coefficients η_s.

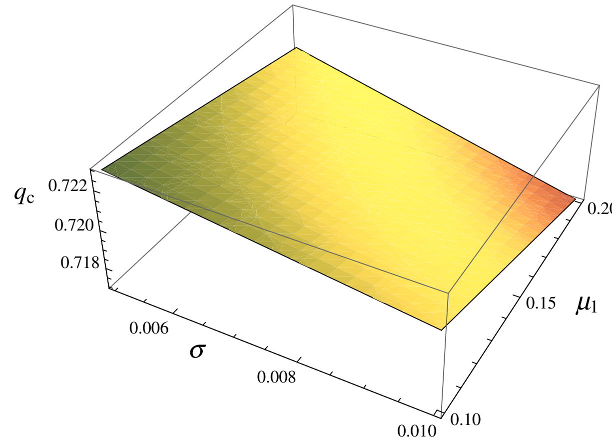

Figure 1. The $A = 0$ graph, which represents the variation of critical value of nonextensive parameter q_c with σ (ion-to-electron temperature ratio) and μ_1 (electron-to-negative dust number density ratio) for $\beta = 0.3$, $\alpha = 0.2$, and $\mu_2 = 0.5$.

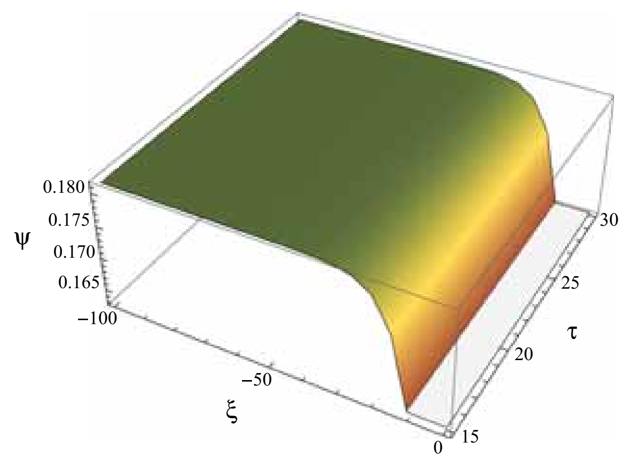

Figure 2. The variation of positive potential shock profile with ξ and τ for a planar ($\nu = 0$) geometry where $q < 0.73$. The other plasma parameters are fixed at $\sigma = 0.01$, $\alpha = 0.2$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$.

4. Results and discussion

The fundamental features of the nonplanar (cylindrical and spherical) DASHWs in an unmagnetized dusty plasma medium consisting of massive, micron-sized, positively and negatively charged inertial dust grains and q-nonextensive distributed electrons and ions are investigated. The well-known reductive perturbation method has been employed to derive the modified Burgers equation and their solutions are also analysed numerically. It is important to note that the modified Burgers equation is not valid [41] for the limit $A = 0$. DASHWs of both positive and negative potential are found below and above the critical value of $q = q_c = 0.73$. The

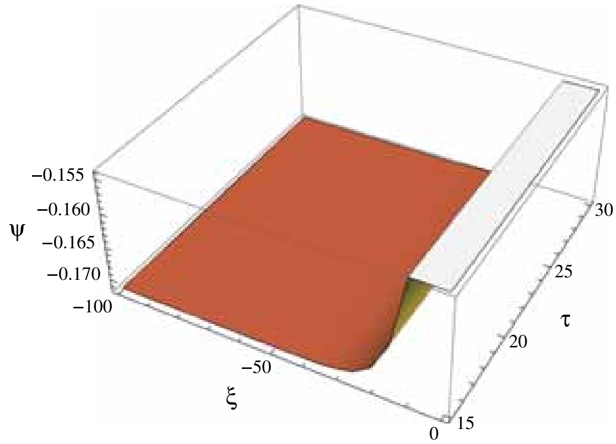

Figure 3. The variation of negative potential shock profile with ξ and τ for a planar ($\nu = 0$) geometry where $q > 0.73$. The other plasma parameters are fixed at $\sigma = 0.01$, $\alpha = 0.2$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$.

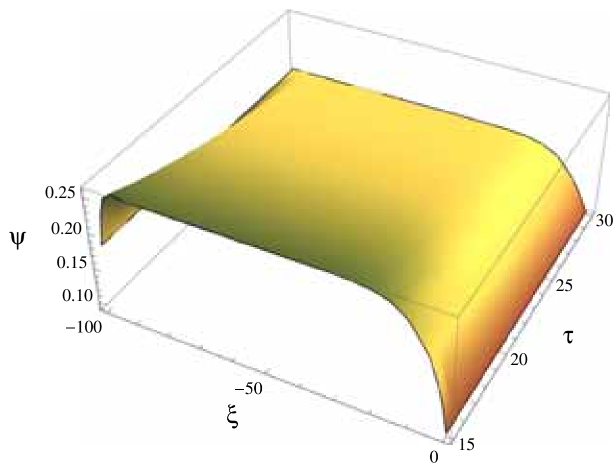

Figure 4. The variation of positive potential shock profile with ξ and τ for a nonplanar cylindrical ($\nu = 1$) geometry where $q < 0.73$. The other plasma parameters are fixed at $\sigma = 0.01$, $\alpha = 0.2$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$.

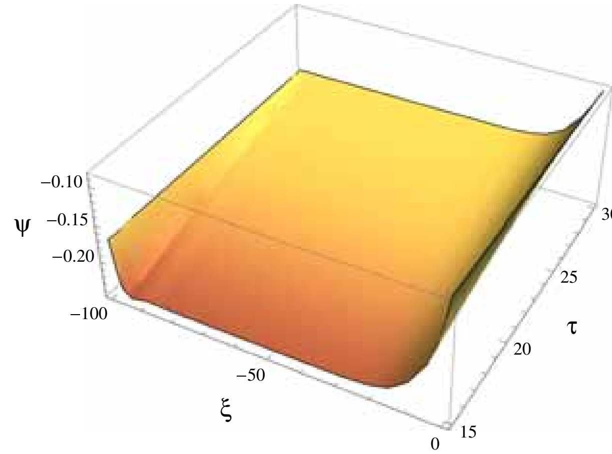

Figure 5. The variation of negative potential shock profile with ξ and τ for a nonplanar cylindrical ($\nu = 1$) geometry where $q > 0.73$. The other plasma parameters are fixed at $\sigma = 0.01$, $\alpha = 0.2$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$.

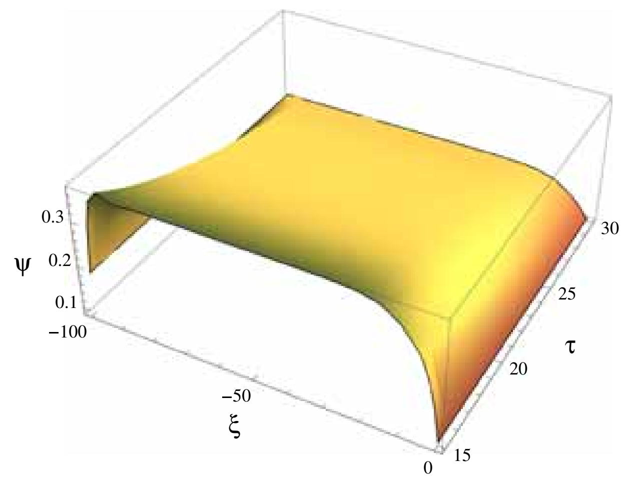

Figure 6. The variation of positive potential shock profile with ξ and τ for a nonplanar spherical ($\nu = 2$) geometry where $q < 0.73$. The other plasma parameters are fixed at $\sigma = 0.01$, $\alpha = 0.2$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$.

results obtained from this investigation can be summarized as follows:

1. The dusty plasma medium with dusts of opposite polarity supports the small but finite-amplitude shock structures, whose basic features (viz., polarity, amplitude, width, etc.) strongly depend on different plasma parameters, e.g., μ_1, μ_2, σ, α, β, q, and η_s.
2. We have found that the critical value of q, i.e., $q_c = 0.73$ for $\beta = 0.3$, $\alpha = 0.2$, and $\mu_2 = 0.5$. The variation of q_c with σ (ion-to-electron temperature ratio) and μ_1 (electron-to-negative dust number density ratio) is shown in figure 1. It is obvious from figure 1 that the possibility for the formation of DASHWs with positive and negative potential increases significantly due to the coexistence of electron and ion nonextensivity.
3. We have observed that when $q > 0.73$, negative (rarefactive) shock profile exists, whereas positive (compressive) shock profile exists when $q < 0.73$. The basic features of the positive and the negative polarity shock profiles are shown in figures 2–7.
4. The amplitude of DASHWs of both positive and negative potential increases with the increasing value of q (shown in figures 8 and 9).
5. The shock wave width Δ is found to increase with the increase of dissipative coefficient η_s but the

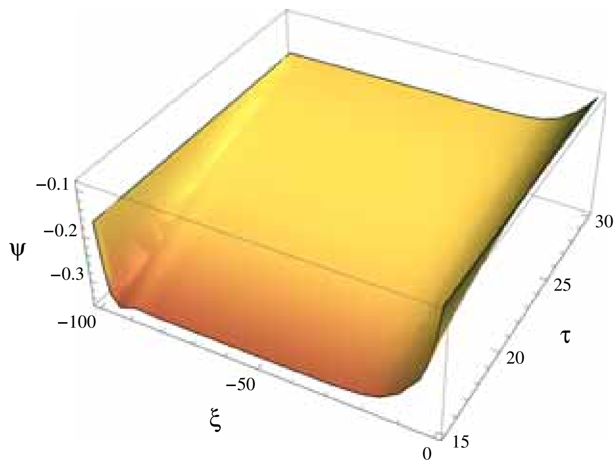

Figure 7. The variation of negative potential shock profile with ξ and τ for a nonplanar spherical ($\nu = 2$) geometry where $q > 0.73$. The other plasma parameters are fixed at $\sigma = 0.01$, $\alpha = 0.2$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$.

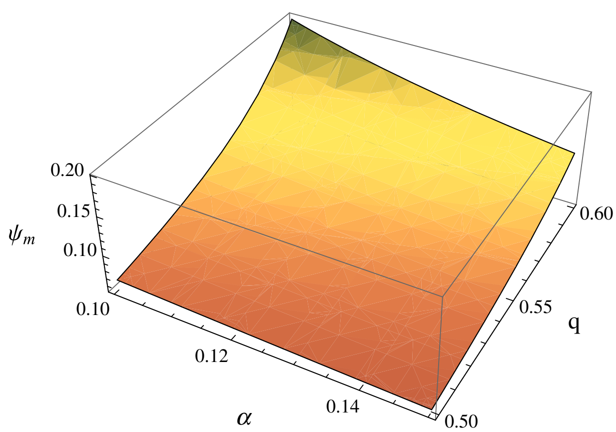

Figure 8. The variation of shock wave amplitude ψ_m with α and q when $q < q_c$. The other plasma parameters are fixed at $\sigma = 0.01$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$.

width decreases with the increase in speed of shock wave U_0 shown in figure 10. In addition, it can also be noted that with the increase in dissipation, the shock structures become more smooth and weak.

6. It may be concluded that the time expansion of nonplanar DASHWs greatly differs from 1D planar DASHWs. The characteristics of the shock waves are found to be influenced by time for both the cylindrical and the spherical cases.
7. By observing figures 2–7 we may conclude that the strength (amplitude) of the shock wave is maximum for spherical geometry, whereas it is minimum for planar geometry and intermediate for cylindrical geometry.

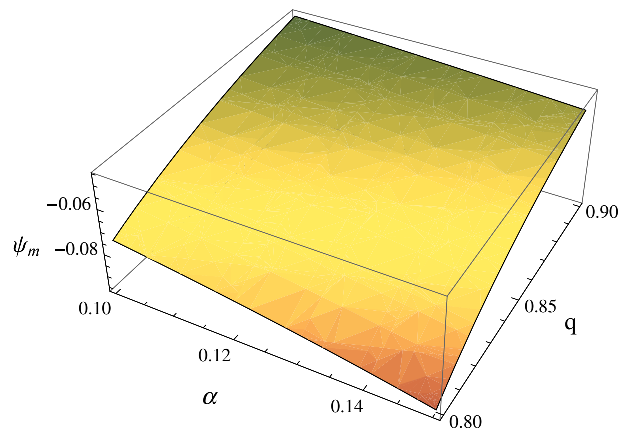

Figure 9. The variation of shock wave amplitude ψ_m with α and q when $q > q_c$. The other plasma parameters are fixed at $\sigma = 0.01$, $\beta = 0.3$, $\mu_1 = 0.2$, $\mu_2 = 0.5$, $\eta_s = 0.1$, and $U_0 = 0.01$.

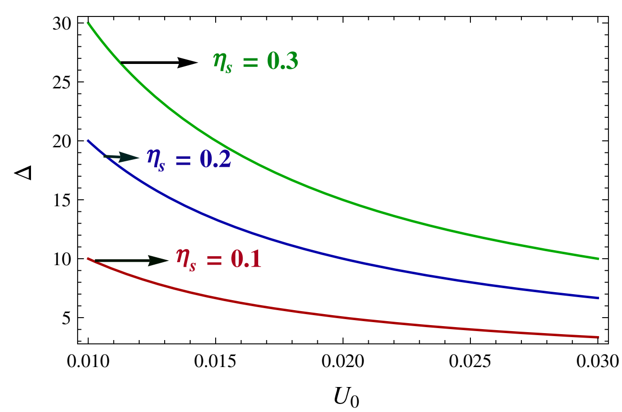

Figure 10. The variation of shock wave width Δ with U_0 for different viscosity coefficients η_s.

The ranges of dusty plasma parameters used in our numerical analysis are $\sigma \to 0.01$–0.1, $\alpha \to 0.1$–0.2, $\mu_1 \to 0.1$–0.2, and $\eta_s \to 0.1$–0.3. We note that, for a very small value of τ, the amplitude of the shock structures becomes very large to break down the validity of the reductive perturbation method used in our investigation and that our theory is no longer valid when $\tau \to 0$. So, we have plotted the figures only for finite τ. Therefore, our numerical observation are relevant to both space and laboratory plasmas [33–39] where nonextensivity plays a vital role. The analysis of nonplanar shock structure will be more general in the presence of external magnetic field, but is beyond the scope of our present work. We hope that the results of this investigation will be helpful for the better understanding of nonplanar geometry in nonextensive distributed laboratory and space plasmas where negatively as well as positively charged dust grains coexist. We, therefore, propose to perform a laboratory experiment which will be able to identify the special new features of our present work.

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