Pramana – J. Phys. (2017) 88: 4
DOI 10.1007/s12043-016-1311-x

On quantum harmonic oscillator being subjected to absolute potential state

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MS received 1 May 2015; accepted 6 May 2016; published online 3 December 2016

Abstract. In a quantum harmonic oscillator (QHO), the energy of the oscillator increases with increased frequency. In this paper, assuming a boundary condition that the product of momentum and position, or the product of energy density and position remains constant in the QHO, it is established that a particle subjected to increasing frequencies becomes gradually subtler to transform into a very high dormant potential energy. This very high dormant potential energy is referred to as 'like-potential' energy in this paper. In the process a new wave function is generated. This new function, which corresponds to new sets of particles, has scope to raise the quantum oscillator energy (QOE) up to infinity. It is proposed to show that this high energy does not get cancelled but remains dormant. Further, it is proposed that the displacement about the equilibrium goes to zero when the vibration of the oscillator stops and then the QOE becomes infinity – this needs further research. The more the QOE, the greater will be the degree of dormancy. A simple mathematical model has been derived here to discuss the possibilities that are involved in the QHO under the above-mentioned boundary conditions.

Keywords. Quantum harmonic oscillator; like-potential energy; wave function; energy density.

PACS Nos 11; 98.80.−k

1. Introduction

The Planck mission [1] team, based on the Standard Model of cosmology, on a mass–energy equivalence basis, observed that the Universe consists of about 26.8% dark matter, 68.3% dark energy (DE) and 4.9% ordinary matter. So, about 95% of the Universe remains elusive to us.

Most quantum field theories predict the magnitude of the vacuum energy density (as in DE), which is derived from the quantum vacuum, to be more than 100 orders [2]. Conventionally, the supersymmetric quantum theories predict to cancel out this enormous energy so that, as we know, the space–time curvature due to energy–momentum tensor in quantum vacuum becomes zero (≈0). However, so far, this theory of supersymmetry is not supported by experimental evidence. For example, no scalar electron has been observed with a mass smaller than 100 GeV [3].

Quantum gravity applied to the isotropic models [4] has already shown that the quantum evolution of a wave function extends through the Big Bang [5]. A quantum mathematical model applied to quantum oscillator energy (QOE) thus has been used in this paper to explain different states of subtle matters both within the Standard Model as well as outside the Standard Model.

In this paper, we propose to study the QOE in a quantum harmonic oscillator (QHO) to generate a wave equation under the boundary condition of constant phase space area. The proposed study states that under these boundary conditions the vacuum energy density becomes extremely dormant (like-potential), and this is the reason why QOE behaves to be almost nonexistent in the nature. Under such circumstances, the QOE does not get cancelled out by some other pair particles, but stays the same.

We considered an ordinary classical harmonic oscillator with one-dimensional phase space. Then we changed the coordinates of the phase space (position and momentum) to quantum mechanical operators following the standard transform method [6]. We thus got energy quantized in the quantum phase space.

The behaviour of the QHO under the above-mentioned boundary conditions will therefore be under some sort of invariance (due to constant phase space area). Thus, if under this constraint the QHO could be mathematically formulated for increasing momentum, then a tapering existence of subtle matters in different stages (including DE and other higher energy levels) can be deduced.

The energy in the system as per this proposition does not stay manifestly active (as in the case of DE etc.) but becomes 'like-potential'. This sort of 'like-potential' energy is extremely active within, though it remains unmanifest in terms of our current methods of experimentally verifying the fundamental particles of modern physics.

Many attempts have been made to find out a single field, out of which all fields have originated, for example the Evans Unified Field Theory [7,8]. But as long as we fail to find a field without vibration, it appears to be impossible to find a single field out of which all other fields emerged. An attempt is made here to show that there is only one field that exists throughout when the QOE goes to infinity. Out of this field, with infinite energy, all other fields (where QOE is less than infinity) have emerged.

Being an initial foray into a vast and open subject for further research, we used the QHO model and focussed on the possibility of mathematical consistency for the existence of such an oscillator that directly corresponds with many subtle matters that exist in the nature.

2. The theory

A classical spring undergoing harmonic oscillation traces an ellipse in the phase space, with momentum and position as the axes (figure 1A). Here, we have taken only two-dimensional 'area' for simplification – one dimension for position and one dimension for momentum. If the mathematical consistency is proved for one dimension, it automatically holds good for higher dimensions.

As per the proposition, the boundary condition is set in such a way that the product of the position and the momentum of the oscillator is always kept constant.

This oscillator will then eventually vibrate with higher and higher energy levels as we pump in more and more energy into the oscillator. At this stage, the ellipse describing the harmonic oscillator in the phase space will be stretched along the momentum axis, while it will shrink very fast along the position axis (as shown in figure 1B).

In this case, though a harmonic oscillator may vibrate with ever increasing frequency, the states in figure 1A and figure 1B will have some sort of similar properties due to the constancy of the phase space area in both the cases. Thus, using the invariance obtained due to the considered boundary condition, we study the state of the system below.

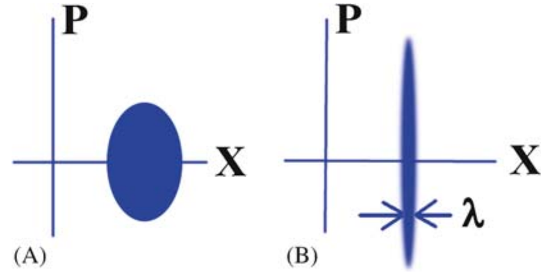

Figure 1. Phase-space area: (**A**) becomes (**B**) having the same phase-space area ($A = \pi x P$), but (**B**) having more energy than (**A**).

Differentiating the relation $A = \pi x P$ with respect to time, we have

$$\frac{\partial A}{\partial t} = \pi \left(x \frac{\partial P}{\partial t} + P \frac{\partial x}{\partial t} \right).$$

As we assumed that phase space area is constant, we get,

$$\frac{\partial A}{\partial t} = 0.$$

This yields,

$$x\dot{P} = -P\dot{x}. \tag{1}$$

Subsequently, as the oscillator vibrates with higher energy, as time evolves, the energy (or momentum) will be quantized. In this case, the classical momentum (P) will be transformed into the momentum operator ($\hat{P} = -i\hbar(\partial/\partial x)$), and the position ($x$) will be transformed into position operator ($\hat{x}$). Note that by applying the above operators, our model changes from classical harmonic oscillator to a QHO.

On the other hand, consider a one-dimensional box (line segment) of length L, and a single particle in the box moving on a line with velocity v, bouncing back and forth (see figure 2). When the particle bounces off the wall it creates a force on the wall (in case of photon it is the radiation pressure); the force then is,

$$F = \frac{\partial P}{\partial t}.$$

Further, the change of momentum of the particle is $2P$, the time interval between successive collisions with the

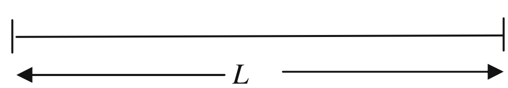

Figure 2. One-dimensional box of length L.

wall, or the momentum per unit time is then $2L/v$: This being the force F on the wall of the box.

This implies that

$$\dot{P} = \frac{\partial P}{\partial t} = F = \frac{2P}{2L/v} = \frac{Pv}{L} = \frac{E}{L} = \rho,$$

where ρ is the energy density.

Now using the above relation in (1) we have

$$x\rho = -P\dot{x}. \tag{2}$$

Replace x with $\hat{x}$, and P with $\hat{P}$, the position and momentum operator in (2). By considering a state of system $|\psi(x)\rangle$ on which the aforementioned operators act, we arrive at the QHO given by

$$\hat{x}\rho\,|\psi(x)\rangle = -\hat{P}\dot{x}\,|\psi(x)\rangle. \tag{3}$$

Simplifying (3) we obtain

$$x\rho\psi(x) = i\hbar\frac{\partial}{\partial x}\left(\dot{x}\psi(x)\right),$$

$$x\rho\psi = i\hbar\frac{\partial\dot{x}}{\partial x}\psi + i\hbar\frac{\partial\psi}{\partial x}\dot{x}$$

$$x\rho\psi = i\hbar\frac{\partial v}{\partial x}\psi + i\hbar\frac{\partial\psi}{\partial t}.$$

Setting $(\partial v/\partial x) = \omega$, we have

$$i\hbar\frac{\partial\psi}{\partial t} = x\rho\psi - i\hbar\omega\psi$$

or

$$\frac{\partial\psi}{\partial t} = \frac{\psi}{i\hbar}\left(x\rho - i\hbar\omega\right). \tag{4}$$

The solution to eq. (3) is given by

$$\psi = \exp\left|\frac{t}{i\hbar}\left(x\rho - i\hbar\omega\right)\right|, \tag{5}$$

$$\log\psi = \frac{t}{i\hbar}\left(x\rho - i\hbar\omega\right)$$

or

$$\rho = \frac{i\hbar}{xt}\left(\log\psi + \omega t\right), \tag{6}$$

where the product $x\rho$ is always a constant and ρ tends to infinity as x tends to zero. From eq. (4) we get

$$\frac{\partial\psi}{\partial x} = \frac{\psi t\rho}{i\hbar}. \tag{7}$$

To normalize ψ,

$$\int \psi^*\psi\,\mathrm{d}t = \int \langle\psi|\psi\rangle\,\mathrm{d}t = 1.$$

Figures 3–8 show various graphs drawn using eqs (4) and (5).

3. Result

From eqs (5)–(7) and the subsequent graphs we can see that when the vibrational energy of the oscillator increases, the position (displacement) decreases. This means that when the oscillator possesses infinite energy, the displacement goes to zero. This zero oscillation state may be correlated to the no-creation state, as the creation implies some sort of oscillations. Further, this state can be defined as the Absolute LIKE-POTENTIAL State, due to its enormous energy with no oscillation. This state can also be considered as a field having no vibration. We hypothesize that this field having no vibration is the field from where all fields have emerged.

On the other hand, it can be observed from the QHO that as the vibration becomes faster and faster, the QOE in each quantized packet increases. These packets may

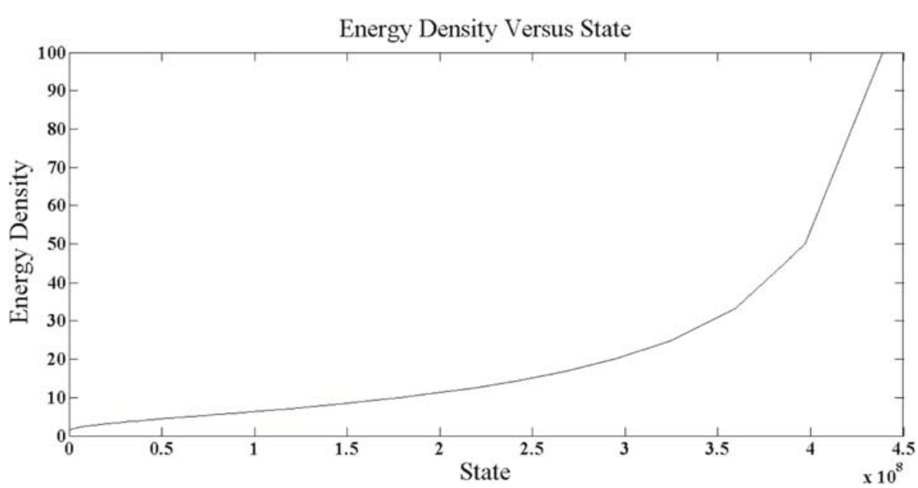

Figure 3. Energy density (ρ) vs. the state vector (ψ) (wave function).

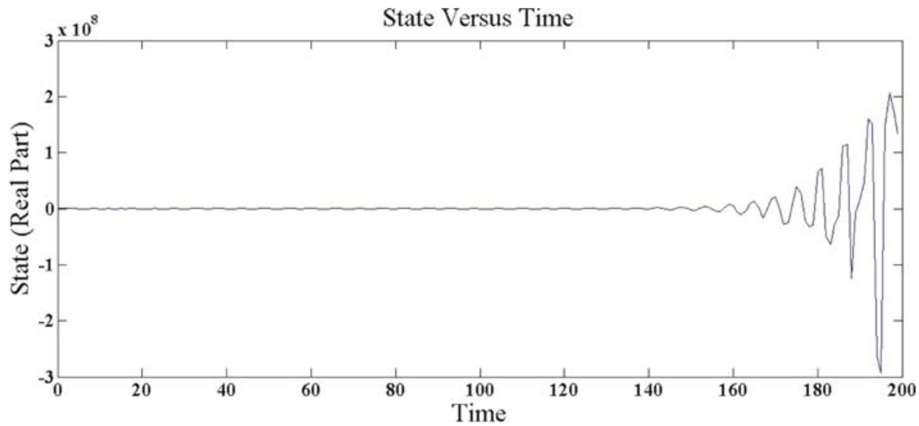

Figure 4. State vector (ψ) vs. time (t) (real part).

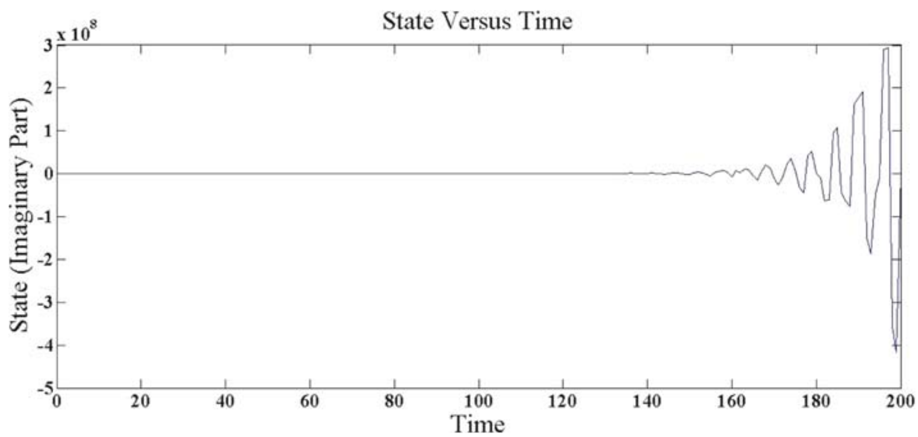

Figure 5. State vector (ψ) vs. time (t) (imaginary part).

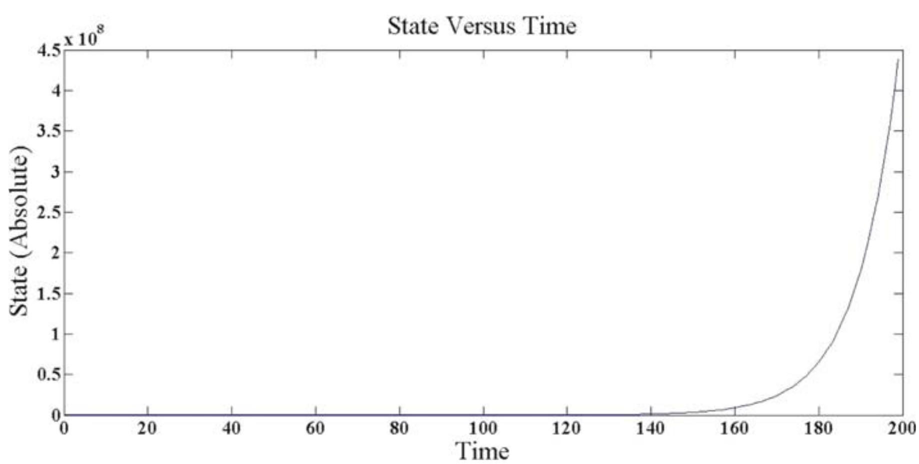

Figure 6. State vector (ψ) vs. time (t) (absolute value, $|\psi|$).

be treated as subtle particles. These subtle particles, after certain range of vibration, can be those beyond the Standard Model.

The existence of new particles, proposed in this paper, is strongly supported by a recent research conducted by the physicists of Large Hadron Collider (LHC). The LHC physicists anticipate the existence of new elementary particles that are beyond those of the Standard Model. Physicist Natalie Wolchover, in her recent article [9], mentioning the existence of new particles with a scope of NEW PHYSICS writes:

"Most of the matter in the universe – the particles that make up "dark matter" – is completely missing from the Standard Model, and what is included seems

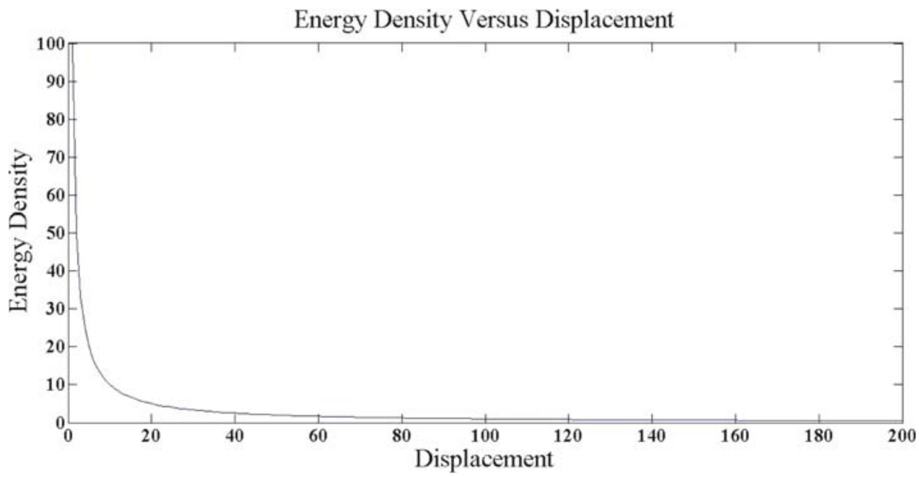

Figure 7. Energy density (ρ) vs. displacement (x).

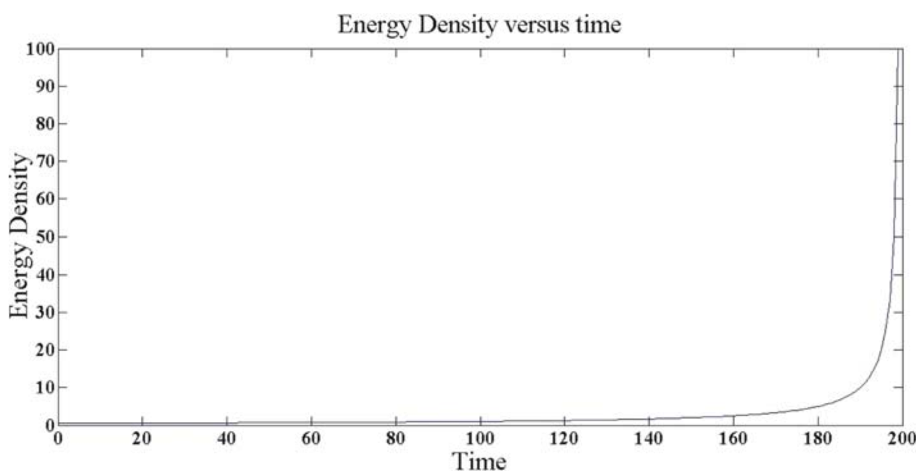

Figure 8. Energy density (ρ) vs. coordinate time (t).

fragmentary and suggestive of a larger pattern. Physicists built the most powerful machine in history to search for signs of those more complete laws of nature. But almost everything about the way particles shape-shifted and shattered during the first round of collisions at the LHC precisely matched Standard Model predictions. In the 3.7-sigma penguin anomaly – as well as another, 2.6-sigma deviation the group detected in a different penguin process – some particle physicists see a sliver of hope that new discoveries lie around the corner.... If some as-yet-undiscovered particle does play a ghostly role in penguin decays, it will spew leptons with unexpected combinations of energies and directions, skewing measurements. Sure enough, the LHCb scientists measured a 3.7-sigma discrepancy with the Standard Model in some outgoing particles' combined energies and directions.... In the related 2.6-sigma anomaly, the scientists found that penguin processes produced more of some leptons than others, violating a Standard Model rule called "lepton universality."

We can figuratively explain the entire phenomena of going from lower to higher energy levels or vice versa, keeping the phase-space area constant. This increase in energy will have the displacement (position) of the oscillator becoming shorter and shorter about the equilibrium position, to form a tapering existence (figure 9); the displacement becomes 0, when $E = \infty$. This model explains the behaviour of the subtle matter like DE, etc. and also the matters subtler which have more energy than DE.

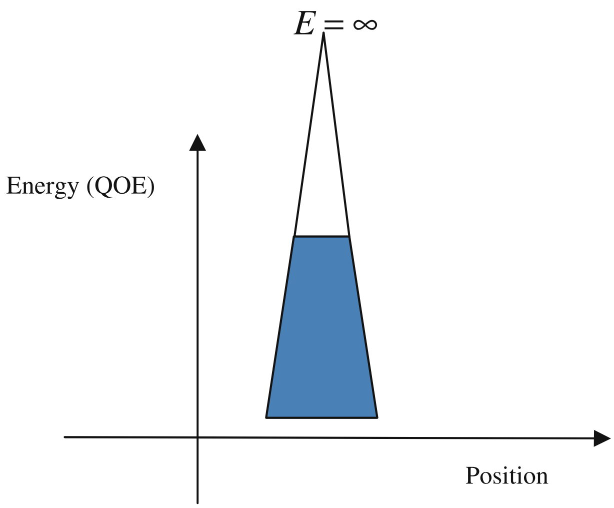

Figure 9. The shaded area is figuratively meant for all the particles within the Standard Model, and the unshaded area indicates the particles beyond the Standard Model.

4. Conclusions

In this paper the considered model gives scope for the existence of many subtler particles outside the Standard Model which fill the Universe. These subtler particles, though having enormously high energy, remain inactive due to certain boundary conditions as stated in the paper. The energy of these particles does not get cancelled out but remains in a very dormant state termed in this paper as 'like-potential' state. A few such particles could be DE and dark matter. There could be even subtler particles than DE, whose existence has recently been anticipated by LHC physicists.

It is hypothesized in this paper that these same subtle particles with their dormant state of energy become the source of other known particles, when the energy and the corresponding frequency of the particle come down to certain limit (shown as the shaded area in figure 9).

The behaviour of these subtle particles is explained in this paper using the QHO under the specified boundary conditions. The state of these particles becomes more and more 'like-potential' and inactive as the displacement of the QHO is decreased with the increase of QOE. This way, when the energy of the QHO reaches infinity, displacement goes to zero. This zero displacement state, which is nothing but no-vibration state, is described here as the pre-creation condition of the Universe. At this state, the energy stays infinity but there is no movement in the oscillator. No creation is possible at this state. At this state, the space-time-vibration (causation) triad comes to absolute halt. Creation begins again when the same no-vibration QHO gets initial start-up to begin its first vibration. The space and the time also get created along with it.

Acknowledgements

The author's first and foremost acknowledgement is due to the Ramakrishna Math and Ramakrishna Mission – the twin organizations where he belongs. This spiritual organization is well known for preserving ancient Indian tradition coupled with modern scientific knowledge – as envisaged by Swami Vivekananda. This unique environment in the organization paved the way for the development and completion of this paper.

The research has been greatly aided with the support of Dr J Vasundhara Devi, Professor, Associate Director, Lakshmikantham Institute for Advanced Studies – Gayatri Vidya Parishad College of Engineering (GVP-LIAS), Visakhapatnam. She has extensively helped to bring this paper to a proper readable shape.

The contribution of Prof. Seetapathi, Retired Professor, Department of Applied Mathematics, Andhra University is also thankfully acknowledged. He diligently studied the mathematical derivations in the paper and made valuable suggestions and corrections wherever needed.

The invaluable support extended by Dr T G K Murty, former Program Director, ISRO, and an outstanding scientist, Bangalore, is also duly acknowledged here. His constant guidance, encouragement and his valuable insights at proper time provided a meaningful direction to the paper.

Acknowledgement is further due to Mr D Ammaji Rao, Student, Andhra University. He made many appropriate suggestions with regard to the MATLAB plots and the mathematical part of this paper.

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