Analysis of the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay

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Abstract. We analysed the process of $B_c^+ \to D_s^+ \bar{K}^{0*}$ using QCD factorization (QCDF) and final-state interaction (FSI) effects. First, the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay is calculated using QCDF method. The value found by using the QCDF method is less than the experimental value. Then we considered FSI effect as a sizable correction where the intermediate state $D^+ \pi^0$ mesons via the exchange of $K^0(K^{0*})$ are produced. To consider the amplitudes of this intermediate state, the QCDF approach was used. The experimental branching ratio of $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay is less than $0.4 \times 10^{-6}$ and our results are $(0.21 \pm 0.04) \times 10^{-7}$ and $(0.37 \pm 0.05) \times 10^{-6}$ from QCDF and FSI, respectively.

Keywords. QCD factorization; final-state interaction; intermediate state; branching ratio.


1. Introduction

The study of the two-body non-leptonic weak decay of $B_c^+ \to D_s^+ \bar{K}^{0*}$ may be useful in the search for new physics beyond the Standard Model. Several useful methods such as the perturbative QCD approach [1], QCDF by Beneke and Neubert [2] and FSI effects by Lu et al [3] are there to calculate the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay. In perturbative QCD, Xiao et al have considered complete twist-3 contributions, and in the QCDF, state-of-the-art analysis was performed according to QCDF by four-parameter scenarios. The branching ratios obtained from these scenarios, considering large arbitrary numbers (the same work has been conducted in scenario 4 in ref. [2]), are smaller than experimental ones. In FSI, the $D_s^+ \pi^0$ mesons have been considered as intermediate states. We mainly used the same framework in the QCDF approach and selected the leading-order Wilson coefficients at the scale $m_b$ [2,4] and estimated the amplitude of the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay while assuming the annihilation contributions. In this case, we obtained $(0.21 \pm 0.04) \times 10^{-7}$ as the branching ratio. Motivated by the above-mentioned study, we contributed FSI corrections in the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay mode. As, in $B$ decays, resonant FSI is expected to be suppressed due to the absence of resonances at energies close to the mass of the $B$ meson, we considered only t-channels and estimated it via the one-particle exchange processes at the hadronic loop level (HLL) as explained in §4. The FSI can give sizable corrections. Rescattering amplitude can be derived by calculating the absorptive part of triangle diagrams. In this decay, intermediate state is $D^+ \pi^0$. Then we calculated the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay using the HLL method. Using the FSI method we obtain the branching ratio of $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay, $(0.37 \pm 0.05) \times 10^{-6}$, and the experimental result of this decay is less than $0.4 \times 10^{-6}$ [5]. We present the calculation of QCDF for the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay in §2. In §3, we calculate the amplitudes of the intermediate state. Then we present the calculation of HLL for $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay in §4. In §5, we give the numerical results, and in §6, we have conclusions.

2. QCD factorization in the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay

In this section, we calculate the amplitude of the $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay by the QCDF method. The $B_c^+ \to D_s^+ \bar{K}^{0*}$ decay has tree, penguin and annihilation diagrams that are shown in figure 1. According to these
Feynman diagrams, we wrote the amplitude of this decay by the QCDF method as follows:
\[
A_{\text{QCD}} = -i \sqrt{2} G_F f_{D_s} f_{\bar{K}^0} (\varepsilon_{K^*} \cdot p_B) A_0^{B K^*} \\
\times \left[ \left( a_4 - \frac{1}{2} a_{10} \right) + r_{K^*}^s \left( a_6 - \frac{1}{2} a_8 \right) \right] \lambda_p \\
- \frac{G_F}{\sqrt{2}} f_{B_c} f_{D_s} f_{K^*} [b_2 (V_{cb} V_{cd}^*)] \\
+ (b_3 - b_{3\text{eW}}) \lambda_p ,
\]
(1)
where \( \lambda_p \) are the products of elements of the quark mixing matrix. Using the unitarity relation
\[
\lambda_p + \lambda_c + \lambda_t = 0,
\]
(2)
we get
\[
\lambda_p = \sum_{p=u,c} V_{pb} V_{pd}^* .
\]
(3)

We also define \( a_i \) coefficients obtained from a combination of Wilson coefficients appeared in decay amplitude as \[6,7\]
\[
a_i = c_i + \frac{1}{N_c} c_{i+1} \quad (i = \text{odd}),
\]
\[
a_i = c_i + \frac{1}{N_c} c_{i-1} \quad (i = \text{even}).
\]
(3)

\( b_2, b_3 \) and \( b_{3\text{eW}} \) correspond to the current–current annihilation, penguin annihilation and electroweak penguin annihilation. These non-singlet annihilation coefficients are given as \[8,9\]
\[
b_2 = \frac{C_F}{N_c^2} c_2 A_2^i ,
\]
\[
b_3 = \frac{C_F}{N_c^2} [c_3 A_1^i + c_5 (A_3^i + A_5^i) + N_c c_6 A_3^f] ,
\]
\[
b_{3\text{eW}} = \frac{C_F}{N_c^2} [c_9 A_1^i + c_7 (A_3^i + A_5^i) + N_c c_8 A_3^f] ,
\]
(4)
where \( c_i \) are the Wilson coefficients, \( N_c \) is the colour number and
\[
A_1^i \approx -A_2^i = 6 \pi \alpha_s \left[ 3 \left( X_A - 4 + \frac{\pi^2}{3} \right) \right. \\
\left. + r_{K^*}^s r_{K^*}^s (X_A^2 - 2 X_A) \right] ,
\]
A_3^f = 0,
\[
A_3^f = 2 \pi \alpha_s (r_{K^*}^s + r_{K^*}^s) (2 X_A^2 - X_A) ,
\]
(5)
\[
C_F = \frac{N_c^2 - 1}{2 N_c} .
\]
(6)

There are large theoretical uncertainties related to the modelling of power corrections corresponding to weak annihilation effects. We parametrize these effects in terms of the divergent integrals \( X_A \) \( (\text{weak annihilation}) \) \[9\]
\[
X_A = (1 + \rho e^{i \phi}) \ln \frac{m_B}{\Lambda_h}, \quad \rho \leq 1, \quad \Lambda_h = 0.5 \text{ GeV} ,
\]
(7)
and for the pseudoscalar mesons of \( D_s \) and \( K^* \), the ratios \( r_{K^*}^s \) and \( r_{K^*}^s \) are defined as
\[
r_{D_s}^s = \frac{2 m_{D_s}^2}{(m_b - m_c)(m_s + m_c)} ,
\]
\[
r_{K^*}^s = \frac{2 m_{K^*} f_{K^*}^s}{m_b f_{K^*}^s} .
\]
(8)

3. Weak amplitudes of intermediate state

To consider the FSI effects in the \( B_c^+ \rightarrow D_s^+ \bar{K}^{0*} \) decay, we must extract the accessible intermediate states and
Figure 2. Feynman diagrams for $B_c^+ \rightarrow D^{\ast +} \pi^0$ decay.

Figure 3. Quark level diagram for $B_c^+ \rightarrow D^{\ast +} \pi^0 \rightarrow D_s^+ \bar{K}^{0\ast}$ decay.

calculate their weak amplitude. According to figure 2, the $D^{\ast +} \pi^0$ mesons can be a product for intermediate state via exchange $\bar{K}^{0\ast}$ meson. Now that the intermediate state is obtained, we can calculate the amplitude of this intermediate state which is produced in $B_c \rightarrow M_1 M_2$ decay modes, where $M_1$ and $M_2$ are the intermediate state mesons. According to Feynman diagrams related to the $B_c^+ \rightarrow D^{\ast +} \pi^0$ decay that are shown in figure 2, we calculated the amplitude of this decay by the QCDF method as follows (figure 3):

$$A_{\text{QCD}} = -i \sqrt{2} G_F f_D [a_2 V_{ub} V_{ud}^* + (a_3 + a_4) \lambda_p]$$

$$- \frac{i G_F}{\sqrt{2}} f_{B_c} f_D [b_2 (V_{ub} V_{ud}^*)]$$

$$+ (b_3 - b_3 \epsilon_{\pi}) \lambda_p].$$

(9)

4. The one-particle exchange method for FSI

It is extremely difficult to calculate the FSI effects, but at the hadronic level formulated as re-scattering processes with s-channel resonances and one-particle exchange in the t-channel can calculate the FSI effects. The s-channel resonant FSI effects in the $B_c^+ \rightarrow D_s^+ \bar{K}^{0\ast}$ decay is expected to be vanished because of the absence of resonances. Therefore, one can model the FSI effects as re-scattering processes of two-body intermediate state with one-particle exchange in the t-channel and compute the absorptive part via the optical theorem [10]. So, according to the hadronic loop level (HLL) diagrams, shown in figure 4, the absorptive part of the amplitude is calculated using the following formula:

$$\text{Abs} M(B_c \rightarrow M_1 M_2) \rightarrow M(p_1) M(p_2) \rightarrow M(p_3) M(p_4)$$

$$= \frac{1}{2} \int \frac{d^3 p_1}{2 E_1 (2\pi)^3} \frac{d^3 p_2}{2 E_2 (2\pi)^3} (2\pi)^4 \delta^4 (p_B - p_1 - p_2)$$

$$\times M(B_c \rightarrow D^{\ast +} \pi^0) G(D^{\ast +} \pi^0 \rightarrow D_s^+ \bar{K}^{0\ast}),$$

(10)

where $M(B_c \rightarrow D^{\ast +} \pi^0)$ is the amplitude of the $B_c \rightarrow D^{\ast +} \pi^0$ decay calculated using the QCDF method and $G(D^{\ast +} \pi^0 \rightarrow D_s^+ \bar{K}^{0\ast})$ denotes the hadronic vertices factor related to the hadronic diagrams shown in figure 4. The hadronic diagrams in figure 4 include vertices up and down and $G$ is the multiplication value between vertices up and vertices down that is defined as

$$\langle K(p_3) K^* (p_2) | i \mathcal{L} | \pi(p_1) \rangle = -i g_{\pi K K^*} e_2 \cdot (p_1 + p_3),$$

$$\langle K^* (p_3) K (p_2) | i \mathcal{L} | \pi(p_1) \rangle = -i \sqrt{2} g_{\pi K K^*} e_{\mu} e_{\nu} e^\mu e^\nu p_1^\alpha p_2^\beta. \quad (11)$$
The dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation [10,11]:

$$\text{Dis} \ M(m_{B_c}^2) = \frac{1}{\pi} \int_{s}^{\infty} \text{Abs} \ M(s') \ ds',$$

(12)

where $s'$ is the square of the momentum carried by the exchanged particle and $s$ is the threshold of intermediate states, and in this case $s \sim m_{B_c}^2$. Unlike the absorptive part, the dispersive contribution suffers from large uncertainties arising from the complicated integration.

So, for figure 4a, the absorptive part of the amplitude of the $B_c^+ \rightarrow D^{+}\pi^0 \rightarrow D_s^+ K^{0*}$ process where $K^0$ meson is exchanged particle at t-channel, is given by

$$\text{Abs} \ 4 \ (a) = \frac{-iG_F}{2\sqrt{2}} \int_{-1}^{1} \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \delta^4 \left(p_{B_c} - p_1 - p_2\right)$$

$$\times \left(p_{B_c} - p_1 - p_2\right) \left(-ig_{D^*K D_i} \varepsilon_3 \cdot (p_1 + q)(-ig_{\pi K K^*}) \varepsilon_2 \cdot (-q)(2m_{D^*}(\varepsilon_2 \cdot p_1) A_0^{D^*}\right)$$

$$\times \left[a_2(V_{ub} V_{ud}^*) + (a_3 + a_4) \lambda_p\right]$$

$$- f_{B_c} f_{D^*} f_{\pi} [b_2(V_{cb} V_{cd}^*)$$

$$+ (b_3 - b_{3ew}) \lambda_p] \frac{F^2(q^2, m_K^2)}{T_1}.$$  

Here

$$H_1 = (\varepsilon_2 \cdot p_1)(\varepsilon_2 \cdot p_4)(\varepsilon_3 \cdot p_1)$$

$$= \left(-p_1 \cdot p_4 + \frac{(p_1 \cdot p_2)(p_2 \cdot p_4)}{m_{B_c}^2}\right) \times \left(\frac{E_1|p_3| - E_3|p_1|\cos \theta}{m_{B_c}|p_3|}\right),$$

$$H_2 = (\varepsilon_3 \cdot p_1)(\varepsilon_2 \cdot p_4) = \left(\frac{E_4|p_2| - E_2|p_4|\cos \theta}{m_{B_c}|p_2|}\right),$$

$$T_1 = (p_1 - p_3)^2 - m_K^2$$

$$= \left(p_1^2 + p_3^2 - 2p_1^0 p_3^0 + 2p_1 \cdot p_3 - m_K^2\right),$$

$$q^2 = m_1^2 + m_3^2 - 2E_1 E_3 + 2|p_1||p_3|\cos \theta$$

$$= m_{D^*}^2 + m_{D_s}^2 - 2p_1^0 p_3^0 + 2|p_1||p_3|\cos \theta.$$  

(14)

$\theta$ is the angle between $p_1$ and $p_3$, $q$ is the momentum of the exchange $K$ meson and $F(q^2, m_K^2)$ is the form factor defined to take care of the off-shell of the exchange particles, which is introduced as [3]

$$F(q^2, m_K^2) = \left(\frac{\Lambda^2 - m_K^2}{\Lambda^2 - q^2}\right)^n.$$  

(15)

The calculations of hadronic diagrams for FSIs involve many theoretical uncertainties. As the particle exchanged in the t-channel is off-shell and as final-state particles are hard mesons, form factors or cut-offs must be introduced to the strong vertices to render the calculation meaningful in perturbation theory. The form factor (i.e. $n = 1$) is normalized to unity at $q^2 = m_K^2$. The monopole behaviour of the form factor (i.e. $n = 1$) is preferred as it is consistent with the QCD sum rule expectation [12]. $m_K$ and $q$ are the physical parameters of the exchange particle and $\Lambda$ is phenomenological parameter.

It is obvious that for $q^2 \rightarrow 0$, $F(q^2, m_K^2)$ becomes a number. If $\Lambda \gg m_K$ then $F(q^2, m_K^2)$ turns to be unity.

Figure 4. t-channel contributions to final-state interaction in the $B_c^+ \rightarrow D^{+}\pi^0 \rightarrow D_s^+ K^{0*}$ decay due to one-particle exchange.

whereas, as \( q^2 \to \infty \) the form factor approaches zero and the distance becomes small and the hadron interaction is no longer valid. As \( \Lambda \) should not be far from \( m_K \) and \( q \), we choose

\[
\Lambda = m_K + \eta \Lambda_{QCD},
\]

where \( \eta \) is the phenomenological parameter and its value in the form factor is expected to be of the order of unity and can be determined from the measured rates. Also the parameter \( \eta \) depends not only on the exchanged particle, but also on the external particles involved in strong interaction. The range of \( \eta \) parameter is from 0.5 to 3 and we used 0.6 to 1.6 in this paper.

Abs 4 “(b)” \[= \frac{-iG_F}{2\sqrt{2}} \int_{-1}^{1} \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \]

\[
\times (2\pi)^4 \delta^4(p_{Bc} - p_1 - p_2) \]

\[
\times (-i\sqrt{2g_{D^*K^+D^}}\epsilon_{\mu\nu\alpha\beta} \epsilon_3^{\mu} \epsilon^{\nu}_{K^+} p_1^\nu p_3^\beta) \]

\[
\times (-i\sqrt{2g_{D^*K^+K^+}}\epsilon_{\rho\sigma\lambda\eta} \epsilon_2^{\rho} \epsilon^{\sigma}_{K^+} p_1^\lambda p_3^\eta) \]

\[
\times (2m_{p^*}(\epsilon_2.p_1) f_{\pi} A_0^{BD^*}) \]

\[
\times [a_2(V_{ub}V_{ud}^*) + (a_3 + a_4)\lambda_p] \]

\[
- f_{Bc} f_{D^*} f_{\pi} [b_2(V_{cb}V_{cd}^*)] \]

\[
+ (b_3 - b_{3ew})\lambda_p] H_4 \]

\[
\frac{F^2(q^2, m_{K^+}^2)}{T_2}. \]

\[
(17)
\]

Here

\[
H_3 = m_3^2(p_1 \cdot p_2) - (p_1 \cdot p_3)(p_2 \cdot p_3) \]

\[
+ \left( \frac{E_2|p_3| - E_3|p_2| \cos \theta}{m_{Bc}|p_3|} \right) \]

\[
\times [(p_{Bc} \cdot p_1)(p_3 \cdot p_4) - (p_{Bc} \cdot p_3)(p_1 \cdot p_4)],
\]

\[
H_4 = \epsilon_{\mu\nu\alpha\beta} \epsilon_{\rho\sigma\lambda\eta} \epsilon_1^{\mu} \epsilon^{\nu}_{K^+} p_1^\nu p_2^\beta \epsilon_2^{\rho} \epsilon^{\sigma}_{K^+} p_3^\lambda p_4^\eta, \]

\[
T_2 = (p_1 - p_3)^2 - m_{K^+}^2 = p_1^2 + p_3^2 - 2p_1^0 p_3^0 \]

\[
+ 2p_1 \cdot p_3 - m_{K^+}^2, \]

\[
q^2 = m_{D^*}^2 + m_{D^*}^2 - 2E_1 E_3 - 2|p_1||p_3| \cos \theta \]

\[
= m_{D^*}^2 + m_{D^*}^2 - 2p_1^0 p_3^0 + 2|p_1||p_3| \cos \theta. \]
Table 1. The branching ratio of $B_c^+ \rightarrow D_s^+ \overline{K}^{0\ast}$ decay, with $\eta = 0.6$–1.4 (in units of $10^{-6}$).

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR</td>
<td>0.01 ± 0.01</td>
<td>0.06 ± 0.02</td>
<td>0.15 ± 0.03</td>
<td>0.37 ± 0.05</td>
<td>0.75 ± 0.07</td>
<td>1.11 ± 0.10</td>
<td>&lt;0.4</td>
</tr>
</tbody>
</table>

The other input parameters used are given by [3,10]

\[ g_{D^\ast K D_s} = 18.37, \quad g_{\pi K K^\ast} = 4.6, \]
\[ g_{D^\ast K^\ast D_s} = 2.79, \quad (26) \]

We calculated the branching ratio using the QCDF method as $(0.21 \pm 0.04) \times 10^{-7}$ which is small compared to the experimental result. Within FSI, the branching ratio is shown in table 1 and if $\eta = 1.2$ is selected, the branching ratio is $(0.37 \pm 0.05) \times 10^{-6}$ which is close to the experimental result.

6. Conclusions

We analysed the $B_c^+ \rightarrow D_s^+ \overline{K}^{0\ast}$ decay in the QCD factorization approach and then we added the final-state interaction effects. For evaluating the FSI effects, we have considered only the absorptive part of the HLL because both hadrons produced via the weak interaction are on their mass shells. The experimental result of this decay is less than $0.4 \times 10^{-6}$. According to QCDF and FSI, our results are: $\text{BR}(B_c^+ \rightarrow D_s^+ \overline{K}^{0\ast}) = (0.21 \pm 0.04) \times 10^{-7}$ and $(0.37 \pm 0.05) \times 10^{-6}$, respectively. The main phenomenological parameter in the FSI effects is $\eta$ which is determined from the measured ratios. Its value in form factor is expected to be of the order of unity. In this work we have considered $\eta = 0.6$–1.4 and the best result is obtained when $\eta = 1.2$.

We observed that the branching ratio, from 0.6 to 1.6, is close to the experimental value. But by increasing $\eta$ from 1.2 to 1.6, we showed that the branching ratio exceeds the experimental value.

References