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Terahertz radiation source using a high-power industrial electron linear accelerator

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Abstract. High-power (∼ 100 kW) industrial electron linear accelerators (linacs) are used for irradiations, e.g., for pasteurization of food products, disinfection of medical waste, etc. We propose that high-power electron beam from such an industrial linac can first pass through an undulator to generate useful terahertz (THz) radiation, and the spent electron beam coming out of the undulator can still be used for the intended industrial applications. This will enhance the utilization of a high-power industrial linac. We have performed calculation of spontaneous emission in the undulator to show that for typical parameters, continuous terahertz radiation having power of the order of μW can be produced, which may be useful for many scientific applications such as multispectral imaging of biological samples, chemical samples etc.

Keywords. Undulator; terahertz radiation; industrial accelerator.

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1. Introduction

Due to several important scientific applications in research and in industry, there is a demand for powerful and tunable sources of terahertz (THz) radiation [1–6]. Direct laser-based sources, e.g., optically pumped lasers and quantum cascade lasers, provide THz radiation with average output power of the order of tens or hundreds of mW [7]. These sources are however inherently not continuously tunable. Conventional sources such as parametric oscillators and time-domain systems give pulsed THz radiation with relatively low average output power (∼ tens of nW) [7]. On the other hand, electron beam-based sources, e.g., backward wave oscillators (BWOs) and free-electron lasers (FELs) are very promising and powerful sources of continuously tunable THz radiation. Commercially available BWOs produce milliwatts of output power with maximum operating frequency upto 1 THz [8]. FELs, where an electron beam is passed through an undulator immersed in a resonating cavity for the generation of coherent radiation, are known for their wide tunability [9]. THz radiation is produced in an FEL because of interaction of the electron beam with the on-axis, static transverse magnetic field, varying sinusoidally along the undulator axis, in the presence of electromagnetic field building up in the resonator cavity. Lasing in these devices however requires high peak current; which needs an electron beam comprising of pulses of very short duration (∼picoseconds) [10]. A high quality electron beam with very low energy spread is required to achieve lasing in an FEL system [11]. The infrastructure needed to meet these requirements is quite expensive and bulky, which makes these devices impractical for table-top experiments. Investigations on the compact electron-beam based sources of THz radiation, such as Smith-Purcell FELs [12,13] and Čerenkov FELs [14,15] are attractive and promising. These devices however require very low emittance electron beam of low energy, which are yet to be demonstrated [16,17].

Recently, there has been a lot of interest in making high average power (up to 100 kW) industrial electron linear accelerator (linac) for various industrial applications such as polymer reforming, material irradiation, and for the pasteurization of food products [18–21]. A high-energy (∼10 MeV) electron beam is favourable for the irradiation processes due to its high penetration depth. Note that a low average power electron beam with very fine energy spread and emittance generates powerful THz radiation in an FEL system, due to coherent stimulated emission. The quality of the electron beam from a typical industrial linac may not be very good for the operation of a free-electron laser, but when such a high average power electron beam passes through an undulator, it can emit useful THz radiation through spontaneous emission. This radiation can fulfill the requirements of many scientific applications, such as imaging of biological samples, inspecting packaging and analysing chemical composition [6,22,23]. Thereafter, the spent electron beam can be used for the intended irradiation application, which gives us twofold advantage. An experimental observation of THz radiation from an undulator through spontaneous emission by using a 2 mA, 7.5 MeV electron beam has been reported recently in ref. [24]. In this paper, we have performed calculations to estimate the power and brightness of the emitted spontaneous radiation, when an electron beam emerging from a high average power industrial linac dedicated for the irradiation application, is passed through an undulator with suitably optimized parameters.

2. Theoretical analysis

A schematic of the proposed device is shown in figure 1. A high-energy electron beam emerging from a powerful industrial linac is allowed to pass through an undulator, before being directed to the target to be irradiated. It will generate a copious amount of THz radiation in the undulator through spontaneous emission. The proposed system is a single-pass system, i.e., without any optical resonator. Hence, we shall perform an analysis for the calculation of power radiated through spontaneous emission in the single-pass operation. The undulator is assumed to consist of N_u number of periods having a period length of λ_u. A detailed analysis of the motion of electron beam in the undulator is already available in [11,25–29], and we can write the expression for the energy radiated per unit frequency width $d\omega$ per unit solid angle $d\Omega$ by a system of N electrons in a bunch, along the direction of the unit vector $\mathbf{n}$ as [26]:

$$\frac{d^2\mathcal{I}}{d\omega d\Omega} = [N + (N^2 - N) f(\omega)] \frac{e^2\omega^2}{16\pi^3\epsilon_0 c^3} \times \left| \int_{-\infty}^{\infty} \mathbf{n} \times (\mathbf{n} \times \mathbf{v}) e^{i(\omega t - k_R \boldsymbol{n}\cdot\boldsymbol{r})} dt \right|^2. \quad (1)$$

Here, $\omega = ck_R$ is the angular frequency of the emitted radiation, k_R is the wavenumber of light, c is the speed of light, e is the electronic charge, ϵ_0 is the permittivity of the free space, $\mathbf{r}$ and $\mathbf{v}$ represent the instantaneous location and velocity respectively of the electron bunch centre that evolves due to the interaction with the undulator field, $f(\omega) = |\int e^{i\omega r/c} S(\mathbf{r}) d^3r|^2$ is a form factor, which describes coherence of the emitted light, and $S(\mathbf{r})$ is a continuous normalized density distribution function of the electron bunch such that the factor $NS(\boldsymbol{r})d^3r$ gives the probability of finding an electron in the region $\mathbf{d}^3r$ around $\mathbf{r}$. The total power radiated by an electron beam passing through an undulator will depend upon the bunch length of the electron beam. If the electron bunch length is significantly greater than the wavelength of light, then the form factor $f(\omega) = 0$ represents the incoherent limit and the emitted power will be N times the result from a single electron [26]. This is known as incoherent spontaneous emission or simply spontaneous emission (SE). For the electron bunch length shorter than the wavelength of light, form factor $f(\omega) = 1$ represents the coherent limit and the power radiated by the electron beam will be N^2 times the result of single electron [26]. This is known as coherent spontaneous emission (CSE). Thus, a prebunched electron beam with bunch length smaller than the wavelength of light can generate high-power,

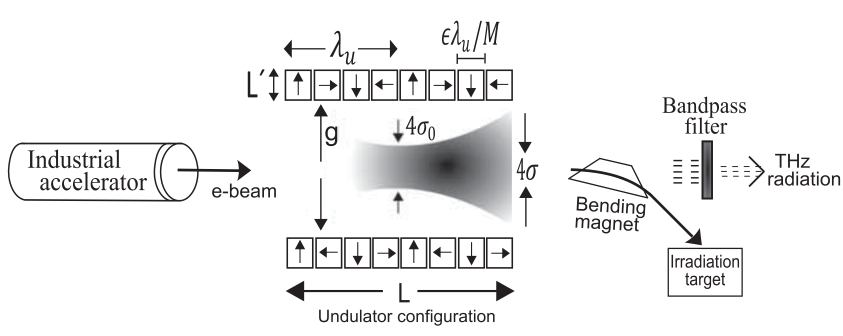

Figure 1. Schematic of a device based on the arrangement of industrial linac and undulator, to produce THz radiation along with the irradiation application.

coherent THz radiation, when it passes through an undulator [30,31]. This scheme has been successfully used in a compact advanced terahertz source (CATS) at ENEA, Italy, to generate THz radiation [32]. However, this approach requires an electron beam with low-energy spread, and an additional RF cavity to produce a prebunched electron beam. Moreover, the performance of the system is critically dependent on the shape of the electron bunch [33,34]. In the scheme proposed in this paper, we are considering the situation where the bunch length of the electron beam is significantly greater than the wavelength of light, which is the case of spontaneous emission.

We now briefly describe the salient features of spontaneous emission in the undulator for a monoenergetic beam, assuming an infinitely long undulator ($N_\mathrm{u} = \infty$), which can be derived using eq. (1). The expression for the wavelength λ of the radiation emitted at an angle θ from the axis of the undulator is given by

$$\lambda = \frac{\lambda_\mathrm{u}}{2n\gamma^2}\left(1+\gamma^2\theta^2+\frac{K^2}{2}\right), \tag{2}$$

where γ is the energy of the electron beam in units of its rest mass energy, n is the harmonic number, $K{=}eB_0/k_\mathrm{u}mc$ is the peak value of the undulator parameter, B_0 represents peak undulator magnetic field, $k_\mathrm{u} = 2\pi/\lambda_\mathrm{u}$, and m is the rest mass of the electron. We are considering $n = 1$ here, which corresponds to the fundamental wavelength. We notice here that the emitted radiation has a broad bandwidth due to the angular dependence of the radiation wavelength. For a finite undulator length, the radiation will be emitted for a finite duration, which will give a natural broadening of $\Delta\omega/\omega = 1/N_\mathrm{u}$, even at a fixed value of $\theta = 0$. If we keep θ confined within $1/\gamma^*\sqrt{N_\mathrm{u}}$, where $\gamma^* = \gamma/\sqrt{1+K^2/2}$, then the broadening due to angular variation will be the same as the natural broadening. Thus, if we collect the radiation only in a central cone with semiangle $\theta_\mathrm{cen} \approx 1/\gamma^*\sqrt{N_\mathrm{u}}$, the radiation will have a narrow relative bandwidth of $\sim 1/N_\mathrm{u}$.

The finite-energy spread of the electron beam also affects the radiation emitted, as can be seen from eq. (2). In the case of lasing in an FEL, the gain resulting due to stimulated emission is very sensitive to the electron energy and therefore, one requires a high-quality electron beam with low-energy spread to maintain the resonance condition of the system [35,36]. In our system, the electron beam emerging from the industrial linac has high relative energy spread (around 10%) and large pulse duration ($\sim$0.5 μs). When this electron beam passes through an undulator, the output radiation here will be generated only through spontaneous emission, unlike in the case of FELs where it is through stimulated emission. It is to be noted that if the energy spread of the electron beam is large, the intensity spectrum in the case of spontaneous emission of radiation will be broad, keeping the total output power nearly the same [37]. The relative frequency width $\delta\omega/\omega$ due to the energy spread is $2\delta\gamma/\gamma$, as can be seen through eq. (2). The total power radiated in all harmonics in an undulator, integrated over all wavelengths and angles, is given by $P_\mathrm{T} = \pi e\gamma^2 I N_\mathrm{u} K^2/3\epsilon_0\lambda_\mathrm{u}$ [25,28], where I is the beam current. For the spectroscopy and imaging related applications, we want narrow spectral bandwidth of the radiation. As already discussed, in an undulator, narrower bandwidth radiation is emitted inside the central radiation cone, which can be selected by inserting a bandpass filter [28] in the path of the output radiation as shown in figure 1. In the central radiation cone, the average power radiated at the fundamental frequency due to the spontaneous emission by the electron beam for an arbitrary K is given by [25,28]

$$P_\mathrm{cen} = \frac{\pi e\gamma^2 I}{\epsilon_0\lambda_\mathrm{u}}\frac{f(K)K^2}{(1+K^2/2)^2}, \tag{3}$$

where $f(K) = [J_0(x) - J_1(x)]^2$, J_0 and J_1 are zeroth- and first-order Bessel function of the first kind respectively, and $x = K^2/(4+2K^2)$. Note that the power emitted in the central radiation cone goes as the square of the beam energy for a relativistic electron beam. Therefore, higher energy beams are preferred for the generation of radiation through spontaneous emission.

Another important quantity of the radiation in the imaging-related applications is spectral brightness, which is defined as the photon flux per unit area and per unit solid angle at the source within a relative bandwidth of 0.1%. The expression for the on-axis spectral brightness of the undulator radiation is given by [25,28]

$$\mathcal{B}_{(\Delta\omega/\omega)} = \frac{7.25\times 10^6 \times \gamma^2 I(\mathrm{A}) N_\mathrm{u}^2 K^2 f(K)}{\sigma_\mathrm{T}^2(\mathrm{mm}^2)\left(1+\sigma_\mathrm{e}'^2/\theta_\mathrm{cen}^2\right)\left(1+K^2/2\right)^2} \times \frac{\mathrm{photons/s}}{\mathrm{mm}^2\,\mathrm{mrad}^2(0.1\%\mathrm{BW})}, \tag{4}$$

where I is in Amperes, $\sigma_\mathrm{T} = \sqrt{\sigma_\mathrm{e}^2+\sigma_0^2}$ is the rms source size of the undulator radiation in millimetres, σ_e is the rms electron beam size, σ_0 is the radiation beam waist, and σ_e' is the rms beam divergence. Note that the above formula is valid for a monoenergetic electron beam having very small divergence, i.e., $\sigma_\mathrm{e}' \ll$

θ_{cen}. When $\sigma_e' \simeq \theta_{cen}$, eq. (4) overestimates the brightness by a factor of 2, and we need to use numerical simulations for more accurate estimate of the spectral brightness [28].

The emitted radiation from the undulator will propagate along the direction of the electron beam. The radiation beam can be out-coupled by putting a window at the end of the interaction region, and can be used in various experiments. Within the undulator, the radiation beam will undergo diffraction in the transverse direction and one has to make sure that the diffracting optical beam should not be distorted by the vacuum chamber of the undulator. A rigorous analysis for the representation of undulator radiation has been recently given by Lindberg and Kim [29], where the radiation beam coming out of the undulator is described as a freely diffracting beam having rms beam waist size $\sigma_0 = \sqrt{\lambda_R L}/2\pi$, and rms beam divergence $\sigma_0' = (1/2)\sqrt{\lambda_R/L}$. Here, L is the length of the undulator and $\lambda_R = \lambda_u(1 + K^2/2)/2\gamma^2$ is the on-axis central wavelength of the radiated spectrum. It can be assumed that the beam waist is formed at the centre of the undulator. As we move away from the centre of the undulator, the beam size increases due to diffraction, which is described here in terms of Rayleigh range $Z_R = L/\pi$ [29]. The rms beam size σ at the exit of the undulator is given by $\sigma = \sigma_0\sqrt{1 + (L/2Z_R)^2}$. Note that this calculation assumes an electron beam of negligible size and divergence. Taking a finite size and divergence for the electron beam, the formulae for the rms beam size at the waist and the rms beam divergence get modified as $\sigma_T = \sqrt{\lambda_R L/4\pi^2 + \sigma_e^2}$, and $\sigma_T' = \sqrt{\lambda_R/4L + \sigma_e'^2}$ respectively [25]. We would like to mention that the effect of finite electron beam size and divergence is not very significant if the unnormalized rms electron beam emittance ε_{un}, which can be understood as the product of rms electron beam waist size and divergence, is much less than $\lambda_R/4\pi$, which is the product of rms optical beam waist size and divergence [25]. In terms of the normalized rms beam emittance $\varepsilon_n = \beta\gamma\varepsilon_{un}$, this criterion can be expressed as $\varepsilon_n \ll \beta\gamma\lambda_R/4\pi$, where β is the speed of electron in units of speed of light.

The undulator gap g has to be chosen sufficiently greater than the total beam diameter at the undulator exit, i.e., four times the rms beam size σ. Here, one has to keep in mind that increase in g will diminish the on-axis undulator magnetic field as it is proportional to $\exp(-k_u g/2)$ [11]. One needs to optimize the magnetic field strength together with the undulator length such that the beam diameter at the undulator exit remains sufficiently less than the minimum value of the undulator gap g. The expression for the peak undulator field B_0 for the Halbach configuration of a pure permanent magnet (PPM)-based undulator is given by [11]

$$B_0 = 2B_{rem}e^{-k_u g/2}(1 - e^{-k_u L'})\frac{\sin(\epsilon\pi/M)}{\pi/M}. \tag{5}$$

Here, B_{rem} is the remnant field of the PPM, and has the value of 1.1 T for NdFeB magnets, M is the number of magnets required to complete one period, L' represents the height of the magnet, and width of the magnet is $\epsilon\lambda_u/M$. In the most common configuration, $M = 4$, $\epsilon = 1$ and $L' = \lambda_u/4$. Using these values in eq. (5), we obtain the peak magnetic field in an undulator as [11]

$$B_0 = 1.57 \times \exp\left(-\frac{\pi g}{\lambda_u}\right). \tag{6}$$

The expression for the peak value of undulator parameter K is given by

$$K = 1.48 \times \lambda_u(\text{cm}) \times \exp\left(-\frac{\pi g}{\lambda_u}\right). \tag{7}$$

The analysis which we have presented in this section for the spontaneous emission of THz radiation in an undulator will be helpful in obtaining the parameters of a practical device, as described in the following section.

3. An example

To perform the calculations, we now take an example of a high average power industrial linac, and optimize the parameters of the undulator in accordance with the analysis described in §2. The parameters considered in our calculations are listed in table 1, which are close to the parameters of the ILU-14 linac operating in Budker Institute of Nuclear Physics, Russia [20,21]. This device is a pulsed linac having the electron beam energy in the range 7.5–10 MeV, average beam current of 10 mA, and average power up to 100 kW. The relative rms energy spread $\delta\gamma/\gamma$ for this system is around

Table 1. Linac parameters used in our calculations.

Electron beam energy	10 MeV
Beam peak current	480 mA
Beam pulse duration	420 μs
Repetition rate	50 Hz
Average beam current (I)	10 mA
Average beam power	100 kW
Relative rms energy spread ($\delta\gamma/\gamma$)	7%
Electron beam diameter ($4\sigma_e$)	2.4 mm
Normalized beam emittance (ε_n)	30 mm-mrad

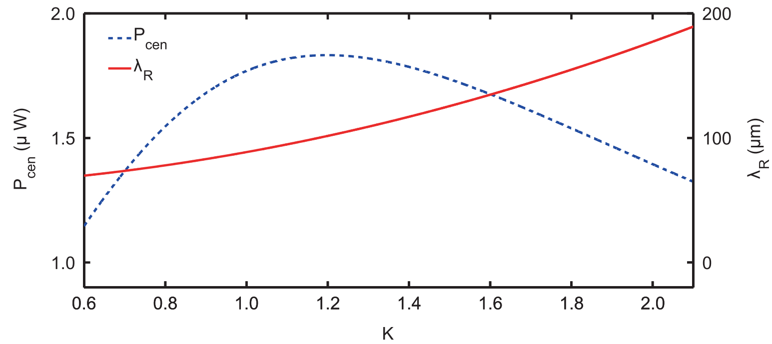

Figure 2. Plot of the output power (dashed) in central cone and operating wavelength (solid) as a function of undulator parameter K for $E = 10$ MeV, $I = 10$ mA, $N_u = 15$, and $\lambda_u = 50$ mm.

7% [20,21]. Note that such a high average power is possible in this linac because it operates with a long electron beam pulse (420 μs) repeating at 50 Hz, giving a large duty factor.

For a given energy spread of the electron beam from the linac, we choose the number of undulator periods $N_u \simeq \gamma/\delta\gamma$. Based on this argument, we have chosen $N_u = 15$ in our calculations. Note that if we take the undulator length more than this value, the spectrum width and the brightness of the emitted radiation will be limited by the energy spread of the electron beam. Also, if we take a longer undulator, the size of the radiation beam at the exit of the undulator will increase and the radiation beam may strike the edge of the vacuum pipe of the undulator. We have taken the undulator period λ_u as 50 mm and the undulator parameter K from 0.6 to 2.1. These values of K can be achieved by choosing the value of g between 40 and 20 mm of an undulator made up of pure permanent magnets, as can be seen using eq. (7) [11].

In figure 2, we have shown the variation of the output power in the central cone and the radiation wavelength as a function of undulator parameter K for a 10 MeV electron beam. The parameters used in the calculations are listed in tables 1 and 2. The radiation wavelength increases with K value, and the output power shows a maximum near $K = 1.2$. The maximum output power in the central radiation cone is obtained as 1.8 μW at 3 THz frequency. The selection of a narrow spectrum around the central wavelength can be made by using a THz bandpass filter in the path of the output radiation. The bandpass filters fabricated from gold-mesh frequency-selective surfaces are commercially available, and have a transmission of about 80% at 3 THz frequency [38]. The filtered radiation will have a continuous average power of 1.5 μW, and can be transported to a nearby experimental station via suitable optical arrangements. We would like to mention that an average power of around tens of nW can be achieved in conventional THz sources such as parametric oscillators and photoconductive antennas [7]. However, all these sources are not continuously tunable. Further, the relative bandwidth of the output radiation at central wavelength in our system is around 14%, which is nearly three times less than the relative bandwidth of the output radiation in the conventional sources described above [7].

Table 2. Parameters of the undulator.

Undulator period (λ_u)	50 mm
Number of periods (N_u)	15
Undulator gap (g)	20–40 mm
Undulator parameter (K)	2.1–0.6
Peak magnetic field (B_0)	0.45–0.13 T
Radiation wavelength (λ_R)	190–70 μm

For a fixed undulator period, the wavelength of the output radiation in the proposed system can be tuned by either changing the electron beam energy or by changing the undulator gap. In our example, we take the energy range from 7.5 MeV to 10 MeV, and the undulator gap g can be changed from 20 mm to 40 mm under normal tuning range. As shown in figure 2, the radiation wavelength can be tuned from 70 μm to 190 μm by varying K from 0.6 to 2.1 for a 10 MeV electron beam. For $E = 10$ MeV and $K = 2.1$, the rms radiation beam waist size σ_T is calculated as 2 mm by taking into account the finite electron beam size as mentioned in the previous section. The radiation beam diameter at the exit of the undulator is obtained as 15.3 mm. It is thus ensured that the maximum value of the optical beam size is sufficiently smaller than the minimum value of the undulator gap. Further increment in the value of K will make the optical beam size comparable

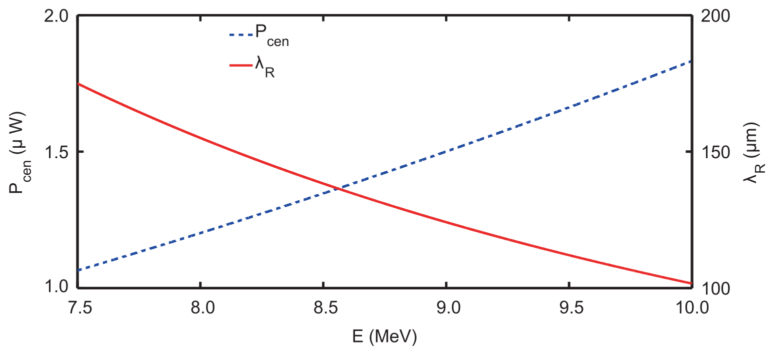

Figure 3. Plot of the output power (dashed) in central cone and operating wavelength (solid) as a function of electron beam energy for $I = 10$ mA, $K = 2.1$, $\lambda_u = 50$ mm, and $N_u = 15$.

or larger than the undulator gap. The radiation wavelength can also be tuned by changing the energy of the electron beam. In figure 3, we have shown such an example. Here, we have considered K=1.2 to keep the output power around the maximum value and varied the electron beam energy in the range 7.5–10 MeV. By using these parameters, we can get output radiation ranging from 1.6 THz to 4.3 THz with an average power of the order of 1 μW.

An approximate value of spectral brightness $\mathcal{B}_{\Delta\omega/\omega}$ of the emitted radiation for $K = 1.2$ and $E = 10$ MeV is estimated, by using eq. (4), to be 9×10^8 photons/s/mm^2/mrad^2 (0.1% BW), which can also be written as $\mathcal{B}_{\Delta\omega/\omega} = 0.6 \times 10^{-6}$ W/mm^2/sr (0.1% of BW). The spectral brightness of the output radiation in the proposed system is comparable to the brightness of coherent synchrotron radiation at the meterology light source [39], and higher than the conventional thermal sources.

We would now like to mention some applications that can be performed by using a continuously tunable radiation in the frequency range 1.6–4.3 THz, having an average power of the order of μW. As discussed in refs [3,40], THz radiation can easily pass through paper envelopes and can partially pass through plastics and ceramics [40]. This has great utility in performing multispectral THz imaging to identify substances without opening the envelopes containing sensitive materials [23]. THz radiation with average power of the order of μW is useful in non-destructive probing of sensitive biological materials and fragile electronic parts [1,2,4], where low average power (from tens of ∼nW up to 1 μW) is required [41]. At present, most of the commercially available conventional THz sources produce tens or hundreds of nW average power [7], which is commonly used in imaging and spectroscopy applications [1–4]. The proposed THz source with an average power of the order of μW and very high brightness will be helpful in obtaining high-resolution images in spectroscopy and imaging-related applications [22]. In these applications, the composition of different components in the biological and chemical samples is determined by measuring dielectric constant by using THz time domain imaging and spectroscopy technique, which is sensitive to the phase of the THz wave [3,22]. The phase-sensitive imaging applications require highly spatial or transverse coherent radiation [5,28,42]. THz undulator radiation in the proposed source has very high transverse coherence because it is generated by an electron beam having small emittance compared to the wavelength of the radiation beam, i.e., $\sigma_e\sigma_e' \ll \lambda/4\pi$, and will be helpful in performing phase-sensitive THz imaging applications.

In our calculations, a moderate value of normalized electron beam emittance is considered, i.e., $\varepsilon_n = 30$ mm-mrad, which can be easily achieved in a typical 10 MeV electron linac [43]. Note that the effect of finite electron beam size and divergence will not be significant as long as the normalized rms electron beam emittance is much less than $\beta\gamma\lambda_R/4\pi$, which is 310 mm-mrad for λ_R=190 μm. We have also found that the space charge effects are negligible for the chosen parameters of the electron beam as the space-charge term turns out to be much smaller than the emittance term in the beam envelope equation [44]. This condition can be expressed as [44]

$$\frac{\sigma_e^2}{2\beta\gamma\varepsilon_n^2}\frac{I_p}{I_A} < 1, \tag{8}$$

where $I_p = 4.8$ A is the peak current of micropulse and $I_A = 17.04$ kA is the Alfvén current. We evaluate left-hand side of the above equation, and obtain 0.003; for which the inequality is satisfied.

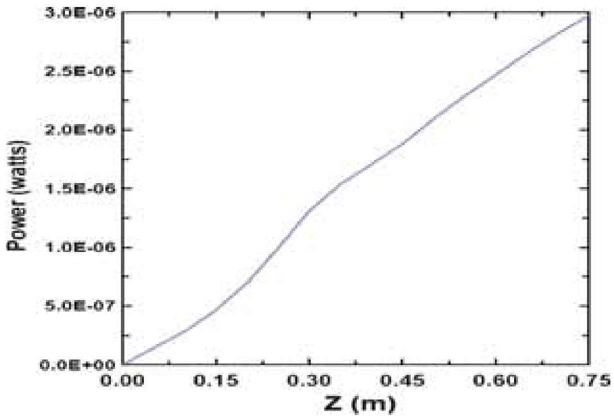

Figure 4. Plot of the power growth as a function of interaction length z for the spontaneous emission in an undulator having $K = 1.2$, $N_u = 15$, and $\lambda_u = 50$ mm. For the input electron beam, energy E is taken as 10 MeV and average current I is taken as 10 mA.

4. Numerical simulations

We have also performed numerical simulations using the computer code GINGER [45] to study the power growth and power spectrum of the spontaneous emission in the optimized undulator. GINGER is a multidimensional (full 3D for particles and 2D ($r - z$) for radiation), time-dependent computer code, which is primarily developed to simulate FELs in various configurations. In addition, it can also simulate the process of spontaneous emission discussed in our paper. It solves KMR [46] undulator-period-averaged equations in slowly-varying envelope approximation. Spontaneous emission evolves from the shot noise, which is modelled in GINGER by appropriately giving a controlled amount of randomness in initial longitudinal phases of particles [47].

Here, we describe the simulation results for one particular case, i.e., beam energy of 10 MeV and $K = 1.2$, which has been discussed in the previous section. The parameters of the electron beam and the undulator are the same as described earlier in the text. Figure 4 shows the evolution of the radiated power along the length of the undulator. It is seen that $\sim 3\ \mu$W of average power is radiated at the undulator exit, which is higher than 1.8 μW, the value obtained using the analytic formula for the power radiated in the central cone. This is because the computer code GINGER integrates the power over a bandwidth, which is larger than the bandwidth of the radiation emitted in the central radiation cone [48], as seen in figure 5. The full spectrum of radiation emitted in the central radiation cone should have wavelengths in the range $100 \pm 14\ \mu$m, whereas the radiation spectrum calculated using GINGER has wavelength even outside this range. The simulation result thus reasonably agrees with the results of the analytical calculations.

We have also performed numerical simulations to study the effect of energy spread and emittance of the electron beam on the power and brightness of the undulator radiation. Figure 6 shows the effect of variation of energy spread, when the beam emittance is kept constant at 30 mm-mrad. As expected, there is only a nominal reduction in the radiated power with energy spread, whereas the brightness decreases significantly with energy spread. This is because as the energy spread increases, there are more particles with reduced energy, resulting in slightly reduced emission of spontaneously emmitted radiation. The bandwidth of the emitted radiation however increases significantly with energy spread as discussed in §2, resulting in significant reduction of brightness. We have then studied the effect of variation in beam emittance as shown in figure 7. Here, as the beam emittance increases, the size of the electron beam increases because of which more number of particles become off-axis and radiate with less efficiency. Also, as the emittance increases, the radiated power decreases because the average longitudinal speed of particles decreases due to the increase in the off-axis component of velocity. This also leads to a spread in the frequency of radiation emitted by different electrons. As can be understood from the formula for the source size described in §2, the source size increases with emittance. Due to the broadening of the radiation spectrum and increase in the source size, the brightness decreases with emittance, as seen

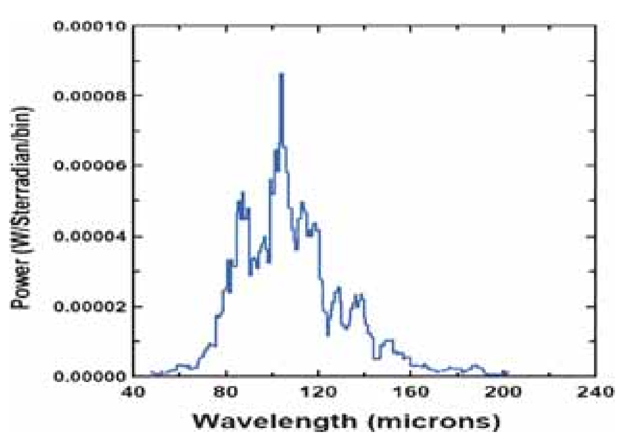

Figure 5. Plot of the power spectrum in spontaneous emission of radiation at the exit of a 0.75 m long undulator having $K = 1.2$, $N_u = 15$ and $\lambda_u = 50$ mm. The input electron beam energy E is taken as 10 MeV and input electron beam current I is taken as 10 mA.

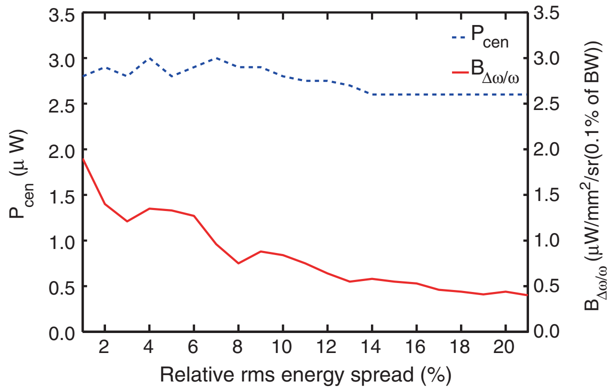

Figure 6. Plot of the output power (dashed) in central cone and spectral brightness (solid) as a function of relative rms beam energy spread for $\varepsilon_n = 30$ mm-mrad, $I = 10$ mA, and $K = 1.2$.

in figure 7. We would like to mention that we have verified in the simulation that the power and brightness are directly proportional to the beam current, which is expected from eqs (3) and (4).

Based on the numerical simulation studies presented in this section, we find that the proposed system will be able to generate ~ 1 μW of terahertz power with 10^9 photons/s/mm^2/mrad^2 (0.1% BW) brightness, including the effect of finite emittance and energy spread of the electron beam. We would like to emphasize that in the case of FEL, the performance of the system has much stronger dependence on energy spread and emittance, unlike in the case of spontaneous emission discussed here. This is because the working of an FEL requires a sharp resonance between the electron speed and the speed of ponderomotive wave, requiring

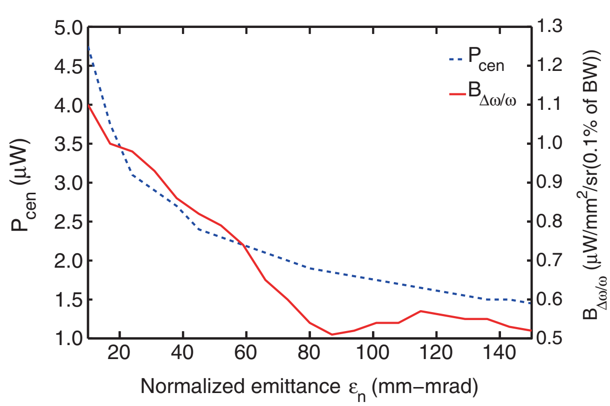

Figure 7. Plot of the output power (dashed) in central cone and spectral brightness (solid) as a function of normalized beam emittance for the undulator parameter $K = 1.2$. Here, the beam current is assumed to be 10 mA and the relative rms energy spread of the electron beam is assumed to be 7%.

a sharp energy spectrum and emittance of the electron beam. As this requirement is not there in the case of spontaneous emission, the industrial linacs, which do not satisfy the stringent criteria on beam quality, can be used for the generation of terahertz radiation through spontaneous emission.

5. Discussions and conclusions

In this paper, we have discussed a proposal to enhance the utilization of a high average power industrial electron linac by using an undulator to generate terahertz radiation, in addition to the irradiation application. In an FEL, the average power of the electron beam is typically low, which is usually of the order of few tens or hundreds of watts. Such an electron beam typically generates THz radiation of sub-nW power through spontaneous emission, which gets enhanced to the order of few tens or hundreds of mW by stimulated emission. To have a strong interaction between the electron beam and the optical beam in an FEL system such that it generates stimulated radiation, the electron beam envelope has to be well inside the optical beam envelope throughout the interaction length, and its average electron beam radius over the length of the undulator needs to be minimum. This requires that the parameters related to the profile of the electron beam are suitably matched at the entrance of the undulator. Also, for the operation in the FEL oscillator configuration, we need to use an achromat to bend the electron beam and match it with the axis of the undulator immersed in the optical cavity. All these are achieved by having a suitable beam transport line between the linac and the undulator. In addition, as discussed, very high quality electron beam with low energy spread, low emittance and high peak current is needed for the generation of stimulated emission in an FEL. These requirements are not necessary, when we use a high average power electron beam for the generation of terahertz radiation by spontaneous emission, as discussed in the paper. This makes the proposed device simpler.

We would like to emphasize that the length of the undulator needs to be chosen in an optimum manner based on the energy spread of the electron beam. We choose the number of undulator periods $N_u \simeq \gamma/\delta\gamma$. A longer undulator will generate brighter radiation according to eq. (4). However, as discussed, eq. (4) is valid for monoenergetic beam. For a beam with finite energy spread, the brightness will reduce because finite energy spread will lead to additional width in radiation spectrum. If we take $\delta\gamma/\gamma > 1/N_u$, the brightness and the spectral width of the emitted radiation will be limited by the energy spread of the electron beam. We therefore choose $N_u \simeq \gamma/\delta\gamma$. Also, the size of the radiation beam at the undulator exit will be large for a longer undulator, and the radiation beam may strike at the edge of the vacuum pipe of the undulator in this case. It is also important to note that as the electron beam propagates down the undulator, an additional energy spread will be induced due to the quantum fluctuations of the spontaneous undulator radiation [49]. For parameters considered in our example, we have calculated the relative energy spread induced by the undulator as 0.015% by following an analysis given in ref. [49]. This value of induced energy spread is quite low compared to the considered initial relative energy spread of 7.0%. Hence, the spent electron beam after emitting the THz radiation can still be used for irradiation.

We would like to mention that our aim in this paper is to show that with a modest investment in terms of putting a small undulator with optimized parameters as an additional component in an existing irradiation facility based on a high average power industrial linac, we can build a useful THz source. In India, 10 MeV, 10 kW industrial linacs have been built at BARC [50,51], and 10 MeV, 6 kW industrial linacs have been built at RRCAT [52]. A 3 MeV, 30 kW accelerator is also being developed at BARC [51]. However, as discussed in §2, the power in the THz radiation in the proposed device is directly proportional to the square of the beam energy. Therefore, a beam energy of ∼10 MeV is preferred over lower beam energy for the THz generation in the proposed scheme. A 10 MeV, 5–10 kW linac developed at BARC and RRCAT can also be used for the THz generation. The analytical calculations and numerical simulations presented in this paper are applicable to these linacs also. Power generated through the spontaneous emission is proportional to the average beam current or the beam power. Hence, for the 5–10 kW linac, the terahertz power generated is proportionately reduced to 50–100 nW, because the energy spread and emittance are expected to be similar to the values taken in this paper. There are proposals to develop high average power electron linacs in future at both these places. If such a project is taken up in future, the concept proposed in this paper may be used to generate THz radiation with an average power of ∼1 μW for various applications.

To conclude, we have presented an analysis of a device, which is based on the arrangement of a powerful industrial linac and an undulator, to produce useful THz radiation, along with the intended irradiation applications. For the analysis of undulator radiation, we followed a recent approach given in ref. [29]. By taking an example of the high average power industrial linac, we have optimized the parameters of an undulator which can be used to produce copious THz radiation. We observed that an undulator with moderate parameters such as 0.75 m length, 50 mm period and K from 0.6 to 2.1, can be used with 7.5 to 10 MeV, 100 kW linac to produce a continuous tunable THz radiation (1.6–4.3 THz) with output power in the central cone of the order of μW and spectral brightness of the order of 10^9 photons/s/mm^2/mrad^2 (0.1% BW). We have also verified these calculations by performing numerical simulations using the computer code GINGER [45]. Thus, the utilization of a high-power electron linac can be enhanced with the help of a short undulator. The device can simultaneously be used for terahertz generation, as well as irradiation applications. The output radiation can be tuned by changing the undulator gap or by changing the electron beam energy. Tunable continuous THz radiation generated using such a device will be very useful in the imaging and spectroscopy-related applications [22,23].

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