Pramana – J. Phys. (2017) 88: 19
DOI 10.1007/s12043-016-1329-0

Separable metrics and radiating stars

G Z ABEBE and S D MAHARAJ*

Astrophysics and Cosmology Research Unit, School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa
*Corresponding author. E-mail: maharaj@ukzn.ac.za

MS received 17 November 2015; accepted 17 June 2016; published online 14 December 2016

Abstract. We study the junction condition relating the pressure to heat flux at the boundary of an accelerating and expanding spherically symmetric radiating star. We transform the junction condition to an ordinary differential equation by making a separability assumption on the metric functions in the space–time variables. The condition of separability on the metric functions yields several new exact solutions. A class of shear-free models is found which contains a linear equation of state and generalizes a previously obtained model. Four new shearing models are obtained; all the gravitational potentials can be written explicitly. A brief physical analysis indicates that the matter variables are well behaved.

Keywords. Relativistic astrophysics; radiating stars; relativistic fluids.

PACS Nos 04.20.Jb; 04.40.Dg; 97.60.Jd

1. Introduction

Relativistic radiating stars with expansion, shear and acceleration are the most general models, and they are very important for investigating physical phenomena such as stellar stability, the Penrose cosmic censorship hypothesis, surface luminosity, effects of relaxation, particle production, causal temperature profiles, and gravitational collapse. The interior space–time of the radiating star should match with the exterior Vaidya [1] solution. The junction condition relating pressure with the heat flux at the boundary of the star, established by Santos [2], should be satisfied; this was generalized by Glass [3] to include the effects of shear. This junction condition must be integrated at the stellar surface. In recent times several particular solutions to the stellar boundary condition have been discovered with a variety of different physical features.

The shear is a desirable feature to be included in our analysis to obtain a wider class radiating models. Naidu *et al* [4] obtained the first exact solution with shear by considering the geodesic motion of fluid particles. Rajah and Maharaj [5] extended this result and obtained new classes of solutions by transforming the junction condition and solving it. These extended classes of solutions are nonsingular at the origin. Abebe *et al* [6] studied a geodesic model using the Lie symmetry approach, and presented several classes of exact solutions. Some of their solutions are of the travelling wave type and can be written in terms of self-similar variables. A comprehensive treatment of the accelerating and expanding model with shear was undertaken by Thirukkanesh *et al* [7]. The effects of anisotropic pressure and shear have been studied by Chan *et al* [8] and Herrera and Santos [9].

Several approaches have been followed in the past to solve the fundamental boundary condition at the stellar surface. A recent approach, that has proved very effective, is the Lie analysis of differential equations in both shear-free and shearing models, as shown by Abebe *et al* [6,10–12]. Another simple possibility is to assume separability in the metric functions. An initially static configuration of gravitational collapse has been studied by Chan [13]. In later treatments, Chan [14–16] studied gravitational collapse with shear and bulk viscosity. The treatment of Pinheiro and Chan [17,18] considers luminosity, viscous effects and other physical features in detail for a shearing fluid. For a shear-free radiation fluid, Tewari [19] obtained a class of particular solutions by assuming separable metric functions. Pant and Tewari [20] studied various parameters including, mass, radius and luminosity of the radiating fluid in the absence of shear. In most of these analyses, the boundary condition yields a nonlinear differential equation which has to be analysed numerically. Exact models with separable metrics have also been analysed. Tewari [21,22] studied a shear-free radiating fluid assuming separable forms of the metric functions and presented classes of exact solutions. Exact solutions may be obtained by a systematic choice for functional forms of the metric functions. Hence, by suitable selection of specific metrics, it is possible to integrate the boundary condition exactly in the presence or absence of shear.

Generating exact solutions for the boundary condition equation of a radiating star using the separability assumption on the metric functions is the main objective of this paper. We present the model of radiating star in §2 and derive the boundary condition for a separable line element. The master equation is a nonlinear ordinary differential equation in time. In §3 we find a particular exact model which has vanishing shear. In §4 we present four new exact models which have nonvanishing shear in the interior of the star. The physical features of the radiating star, for a particular metric, is considered in §5. Some concluding remarks are made in §6.

2. The model

We consider the general case of a spherically symmetric radiating star with acceleration, expansion and shear. The line element for such stars is given by

$$\mathrm{d}s^2 = -A^2\mathrm{d}t^2 + B^2\mathrm{d}r^2 + Y^2\mathrm{d}\Omega^2, \tag{1}$$

where the metric functions $A = A(r,t)$, $B = B(r,t)$ and $Y = Y(r,t)$ are functions of the coordinate radius r, temporal time variable t and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2$. The fluid four-velocity $\mathbf{u}$ is comoving and is given by $u^a = (1/A)\delta^a_0$. We consider a simple separable form of the metric functions that describes a radiating star. The line element (1), for such a configuration, reduces to

$$\mathrm{d}s^2 = -f^2(t)\alpha^2(r)\mathrm{d}t^2 + g^2(t)\alpha'^2(r)\mathrm{d}r^2 + h^2(t)\alpha^2(r)\mathrm{d}\Omega^2, \tag{2}$$

where the prime denotes differentiation with respect to r.

The acceleration $\dot{u}^a$, the expansion scalar Θ and the magnitude of the shearing scalar σ for the line element (2) are given by

$$\dot{u}^a = \left(0, \frac{1}{g^2\alpha\alpha'}, 0, 0\right), \tag{3a}$$

$$\Theta = \frac{1}{f\alpha}\left(\frac{\dot{g}}{g} + \frac{2\dot{h}}{h}\right), \tag{3b}$$

$$\sigma = \frac{1}{3f}\left(\frac{\dot{h}}{h\alpha} - \frac{\dot{g}}{g\alpha}\right), \tag{3c}$$

and dots denote differentiation with respect to t.

The Einstein field equations for the interior matter distribution become

$$\mu = \frac{f^2(g^2-h^2) + g\dot{h}(2h\dot{g} + g\dot{h})}{f^2g^2h^2\alpha^2}, \tag{4a}$$

$$p_\parallel = -\frac{fg^2(2h\ddot{h} + \dot{h}^2) - 2g^2h\dot{f}\dot{h} + f^3(g^2-3h^2)}{f^3g^2h^2\alpha^2}, \tag{4b}$$

$$p_\perp = \frac{g\dot{f}(h\dot{g} + g\dot{h}) - fg(h\ddot{g} + \dot{g}\dot{h} + g\ddot{h}) + f^3h}{f^3g^2h\alpha^2}, \tag{4c}$$

$$q = -\frac{2\dot{g}}{fg^3\alpha^2\alpha'}, \tag{4d}$$

for the line element (2). In the above, μ is the energy density, $p_\parallel$ is the radial pressure, $p_\perp$ is the tangential pressure and q is the magnitude of the heat flux.

The boundary Σ of a spherically symmetric accelerating and expanding radiating star with shear divides the entire space–time into two distinct regions: the interior and the exterior space–time. The interior space–time (1) has to be matched along the boundary of the star to the exterior Vaidya space–time

$$\mathrm{d}s^2 = -\left(1 - \frac{2m(v)}{R}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}R + R^2\mathrm{d}\Omega^2, \tag{5}$$

where $m(v)$ is the mass of the star. The matching of the line elements (1) and (5) implies the existence of the particular junction condition

$$(p_\parallel)_\Sigma = (Bq)_\Sigma, \tag{6}$$

at the boundary of the radiating sphere. Equation (6), with (4b) and (4d), leads after simplification to the condition

$$\dot{g} - \left[\frac{(2h\ddot{h} + \dot{h}^2)f - 2h\dot{h}\dot{f} + f^3}{2f^2h^2}\right]g^2 + \frac{3}{2}f = 0. \tag{7}$$

Equation (7) determines the gravitational behaviour of the radiating anisotropic star for the line element (2). In this section, we have generated the boundary condition (7) for the separable line element (2). In the Appendix, we show how the master equation is obtained for the general spherically symmetric line element (1).

3. Shear-free models

We can find a simple class of models by relating the functions $g(t)$ and $h(t)$. We set

$$g(t) = \sqrt{3}h, \tag{8}$$

then eq. (7) becomes

$$6\dot{h}h\dot{f} - 3(2h\ddot{h} + \dot{h}^2)f + 2\sqrt{3}\dot{h}f^2 = 0. \tag{9}$$

This is a Bernoulli equation in f and can be integrated to give

$$f(t) = \frac{\sqrt{3}}{2}\frac{\dot{h}\sqrt{h}}{a + \sqrt{h}}. \tag{10}$$

In this case h is arbitrary and a is a constant. Note that any function h yields a solution to (7) as long as (8) is valid.

The line element (2) becomes

$$\mathrm{d}s^2 = -\frac{3}{4}\left[\frac{\dot{h}(t)\sqrt{h(t)}}{a + \sqrt{h(t)}}\alpha(r)\right]^2 \mathrm{d}t^2 + h^2(t)\{3\alpha'^2(r)\mathrm{d}r^2 + \alpha^2(r)\mathrm{d}\Omega^2\}. \tag{11}$$

This is a simple model that depends on the functions $h(t)$ and $\alpha(r)$ and the constant a. Note that the space–time (11) is expanding ($\Theta \neq 0$), accelerating ($\dot{u}^a \neq 0$) but is shear-free ($\sigma = 0$). A general class of shear-free radiating stars was obtained by Abebe *et al* [11] using the Lie method of infinitesimal operators. The line element (11) is not contained in that class; it arises from separability of the line element and not the existence of a Lie symmetry. However, if we set the arbitrary function $\alpha(r) = r^{\pm\sqrt{3}/3}$ then the line element (11) becomes

$$\mathrm{d}s^2 = -\frac{3}{4}\left[\frac{\dot{h}(t)\sqrt{h(t)}}{a + \sqrt{h(t)}}r^{\pm(\sqrt{3}/3)}\right]^2 \mathrm{d}t^2 + h^2(t)r^{\pm 2(\sqrt{3}/3)-2}\{\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\}. \tag{12}$$

This solution has been previously obtained by [11] based on their Lie symmetry analysis of a shear-free model. It is interesting to see that our solution contains a previously obtained solution as a special case. The characteristic equations that arise in the Lie analysis fix the radial function $\alpha(r)$ to a specific power law form. Our line element (11) is a generalization.

It is interesting to note that for this category of solutions

$$\mu = \frac{14}{3h^2(t)\alpha^2(r)}, \qquad p_{\parallel} = -\frac{4}{3h^2(t)\alpha^2(r)}, \tag{13}$$

for the line element (11). Without any lose of generality, we have set $a = 0$. Therefore, the shear-free star satisfies the equation of state

$$p_{\parallel} = \lambda\mu, \quad \lambda = -\frac{2}{7}, \tag{14}$$

which is linear and barotropic. The equation of state in (14) also arises in shear-free models with Lie symmetries as shown by Abebe *et al* [11].

4. Shearing models

The boundary condition (6) has been reduced to the nonlinear ordinary differential equation (7). To complete the model, with shear, we need to integrate (7) and find functional forms for the functions f, g and h. It is not possible to solve, (7) in general. However, particular solutions can be found. Note that we are treating (7) as a first-order equation in g; in general, (7) is a Riccati equation which does not fall into any of the standard cases. Particular solutions are listed below.

4.1 *Case* I

If we choose

$$f(t) = 1, \quad h(t) = t^{2/3}, \tag{15}$$

then (7) becomes

$$\dot{g} - \frac{1}{2t^{4/3}}g^2 + \frac{3}{2} = 0. \tag{16}$$

Equation (16) can be integrated to give

$$g(t) = \frac{2t^{1/3}}{3} - \frac{\sqrt{3}t^{2/3}\left[a\coth\left(\frac{3\sqrt{3}}{2}t^{1/3}\right) + 1\right]}{a + \coth\left(\frac{3\sqrt{3}}{2}t^{1/3}\right)}, \tag{17}$$

where a is a constant of integration. In this category of solution, the functions f, g and h are given in terms of elementary functions.

4.2 *Case* II

We assume that

$$\frac{(2h\ddot{h} + \dot{h}^2)f - 2h\dot{h}\dot{f} + f^3}{2f^2h^2} = \frac{3}{2}kf, \tag{18}$$

and k is a nonzero arbitrary constant. Then (18) can be written as

$$2h\dot{h}\dot{f} - (2h\ddot{h} + \dot{h}^2)f + (3kh^2 - 1)f^3 = 0, \tag{19}$$

which is a Bernoulli equation in f. We can integrate (19) to obtain

$$f(t) = \dot{h}\sqrt{\frac{h}{a + kh^3 - h}}, \tag{20}$$

where a is an integration constant.

With the assumption (18) we can write (7) in the form

$$\frac{\dot{g}}{kg^2 - 1} = \frac{3}{2}f. \tag{21}$$

We can solve (21) to obtain

$$g(t) = \frac{\tan\left(b - \frac{3\sqrt{k}}{2}\int f(t)\,\mathrm{d}t\right)}{\sqrt{k}}, \tag{22}$$

where b is a constant of integration. For this class of exact solutions, the functions f and g are given in terms of the function h; the function h is arbitrary.

4.3 *Case* III

Equation (7) may also be integrated if we assume

$$\tilde{a} = \frac{(2h\ddot{h} + \dot{h}^2)f - 2h\dot{h}\dot{f} + f^3}{2f^2h^2}, \tag{23a}$$

$$a = \frac{3}{2}f \quad \text{or} \quad f = \frac{2a}{3}, \tag{23b}$$

where $\tilde{a}$ and a are nonzero constants. Equations (23b) and (23a) imply

$$18h\ddot{h} + 9\dot{h}^2 - 12\tilde{a}ah^2 + 4a^2 = 0. \tag{24}$$

Note that (24) yields solutions in terms of a complex variable which is not useful in our analysis. However, (24) does contain real solutions if $\tilde{a}$ and a related as

$$\tilde{a} = a. \tag{25}$$

Then (24) becomes

$$18h\ddot{h} + 9\dot{h}^2 - 12a^2h^2 + 4a^2 = 0. \tag{26}$$

This equation can be integrated to give

$$\dot{h}^2h + \frac{4a^2}{9}h(1 - h^2) + b = 0, \tag{27}$$

which is a first-order nonlinear ordinary differential equation where b is a constant of integration. Equation (27) can be solved in parametric form but the functional form obtained is complicated. A particular choice for the constant b leads to a model expressible in terms of elementary functions. If we set $b = 0$ then (27) becomes separable and can be integrated to give

$$h(t) = \frac{1}{2}\left[\exp\left(c - \frac{2a}{3}t\right) + \exp\left(\frac{2a}{3}t - c\right)\right], \tag{28}$$

where c is a constant of integration.

With the assumption (23), eq. (7) becomes

$$\dot{g} - ag^2 + a = 0, \tag{29}$$

which can be integrated to give

$$g(t) = \frac{1 - \exp(2at + d)}{1 + \exp(2at + d)}, \tag{30}$$

where d is a constant of integration. Here the functions f, g and h are expressed in terms of elementary functions.

4.4 *Case* IV

For this class of models we set

$$2h\dot{h}\dot{f} - (2h\ddot{h} + \dot{h}^2)f - f^3 = 0. \tag{31}$$

Equation (31) is a Bernoulli equation in f that can be integrated to give

$$f(t) = \dot{h}\sqrt{\frac{h}{a - h}}, \tag{32}$$

where a is a constant of integration. Substituting (32) into (7) we have

$$\dot{g} + \frac{3\dot{h}}{2}\sqrt{\frac{h}{a - h}} = 0, \tag{33}$$

which is linear in g. On integration we have

$$g(t) = \frac{3}{2}\left[\sqrt{ah - h^2} - a\arctan\left[\sqrt{\frac{h}{a - h}}\right] + b\right], \tag{34}$$

where b is an arbitrary constant of integration. The function h is arbitrary in this case as well.

5. Physical analysis

Here we briefly comment on the physical features of the models found in this paper. We have already shown that the shear-free radiating model in §3 satisfies a linear equation of state. For a shearing radiating model we consider the metric in Case III of the solution in §4. In this case, the line element (2) becomes

$$
\begin{aligned}
ds^2 = &-\frac{4}{9}a^2\alpha^2(r)dt^2 + \left\{\left[\frac{1-\exp(2at+d)}{1+\exp(2at+d)}\right]\right. \\
&\left.\times\,\alpha'(r)\right\}^2 dr^2 + \frac{1}{4}\left\{\left[\exp\left(c-\frac{2a}{3}t\right)\right.\right. \\
&\left.\left.+\exp\left(\frac{2a}{3}t-c\right)\right]\alpha(r)\right\}^2 d\Omega^2. \qquad (35)
\end{aligned}
$$

The kinematical quantities have the form

$$
\dot{u}^a = \left(0, \frac{(\exp(2at+d)+1)^2}{\alpha(r)\,(\exp(2at+d)-1)^2\,\alpha'(r)}, 0, 0\right), \qquad (36a)
$$

$$
\begin{aligned}
\Theta = &\left[2\left(-\exp(2(2at+c+d)) + 3\exp(2at+2c+d)\right.\right. \\
&+3\exp\left(\frac{10at}{3}+d\right)+\exp\left(\frac{16at}{3}+2d\right) \\
&\left.\left.-\exp\left(\frac{4at}{3}\right)+\exp(2c)\right)\right]\left[\alpha(r)\left(\exp\left(\frac{4at}{3}\right)\right.\right. \\
&\left.+\exp(2c)\right)(\exp(2at+d)-1) \\
&\left.\times(\exp(2at+d)+1)\right]^{-1}, \qquad (36b)
\end{aligned}
$$

$$
\begin{aligned}
\sigma = &\left[2\left(-\exp(2(2at+c+d)) + 3\exp(2at+2c+d)\right.\right. \\
&+3\exp\left(\frac{10at}{3}+d\right)+\exp\left(\frac{16at}{3}+2d\right) \\
&\left.\left.-\exp\left(\frac{4at}{3}\right)+\exp(2c)\right)\right]\left[\alpha(r)\left(\exp\left(\frac{4at}{3}\right)\right.\right. \\
&\left.+\exp(2c)\right)(\exp(2at+d)-1) \\
&\left.\times(\exp(2at+d)+1)\right]^{-1}. \qquad (36c)
\end{aligned}
$$

It is clear from (36) that the acceleration, expansion and shear are nonvanishing. For suitable choices of the function $\alpha(r)$, away from the centre, the kinematical quantities may be small and the space–time approaches asymptotic flatness. This property is expected in a radiating star as it radiates energy away.

The matter variables are given as

$$
\begin{aligned}
\mu = &\left[8\exp(2at+d)\left(-2\exp(2at+2c+d)\right.\right. \\
&\left.\left.+\exp\left(\frac{10at}{3}+d\right)-2\exp\left(\frac{4at}{3}\right)+\exp(2c)\right)\right] \\
&\times\left[\alpha^2(r)\left(\exp\left(\frac{4at}{3}\right)+\exp(2c)\right)\right. \\
&\times(\exp(2at+d)-1)^2 \\
&\left.\times(\exp(2at+d)+1)\right]^{-1}, \qquad (37a)
\end{aligned}
$$

$$
p_{\|} = \frac{12\exp(2at+d)}{\alpha^2(r)\,(\exp(2at+d)-1)^2}, \qquad (37b)
$$

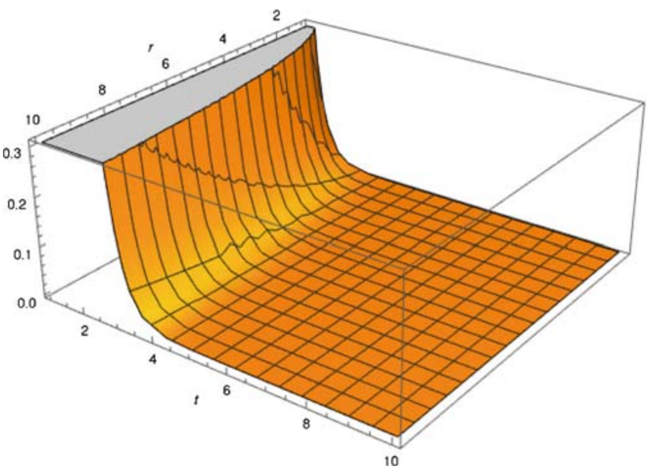

Figure 2. Radial pressure.

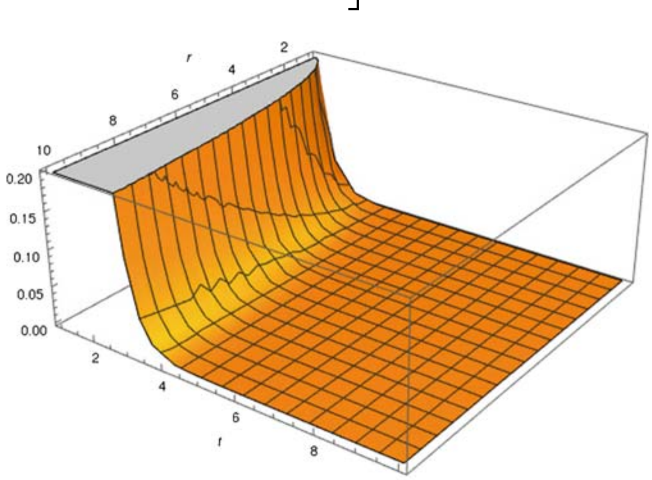

Figure 1. Energy density.

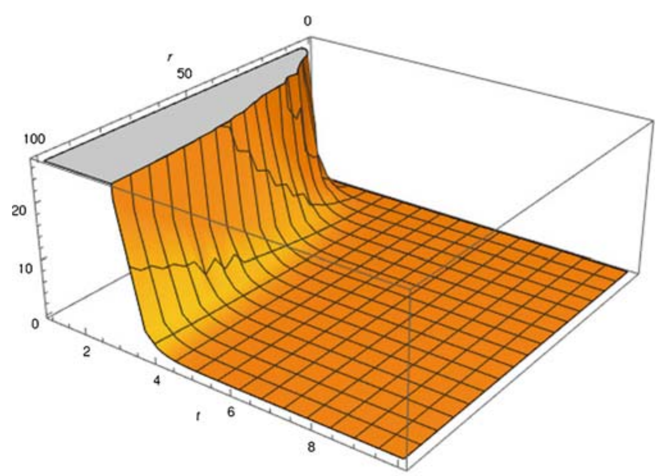

Figure 3. Tangential pressure.

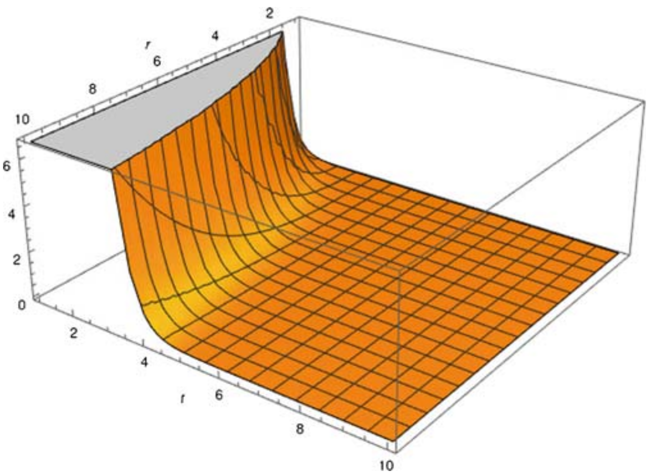

Figure 4. Heat flux.

$$p_{\perp} = \Bigg[4\exp(2at+d)\Bigg(7\exp(4at+2c+2d) - 7\exp(2at+2c+d) - 7\exp\left(\frac{10at}{3}+d\right) + 4\exp\left(\frac{16at}{3}+2d\right) + 7\exp\left(\frac{4at}{3}\right) + 4\exp(2c)\Bigg)\Bigg]\Bigg[\alpha^2(r)\left(\left(\frac{4at}{3}\right)+\exp(2c)\right) \times (\exp(4at+2d)-1)^2\Bigg]^{-1}, \tag{37c}$$

$$q = -\frac{12\exp(2at+d)\,(\exp(2at+d)+1)}{\alpha'(r)\alpha^2(r)\,(\exp(2at+d)-1)^3}, \tag{37d}$$

Table 1. Models, the functions f, g and h and the resulting gravitational potentials.

Models	The functions f, g and h	Gravitational potentials
Model 1 ($\dot{u}^a, \Theta \neq 0, \sigma = 0$)	$f = \frac{\sqrt{3}}{2}\frac{\dot{h}(t)\sqrt{h(t)}}{a+\sqrt{h(t)}}$ $g = \sqrt{3}h(t)$ $h = h(t)$	$A = \frac{\sqrt{3}}{2}\frac{\dot{h}(t)\sqrt{h(t)}}{a+\sqrt{h(t)}}\alpha(r)$ $B = \sqrt{3}h(t)\alpha'(r)$ $Y = h(t)\alpha(r)$
Model 2 ($\dot{u}^a, \Theta \neq 0, \sigma \neq 0$)	$f = 1$ $g = \frac{2t^{1/3}}{3} - \frac{\sqrt{3}t^{2/3}\left(a\coth\left(\frac{3\sqrt{3}}{2}t^{1/3}\right)+1\right)}{a+\coth\left(\frac{3\sqrt{3}}{2}t^{1/3}\right)}$ $h = t^{2/3}$	$A = \alpha(r)$ $B = \alpha'(r)\left[\frac{2\sqrt[3]{t}}{3} - \frac{\sqrt{3}t^{2/3}\left(a\coth\left(\frac{3\sqrt{3}}{2}t^{1/3}\right)+1\right)}{a+\coth\left(\frac{3\sqrt{3}}{2}t^{1/3}\right)}\right]$ $Y = t^{2/3}\alpha(r)$
Model 3 ($\dot{u}^a, \Theta \neq 0, \sigma \neq 0$)	$f = \dot{h}(t)\sqrt{\frac{h(t)}{a+kh^3(t)-h(t)}}$ $g = \frac{1}{\sqrt{k}}\tan\left[b - \frac{3\sqrt{k}}{2}\int f(t)\,\mathrm{d}t\right]$ $h = h(t)$	$A = \dot{h}(t)\sqrt{\frac{h(t)}{a+kh^3(t)-h(t)}}\alpha(r)$ $B = \frac{1}{\sqrt{k}}\alpha'(r) \times \tan\left[b - \frac{3\sqrt{k}}{2}\int f(t)\,\mathrm{d}t\right]$ $Y = h(t)\alpha(r)$
Model 4 ($\dot{u}^a, \Theta \neq 0, \sigma \neq 0$)	$f = \frac{2a}{3}$ $g = \frac{1-\exp(2at+d)}{1+\exp(2at+d)}$ $h = \frac{1}{2}\left[\exp\left(c-\frac{2a}{3}t\right) + \exp\left(\frac{2a}{3}t-c\right)\right]$	$A = \frac{2a}{3}\alpha(r)$ $B = \alpha'(r)\left[\frac{1-\exp(2at+d)}{1+\exp(2at+d)}\right]$ $Y = \frac{1}{2}\left[\exp\left(c-\frac{2a}{3}t\right) + \exp\left(\frac{2a}{3}t-c\right)\right]\alpha(r)$
Model 5 ($\dot{u}^a, \Theta \neq 0, \sigma \neq 0$)	$f = \dot{h}(t)\sqrt{\frac{h(t)}{a-h(t)}}$ $g = \frac{3}{2}\left[\sqrt{ah(t)-h^2(t)} - a\arctan\left[\sqrt{\frac{h(t)}{a-h(t)}}\right] + b\right]$ $h = h(t)$	$A = \dot{h}(t)\sqrt{\frac{h(t)}{a-h(t)}}\alpha(r)$ $B = \frac{3}{2}\alpha'(r)\left[\sqrt{ah(t)-h^2(t)} - a\arctan\left[\sqrt{\frac{h(t)}{a-h(t)}}\right] + b\right]$ $Y = h(t)\alpha(r)$

for the line element (35). In general, we observe qualitatively that μ, $p_\parallel$, $p_\perp$ and q approach static limit for large time. This is a desirable feature for a radiating body in a closed system. We plot the matter variables μ, $p_\parallel$, $p_\perp$ and q in figures 1–4 by setting $\alpha(r) = 1/r$, $a = 3/2$ and $d = c = 0$. The matter quantities μ, $p_\parallel$, $p_\perp$ and q are well behaved in the interior of the radiating sphere surrounding the core. The plots represent the profiles which are regular over the interval plotted. The expressions for the matter variables in (37) are complicated. However, there exist regions in space–time, with relevant choices of the parameters, in which the density, the radial pressure and the tangential pressure are decreasing functions. It is also pleasing to observe that heat flux is decreasing in this interval for different values of temporal coordinates. This suggests that the star is losing energy via radiation loss across the boundary as it approaches a static configuration.

In this example we observe the relationship

$$\begin{aligned} p_\parallel &= \beta\mu, \\ \beta &= \left[3\left(\exp\left(\frac{4at}{3}\right)+\exp(2c)\right)(\exp(2at+d)+1)\right] \\ &\quad\times\left[2\left(-2\exp(2at+2c+d)+\exp\left(\frac{10at}{3}+d\right)\right.\right. \\ &\quad\left.\left.-2\exp\left(\frac{4at}{3}\right)+\exp(2c)\right)\right]^{-1}. \end{aligned} \tag{38}$$

Hence the ratio $p_\parallel/\mu$ is independent of the radial coordinate r. This implies that the relation is independent of position in the interior of the star.

6. Conclusion

We considered a shearing relativistic radiating star with acceleration and expansion. We studied the junction condition by assuming that the metric functions are separable. We derived a simplified form for the boundary condition; this is a nonlinear ordinary differential equation in general. It is interesting to note that in several of the models, the master equation reduces to a Bernoulli equation. A class of shear-free models was found which contains a linear barotropic equation of state. This class contains a shear-free metric of Abebe *et al* [11] obtained using the Lie method of symmetry operators. In addition, four new shearing models were found by integrating the master equation. The results have been summarized in table 1 which gives the various models, the conditions on the separable functions f, g and h and the gravitational potentials. In all cases, we have explicit expressions for the metric functions which simplify the physical analysis. The physical features of a particular shearing metric was studied in §5. This line element is expanding, accelerating and shearing. The matter variables approach a static limit for large time. A graphical analysis shows that μ, $p_\parallel$, $p_\perp$ and q are well behaved in the interior of the star.

Acknowledgements

GZA thanks the University of KwaZulu-Natal for continuing support. SDM acknowledges that this work is based on research supported by the South African Research Chair Initiative of the Department of Science and Technology and the National Research Foundation.

References

[1] P C Vaidya, *Proc. Ind. Acad. Sci. A* **33**, 264 (1951)
[2] N O Santos, *Mon. Not. R. Astron. Soc.* **216**, 403 (1985)
[3] E N Glass, *Gen. Relativ. Gravit.* **21**, 733 (1989)
[4] N F Naidu, M Govender and K S Govinder, *Int. J. Mod. Phys. D* **15**, 1053 (2006)
[5] S S Rajah and S D Maharaj, *J. Math. Phys.* **49**, 012501 (2008)
[6] G Z Abebe, S D Maharaj and K S Govinder, *Gen. Relativ. Gravit.* **46**, 1650 (2014)
[7] S Thirukkanesh, S S Rajah and S D Maharaj, *J. Math. Phys.* **53**, 032506 (2012)
[8] R Chan, M F A Da Silva and J F V Da Rocha, *Int. J. Mod. Phys. D* **12**, 347 (2003)
[9] L Herrera and N O Santos, *Mon. Not. R. Astron. Soc.* **287**, 161 (1997)
[10] G Z Abebe, K S Govinder and S D Maharaj, *Int. J. Theor. Phys.* **52**, 3244 (2013)
[11] G Z Abebe, S D Maharaj and K S Govinder, *Eur. Phys. J. C* **75**, 496 (2015)
[12] G Z Abebe, S D Maharaj and K S Govinder, *Gen. Relativ. Gravit.* **46**, 1733 (2014)
[13] R Chan, *Mon. Not. R. Astron. Soc.* **288**, 589 (1997)
[14] R Chan, *Mon. Not. R. Astron. Soc.* **316**, 588 (2000)
[15] R Chan, *Astron. Astrophys.* **368**, 325 (2001)
[16] R Chan, *Int. J. Mod. Phys. D* **12**, 1131 (2003)
[17] G Pinheiro and R Chan, *Gen. Relativ. Gravit.* **40**, 2149 (2008)
[18] G Pinheiro and R Chan, *Int. J. Mod. Phys. D* **19**, 1797 (2010)
[19] B C Tewari, *Astrophys. Space Sci.* **306**, 273 (2006)
[20] N Pant and B C Tewari, *Astrophys. Space Sci.* **331**, 645 (2011)
[21] B C Tewari, *Gen. Relativ. Gravit.* **45**, 1547 (2013)
[22] B C Tewari, *Astrophys. Space Sci.* **342**, 73 (2012)