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Nonlinear waves in electron–positron–ion plasmas including charge separation

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Abstract. Nonlinear low-frequency electrostatic waves in a magnetized, three-component plasma consisting of hot electrons, hot positrons and warm ions have been investigated. The electrons and positrons are assumed to have Boltzmann density distributions while the motion of the ions are governed by fluid equations. The system is closed with the Poisson equation. This set of equations is numerically solved for the electric field. The effects of the driving electric field, ion temperature, positron density, ion drift, Mach number and propagation angle are investigated. It is shown that depending on the driving electric field, ion temperature, positron density, ion drift, Mach number and propagation angle, the numerical solutions exhibit waveforms that are sinusoidal, sawtooth and spiky. The introduction of the Poisson equation increased the Mach number required to generate the waveforms but the driving electric field E_0 was reduced. The results are compared with satellite observations.

Keywords. Nonlinear waves; low frequency; ion-acoustic waves.

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1. Introduction

The existence of electron–positron–ion (e–p–i) triplets in most astrophysical environments attracts the attention of many researchers because of their potential relevance to space plasmas [1–4]. The study of both linear and nonlinear wave propagation in e–p–i plasmas plays a vital role in understanding different types of collective processes in space plasmas. The properties of wave motion in the presence of heavy ions are significantly different from those in electron–positron (e–p) plasmas [5,6]. The presence of ions leads to the existence of several low-frequency waves which otherwise do not propagate in e–p plasmas [7].

Recently, it has been suggested that the nonlinear study of wave propagation can be helpful to understand nonlinear structures similar to the broadband electrostatic noise (BEN) observed in the Earth's magnetosphere by numerous satellites (spacecrafts) such as Geotail [8], Polar [9],Viking [10], FAST [11] and Cluster [12]. The waveform observations with the high time resolution confirm that one of the common features of these waves is the burstiness, i.e. rapid changes in the amplitudes or frequencies of the order of a few milliseconds to a few hundreds of milliseconds [8].

Further, the observations show that the electrostatic solitary waves (ESW) identified are parallel propagating waves relative to the ambient magnetic field and exhibit large amplitude, spiky behaviour. It was shown that the nonlinear coupling between the ion cyclotron and ion acoustic modes lead to the generation of parallel electric fields with the periods of the waves varying from the ion cyclotron range to the ion acoustic range [13,14]. Ion-acoustic solitons in e–p–i plasmas under different regimes and models have been studied by different researchers. Kourakis *et al* [15,16] have shown the existence of envelope structures of solitons and holes in e–p–i plasmas. Dubinov and Sazonkin [17] investigated the nonlinear theory of ion-acoustic waves in plasmas with cold ions and inertialess isothermal electrons and positrons. By using a gas dynamic approach and the Bernouilli pseudopotential technique, they reported that the propagation velocity of a solitary wave is always higher than the linear ion sound velocity. Mahmood *et al* [18,19] studied arbitrary-amplitude solitons propagating obliquely with respect to an external magnetic field in a homogeneous magnetized e–p–i plasma and found that the amplitude of solitary structures increases with increase in positrons. Nejoh [20] investigated the effect of ion temperature on the large-amplitude ion-acoustic waves in e–p–i plasmas and observed that the ion temperature decreased the amplitude and increased the maximum Mach number of the ion-acoustic waves.

Using fluid theory, Reddy *et al* [14] and Bharuthram *et al* [21] studied nonlinear low-frequency waves in an e–i plasma. Moolla *et al* [22] studied high-frequency nonlinear waves in the Earth's magnetosphere and showed that a nonlinear coupling between large-amplitude electron cyclotron and electron-acoustic waves can account for the high-frequency component of the field-aligned bipolar electric field pulses observed within the broadband electrostatic noise in the auroral, polar and magnetotail regions of the Earth's magnetosphere. The sawtooth and spiky structures found are in agreement with the observations of Ergun *et al* [11]. Moolla *et al* [23] studied nonlinear low-frequency structures in the auroral plasma in the presence of an oxygen beam including charge separation. The inclusion of charge separation effect tends to, in most cases, increase the frequency of oscillation of the nonlinear structures. It was shown that for a weakly magnetized plasma, the amplitude of the oscillations are found to be constant while they are modulated for strongly magnetized plasmas.

Later, Moolla *et al* [24] studied nonlinear low-frequency structures in an e–p–i plasma. The nonlinear electric field structures found were based on the quasineutrality approximation. Bharuthram *et al* [25] studied the evolution of nonlinear waves in different plasmas and showed that the nonlinear waves evolve in a consistent fashion irrespective of the plasma composition.

In this paper, we extend the work of Moolla *et al* [24] by including the Poisson equation, thereby allowing for the charge separation effect and we numerically solve the resulting set of coupled nonlinear equations. The organization of the paper is as follows: In §2, we present the model and the basic equations, while the numerical solutions are discussed in §3. Finally, in §4, the results are briefly summarized.

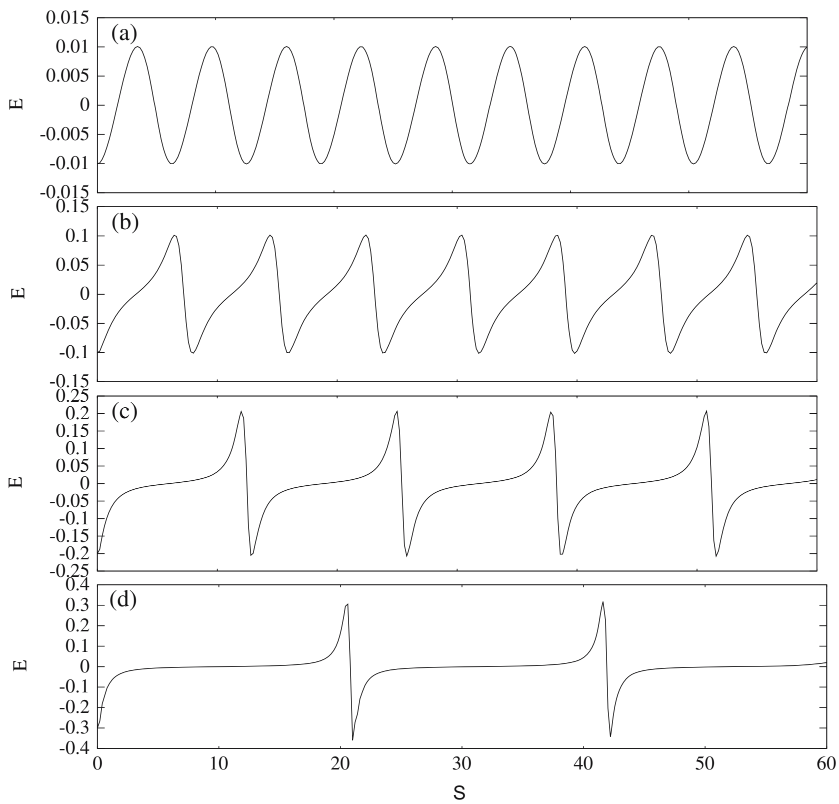
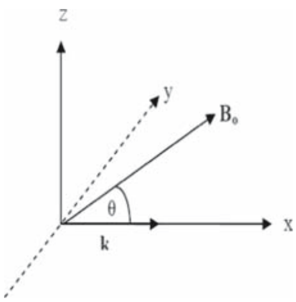


Figure 1. Numerical solution of the normalized parallel electric field for $M = 2.5$, $\delta_i = 0.0$, $(n_{i0}/n_{e0}) = 0.5$, $(T_i/T_h) = 0.0$, $R = 3.0$, $\theta = 2°$ and $E_0 = 0.01$ (**a**), 0.1 (**b**), 0.2 (**c**) and 0.3 (**d**).

2. Model and basic equations

We consider a collisionless, magnetized three-component plasma consisting of warm ions (i), hot positron (ph) and hot electron species (eh). The ion species are drifting along the magnetic field with speed v_0 and the wave propagation is taken to be in the x direction at an angle θ to the magnetic field B_0, which is assumed to be in the x–z plane, as shown here.



The continuity and momentum equations for the positive ions are expressed as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i v_{ix}}{\partial x} = 0, \tag{1}$$

$$\frac{\partial v_{ix}}{\partial t} + v_{ix}\frac{\partial v_{ix}}{\partial x} + \frac{1}{n_i m_i}\frac{\partial p_i}{\partial x} = -\frac{\varepsilon_i e}{m_i}\frac{\partial \varphi}{\partial x} + \varepsilon_i \Omega_i v_{iy} \sin\theta, \tag{2}$$

$$\frac{\partial v_{iy}}{\partial t} + v_{ix}\frac{\partial v_{iy}}{\partial x} = \varepsilon_i \Omega_i v_{iz} \cos\theta - \varepsilon_i \Omega_i v_{ix} \sin\theta, \tag{3}$$

$$\frac{\partial v_{iz}}{\partial t} + v_{ix}\frac{\partial v_{iz}}{\partial x} = -\varepsilon_i \Omega_i v_{iy} \cos\theta, \tag{4}$$

$$\frac{\partial p_i}{\partial t} + v_{ix}\frac{\partial p_i}{\partial x} + 3p_i\frac{\partial v_{ix}}{\partial x} = 0. \tag{5}$$

The electrons and positrons are assumed to follow Boltzmann distribution with equal temperature T_h and equilibrium densities n_{e0} and n_{p0}, respectively, as

$$n_{eh} = n_{e0} \exp\left(\frac{e\varphi}{T_h}\right), \tag{6}$$

$$n_{ph} = n_{p0} \exp\left(-\frac{e\varphi}{T_h}\right), \tag{7}$$

Equations (1)–(7) are closed with the Poisson equation

$$\varepsilon_0 \frac{\partial^2 \varphi}{\partial x^2} = -e(n_{ph} - n_{eh} + n_i). \tag{8}$$

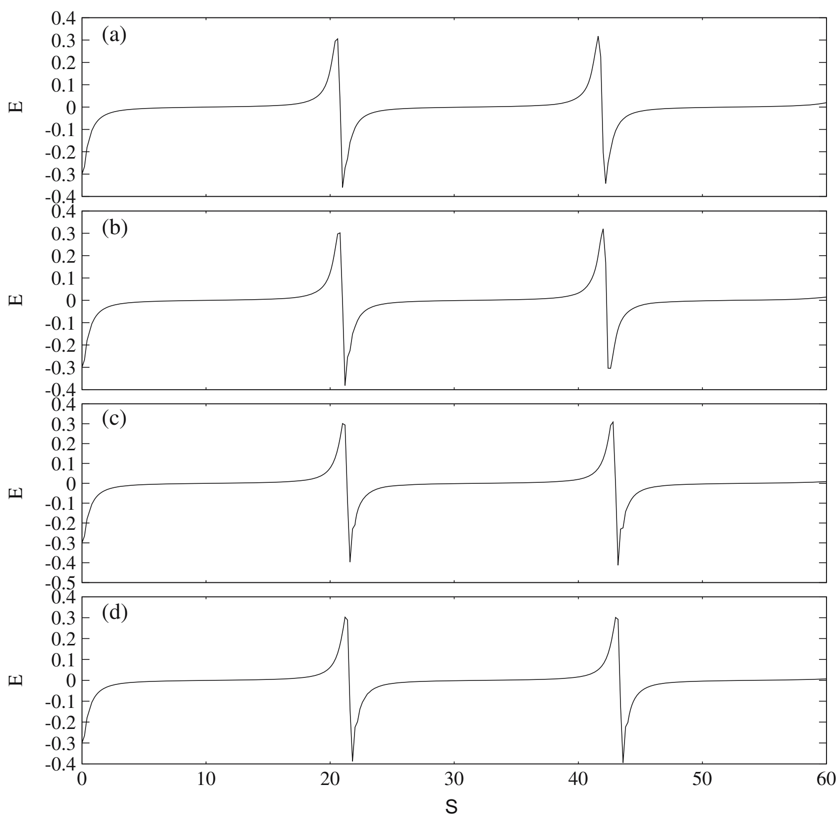


Figure 2. Numerical solution of the normalized parallel electric field for $E_0 = 0.3$, $M = 2.5$, $R = 3.0$, $\delta_i = 0.0$, $(n_{i0}/n_{e0}) = 0.5$, $\theta = 2°$ and $(T_i/T_h) = 0.0$ (**a**), 0.05 (**b**), 0.15 (**c**) and 0.2 (**d**).

In eqs (1)–(8), $\Omega_i = eB_0/m_i$ is the ion cyclotron frequency, $\varepsilon_i = +1(-1)$ for $i = c(h)$, n_i is the ion density, v_{ix}, v_{iy} and v_{iz} are the components of the ion velocity along the x, y and z directions, respectively, p_i is the ion pressure, φ is the electrostatic potential and m_i is the ion mass.

2.1 *Linear analysis*

First, we investigate the linear modes of our system. Equations (1)–(8) are linearized and yield the following dispersion relation:

$$\omega^2 = \frac{1}{2}(\Omega_i^2 + A^2) \pm \frac{1}{2}[(\Omega_i^2 + A^2)^2 - 4A^2\Omega_i^2 \cos^2\theta]^{1/2}, \tag{9}$$

where

$$A^2 = 3k^2 v_{ti}^2 + \frac{k^2 v_{ia}^2}{k^2\lambda_{Dh}^2 + (n_{p0}/n_{e0}) + 1},$$

$v_{ia} = (n_{i0}m/n_{e0}m_i)^{1/2} v_{th}$ is the ion acoustic speed, $\lambda_{Dh} = (\varepsilon_0 T_h/n_{e0}e^2)^{1/2}$ is the electron Debye length and $v_{ti} = \sqrt{T_i/m_i}$ is the ion thermal speed.

In the limit,

$$\left(\Omega_i^2 + A^2\right)^2 \gg 4A^2\Omega_i^2 \cos^2\theta, \tag{10}$$

we obtain two modes from eq. (9). The first mode is the ion cyclotron mode given by

$$\omega_+^2 = \Omega_i^2 + A^2 - \frac{A^2\Omega_i^2\cos^2\theta}{\Omega_i^2 + A^2} \tag{11}$$

and the second mode is the ion-acoustic mode given by

$$\omega_-^2 = \frac{A^2\Omega_i^2\cos^2\theta}{\Omega_i^2 + A^2}. \tag{12}$$

The two modes obtained for this e–p–i plasma have similar characteristics as the electron cyclotron and electron acoustic waves investigated by Lazarus *et al* [26] for a four-component, two-temperature electron–positron plasma.

In the long wavelength limit ($k^2\lambda_{Dh}^2 \ll 1$), the parameter A^2 can be reduced to

$$A^2 = 3k^2 v_{ti}^2 + \left(\frac{n_{e0}}{n_{p0} + n_{e0}}\right) k^2 v_{ia}^2 \tag{13}$$

and the dispersion relation (9) is consistent with the findings of Moolla *et al* [24], where they studied a three-component plasma by using quasineutrality approximation.

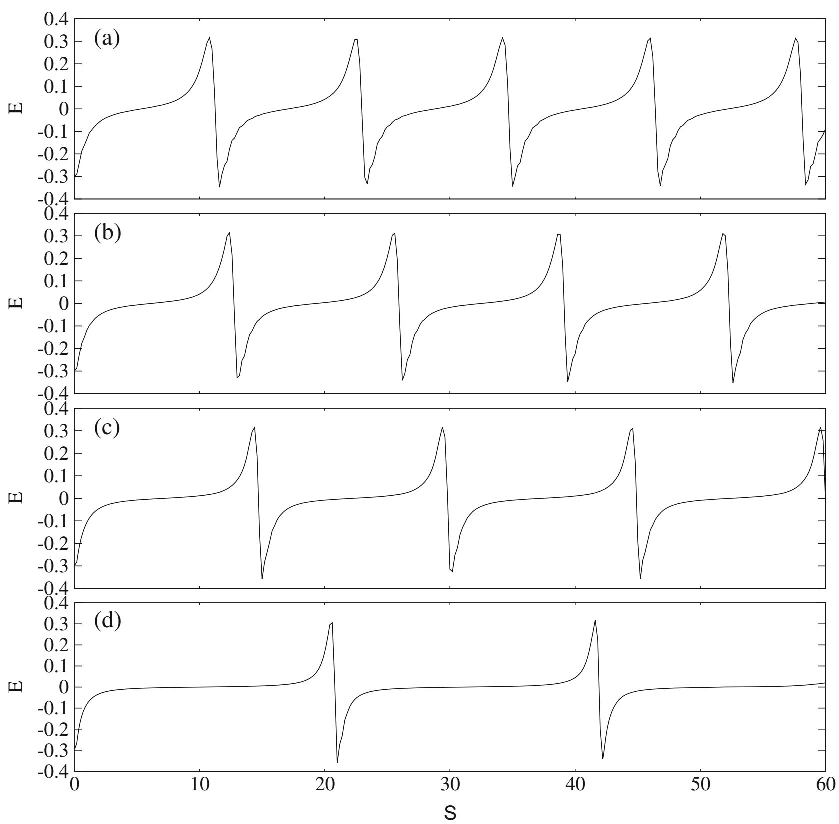


Figure 3. Numerical solution of the normalized parallel electric field for $E_0 = 0.3$, $M = 2.5$, $R = 3.0$, $\delta_i = 0.0$, $\theta = 2°$, $(T_i/T_h) = 0.0$ and $(n_{p0}/n_{e0}) = 0.3$ (**a**), 0.35 (**b**), 0.4 (**c**) and 0.5 (**d**).

2.2 *Nonlinear analysis*

In the nonlinear regime, we transform eqs (1)–(5) and (8) to a stationary frame through a variable $s = (x - vt)/(v/\Omega_i)$. We replace $\partial/\partial t$ by $-\Omega_i(\partial/\partial s)$ and $\partial/\partial x$ by $(\Omega_i/v)(\partial/\partial s)$ in eqs (1)–(5) and (8), and we define the electric potential $\psi = e\varphi/T_h$ and electric field $E = -\partial\psi/\partial s$. In addition, we assume that point quasineutrality $(n_{i0} + n_{p0} = n_{e0})$ is applicable only at $s = 0$. Before proceeding with the analysis, all parameters are normalized as follows: the velocities with respect to the ion thermal velocity $C_s = \sqrt{T_h/m_i}$, the densities with respect to the total unperturbed electron density n_{e0}, pressures with respect to $n_{e0}T_h$ and potential with T_h/e. This results in the following set of differential equations:

$$\frac{\partial\psi}{\partial s} = -E, \tag{14}$$

$$\frac{\partial E}{\partial s} = M^2R^2\frac{n_{e0}}{n_{i0}}\left(\frac{n_{p0}}{n_{e0}}\mathrm{e}^{-\psi} - \mathrm{e}^{\psi} + n_{in}\right), \tag{15}$$

$$\frac{\partial n_{in}}{\partial s} = \frac{n_{in}^3[-E - Mv_{iyn}\sin\theta]}{(n_{i0}/n_{e0})^2\,(M-\delta_i)^2 - 3n_{in}p_{in}}, \tag{16}$$

$$\frac{\partial p_{in}}{\partial s} = \frac{3p_{in}n_{in}^2[-E - Mv_{iyn}\sin\theta]}{(n_{i0}/n_{e0})^2\,(M-\delta_i)^2 - 3n_{in}p_{in}}, \tag{17}$$

$$\frac{\partial v_{iyn}}{\partial s} = \left(\frac{n_{e0}}{n_{i0}}\right)\frac{n_{in}M}{(M-\delta_i)} \times\left[\sin\theta\left(M - \left(\frac{n_{i0}}{n_{e0}}\right)\frac{(M-\delta_i)}{n_{in}}\right) - v_{izn}\cos\theta\right], \tag{18}$$

$$\frac{\partial v_{izn}}{\partial s} = \frac{n_{in}Mv_{iyn}\cos\theta}{(n_{i0}/n_{e0})(M-\delta_i)}, \tag{19}$$

where $\delta_i = v_0/C_s$ is the ion drift, $M = v/C_s$ is the Mach number and $R = \omega_{pi}/\Omega_i$. The additional subscript n introduced in eqs (15)–(19) indicates normalized quantities.

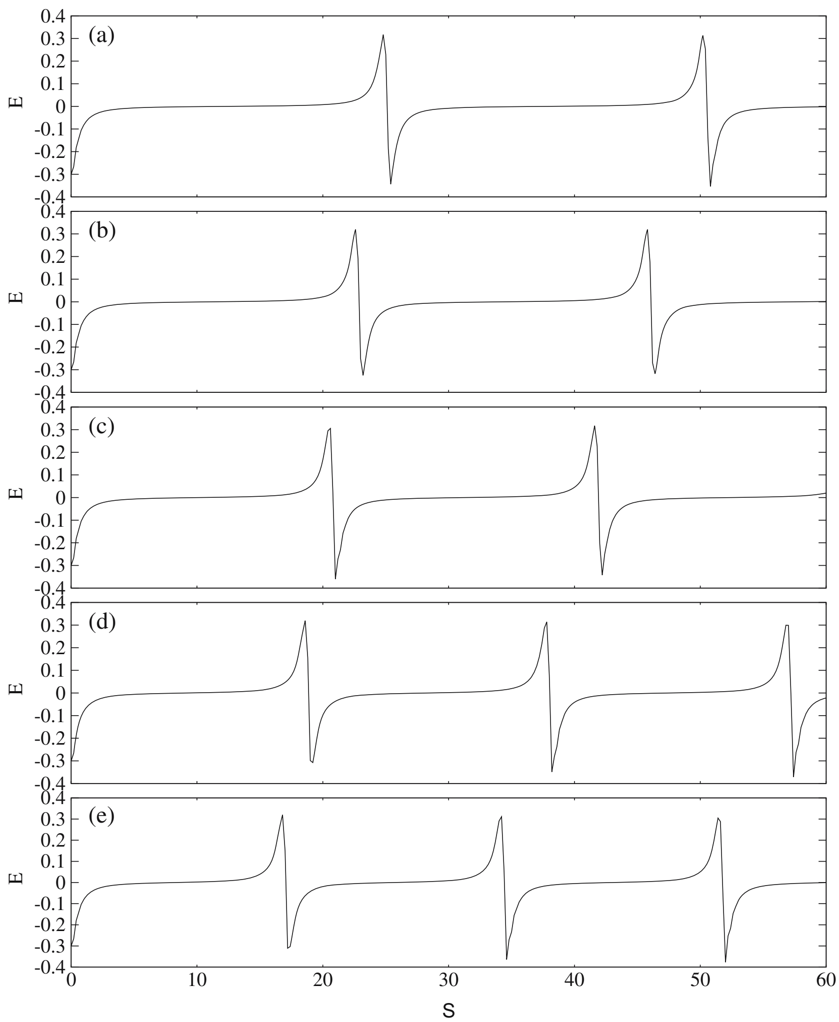


Figure 4. Numerical solution of the normalized parallel electric field for $E_0 = 0.3$, $M = 2.5$, $R = 3.0$, $\theta = 2°$, $(T_i/T_h) = 0.0$, $(n_{i0}/n_{e0}) = 0.5$ and $\delta_i = -0.20$ (**a**), -0.10 (**b**), 0.0 (**c**), 0.10 (**d**) and 0.20 (**e**).

3. The numerical results

Using the Runge–Kutta method, the set of nonlinear differential equations (14)–(19) is solved numerically. The following initial conditions are used:

At $s = 0$, $\psi = 0$, $E = E_0$, $n_{in} = n_{i0}/n_{e0}$, $p_{in} = n_{i0}T_i/n_{e0}T_h$, but v_{iyn0} and v_{izn0} are calculated self-consistently. The results are discussed next.

3.1 *Effect of the driving amplitude, E_0*

For fixed parameters $M = 2.5$, $\delta_i = 0.0$, $(n_{i0}/n_{e0}) = 0.5$, $(T_i/T_h) = 0.0$, $R = 3.0$ and $\theta = 2°$, we vary the driving electric field. The results are shown in figure 1. By increasing the driver strengths, the period of oscillations increases from $1.06\tau_{ci}$ to $3.44\tau_{ci}$, where $\tau_{ci} = 2\pi/\Omega_i$ is the ion cyclotron period. We observe the transition from ion-cyclotron waves to ion-acoustic waves. Bharuthram *et al* [21] and Moolla *et al* [24] found that the driving field strength for the onset of spiky structures were 1.1 and 0.3, respectively and in our study this value is reduced to 0.2. Thus, the introduction of Poisson equation increases the Mach number to generate waveforms, but for a slightly lower driving amplitude, to obtain spiky structures.

3.2 *Effect of ion temperature*

The effect of ion–electron temperature ratio T_i/T_h on the parallel electric field for the following fixed parameters $M = 2.5$, $\delta_i = 0.0$, $(n_{i0}/n_{e0}) = 0.5$, $R = 3.0$ and $\theta = 2°$ is represented in figure 2. It is found that increasing T_i/T_h from 0.0 to 0.2, results in the period of the waves increasing from $3.44\tau_{ci}$ to $3.62\tau_{ci}$. The graphs (a)–(d) in figure 2 show that the variation of ion–electron temperature ratio does not affect the nonlinearity of the waves.

3.3 *Effect of positron density*

We next investigate the effect of positron density on the electric field. It is seen from figure 3 that as the positron density increases from 0.3 to 0.5, the waveforms become more nonlinear and the period of these waveforms increases from $1.94\tau_{ci}$ to $2.83\tau_{ci}$. Similar results have been found by Moolla *et al* [24]. Thus, for both quasineutral and charge separated plasma of this

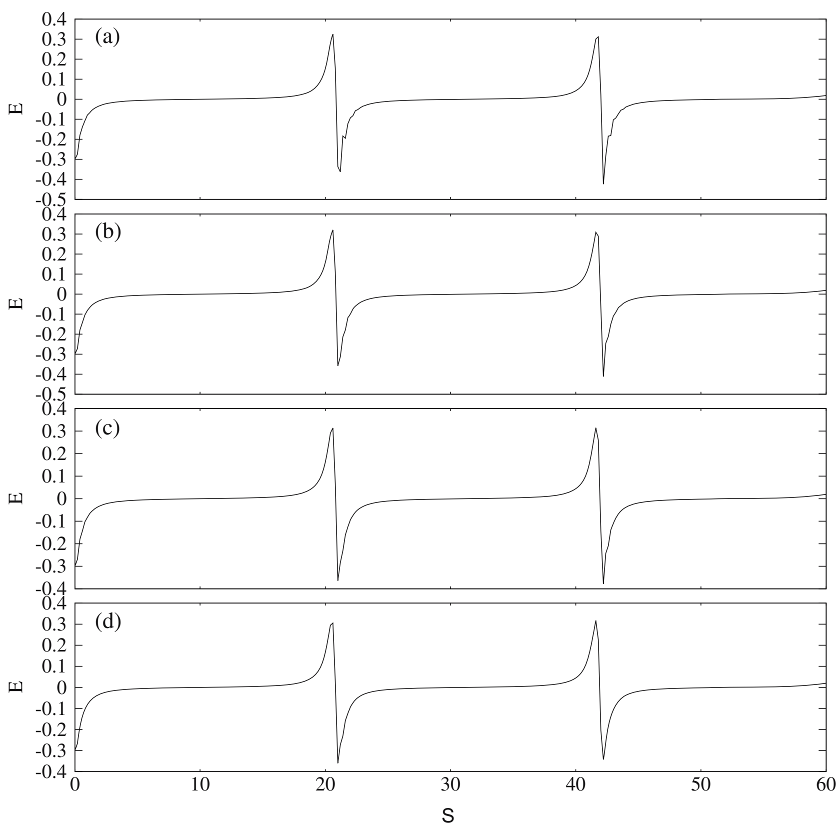


Figure 5. Numerical solution of the normalized parallel electric field for $E_0 = 0.3$, $R = 3.0$, $\delta_i = 0.0$, $\theta = 2°$, $(T_i/T_h) = 0.0$, $(n_{i0}/n_{e0}) = 0.5$ and $M = 2.2$ (**a**), 2.3 (**b**), 2.4 (**c**) and 2.5 (**d**).

type, the increase in positron density enhances nonlinearity making spiky structures easier to generate. This behaviour can be due to the fact that the positrons are much lighter than the ions, resulting in a plasma of less heavier particles. Hence, a weaker driving electric field is required to accelerate lighter particles to the nonlinear regime.

3.4 *Effect of ion drift*

The effect of ion drift velocity on the electric field structures is shown in figure 4. It is seen that when $\delta_i < 0$ (antiparallel ion drift), the periods of the waves are significantly higher than when $\delta_i > 0$ (parallel drift). However, the nonlinearity is unaffected and all waveforms are spiky in nature. These results are similar to those of Moolla *et al* [24] and agree with the satellite observations. These observations show that the periods of the BEN structures change significantly, implying that the ion drift changes rapidly. This means that they are accelerated in bursts leading to rapid changes in their periods (from parallel to antiparallel drift), consistent with the features of BEN [22,27].

3.5 *Effect of Mach number*

Figure 5 shows the effect of Mach number M on the electrostatic waves when $E_0 = 0.3$, $R = 3.0$, $\delta_i = 0.0$, $\theta = 2°$, $(T_i/T_h) = 0.0$ and $(n_{i0}/n_{e0}) = 0.5$. With a variation of M from 2.2 to 2.5, the period of oscillations slightly decreases from $3.45\tau_{ci}$ to $3.36\tau_{ci}$ and the nonlinearity of the waves is not affected. This trend can be explained by the fact that increasing the wave speed causes an increase in the wave frequency and consequently a decrease in the wave period.

3.6 *Effect of propagation angle*

For $E_0 = 0.3$, $R = 3.0$, $\delta_i = 0.0$, $M = 2.5$, $(T_i/T_h) = 0.0$ and $(n_{i0}/n_{e0}) = 0.5$, the curves representing different values of propagation angle θ are represented in figure 6. This figure shows that the period of oscillations slightly decreases from $3.45\tau_{ci}$ to $3.34\tau_{ci}$ with increasing θ. The observed increase in frequency with increasing θ is due to the transition from the ion-acoustic to the higher frequency ion-cyclotron mode. Here, we notice that the variation of propagation angle from 2° to 80° does not affect the nonlinearity of the wave structures.

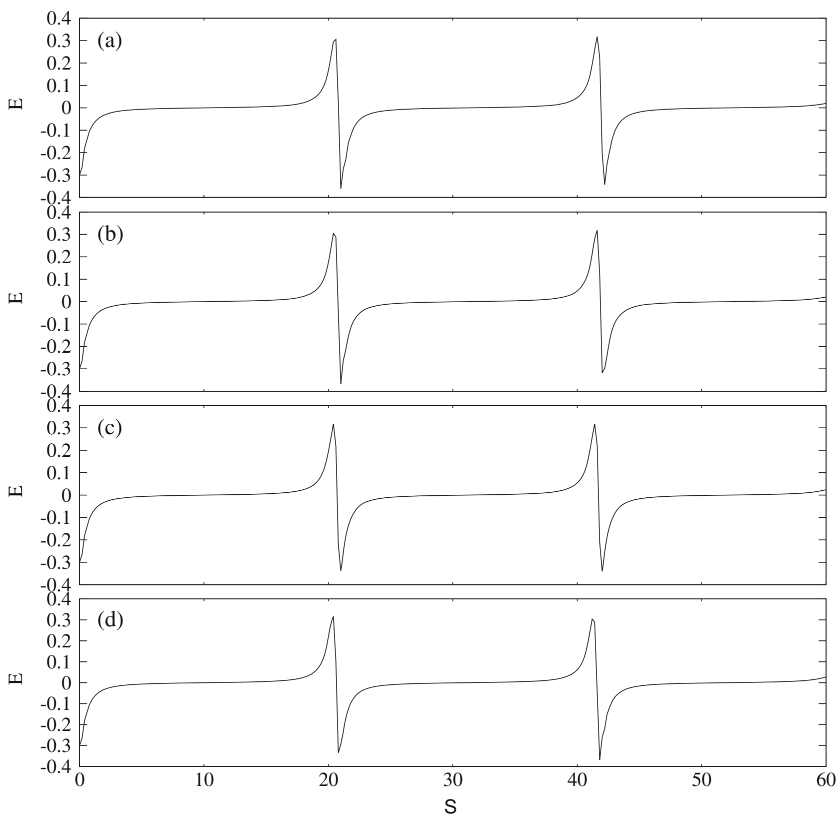


Figure 6. Numerical solution of the normalized parallel electric field for $E_0 = 0.3$, $M = 2.5$, $R = 3.0$, $(T_i/T_h) = 0.0$, $(n_{i0}/n_{e0}) = 0.5$, $\delta_i = 0.0$ and $\theta = 2°$ (**a**), 25° (**b**), 50° (**c**) and 80° (**d**).

4. Summary

We have studied nonlinear low-frequency waves in e–p–i plasma including charge separation. By using fluid equations for the warm ions with Poisson's equation, nonlinear electrostatic waves have been investigated in a plasma consisting of Boltzmann electrons and positrons. The effects of driving electric field, ion temperature, positron density, ion drift velocity, Mach number and propagation angle were studied. The electric fields of the nonlinear waves were investigated and we have shown that for high positron density, the spiky structures are easier to generate. Our model also shows that the Mach number and the angle of propagation do not affect the nonlinearity of wave structures.

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