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Recent observational constraints on generalized Chaplygin gas in UDME scenario

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Abstract. Recent observational predictions suggest that our Universe is passing through an accelerating phase in the recent past. This acceleration may be realized with the negatively pressured dark energy. Generalized Chaplygin gas may be suitable to describe the evolution of the Universe as a candidate of unified dark matter energy (UDME) model. Its EoS parameters are constrained using (i) dimensionless age parameter (H_0t_0) and (ii) the observed Hubble ($H(z)-z$) data (OHD) + baryon acoustic oscillation (BAO) data + cosmic microwave background (CMB) shift data + supernovae (Union2.1) data. Dimensionless age parameter puts loose bounds on the EoS parameters. Best-fit values of the EoS parameters H_0, A_s and α (A_s and α are defined in the energy density for generalized Chaplygin gas (GCG) and in EoS) are then determined from OHD + BAO + CMB + Union2.1 data and contours are drawn to obtain their allowed range of values. The present age of the Universe (t_0) and the present Hubble parameter (H_0) have been estimated with 1σ confidence level. Best-fit values of deceleration parameter (q), squared sound speed (c_s^2) and EoS parameter (ω) of this model are then determined. It is seen that GCG satisfactorily accommodates an accelerating phase and structure formation phase.

Keywords. Observational cosmology; dark energy; accelerating Universe.

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1. Introduction

Recent cosmological observations from supernovae [1–4], WMAP [5–9] and baryon acoustic oscillation (BAO) data [10] predict that the present Universe is passing through a phase of accelerating expansion. This late acceleration cannot be realized in the framework of Standard Model of particle physics. In the literature, theories are put forward to accommodate late acceleration either (i) by a modification of the matter sector in the Einstein–Hilbert action or (ii) by a modification of the gravitational sector. Universe accelerates when matter sector is modified with the introduction of a new kind of matter called exotic matter (dark energy). It has a negative pressure with gravitational repulsion. As the Universe expands, dark energy stays at nearly constant energy density and, as the matter in the Universe thins out, the dark energy begins to dominate. The most simple candidate for these uniformly distributed (i.e. unclustered) dark energy is considered to be in the form of vacuum energy density or cosmological constant (Λ). The model with cosmological constant is entangled with (a) fine tuning problem (present amount of the dark energy is so small compared to the fundamental scale) and (b) coincidence problem (dark energy density is comparable with critical density today). The other choices are: (i) a light homogeneous scalar field ϕ, whose effective potential $V(\phi)$ leads to an accelerated phase at a later stage of the Universe [11,12], (ii) an X-matter component, which is characterized by an equation of state $p=\omega\rho$, where, $-1\leq\omega<0$ [13], (iii) effects from extra dimensions [14,15], (iv) an exotic fluid, Chaplygin gas etc.

Recently, it has been shown that Chaplygin gas (CG) may be useful for describing dark energy because of its negative pressure. Although it has positive energy density, it carries a negative pressure because of which it is referred to as an exotic fluid. Initial idea of CG originated in aerodynamics [16] and it may be considered as an alternative to quintessence [17]. The equation of state (henceforth, EoS) for CG is

$$p=-\frac{A}{\rho}, \tag{1}$$

where A is a positive constant. In the context of string theory, Chaplygin gas emerges from the dynamics of a generalized d-brane in a (d+1,1) space–time. It can be described by a complex scalar field which is obtained from a generalized Born–Infeld action. But cosmological models with CG is not consistent with observational data of SNIa, BAO, cosmic microwave background (CMB) and so on [18,19]. Subsequently, generalized Chaplygin gas is introduced (in short, GCG) [20,21] whose EoS is given by

$$p = -\frac{A}{\rho^{\alpha}} \tag{2}$$

with $0 < \alpha \leq 1$. In the above EoS, A and α are free parameters that will be reduced to that of CG for $\alpha = 1$. This GCG can describe matter-dominated ($p = 0$) early Universe to dark energy-dominated ($p =$ negative) present Universe consistently. This GCG model has successfully confronted with various observational tests: high-precision cosmic microwave background radiation data [22], supernova data [23] and gravitational lensing [24]. It is capable of explaining background dynamics [25] and various other features of a homogeneous isotropic Universe satisfactorily.

Despite all these pleasing features, there are certain issues of structure formation that plague the dark matter sector of this model [26]. Some alternate views are also suggested in this context [27]. However, here the emphasis is on the dark energy part of the model that is responsible for the recent accelerated cosmic expansion.

The possibility of describing dark energy via GCG model has generated interest to constrain the model using the observational data [18,19,28]. Here attempts have been made to constrain the GCG model parameters with the recent sets of OHD + BAO + CMB + Union2.1 data and the age parameter ($H_0 t_0$).

Dimensionless age parameter ($H_0 t_0$) is used here to get an estimation of the EoS parameters of GCG. Then, thirty-eight observational Hubble parameter data (OHD), seven BAO data, one CMB shift parameter and 580 union compilation (2.1) data are used for the background test.

In the analysis, total chi-square is constituted using all the background data. The best-fit values of the model parameters are then determined from the chi-square function to study the evolution of the Universe.

The paper is organized as follows: In §2, Einstein's field equations are constituted. In §3, constraints are obtained on the EoS parameters from the dimensionless age parameter of the Universe. In §4, constraints are obtained from observational data of the background test. In §5, results are given. Finally, in §6, a brief discussion is given.

2. Einstein's field equations

Einstein's field equation, in general theory of relativity (GTR) relating matter with space–time curvature (geometry), is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{3}$$

where $R_{\mu\nu}$ represents the Ricci tensor, R represents the Ricci scalar, $T_{\mu\nu}$ represents the energy–momentum tensor and $g_{\mu\nu}$ represents the metric tensor in four-dimensions. The Friedmann–Robertson–Walker (FRW) metric which is considered here is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t) \times \left[\frac{\mathrm{d}r^2}{1 - kr^2} + r^2 (\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2) \right], \tag{4}$$

where $k = 0, +1(-1)$ is the curvature parameter in the spatial section representing flat, closed (open) Universe, $a(t)$ is the scale factor of the Universe and r, θ, ϕ are the co-moving coordinates.

Now using the FRW metric (4) in the Einstein's field eq. (3), the following equations are obtained:

$$3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = 8\pi G \rho, \tag{5}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi G p, \tag{6}$$

where ρ and p represent the energy density and pressure respectively. Considering the conservation equation which is given by

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + 3H(\rho + p) = 0, \tag{7}$$

and using EoS for GCG as given by eq. (2) energy density for GCG is obtained as

$$\rho_{\mathrm{GCG}} = \rho_0 \left[A_{\mathrm{s}} + \frac{1 - A_{\mathrm{s}}}{a^{3(1+\alpha)}} \right]^{1/(1+\alpha)}, \tag{8}$$

where $H = \dot{a}/a$ is the Hubble parameter, $A_{\mathrm{s}} = A/\rho_0^{\alpha+1}$ and ρ_0 is a positive integration constant. The scale factor of the Universe (a) is related to the redshift parameter (z) as $(a/a_0) = 1/(1+z)$, where the present scale factor of the Universe is chosen as $a_0 = 1$ for convenience. From eq. (8) it is evident that the positivity condition of the energy density is ensured when $0 \leq A_{\mathrm{s}} \leq 1$. Considering that the total energy density comprises baryon (ρ_{b}) and GCG energy density as

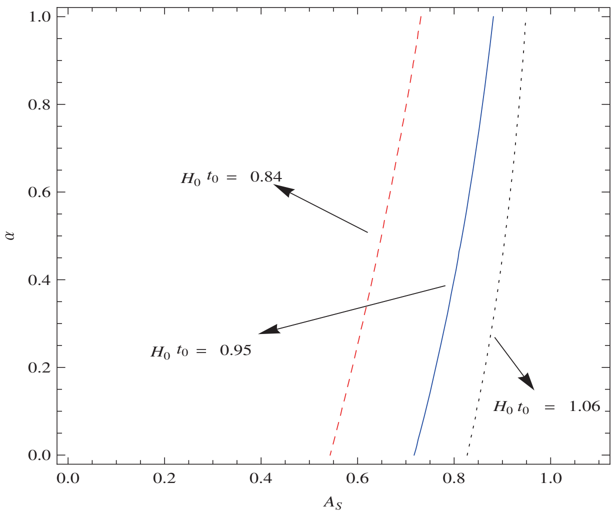

Figure 1. Contours of A_s and α at 68.3% confidence limit.

$\rho = \rho_b + \rho_{GCG}$ and using the field eq. (5) the Hubble parameter can be expressed for a flat Universe as a function of redshift as

$$H(z) = H_0[\Omega_{b0}(1+z)^3 + (1-\Omega_{b0}) \times [A_s + (1-A_s)(1+zz)^{3(1+\alpha)}]^{1/(1+\alpha)}]^{1/2}, \quad (9)$$

where Ω_{b0} and H_0 are the present dimensionless baryon energy density and present Hubble parameter respectively. In this model, it is considered that the baryons (ρ_b) do not interact with the generalized Chaplygin gas (ρ_{GCG}). In terms of scale factor, $H(a)$ is given as

$$H(a)=H_0\left[\frac{\Omega_{b0}}{a^3}+(1-\Omega_{b0})\left[A_s+\frac{(1-A_s)}{a^{3(1+\alpha)}}\right]^{1/(1+\alpha)}\right]^{1/2}. \quad (10)$$

Another important parameter is the adiabatic squared sound speed (c_s^2) whose positive value indicates the structure formation at the early Universe. This squared sound speed is defined as

$$c_s^2 = \frac{\delta p}{\delta \rho} = \frac{\dot{p}}{\dot{\rho}}. \quad (11)$$

Using the EoS for GCG (eq. (2)), expression for energy density (eq. (8)) and $A_s = A/\rho_0^{\alpha+1}$, it can be expressed in terms of model parameters as,

$$c_s^2 = \frac{A_s\alpha}{\left[A_s + (1-A_s)(1+z)^{3(1+\alpha)})\right]}. \quad (12)$$

In terms of equation of state ($\omega = p/\rho$) it is

$$c_s^2 = -\alpha\omega \quad (13)$$

where

$$\omega = -\frac{A_s}{\left[A_s + (1-A_s)(1+z)^{3(1+\alpha)})\right]}. \quad (14)$$

It may be mentioned here that for causality and stability under perturbation, the inequality condition $0 \le c_s^2 \le 1$ is to be satisfied [43].

3. Age of the Universe as a constraining tool

The present age of the Universe (t_0) is obtained from the definition of Hubble parameter ($H = \dot{a}/a$) as

$$t_0 = \int_0^1 \left[\frac{\mathrm{d}a}{aH(a)}\right], \quad (15)$$

where the scale factor of the Universe (a) is related to red-shift as

$$\frac{a}{a_0} = \frac{1}{1+z}$$

and $H(a)$ is given by eq. (10). The limit on the scale factor runs from 0 to 1 (i.e. from the beginning to the present state of the Universe). The predicted age of the Universe in GCG model becomes

$$t_0 = \frac{1}{H_0}\int_0^1 \left[\frac{\mathrm{d}a}{af(a,\Omega_{b0},A_s,\alpha)}\right] \quad (16)$$

with

$$f(a,\Omega_{b0},A_s,\alpha) = \frac{H(a)}{H_0}. \quad (17)$$

The age parameter (H_0t_0) is dimensionless and a constant irrespective of the model considered. From experimental facts the value of H_0t_0 lies in the range $H_0t_0 =$

Table 1. OHD data.

z	$H(z)$	σ	Ref.	z	$H(z)$	σ	Ref.
0.070	69.0	±19.6	[32]	0.57	96.8	±3.4	[39]
0.09	69.0	±12.0	[29]	0.593	104.0	±13.0	[31]
0.12	68.6	±26.2	[32]	0.6	87.9	±6.1	[35]
0.17	83.0	±8.0	[29]	0.68	92.0	±8.0	[31]
0.179	75.0	±4.0	[31]	0.73	97.3	±7.0	[35]
0.199	75.0	±5.0	[31]	0.781	105.0	±12.0	[31]
0.20	72.9	±29.6	[32]	0.875	125.0	±17.0	[31]
0.240	79.69	±2.99	[34]	0.88	90.0	±40.0	[30]
0.27	77.0	±14.0	[29]	0.9	117.0	±23.0	[29]
0.28	88.8	±36.6	[32]	1.037	154.0	±20.0	[31]
0.30	81.7	±6.22	[40]	1.3	168.0	±17.0	[29]
0.34	83.8	±3.66	[34]	1.363	160.0	±33.6	[33]
0.35	82.7	±9.1	[37]	1.43	177.0	±18.0	[29]
0.352	83.0	±14.0	[31]	1.53	140.0	±14.0	[29]
0.4	95.0	±17.0	[29]	1.75	202.0	±40.0	[29]
0.43	86.45	±3.97	[34]	1.965	186.5	±50.4	[33]
0.44	82.6	±7.80	[35]	2.3	224.0	±8.6	[36]
0.48	97.0	±62.0	[30]	2.34	222.0	±8.5	[41]
0.57	87.6	±7.8	[38]	2.36	226.0	±9.3	[42]

0.95 ± 0.11 [44]. Imposing this constant age parameter, the effective ranges of values of the free parameters in this model can be determined. It is taken into consideration that parameters A_s, α have some preferred range of values. At 68.3% confidence limit, the value of A_s varies between 0.539 and 0.951 and α varies between 0 and 1 (figure 1).

4. Observational constraints

The following observational data of the background test have been used for the analysis:

1. The observed Hubble data (OHD) ($H(z) - z$) (table 1)
2. Baryon acoustic oscillation data (BAO) (table 2)
3. Cosmic microwave background (CMB) shift parameter
4. The SNIa data [47].

In the analysis, H_0 is treated as free parameter along with A_s and α. So, the GCG model now has three free EoS parameters (H_0, A_s and α) as present dimensionless baryon energy density is considered from observations in the expression of Hubble parameter (eq. (9)). Chi-square function is defined in terms of EoS parameters of the model for the four different background tests (mentioned above) and then numerical analysis is done.

4.1 χ^2-*function for the observed Hubble data (OHD) (*table 1*)*

Case I: *For OHD.* OHD can be used to constrain cosmological parameters because they are obtained from model-independent direct observations. Thirty-eight data points [29–42] are presented in table 1. Until now, two methods have been developed to measure OHD: galaxy differential age method and radial BAO size methods. The data obtained in the first approach are in refs [29–33] and in second method are in refs [34–42]. To analyse, first chi-square (χ^2_{H-z}) function for the Hubble parameter is defined as

$$\chi^2_{H-z}(H_0, A_s, \alpha) = \sum \frac{(H(H_0, A_s, \alpha, z) - H_{\text{obs}}(z))^2}{\sigma_z^2}, \tag{18}$$

where $H_{\text{obs}}(z)$ is the observed Hubble parameter at redshift z and σ_z is the error associated with that particular observation as shown in table 1.

Case II: *For BAO data.* A model-independent BAO peak parameter for low redshift z_1 measurements in a flat Universe is given by [10]

$$\mathcal{A}(A_s, \alpha, z_1) = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left(\frac{\int_0^{z_1} (dz/E(z))}{z_1} \right)^{2/3}, \tag{19}$$

where Ω_m is the dimensionless matter density parameter for the Universe and $E(z) = H(z)/H_0$. The chi-square function in this case is defined as

$$\chi^2_{\text{BAO}}(A_s, \alpha) = \sum \frac{(\mathcal{A}(A_s, \alpha, z_1) - \mathcal{A}_{\text{obs}}(z_1))^2}{(\sigma_{\mathcal{A}})^2}. \tag{20}$$

The BAO data are given in table 2.

Case III: *For CMB.* The *CMB* shift parameter ($\mathcal{R}$) is given by [48]

$$\mathcal{R}(A_s, \alpha) = \sqrt{\Omega_m} \int_0^{z_{ls}} \frac{dz'}{H(z')/H_0}, \tag{21}$$

where z_{ls} is the z at the surface of last scattering. The WMAP7 data predict $\mathcal{R} = 1.726 \pm 0.018$ at $z = 1091.3$. Chi-square function is defined as

$$\chi^2_{\text{CMB}}(A_s, \alpha) = \frac{(\mathcal{R}(A_s, \alpha) - 1.726)^2}{(0.018)^2}. \tag{22}$$

Case IV: *For supernovae data.* The distance modulus function (μ) is defined in terms of the luminosity distance (d_L) as [49]

$$\mu(H_0, A_s, \alpha, z) = m - M = 5 \log_{10}(d_L/\text{Mpc}) + 25, \tag{23}$$

where m and M are the apparent and absolute magnitudes and d_L is given [49] as

$$d_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}. \tag{24}$$

Table 2. BAO data.

z_1	$\mathcal{A}$	$\sigma_{\mathcal{A}}$	Ref.
0.106	0.526	0.028	[45]
0.20	0.488	0.016	[45]
0.35	0.484	0.016	[45]
0.44	0.474	0.034	[46]
0.57	0.436	0.017	[38,45]
0.60	0.442	0.020	[46]
0.73	0.424	0.021	[46]

In this case, the chi-square (χ^2_μ) function is defined as

$$\chi^2_\mu(H_0, A_s, \alpha) = \sum \frac{(\mu(H_0, A_s, \alpha, z) - \mu_{obs}(z))^2}{\sigma_z^2}, \tag{25}$$

where $\mu_{obs}(z)$ is the observed distance modulus at redshift z and σ_z is the corresponding error for the observed data [47].

4.2 *Combined χ^2 and likelihood function*

Finally, total χ^2-function is defined as follows:

$$\chi^2_{back}(H_0, A_s, \alpha) = \chi^2_{OHD}(H_0, A_s, \alpha) + \chi^2_{BAO}(A_s, \alpha) + \chi^2_{CMB}(A_s, \alpha) + \chi^2_\mu(H_0, A_s, \alpha). \tag{26}$$

Likelihood function L is related with the chi-square as $L \propto \exp(-\chi^2_{tot}/2)$. To get the best-fit values of the EoS parameters of the GCG model, likelihood function may be maximized or the chi-square function may be minimized. Best-fit values are thereafter utilized to draw the contours with the help of the χ^2-function.

5. Results of the analysis

In this analysis, the value of Ω_{b0} is taken as 0.047 as the simple average of $\Omega_{b0} = 0.0464$ as per the WMAP data [45] and $\Omega_{b0} = 0.0485$ as per the Planck [50,51] data. Using the OHD + BAO + CMB + Union2.1 data, contours of H_0–α, H_0–A_s and A_s–α are drawn in figures 2a, 2b and 2c respectively. Plot of likelihood functions are given in figure 3 for H_0, A_s and α. The permitted range of EoS parameters of the model at 68.3% confidence level (1σ) obtained from background data are $H_0 = 70.23^{+0.35}_{-0.36}$, $\alpha = 0.023^{+0.034}_{-0.034}$ and $A_s = 0.772^{+0.021}_{-0.023}$ (tables 3–5).

In table 6, ranges of values of other EoS parameters (q, c_s^2, ω) are shown. In table 7, parameters from other observations have been shown along with our findings. In figure 4, variation of deceleration parameter (q), squared sound speed (c_s^2) and equation of state (ω) are plotted with z from those data.

6. Discussion

Here cosmological model with GCG as a candidate of unified dark matter energy model (UDME) is presented

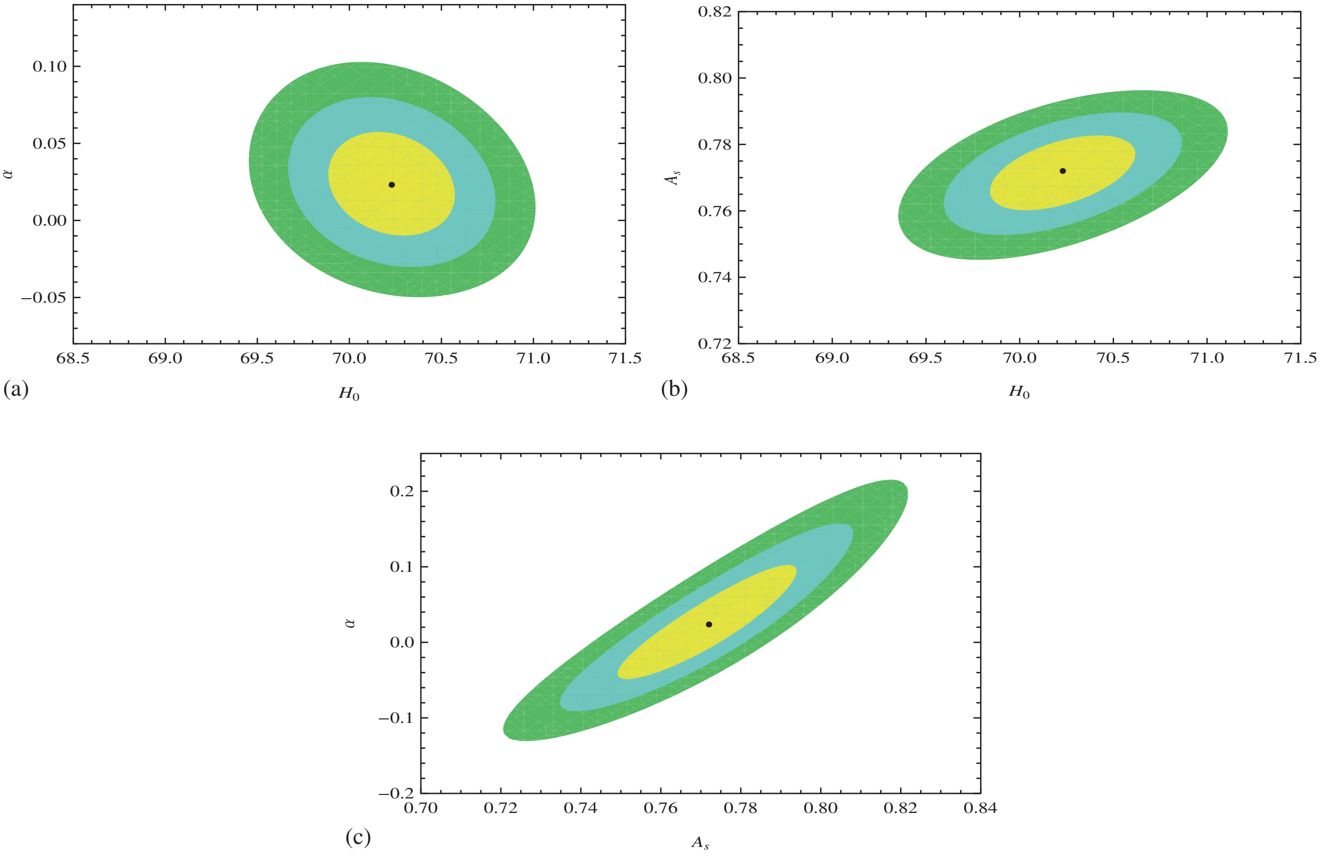

Figure 2. Contours of (**a**) H_0–α, (**b**) H_0–A_s and (**c**) A_s–α from OHD + BAO + CMB + Union2.1 data at 68.3% (yellow), 95.4% (blue) and 99.7% (green) confidence limits.

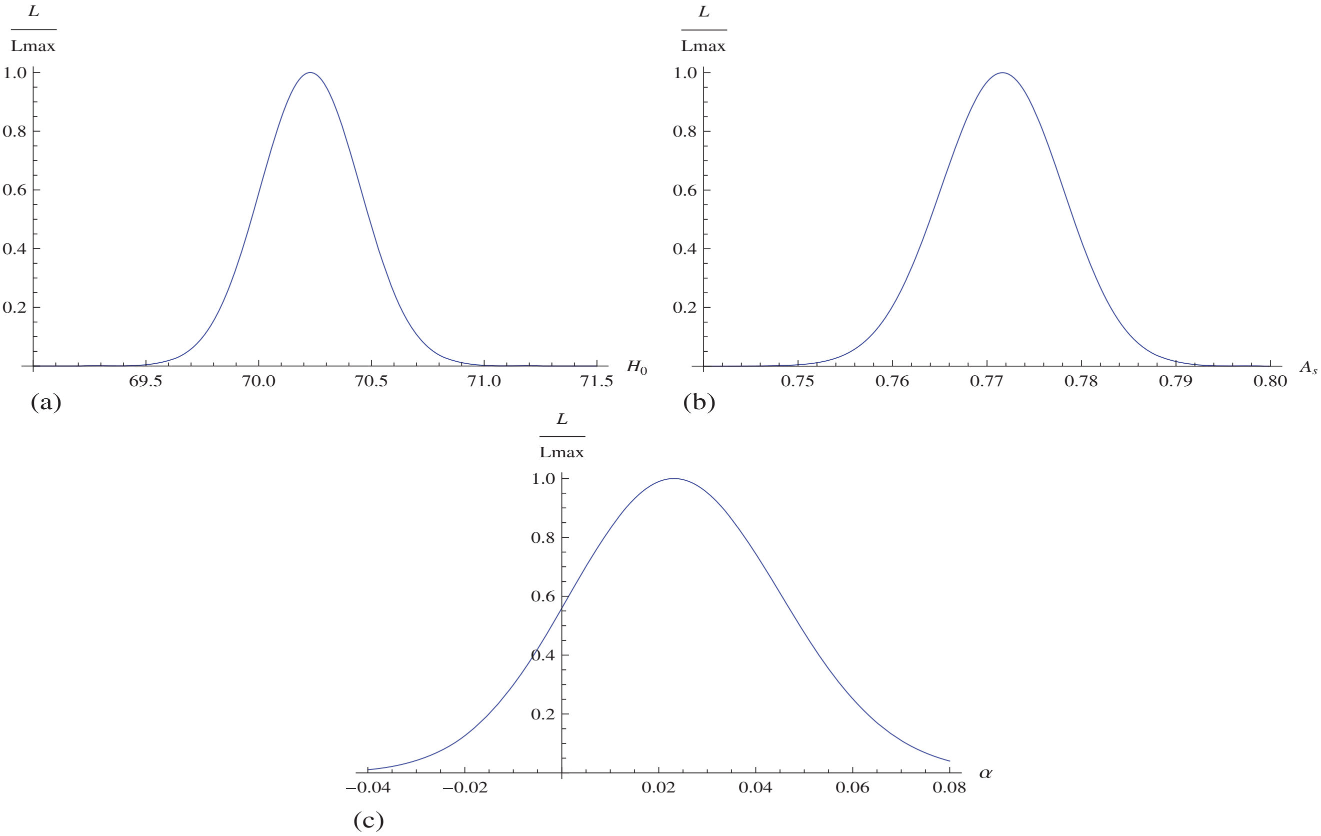

Figure 3. Likelihood contours of (**a**) H_0, (**b**) A_s and (**c**) α from OHD + BAO + CMB + Union2.1 data.

Table 3. Range of values of the EoS parameters, where OU = OHD + BAO + CMB + Union2.1.

Data	CL (%)	H_0	α
OU	68.3	(69.87, 70.58)	(−0.011, 0.057)
OU	95.4	(69.65, 70.80)	(−0.032, 0.080)
OU	99.7	(69.45, 71.01)	(−0.049, 0.103)

Table 4. Range of values of the EoS parameters, where OU = OHD + BAO + CMB + Union2.1.

Data	CL (%)	H_0	A_s
OU	68.3	(69.83, 70.62)	(0.760, 0.783)
OU	95.4	(69.59, 70.87)	(0.752, 0.789)
OU	99.7	(69.34, 71.11)	(0.745, 0.796)

Table 5. Range of values of the EoS parameters, where OU = OHD + BAO + CMB + Union2.1.

Data	CL (%)	A_s	α
OU	68.3	(0.749, 0.793)	(−0.049, 0.103)
OU	95.4	(0.735, 0.809)	(−0.092, 0.159)
OU	99.7	(0.720, 0.822)	(−0.130, 0.215)

Table 6. EoS parameters at different redshifts where OU = OHD + BAO + CMB + Union2.1.

Data	z	q	c_s^2	ω
OU	0	−0.597	0.018	−0.775
OU	$z_t = 0.787$	0	0.009	−0.377

and the allowed ranges of values of the EoS parameters are determined using observational data. First, $H_0 t_0$ of the Universe is used to estimate the EoS parameters. At 68.3% confidence limit, the value of A_s varies between 0.539 and 0.951 and α varies between 0 and 1.

The best-fit values of the EoS parameters H_0, A_s and α obtained from OHD + BAO + CMB + Union2.1 data are $H_0 = 70.229$, $A_\mathrm{s} = 0.772$ and $\alpha = 0.023$. The allowed ranges of values of H_0 and α at 99.7% confidence limit are (69.45, 71.01) and (−0.049, 0.103) (table 3) whereas H_0 and A_s lie between (69.34, 71.11) and (0.745, 0.796) (table 4). A_s and α lie in the range (0.720, 0.822) and (−0.130, 0.215) (table 5) at 99.7% confidence limit.

Table 7. Comparison of the values of EoS parameters with other GCG models where OU = OHD + BAO + CMB + Union2.1.

Model	Data	A_s	α	B	Ref.
GCG	Supernovae	0.6–0.85	–	0.0	[52]
GCG	CMBR	0.81–0.85	0.2–0.6	0.0	[53]
GCG	WMAP	0.78–0.87	–	0.0	[54]
GCG	CMBR + BAO	$\approx$0.77	$\leq$0.1	0.0	[55]
GCG	OU	0.772	0.023	0.0	This paper

Present age of the Universe according to this model is $t_0 = 13.53$ Gyr [44] whereas the present Hubble parameter is $H_0 = 70.23$. It is found that ω is -0.772 and deceleration parameter (q) is -0.597 at the present epoch ($z = 0$). The value of the transition redshift at which Universe re-entered the accelerating phase is $z_t = 0.787$ (table 6). It is found that c_s^2 varies between 0.018 and 0.009 in the redshift range (0, 0.787). From table 7 it is clear that the values of the EoS parameters for the present analysis are in good agreement with earlier GCG models.

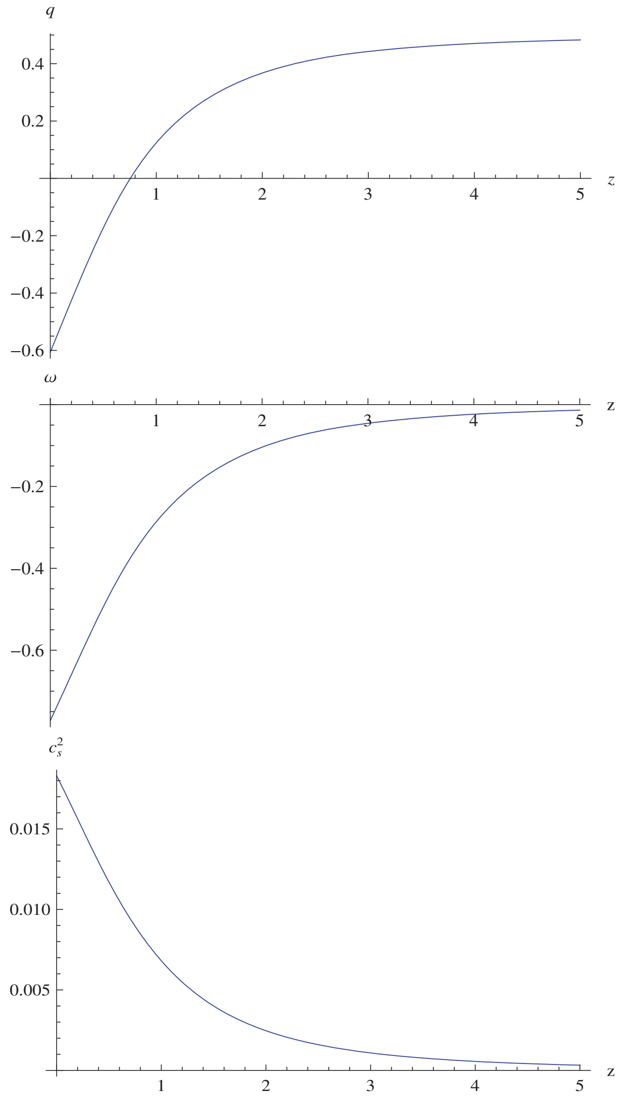

Figure 4. q, ω and c_s^2 at best-fit values: $H_0 = 70.229$, $A_s = 0.772$, $\alpha = 0.023$.

In the case of dimensionless age parameter ($H_0 t_0$) the constraints on the EoS parameters A_s and α are wider compared to that in OHD + BAO + CMB + Union2.1 data. It is noted that values of EoS parameters obtained in the latter background test falls within the range specified by dimensionless age parameter.

The equation of state at present which satisfies the limit $\omega < -1/3$ needed for the accelerating phase of the Universe is well supported by observational data. The squared sound speed obtained in the model is small and positive that permits structure formation. Thus, a satisfactory cosmological model with GCG permitting the present accelerating phase and early structure formation era can be described in the UDME scenario.

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