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Search for $^{12}C+^{12}C$ clustering in ^{24}Mg ground state

B N JOSHI*, ARUN K JAIN, D C BISWAS, B V JOHN, Y K GUPTA, L S DANU, R P VIND, G K PRAJAPATI, S MUKHOPADHYAY and A SAXENA

Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai 400 085, India
*Corresponding author. E-mail: bnjoshi@barc.gov.in

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Abstract. In the backdrop of many models, the heavy cluster structure of the ground state of ^{24}Mg has been probed experimentally for the first time using the heavy cluster knockout reaction $^{24}Mg(^{12}C, 2^{12}C)^{12}C$ in the quasifree scattering kinematic domain. In the $(^{12}C, 2^{12}C)$ reaction, the direct ^{12}C-knockout cross-section was found to be very small. Finite-range knockout theory predictions were much larger for $(^{12}C, 2^{12}C)$ reaction, indicating a very small $^{12}C-^{12}C$ clustering in $^{24}Mg_{(g.s.)}$. Our present results contradict most of the proposed heavy cluster $(^{12}C+^{12}C)$ structure models for the ground state of ^{24}Mg.

Keywords. Direct nuclear reactions; heavy cluster knockout; structure of $^{24}Mg_{(g.s)}$; C–C optical potential.

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1. Introduction

Nucleus, being a quantum mechanical many-body system, requires simplified models which can describe their gross properties. One such model is the α-cluster model. The α-cluster model is one of the earliest models to describe the nucleus in terms of clusters. Numerous experiments have been performed to identify α-clusters in light and medium mass nuclei [1–7]. Experimental evidence for the existence of clusters in nuclei has been found in the past with cluster binding energies ranging from 5 to 25 MeV [8] in knockout reactions on various light to medium mass nuclei. Absolute α-spectroscopic factors were obtained from the knockout reactions, using various projectiles such as α and protons to study α-clustering in many nuclei in the past.

In the last few decades, heavy cluster structure prescriptions have been proposed [9–11], to describe the ground state as well as the excited states of light–medium mass nuclei. The key feature of these theoretical heavy cluster models [9,12–15] is that the nucleus is described in terms of two subnuclei. For example, ^{24}Mg has been described in terms of $^{12}C+^{12}C$ and $^{16}O+^{8}Be$ structures besides of course, the α-cluster with ^{20}Ne. The heavy cluster structures of the high-lying excited states [11] of ^{24}Mg are studied by inelastic scattering and transfer reactions. On the other hand, the α-cluster structure in the ground state of ^{24}Mg has been studied with the knockout reactions [16]. In the present paper, we study the $^{12}C+^{12}C$ heavy cluster structure of $^{24}Mg_{(g.s.)}$ using ^{12}C-cluster knockout reaction, which exclusively study the ground-state nuclear structures. This is to verify the theoretical predictions of heavy cluster models for the ground state of ^{24}Mg.

Microscopic α-cluster model calculations [12] using simple α–α interactions concluded that both $^{12}C+^{12}C$ and $^{16}O+^{8}Be$ structures show up only in the excited ^{24}Mg nucleus. The $^{12}C+^{12}C$ heavy cluster model calculations [14], however, failed to obtain the ground-state results accurately [15]. On the other hand, the $^{16}O+^{8}Be$ cluster model [13] produced good results not only for the ground state but also for the low-lying positive-parity spectrum of ^{24}Mg.

Transition from a clustered nuclear state to the point at which the two cluster fragments separate, or the fusion of clusters to form a composite state has been connected through the two centre shell model [17] or through a simplification of it by Harvey prescription [9]. In the Harvey prescription, the system of two-clusters is treated as in constrained Hartree–Fock (CHF) model where the system is 'frozen' at each

stage in the separation of clusters. In such a situation, the Pauli principle has the maximum effect. Here the two nuclei at large separations are considered as separate oscillator wells where nucleons occupy the lowest quantum states. As the wells are brought together along the z-axis, the x and y degrees of freedom are not affected and the x and y quanta remain unchanged, while the quanta on the z-direction change to satisfy the Pauli principle. The prediction is that $^{24}\mathrm{Mg}_{(\mathrm{g.s.})}$ can separate into two $^{12}\mathrm{C}_{(\mathrm{g.s.})}$ clusters while these two oblate $^{12}\mathrm{C}_{(\mathrm{g.s.})}$ nuclear clusters will have their planes perpendicular to each other as seen in figure 1 for the Harvey diagram.

All the heavy cluster models as well as the full $(sd)^8$-space shell model [18] have witnessed success and failures of similar measures. This structural degeneracy for the ground state of ^{24}Mg is understandable from the energetics which are predominantly governed by the high-density nuclear interior region where all these descriptions with full antisymmetrization have large overlaps.

With confusion and contradictions existing in the structure theoretical models we provide the first experimental results for the heavy cluster structure of the ground state of ^{24}Mg in terms of $^{12}\mathrm{C}_{(\mathrm{g.s.})}+^{12}\mathrm{C}_{(\mathrm{g.s.})}$ using the ^{12}C cluster knockout reaction.

Transfer reactions have been used for a long time to obtain spectroscopic factors for the light cluster transfers because of the relative simplicity of 2-body kinematics experiments and involvement of simple DWBA analyses. However, absolute spectroscopic factors are not available from this method due to considerable momentum mismatch and complicated finite-range calculations involving phenomenological residual interactions. A major setback for the reliability of transfer reactions is due to the overlap integration of the residual interaction with the wave function of particles at the transition vertex. The situation is better with the alternative use of knockout reactions to obtain the absolute spectroscopic factor because here one uses the full interaction at the transition vertex under the impulse approximation. However, the knockout reactions also have met with some inherent problems as the spectroscopic factor varied as a function of energy as well as its projectile dependence [3,19,20]. These strong dependencies, especially the large anomalies in the α-particle knockout reactions, have been satisfactorily settled recently [21]. To overcome the shortcomings of the knockout reaction theory, more sophisticated formulation of the finite-range nature has been reported by us, which eliminates the shortcomings leading to better predictions of the heavy cluster structure of the medium to heavy mass nuclei. The cluster knockout reactions thus are analysed using the recently developed finite-range distorted wave impulse approximation (FR-DWIA) formalism to extract the absolute cluster spectroscopic factors.

It is well known [22,23] that knockout reaction cross-section values are most reliable for nuclear studies in the low-momentum region. This is why for the study of exotic borromean nuclei [24], in the RIB facilities, the knockout reactions are considered to be more sensitive, especially to the low momentum components of the bound wave functions. Similarly, the ^{12}C-knockout reaction from $^{24}\mathrm{Mg}_{(\mathrm{g.s.})}$ in the knockout

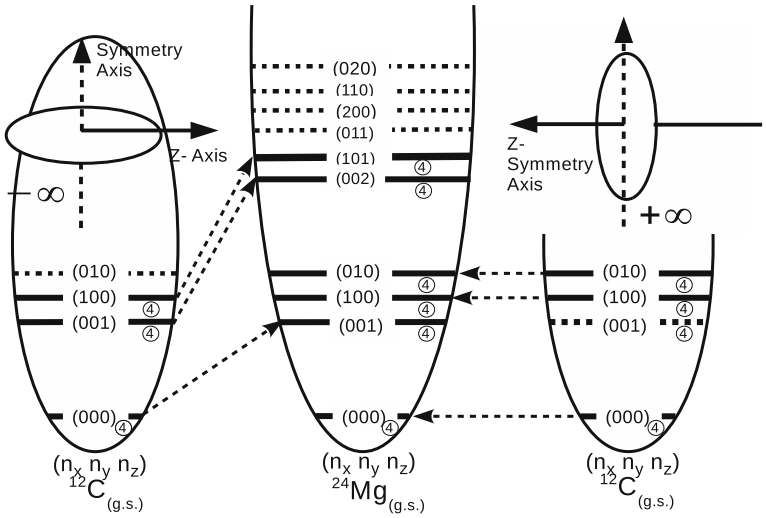

Figure 1. Harvey prescription for the $^{24}\mathrm{Mg}_{(\mathrm{g.s.})}$ configuration reached by two interacting oblate $^{12}\mathrm{C}_{(\mathrm{g.s.})}$ nuclei with their symmetry axes perpendicular to each other.

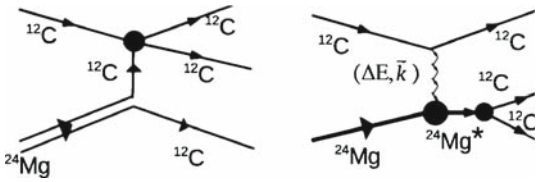

Figure 2. Schematic diagram of ^{24}Mg(^{12}C, 2^{12}C)^{12}C reaction. (**a**) Knockout and (**b**) resonance breakup.

kinematic domain is the best choice to find intrinsic ^{12}C+^{12}C clustering in the structure of ^{24}Mg$_{(g.s.)}$. The sensitivity of the knockout reactions to the low-momentum components arises because of the special choice of kinematics (known as the quasifree knockout kinematics) for the exclusive measurements of 3-body final state. In the quasifree knockout domain, a small change in the recoil momentum (from $k_B \sim 0$) corresponds to a significant measurable variation of the kinetic energy of the detected particles. In the impulse approximation knowledge of the recoil momentum, $\vec{k}_B$, in a direct knockout reaction leads to the momentum of the knockedout particle, $\vec{Q}$ ($=-\vec{k}_B$), before its removal from the ground state of the target nucleus. Conceptually however, similarly abbreviated breakup reaction [10] involves a sequential decay from an intermediate (usually) inelastic excitation (schematically shown in figure 2b) while knockout involves a direct knockout from the target ground state leading to a similar 3-body final state without involving any intermediate excitation (schematically shown in figure 2a). Thus, the knockout provides information about the ground state while the breakup provides information about the excited resonant state.

2. Motivation

The present ^{24}Mg(^{12}C, 2^{12}C)^{12}C knockout reaction experiment aims to extract the most probable intrinsic clustering in terms of ^{12}C$_{(g.s.)}$+^{12}C$_{(g.s.)}$ in the ground state of ^{24}Mg nucleus. The finite range-distorted wave impulse approximation (FR-DWIA) is used for the theoretical analysis of the cluster knockout data [21,25]. The FR-DWIA analyses [21] were found to reproduce the absolute cross-section observations exceptionally well in comparison to the prevalent and conventional zero-range (ZR)-DWIA analyses [22] (which underestimated the data by orders of magnitude).

In ^{24}Mg(^{12}C, 2^{12}C)^{12}C heavy cluster knockout reaction, the final 3-body system has two ^{12}C+^{12}C pair systems between the detected outgoing particles with the residual nucleus. Both these pairs may form compound systems corresponding to ^{24}Mg in the excited states. In fact, a large number of nuclear molecular resonances are seen to exist in ref. [11] (and references therein) for these ^{24}Mg compound systems. The incident energies and angles of the two detected particles of the 3-body final state are chosen such that corresponding to the zero recoil momentum, the two detected particles do not form resonances with the undetected recoiling residual nucleus (B). Now the events, for a kinematics where there are no resonances for the outgoing particles with the residual nucleus and where the zero recoil momentum position (for bound $\ell = 0$ direct knockout is expected to peak) occurs, will correspond to the direct knockout reaction. Comparison of these data with the FR-DWIA predictions will indicate the amount of ^{12}C+^{12}C clustering in the ground state of ^{24}Mg nucleus.

3. Experiment

Experiment was performed using the BARC–TIFR pelletron LINAC Facility at Mumbai. For ^{24}Mg(^{12}C, 2^{12}C)^{12}C reaction, the incident ^{12}C beam energy was chosen to be 104 MeV. With this choice and with coplanar symmetric quasifree knockout kinematics of symmetric detection angle pair $\theta_1 = -\theta_2 = 40.5°$ there occurs the minimum recoil momentum ($q_B = 0$) condition. The equal relative energies E_{1B} and E_{2B} for the detected ^{12}C's with respect to the recoiling ^{12}C, with the zero recoil momentum condition $q_B^m \sim 0$, corresponds to (with Q-value $= -13.92$ MeV) $E_{1B} = E_{2B} = (104 - 13.92)/4 \simeq 22.52$ MeV. This relative energy corresponds to the excitation energy of ^{24}Mg*, $E_x^* = 22.52 + 13.92 = 36.44$ MeV which corresponds to the dip position between the 14^+ and 16^+ resonance peaks of figure 43 of ref. [11].

For the ^{12}C knockout experiment, 104 MeV ^{12}C's, with an average beam current of 3.5 pnA, bombarded a self-supporting natural Mg target of 400 μg/cm^2 thickness. The two outgoing ^{12}C's were detected in coincidence using two $\Delta E(35\,\mu\text{m})$–$E(300\,\mu\text{m})$ silicon surface barrier detector telescopes each with angular coverage, $\delta\theta_1 = \delta\theta_2 = \pm 1.5°$ and the solid angles, $\Delta\Omega_1 = \Delta\Omega_2 = 2.3$ msr. The energy resolutions of the telescopes were found to be $\sim$1 MeV.

The summed energy, (E_1+E_2), spectrum [26] for $\theta_1 = -\theta_2 = 40.5°$, shown in figure 3, represents the coincidence events. This spectrum shows a clear peak at $E_1+E_2 \sim 90$ MeV corresponding to a Q-value of -13.92 MeV for the removal of ^{12}C$_{(g.s.)}$ from

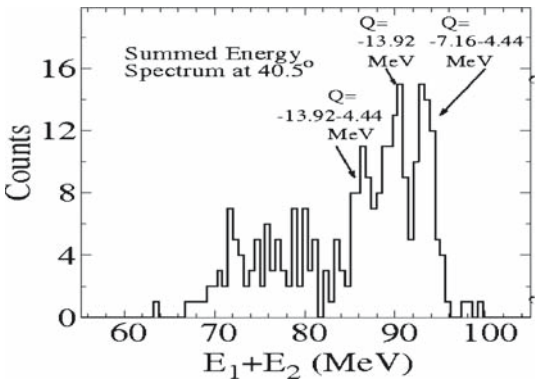

Figure 3. Summed energy spectrum for the ^{24}Mg(^{12}C, 2^{12}C)^{12}C reaction at $\theta_1 = -\theta_2 = 40.5°$ at 104 MeV.

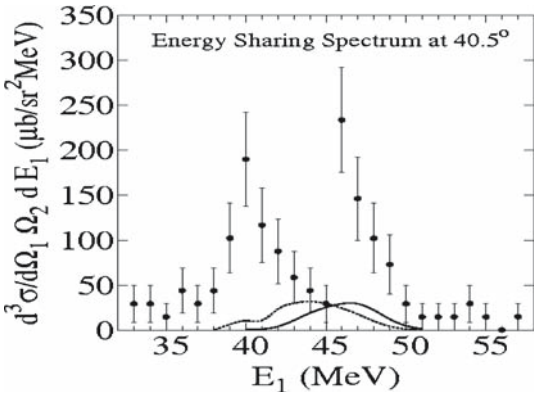

Figure 4. Energy sharing spectrum for the ^{24}Mg(^{12}C, 2^{12}C)^{12}C reaction at $\theta_1 = -\theta_2 = 40.5°$ at 104 MeV is compared with the normalized FR-DWIA predictions solid line and dotted line using repulsive core (R+A) and all-through attractive (A), ^{12}C−^{12}C potentials respectively for the t-matrix generation.

^{24}Mg$_{(g.s.)}$. The peak at E_1+$E_2 \sim 85.5$ MeV corresponds to the 4.44 MeV first excited 2^+ state of ^{12}C produced through the ^{24}Mg(^{12}C, ^{12}C^{12}C$^*_{(4.44)}$)^{12}C reaction, leading to a shift of ∼4.5 MeV from the 90 MeV position where all the three ^{12}C's are produced in their ground states. The Q-values for ^{24}Mg(^{12}C, ^{12}C^{13}C)^{11}C and ^{24}Mg(^{12}C, ^{12}C^{14}C)^{10}C reactions are −27.7 and −32.65 MeV respectively. Hence, even if ^{13}C and ^{14}C are not mass resolved in our ΔE detector, the large Q-value differences for these reactions are well resolved in the E-detector. Such events are thus eliminated by putting appropriate gates in the summed energy spectrum. Even in the case of ^{25}Mg(^{12}C, ^{12}C^{13}C)^{12}C (Q value of −16.3 MeV) and ^{26}Mg(^{12}C, ^{12}C^{14}C)^{12}C (Q value of −19.23 MeV), ^{26}Mg(^{12}C, ^{12}C^{13}C)^{13}C (Q value of −22.4 MeV) reactions are not expected to contribute when ^{13}C and ^{14}C are not mass resolved in our detector telescopes. This arises because of the differences in the Q-values which we are able to resolve nicely.

The peak at E_1+$E_2 \sim 93$ MeV has been identified to be arising from the Q-value of −11.6 (= −7.16 −4.44) MeV from the reaction ^{16}O(^{12}C, ^{12}C^{12}C$^*_{(4.44)}$)^{4}He. This ^{16}O peak arises because of the oxidation of the self-supporting natural Mg target. The favourable angles of $\theta_1 = -\theta_2 = 42.9°$ for ^{16}O(^{12}C, ^{12}C^{12}C)^{4}He reaction, corresponding to the Q-value of −7.16 MeV, remained outside the angular coverage $d\theta_1 = d\theta_2 = 1.5°$, of our detector telescopes and that is why no corresponding peak is seen in the summed energy spectrum of figure 3. Therefore, the ^{16}O(^{12}C, 2^{12}C)^{12}C reaction will not affect our ^{24}Mg(^{12}C, 2^{12}C)^{12}C data at all.

Counts below E_1+$E_2 \sim 85$ MeV correspond to events with two or three ^{12}C's excited to their 4.44 MeV or even higher excited states of ^{12}C. In natural Mg there exist 79% ^{24}Mg, 10% ^{25}Mg and 11% ^{26}Mg isotopes. The (^{12}C, 2^{12}C) reaction Q-values for these isotopes are −13.92, −16.3 and −19.2 MeV respectively. The peak in the $\theta_1 = -\theta_2 = 40.5°$ summed energy spectrum of figure 3 at ∼90 MeV is identified to be corresponding to the Q-value of ~ -14 MeV. Now for the generation of our $\theta_1 = -\theta_2 = 40.5°$ energy sharing spectrum of figure 4 we have considered counts only from the 90 ± 2 MeV peak of the summed energy spectrum of figure 3. Therefore, contributions from the reactions ^{25}Mg(^{12}C, 2^{12}C)^{13}C and ^{26}Mg(^{12}C, 2^{12}C)^{14}C will remain outside the selected peak range of the summed energy spectrum of figure 3. Therefore, the energy sharing spectrum of figure 4 should not have contributions from ^{25}Mg and ^{26}Mg.

As the peak at $E_1 + E_2 \sim 90$ MeV of figure 3 for $\theta_1 = -\theta_2 = 40.5°$ belongs to the three ^{12}C's in their ground state, the knockout events sought in the energy sharing spectrum [26] of figure 4 are generated from the events in this peak. This E_1 vs. ($d^3\sigma/d\Omega_1 d\Omega_2 dE_1$) cross-section, plotted in figure 4, is always asymmetric about $q_B \simeq 0$ due to the recoil energy, has two peaks one at $E_1 \sim 41$ MeV and the other at ∼47 MeV. However, at $E_1 \sim 45$ MeV, one expects a peak from the direct knockout (corresponding to $q_B \simeq 0$) of the $L = 0$ bound ^{12}C$_{(g.s.)}$ in ^{24}Mg$_{(g.s.)}$. Here one gets a very small cross-section of $\sim 43.8 \pm 25.3$ μb/sr^2 MeV. The peaks on either side of the minimum, i.e. the one at $E_1 \sim 41$ MeV may be ascribed to the 16^+ ^{24}Mg* resonance of $E^*_x \sim 38.5$ MeV with FWHM $\simeq 2.5$ MeV in E_{2B}, the relative $2 - B$ energy. Similarly, the peak at $E_1 \sim 47$ MeV may be ascribed to the same 16^+ resonance in E_{1B}, the relative $1 - B$ energy. These peaks indicate the sequential decay resonance breakup contributions of the type shown in figure 2b over and

above the direct knockout component of figure 2a. At $E_1 = 45$ MeV the tails of the two 2-body 16^+ resonances may contribute. For this, figure 5 shows that incident partial waves $\ell_i = 0$ and $\ell_i = 32$ only can contribute at this E_1. However, $\ell_i = 0$ is likely to be strongly absorbed while $\ell_i = 32$ is too peripheral to contribute. In contrast to this, the resonance peak regions, i.e. $E_1 \sim 39$–43 MeV and $46-49$ MeV have contributions from all possible ℓ_i's. Thus, in the region $E_1 \sim 44$–45 MeV, direct knockout mainly is expected to occur.

In figures 6–8 summed spectra obtained at $\theta_1 = -\theta_2 = 36.7°$, $\theta_1 = -\theta_2 = 33.9°$ and $\theta_1 = -\theta_2 = 49.6°$ respectively are shown. In the $\theta_1 = -\theta_2 = 36.7°$ summed spectrum, the counts ~ 90 MeV correspond to a situation

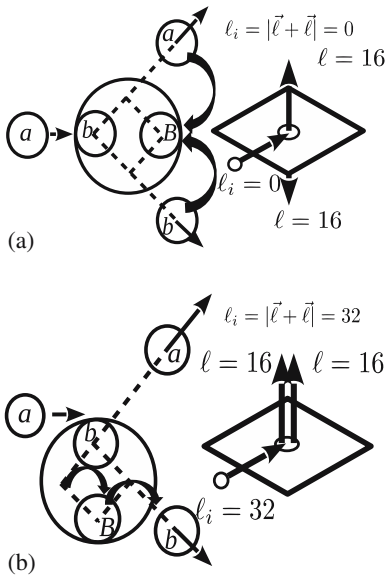

Figure 5. Resonance contributions for $A(a, ab)B$ reaction from (**a**) $\ell_i = 0$ and (**b**) $\ell_i = 32$ at $E_1 = 45$ MeV of figure 4.

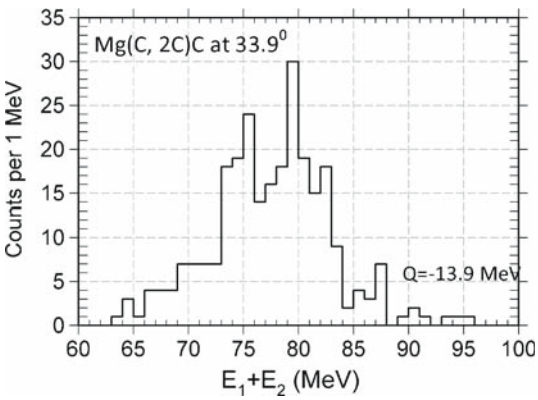

Figure 6. Summed energy spectrum for ^{24}Mg(^{12}C, 2^{12}C)^{12}C reaction at $\theta_1 = -\theta_2 = 33.9°$ at 104 MeV. Counts around $E_1 + E_2 \sim 90$ MeV belong to $Q = -13.92$ MeV.

where the two 2-body 16^+ 39.4 MeV ^{24}Mg* resonance peaks (which were occurring at $E_1 \sim 41$ MeV and $E_1 \sim 47$ MeV in the $\theta_1 = -\theta_2 = 40.5°$ case above) overlap in the 3-body final state. In the $\theta_1 = -\theta_2 = 33.9°$, $\theta_1 = -\theta_2 = 49.6°$ summed spectra the counts ~ 90 MeV correspond to the dip positions between the 16^+ and 18^+ and 12^+ and 14^+ ^{24}Mg resonances respectively (see [11] and references there in). It is seen that at these angle pairs there are very few counts which do not yield any sensible energy sharing distribution.

4. FR-DWIA theory

Theoretical analysis is carried out using the recently developed FR-DWIA formalism [21]. The $(d^3\sigma/d\Omega_1 d\Omega_2 dE_1)$ of the energy sharing distribution for a knockout reaction $A(a_0, a_1 b_2)B$ is expressed in terms of a finite-range transition amplitude T_{FR}, a

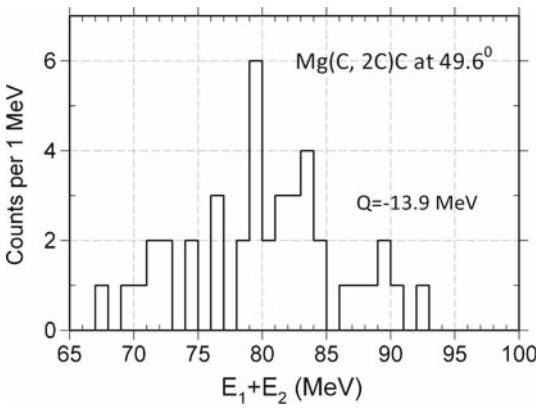

Figure 7. Summed energy spectrum for ^{24}Mg(^{12}C, 2^{12}C)^{12}C reaction at $\theta_1 = -\theta_2 = 49.6°$ at 104 MeV. The counts around $E_1 + E_2 \sim 90$ MeV belong to $Q = -13.92$ MeV.

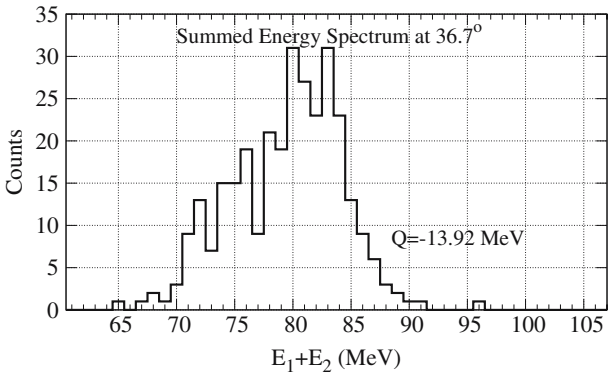

Figure 8. Summed energy spectrum for ^{24}Mg(^{12}C, 2^{12}C)^{12}C reaction at $\theta_1 = -\theta_2 = 36.7°$ at 104 MeV. The counts around $E_1 + E_2 \sim 90$ MeV belong to $Q = -13.92$ MeV.

kinematic factor $F_{\rm kin}$ and a spectroscopic factor S_b^L for $A \to b_2 + B$ as

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\Omega_1 \mathrm{d}\Omega_2 \mathrm{d}E_1} = F_{\rm kin} \cdot S_b^L \cdot \sum_{\wedge} |T_{\rm FR}^{L\wedge}(\vec{k}_f, \vec{k}_i)|^2. \quad (1)$$

The transition matrix element $T_{\rm FR}^{L\wedge}(\vec{k}_f, \vec{k}_i)$ for the b_2 knockout, which involves the a_1–b_2 t-matrix effective interaction, is written as

$$T_{\rm FR}^{L\wedge}(\vec{k}_f, \vec{k}_i) = \int \chi_1^{(-)*}(\vec{r}_{1B}) \chi_2^{(-)*}(\vec{R}) t_{12}(\vec{r}) \chi_0^{(+)}(\vec{r}_{1A}) \times \varphi_{L\wedge}(\vec{R}) \mathrm{d}\vec{r} \mathrm{d}\vec{R}.$$

Here $\vec{r} \equiv \vec{r}_{12}$, $\vec{R} \equiv \vec{R}_{2B}$ (see figure 9 for various coordinates used here) and $t_{12}(\vec{r})$ is the a–b scattering t-matrix effective interaction, which can be derived from a given a–b interaction, described in refs [21,27].

The initial and final states are written as

$$|\chi_i^{(+)}\rangle = |\chi_0^{(+)}(\vec{k}_{1A}, \vec{r}_{1A}) \varphi_{L\wedge}(\vec{R}) \Psi(b_2, B)\rangle$$

and

$$\langle\chi_f^{(-)}| = \langle\chi_1^{(-)}(\vec{k}_{1B}, \vec{r}_{1B}) \chi_2^{(-)}(\vec{k}_{2B}, \vec{R}) \Psi(b) \Psi(B) \Psi(a)|.$$

Here the antisymmterization of a with A in the initial state has been neglected. The contributions from such exchange terms are expected to be small as discussed in ref. [28]. It is difficult to define scattering wave function, for the optical potential scattering, for exchange terms, because the reference coordinates for such a system are different from that of the direct terms, while fitting the scattering data in terms of the optical model exchange terms are not explicitly considered normally. However, it is expected that the gross effects of the exchange terms are already included in the optical potentials. $\Psi(x)$ is the internal wave function of the corresponding particle x. Here we assume that the operator $t_{a_1 b_2}$ does not depend significantly on the internal structure of a_1 and b_2, and depends only on the relative coordinates of a_1 and b_2. Integration over the internal coordinates of particles a_1, b_2 and B lead to

$$T_{fi} = \langle\chi_1^{(-)}(\vec{k}_{1B}, \vec{r}_{1B}) \chi_2^{(-)}(\vec{k}_{2B}, \vec{R}) | t(\vec{r}_{a_1 b_2}) \times | \varphi_{L\wedge}(\vec{r}_{b_2 B}) | \chi_0^{(+)}(\vec{k}_{1A}, \vec{r}_{1A})\rangle.$$

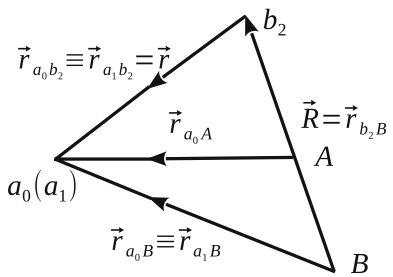

Figure 9. Vector diagram of relative coordinates appearing in the FR-DWIA analysis.

By looking at the vector diagram (figure 9), $\vec{r}_{1A}$ and $\vec{r}_{2B}$ can be expressed as

$$\vec{r}_{1A} = \vec{r}_{a_1 b_2} + \vec{r}_{b_2 A} = \vec{r}_{a_1 b_2} + \frac{m_B}{m_A} \vec{R}_{b_2 B} = \vec{r}_{a_1 b_2} + \epsilon \vec{R},$$

$$\vec{r}_{1B} = \vec{r}_{a_1 b_2} + \vec{R}_{b_2 B},$$

where $\epsilon = m_B/m_A$. The incident and outgoing wavefunctions are written as, $\chi_0^{(+)}(\vec{k}_{1A}, \vec{r}_{a_1 b_2} + \epsilon \vec{R})$ and $\chi_2^{(-)}(\vec{k}_{1B}, \vec{r}_{a_1 b_2} + \vec{R}_{b_2 B})$.

Hence T_{fi} is written as

$$T_{fi} = \langle\chi_1^{(-)}(\vec{k}_{1B}, \vec{r}_{a_1 b_2} + \vec{R}) \chi_2^{(-)}(\vec{k}_{2B}, \vec{R}) | t(\vec{r}_{a_1 b_2}) \times | \varphi_{L\wedge}(\vec{R}) | \chi_0^{(+)}(\vec{k}_{1A}, \vec{r}_{a_1 b_2} + \epsilon \vec{R})\rangle.$$

This is the exact FR-DWIA expression for the knockout reaction matrix element. It is a six-dimensional integral, with integration over $\vec{r}_{a_1 b_2}$ $(=\vec{r})$ and $\vec{r}_{b_2 B}$ $(=\vec{R})$.

The t-matrix effective interaction (a function of relative energy E and $\vec{r}$) is written as

$$t_{12}^{+}(E, \vec{r}) = \mathrm{e}^{-ikz} V(\vec{r}) \Psi_{12}^{(+)}(\vec{r}) \equiv \sum_{L=0,1,2,\ldots} t_L(E, r) P_L(\hat{r}), \quad (2)$$

where $\Psi_{12}^{(+)}(\vec{k}, \vec{r})$ is expanded in terms of partial waves as

$$\Psi_{12}^{(+)}(\vec{k}, \vec{r}) = \sum_{\ell=0,2,4,\ldots} i^\ell (2\ell + 1) \frac{u_\ell(kr)}{kr} \mathrm{e}^{i\sigma_\ell} P_\ell(\hat{r}).$$

Here ℓ is summed for even values of partial waves so that the wave function for two-bosons such as α–α and ^{12}C–^{12}C wave functions, $\Psi_{12}^{(+)}(\vec{k}, \vec{r})$ are symmetrized properly in evaluating $t_{12}^{+}(E, \vec{r})$. As discussed in ref. [27], the Lth-multipole component of $t_{12}^{+}(E, \vec{r})$ can be written as

$$t_L(E, r) = \frac{2L+1}{2} \sum_{\ell, n} V_\ell(r) i^{(\ell - n)} (2\ell + 1) \frac{u_\ell(kr)}{kr} J_n(kr) \times (2n+1) \mathrm{e}^{i\sigma_\ell} \int_{-1}^{+1} P_L^*(t) P_\ell(t) P_n(t) \mathrm{d}(t). \quad (3)$$

$\chi_0(\vec{r}_{0A})$, $\chi_1(\vec{r}_{1B})$ and $\chi_2(\vec{R}_{2B})$ are the distorted waves describing the incident and the two final scattering states. These are solutions of the scattering state Schrödinger equations with respective channel optical potentials [29] and relative energies and are given in table 1. Conventional Woods–Saxon form of the optical potentials has been used to generate distorted waves

Table 1. Optical potentials used for the FR-DWIA analysis of the 104 MeV ^{24}Mg(C, 2C)^{12}C knockout reaction at $\theta_1 = -\theta_2 = 40.5°$.

	Real part			Imaginary part			Coulomb radius
Reaction	$V(R)$ (MeV)	r (fm)	a (fm)	$W(R)$ (MeV)	r (fm)	a (fm)	R_c (fm)
^{12}C–^{24}Mg	210	0.98	1.15	10.19	2.114	0.86	0.988
^{12}C–^{12}C [30]	225	1.925	0.385	16.27	2.825	0.16	2.825

$V(R)$ is the real part of the optical potential and $W(R)$ is the imaginary part of the optical potential.

at the appropriate relative energies. $\varphi_{M\wedge}(\vec{R})$ is the relative motion bound wave function with orbital angular momentum L ($\wedge$ its azimuthal projection) for clusters b and B in the target nucleus A (or more appropriately the projection of the target state A on to the product state of b and B). The radial part of the bound cluster wave function $\varphi_L(R)$ is the solution of the Schrödinger equation in a potential well for relative energy equal to the separation energy of b and B in A. Solution of the bound-state radial Schrödinger equation is obtained, which satisfies, for the orbital angular momentum L, the Wildermuth condition. Here the number of nodes in the bound wave function for the $^{12}C_{(g.s.)} + ^{12}C_{(g.s.)}$ description of $^{24}Mg_{(g.s.)}$ is given as

$$2(N-1)+L=\sum_{i=1}^{12}[2(n_i-1)+\ell_i],$$

where N is the principal quantum number for the bound cluster wave function.

The distorted waves $\chi_0(\vec{r}_{0A})$ and $\chi_1(\vec{r}_{1B})$ couple the coordinates $\vec{r}$ and $\vec{R}$ in the integral of eq. (2) leading to a six-dimensional integral. The spectroscopic factor S_b^L is obtained as the ratio of the experimentally found $d^3\sigma/d\Omega_1 d\Omega_2 dE_1$ and the theoretically estimated $F_{\rm kin}\sum_{\wedge}|T_{\rm FR}^{L\wedge}(\vec{k}_f,\vec{k}_i)|^2$. In the conventional ZR-DWIA the effective interaction is assumed to be a δ-function as $t_0(E_f,\vec{k}_f)\delta(\vec{r}_{12})$ leading to a reduction of the six-dimensional integral of eq. (2) to a three-dimensional integral.

5. Results and discussion

The FR-DWIA calculations were performed for $d^3\sigma/d\Omega_1 d\Omega_2 dE_1$ for the present 104 MeV ^{24}Mg(^{12}C, 2^{12}C)^{12}C reaction and the corresponding energy sharing spectra were generated for the all-through attractive (A) and for the ℓ-dependent repulsive core ($R+A$) ^{12}C–^{12}C potentials as was done in ref. [25] for the FR-DWIA analysis of 120 MeV ^{16}O(^{12}C, 2^{12}C)^{4}He reaction. Identical bosonic wave functions are symmetrized. The ℓ-dependent repulsive core potentials are generated by matching the real and imaginary phase shifts with those obtained from the all-through attractive optical potentials. The FR-DWIA estimate for the ^{12}C direct knockout using repulsive core ($R+A$) ^{12}C–^{12}C interaction potentials (same were also used to fit the 120 MeV ^{16}O(^{12}C, 2^{12}C)^{4}He reaction data [25]) results in a very large cross-section value of $\sim$1840 μb/sr^2 MeV. The resulting very small spectroscopic factor of $S^0_{^{12}C_{(g.s.)}} \sim 0.024$, corresponds to a negligible $^{12}C_{(g.s.)} + ^{12}C_{(g.s.)}$ content in $^{24}Mg_{(g.s.)}$.

Present results, summarized in table 2 provide negligible $^{12}C_{(g.s.)}$–$^{12}C_{(g.s.)}$ clustering in $^{24}Mg_{(g.s.)}$ if we uphold our earlier finding about the presence of a repulsive core in the ^{12}C–^{12}C interaction potentials at least in the $E_{\rm cm} \sim$ 4–5 MeV/u range. On the other hand, if we contradict our earlier finding about the repulsive core in the ^{12}C–^{12}C interaction potential then the present results give a feeling of a somewhat better agreement with the attractive ^{12}C–^{12}C potential. However, with large error bars, small cross-section values, a large spectroscopic factor of 3.2 and

Table 2. Comparison of the spectroscopic factors, $S_b^{L=0}$ for $^{12}C_{(g.s.)} - ^{12}C_{(g.s.)}$ in the ground state of ^{24}Mg obtained from the FR-DWIA analysis of $^{24}Mg(^{12}C, 2^{12}C_{(g.s.)})^{12}C_{(g.s.)}$ reaction.

	μb/sr^2 MeV			S_b^0	
		FR-DWIA			
E_i (MeV)	Expt.	(A)	($R+A$)	(A)	($R+A$)
104	43.8 ± 25.3	13.9	1840	3.2 ± 1.8	0.024 ± 0.014

above all a disagreement with the earlier findings, will not allow us to draw a firm conclusion of an attractive ^{12}C–^{12}C interaction at the knockout vertex from the present data. Repulsive core ^{12}C–^{12}C interaction potential does not violate the earlier findings (an unambiguous choice of the presence of repulsion in ^{12}C–^{12}C interaction potential from the analysis of the 120 MeV $^{16}O(^{12}C, 2^{12}C)^4He$ reaction data [25]) as also the findings of Marsh and Rae [12] as enunciated by Martin Freer on page 33 of ref. [31] by saying that both the α-cluster model and the harmonic oscillator densities suggest that the ^{12}C–^{12}C cluster structure is suppressed in the ground state of ^{24}Mg. It is thus strongly indicated that there is a repulsive core in the ^{12}C–^{12}C interaction potential and hence there is negligible $^{12}C_{(g.s.)}$–$^{12}C_{(g.s.)}$ cluster content in the ground state of ^{24}Mg. This repulsive core may be understood to be arising mainly due to Pauli blocking. The heavy ion interaction potentials were generally considered to be attractive as predicted by the single and double folding theoretical models previously. Some density-dependent effective nucleon–nucleon (N–N) interactions may provide a repulsive core in the double folding model optical potentials. The phenomenological nature of the density dependence of the effective N–N interactions as well as a sudden change from repulsive core to all through attraction, at some relative energy (arising from the shell effects in the resonating group method (RGM) discussed in connection with the (α, 2α) reactions in ref. [21]) needs to be addressed properly.

6. Conclusions

In conclusion, the present heavy cluster knockout experiments investigated the heavy cluster structure of $^{24}Mg_{(g.s.)}$ for the first time. We found $^{12}C_{(g.s.)}$–$^{12}C_{(g.s.)}$ clustering to be negligible in $^{24}Mg_{(g.s.)}$. This finding of the heavy cluster spectroscopic factors does not provide support to the Harvey prescription [9], which was used to predict the behaviour of many of the *sd*-shell nuclei, as it indicated the possibility of $^{12}C_{(g.s.)}$–$^{12}C_{(g.s.)}$ in $^{24}Mg_{(g.s.)}$. Our findings support the α-cluster model (ACM) [12] which predicted that there was no $^{12}C_{(g.s.)}$–$^{12}C_{(g.s.)}$ clustering in the ground state of ^{24}Mg. It would be interesting to find out $^{16}O_{(g.s.)}$–$^{8}Be_{(g.s.)}$ clustering in $^{24}Mg_{(g.s.)}$ through some $^{16}O_{(g.s.)}$ knockout reactions. The disagreement of our findings with most of the theoretical models may be sought in terms of different regions of sensitivity of our experiments and the regions of sensitivity of the various models, mostly representing the dense part of the nuclei. The disagreement may also be associated with the use of oversimplified α–α interactions in the calculations as well as a simplistic view of the Pauli principle in the deformed potentials of the Nilsson model. The present results in conjunction with the results of the fairly sharp energy dependence in α–α optical potentials observed in the FR-DWIA analyses of (α, 2α) knockout reactions calls for some scrutiny in the working of the double folding model. Here the folding prescriptions have to take some cues on the lines of the RGM. Besides that, a proper study of the nature and energy dependence of the effective N–N interactions (derived from the more microscopic dynamic solutions) are warranted. Present experiments as well as their FR-DWIA analyses open up new avenues to study the heavy cluster structure of medium to heavy mass nuclei as also looking for the possibility of deriving interesting predictions and conclusions from the core knockout reactions for the study of weakly bound halo nuclei as also the borromean nuclei in the domain of heavy cluster knockout reactions.

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