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Quantum effects on the Rayleigh–Taylor instability of stratified plasma in the presence of suspended particles

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Abstract. The effects of quantum correction on the Rayleigh–Taylor instability (RTI) in stratified plasma layer have been investigated in the presence of suspended particles. A general dispersion relation is obtained from the linearized set of quantum hydrodynamic (QHD) equations. Two particular cases of suspended particle parameters (f^* and α_0) with and without quantum corrections are analysed. The condition of RTI is derived while the stability of the system is discussed by applying Routh–Hurwitz (RH) criterion in the polynomial equation. The results show that, in the absence of quantum term, the relaxation frequency of the suspended particles has a destabilizing effect, while the mass concentration of the suspended particles has a stabilizing effect on the growth rates of RTI. In the presence of the quantum term, the relaxation frequency of the suspended particle yields to the stability behaviour on the growth rates of RTI.

Keywords. Rayleigh–Taylor instability; quantum plasma; suspended particles.

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1. Introduction

Recently, quantum plasma has attracted wide attention and has become an interesting field of research in plasma physics [1–5] due to its broad applications in ultrasmall electronic devices [6], dense astrophysical plasmas [7–9], laser fusion plasma and in the excitation of linear and nonlinear waves [10]. Quantum plasma is characterized by high densities and low temperatures contrary to ordinary plasma. The well-known quantum hydrodynamics (QHD) model has been used by Gardner and Ringhofer [11] to study electron–hole dynamics in semiconductors. The quantum effects emerge when the de Broglie thermal wavelength λ_D of electrons become comparable to the interparticle distances [12], and this condition holds true for metallic and semiconductor compounds. The state of quantum plasma, where the thermal energy of the electron is much less than its Fermi energy, is governed by the Fermi statistics and this state is called the degenerate state. In a normal metal, at room temperature, the conduction electrons are mostly in degenerate state. Such conditions can also be found in laser-produced plasmas in a laboratory [13,14].

Several studies have been carried out both analytically and numerically in plasmas with quantum corrections in hydrodynamic Rayleigh–Taylor instability (RTI). For example, the quantum effects on the internal waves and the RTI in plasma are studied by Vitaly *et al* [15]. The effect of quantum corrections on the RTI in the presence of horizontal magnetic field was studied by Cao *et al* [16]. Hoshoudy [17,18] studied the same model in the presence of vertical magnetic field and in the absence (or presence) of viscosity, respectively. The effects of quantum term on the RTI in the presence of vertical or horizontal magnetic field of the rotating plasma were studied by Hoshoudy [19,20]. RTI in quantum plasmas with para- and ferromagnetic properties is studied by Modestov *et al* [21]. The effect of quantum term on the RTI of stratified plasma layer through porous medium was studied by Hoshoudy [22,23]. The RTI was studied in a non-uniform dense quantum magnetoplasma by Ali *et al* [24]. Small-amplitude quantum ion-acoustic solitary waves in an unmagnetized two-species relativistic quantum plasma system, comprised of electrons and ions were studied by Sahu [25]. The head-on collisions between dust-ion-acoustic (DIA) wave in quantum electron-dust-ion plasmas are studied by Chatterjee *et al* [26].

The effect of suspended particles is widely considered in space and astrophysical plasmas. It has been observed that interstellar media contain grains which are small particles formed in the outer atmosphere of stars and ejected into the medium. The problem of suspended particles in gas–particle medium has attracted wide attention due to its relevance in many astrophysical situations and laboratory problems. This problem was discussed by Alfven and Carlqvis [27] to analyse the formation of stars through Jeans instability. The effects of suspended particles on gas–particle medium have been studied with other different factors by several researchers [28–33]. Sharma [28] studied the effect of suspended particles on the gravitational instability of an infinite homogeneous gas–particle medium. The gravitational instability of a finitely conducting, rotating gas–particle medium in the presence of uniform vertical magnetic field and suspended particles has been studied by Sharma and Sharma [29]. The gravitational instability of an infinite homogeneous and infinitely conducting self-gravitating gas–particle medium with suspended particles of Hall plasma in the presence of vertical magnetic field and vertical rotation, respectively, have been considered by Sharma [30,31]. The same model, where the effect of rotation is considered along and perpendicular to the direction of the magnetic field has been studied by Chhajlani and Sanghvi [32]. The effects of suspended particles and finite thermal and electrical conductivities on the magnetogravitational instability of an ionized rotating plasma flowing through a porous medium have been studied by Vyas and Chhajlani [33].

In many astrophysical situations, the plasma is often not pure but contains some finite-sized suspended dust particles. The presence of suspended dust particles is more realistic in astrophysical situations. These particles play important roles in hydrodynamic fluid stabilities, viz. RTI, gravitational instability and Kelvin–Helmholtz instability (KHI). Several attempts are made to determine the effects of suspended dust particles on the RTI including various physical parameters. Zadoff and Begun [34] have treated the case of two incompressible fluids separated by a horizontal boundary in the presence of a uniform horizontal magnetic field. They have discussed the effects of finite resistivity and viscosity of the medium on the growth rate of RT modes, and have shown that finite resistivity does not affect the growth rate of unstable modes when the wave vector is perpendicular to the magnetic field, but that it does increase the growth rate when the wave vector is parallel to the magnetic field. Prajapati and Chhajlani [35] have discussed the KHI and RTI in streaming fluids with suspended dust particles flowing through porous media. Prajapati *et al* [36] have also discussed the effect of magnetic field on the combined KHI and RTI of two superimposed fluids. Sharma *et al* [37] have studied the effect of suspended particles in an infinitely conducting gas layer in hydromagnetics where the destabilizing influence of these particles is pointed out. Sharma and Sharma [38] have also analysed the RTI of two superimposed fluids in the presence of suspended particles and derived the criterion of RTI. Sanghvi and Chhajlani [39] have incorporated the finite resistivity effect on the RT configuration of a stratified plasma in the presence of suspended particles, and found that the particles have stabilizing as well as destabilizing influences on the system under certain conditions. Gupta and Bhatia [40] have discussed the RTI of two superposed magnetized viscous fluids with neutral particles. The effects of rotation and FLR corrections on the RTI of two superposed magnetized conducting fluids with suspended dust particles were analysed by Sharma and Chhajlani [41,42]. Kumar and Singh [43] have investigated the RTI of two superposed Rivlin–Eriksen viscoelastic fluids in the presence of suspended dust particles. Also, it is well known that the quantum effects are important in the behaviour of charged plasma particles when the de Broglie wavelength of the charge carriers becomes equal to or greater than the dimension of the quantum plasma system [44].

Therefore, the aim of the present paper is to investigate the effects of suspended dust particles on the RTI of finite incompressible plasma layer in the presence of quantum correction. The role of suspended particles in the presence (or absence) of the quantum effect has been determined.

2. Formulation of the problem and perturbations equations

We consider an incompressible quantum plasma consisting of electrons and single charged ions fluids. The plasma is permeated with suspended dust particles of uniform shape and size. Figure 1 shows the schematic representation of the considered system. In the Fermi degenerate quantum plasma, the Bohm force significantly modifies the dynamics of the charged particles. Madelung [45] and Bohm and Vigier [46] have

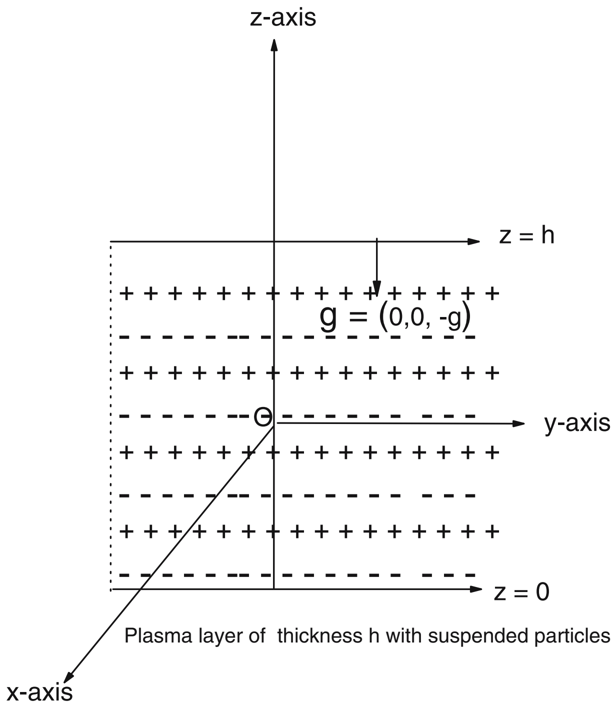

Figure 1. Schematic diagram of the configuration.

represented mathematical function for such a quantum system in the context of classical dynamical laws given by

$$\Psi_N(\{r_i\}, t) = A(\{r_i\}, t) \exp\left[\frac{i}{\hbar} S(\{r_i\}, t)\right],$$

where $(\{r_i\}, t)$ is the real amplitude function and $S(\{r_i\}, t)$ is the real phase. It is inserted into the Schrödinger equation and real and imaginary parts are separated, and there arises a new term known as quantum Bohm potential. The quantum part of the electron is represented by quantum force $-\nabla\varphi_B$, where

$$\varphi_B = -(\hbar^2/2m_e\sqrt{n_e})\nabla^2\sqrt{n_e}.$$

In a more convenient form, the Bohm quantum force term for the quantum plasma is given by

$$F_Q = \frac{\hbar^2\rho}{2m_e m_i}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right),$$

where $\rho = m_e n_e + m_i n_i$ is the global mass density.

The limitation of this model is that it is only applicable for quantum plasma where the de Broglie wavelength of charge particle is comparable to the dimension of the system.

Under the foregoing assumptions, equations of momentum and continuity can be written as (see refs [15–24,26–30]

$$\rho\left(\frac{\partial}{\partial t} + \vec{U}\cdot\vec{\nabla}\right)\vec{U} = -\vec{\nabla}P + \rho\vec{g} + KN(\vec{V} - \vec{U}) + \frac{\hbar^2}{2m_e m_i}\rho\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right), \quad (1)$$

$$\vec{\nabla}\cdot\vec{U} = 0. \quad (2)$$

Here $\vec{U}(x, y, x, t)$ is the fluid velocity, ρ is the fluid density, p is the thermal pressure and $\vec{g}(0, 0, -g)$ is the gravitational acceleration. $\vec{V}(x, y, x, t)$ and $\vec{N}(x, y, x, t)$ denote the velocity and number density of the suspended particles. The Stoke's drag coefficient is expressed as $K = 6\pi\rho\nu a$, where ν and a are the kinematic viscosity of the clean fluid and particle radius, respectively. $\hbar$ is the Planck's constant (h) divided by 2π and $m_{i(e)}$ is the ion (electron) mass.

In the equations of motion (1), by assuming a uniform spherical particle shape and small relative velocities between the fluid and the particles, the presence of particles adds an extra force term proportional to the velocity difference between the particles and the fluid. As the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion of the particles. The distances between particles are assumed to be quite large compared to their diameter, so that interparticle reactions are ignored. The effects of pressure and gravity force on the suspended particles are negligibly small and therefore ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions are

$$mN\left[\frac{\partial}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V}\right] = KN(\vec{U} - \vec{V}), \quad (3)$$

$$\frac{\partial N}{\partial t} + \vec{\nabla}\cdot(N\vec{V}) = 0. \quad (4)$$

Now, in order to investigate the stability of hydrodynamic quantum fluid, we ask how the motion responds to a small fluctuation in the value of any of the flow variables appearing in the Euler equations. If the fluctuation grows in amplitude so that the flow never returns to its initial state, we say that the flow is unstable with respect to the fluctuations of that type. Accordingly, we replace the variables in eqs (1)–(4) as follows:

$$\vec{U} = \vec{U}_0 + \vec{U}_1, \quad \vec{V} = \vec{V}_0 + \vec{V}_1,$$
$$\rho = \rho_0 + \rho_1, \quad N = N_0 + N_1$$

and

$$p = p_0 + p_1.$$

The quantities with subscript '0' represent the unperturbed, or 'zeroth-order' motion of the fluid, while the quantities with subscript '1' represent a small perturbation about the zeroth-order quantities (first-order or linearized quantities). Substituting the above perturbations into eqs (1)–(4) and in particular example of RTI we consider that the fluid is initially at rest (that is, $\vec{U}_0 = 0$ and $\vec{V}_0 = 0$). Then the relevant linearization perturbation equations may be written from eqs (1)–(4) as

$$\rho_0 \frac{\partial \vec{U}_1}{\partial t} = -\vec{\nabla} p_1 + \rho_1 \vec{g} + K N_0 (\vec{V}_1 - \vec{U}_1) + \frac{\hbar^2}{2 m_e m_i} \left\{ \begin{array}{l} \frac{1}{2}\nabla(\nabla^2 \rho_1) - \frac{1}{2\rho_0}\nabla\rho_1 \nabla^2 \rho_0 \\ - \frac{1}{2\rho_0}\nabla\rho_0 \nabla^2 \rho_1 \\ + \frac{\rho_1}{2\rho_0^2}\nabla\rho_0\nabla^2\rho_0 - \frac{1}{2\rho_0}\nabla(\nabla\rho_0 \cdot \nabla\rho_1) \\ + \frac{\rho_1}{4\rho_0^2}\nabla(\nabla\rho_0)^2 \\ + \frac{1}{2\rho_0^2}(\nabla\rho_0)^2\nabla\rho_1 + \frac{1}{\rho_0^2}(\nabla\rho_0 \cdot \nabla\rho_1)\nabla\rho_0 \\ - \frac{\rho_1}{\rho_0^3}(\nabla\rho_0)^2\nabla\rho_0 \end{array} \right\}, \tag{5}$$

$$\vec{\nabla} \cdot \vec{U}_1 = 0, \tag{6}$$

$$\left(\tau \frac{\partial}{\partial t} + 1\right) \vec{V}_1 = \vec{U}_1, \tag{7}$$

$$\frac{\partial M_1}{\partial t} + \vec{\nabla} \cdot \vec{V}_1 = 0. \tag{8}$$

Here $\tau = m/K$ (relaxation time for the suspended dust particles) and $M_1 = N_1/N_0$.

The zeroth-order equation in the limit $\partial/\partial t = 0$ represents the static equilibrium which may be relevant and applicable in Z-pinch and in inertial confinement fusion (ICF). In the dense plasma focus device, this condition can be achieved for stable RT configurations.

We now consider the fact that, in many situations of ICF, unstable flow occurs at velocities much smaller than the local sound speed because of which accelerations in the flow are not strong enough to change the density of a fluid element significantly. So the fluid moves without compressing or expanding. In such a situation, we treat the flow of fluid to be incompressible. If we are well away from shock waves or centres of convergence, the assumption of incompressible flow is often valid. To say that fluid elements move without changing density is to say that the Lagrangian total derivative of density is zero, that is,

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \left(\frac{\partial}{\partial t} + \vec{U} \cdot \vec{\nabla}\right) \rho = 0. \tag{9}$$

We also linearize this equation, where the first-order quantities become

$$\frac{\partial \rho_1}{\partial t} + (\vec{U}_1 \cdot \vec{\nabla})\rho_0 = 0. \tag{10}$$

Now, let $\vec{U}_1 = (u_{x1}, u_{y1}, u_{z1})$ and $\vec{V}_1 = (v_{x1}, v_{y1}, v_{z1})$. The fluid is arranged in horizontal strata, then ρ_0 is a function of the vertical coordinate only ($\rho_0 = \rho_0(z)$) and $p_0 = p_0(z)$. So, the system of eqs. (5)–(7) and (10) become

$$\rho_0 \frac{\partial u_{x1}}{\partial t} = -\frac{\partial p_1}{\partial x} + K N_0 \{v_{x1} - u_{x1}\} + \frac{\hbar^2}{2 m_e m_i} \frac{\partial}{\partial x} \times \left\{ \begin{array}{l} \frac{1}{2}\frac{\mathrm{d}^2\rho_1}{\mathrm{d}z^2} - \frac{1}{2\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}\rho_1}{\mathrm{d}z} \\ + \left\{\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{1}{2\rho_0}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} \frac{1}{2\rho_0^2}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^2\right\}\rho_1 \end{array} \right\}, \tag{11}$$

$$\rho_0 \frac{\partial u_{y1}}{\partial t} = -\frac{\partial p_1}{\partial y} + K N_0 \{v_{y1} - u_{y1}\} + \frac{\hbar^2}{2 m_e m_i} \frac{\partial}{\partial y} \times \left\{ \begin{array}{l} \frac{1}{2}\frac{\mathrm{d}^2\rho_1}{\mathrm{d}z^2} - \frac{1}{2\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}\rho_1}{\mathrm{d}z} \\ + \left\{\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{1}{2\rho_0}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} \frac{1}{2\rho_0^2}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^2\right\}\rho_1 \end{array} \right\}, \tag{12}$$

$$\rho_0 \frac{\partial u_{z1}}{\partial t} = -\frac{\partial p_1}{\partial z} + K N_0 \{v_{z1} - u_{z1}\} - \rho_1 g + \frac{\hbar^2}{2 m_e m_i} \times \left\{ \begin{array}{l} \frac{1}{2}\frac{\mathrm{d}^3\rho_1}{\mathrm{d}z^3} - \frac{1}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^2\rho_1}{\mathrm{d}z^2} \\ + \left\{\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{1}{\rho_0}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} + \frac{3}{2\rho_0^2}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^2\right\}\frac{\mathrm{d}\rho_1}{\mathrm{d}z} \\ + \left\{-\frac{1}{2\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{1}{2\rho_0^2}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} - \frac{1}{\rho_0^3}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^3\right\}\rho_1 \end{array} \right\}, \tag{13}$$

$$\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{y1}}{\partial y} + \frac{\partial u_{z1}}{\partial z} = 0, \tag{14}$$

$$\frac{\partial \rho_1}{\partial t} + u_{1z}\frac{\mathrm{d}\rho_0}{\partial z} = 0, \tag{15}$$

$$\left\{\tau\frac{\partial}{\partial t} + 1\right\} v_{x1} = u_{x1}, \quad \left\{\tau\frac{\partial}{\partial t} + 1\right\} v_{y1} = u_{y1}, \quad \left\{\tau\frac{\partial}{\partial t} + 1\right\} v_{z1} = u_{z1}. \tag{16}$$

Now, we assume that the perturbation in any physical quantity is dependent on the space coordinate (x, y, z) and time t as follows:

$$\psi(x, y, z, t) = \psi(z)\exp(ik_x x + ik_y y - i\omega t), \tag{17}$$

where $k_x,\ k_y\ (k^2 = k_x^2 + k_y^2)$ are horizontal wave numbers and n denotes the rate at which the system departs from the equilibrium. Using expression (17) in the system of eqs (11)–(16), we have

$$-i\rho_0\omega u_{x1} = -ik_x p_1 + KN_0\{v_{x1} - u_{x1}\} + \frac{\hbar^2 k_x}{2\omega m_e m_i} \times \left\{ \begin{array}{l} \frac{1}{2}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^2 u_{z1}}{\mathrm{d}z^2} + \left(\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} - \frac{1}{2\rho_0}\left(\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2}\right)^2\right)\frac{\mathrm{d}u_{z1}}{\mathrm{d}z} \\ + \left(\frac{1}{2}\frac{\mathrm{d}^3\rho_0}{\mathrm{d}z^3} - \frac{1}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} - \frac{k^2}{2}\frac{\mathrm{d}\rho_0}{\mathrm{d}z} + \frac{1}{2\rho_0^2}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^3\right) u_{z1} \end{array} \right\}, \tag{18}$$

$$-i\rho_0\omega u_{y1} = -ik_y p_1 + KN_0\{v_{y1} - u_{y1}\} + \frac{\hbar^2 k_y}{2\omega m_e m_i} \times \left\{ \begin{array}{l} \frac{1}{2}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^2 u_{z1}}{\mathrm{d}z^2} + \left(\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} - \frac{1}{2\rho_0}\left(\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2}\right)^2\right)\frac{\mathrm{d}u_{z1}}{\mathrm{d}z} \\ + \left(\frac{1}{2}\frac{\mathrm{d}^3\rho_0}{\mathrm{d}z^3} - \frac{1}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} - \frac{k^2}{2}\frac{\mathrm{d}\rho_0}{\mathrm{d}z} + \frac{1}{2\rho_0^2}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^3\right) u_{z1} \end{array} \right\}, \tag{19}$$

$$-i\rho_0\omega u_{z1} = -\frac{\mathrm{d}p_1}{\mathrm{d}z} + KN_0\{v_{z1} - u_{z1}\} - \rho_1 g + \frac{\hbar^2}{2\omega m_e m_i} \times \left[\begin{array}{l} \frac{1}{2}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^3 u_{z1}}{\mathrm{d}z^3} + \left(\frac{3}{2}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} - \frac{1}{\rho_0}\left(\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2}\right)^2\right)\frac{\mathrm{d}^2 u_{z1}}{\mathrm{d}z^2} \\ + \left(\frac{3}{2}\frac{\mathrm{d}^3\rho_0}{\mathrm{d}z^3} - \frac{3}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} - \frac{k^2}{2}\frac{\mathrm{d}\rho_0}{\mathrm{d}z} + \frac{3}{2\rho_0^2}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^3\right)\frac{\mathrm{d}u_{z1}}{\mathrm{d}z} \\ + \left(\begin{array}{l}\frac{1}{2}\frac{\mathrm{d}^4\rho_0}{\mathrm{d}z^4} - \frac{1}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{d}^3\rho_0}{\mathrm{d}z^3} - \frac{k^2}{2}\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} - \frac{1}{\rho_0}\left(\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2}\right)^2 \\ + \frac{5}{2\rho_0^2}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^2\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2} + \frac{k^2}{2\rho_0}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^2 - \frac{1}{\rho_0^3}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^4\end{array}\right) u_{z1} \end{array} \right], \tag{20}$$

$$ik_x u_{x1} + ik_y u_{y1} + Du_{z1} = 0, \tag{21}$$

$$\omega\rho_1 + u_{z1}\frac{\mathrm{d}\rho_0}{\partial z} = 0, \tag{22}$$

$$\{1 - i\omega\tau\}v_{x1} = u_{x1}, \quad \{1 - i\omega\tau\}v_{y1} = u_{y1},$$
$$\{1 - i\omega\tau\}v_{z1} = u_{z1}. \tag{23}$$

Now, we eliminate some variables from the system of eqs (18)–(23) and get the following differential equation in u_{z1}:

$$\left\{\omega^2\rho_0[1 - i\omega\tau] + \omega^2 mN_0 - \frac{\hbar^2 k^2}{4m_e m_i}\frac{1}{\rho_0}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^2[1 - i\omega\tau]\right\}\frac{\mathrm{d}^2 u_{z1}}{\mathrm{d}z^2}$$
$$+ \left\{ \begin{array}{l} \omega^2\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)[1 - i\omega\tau] + \omega^2\left(\frac{\mathrm{d}(mN_0)}{\mathrm{d}z}\right) \\ + \frac{\hbar^2}{4m_e m_i}\frac{1}{\rho_0^2}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right) \\ \times \left\{\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)^2 - 2\rho_0\frac{\mathrm{d}^2\rho_0}{\mathrm{d}z^2}\right\}[1 - i\omega\tau] \end{array} \right\}\frac{\mathrm{d}u_{z1}}{\mathrm{d}z}$$
$$- \left\{k^2\left[\rho_0\omega^2 + g\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right) - \frac{\hbar^2}{4m_e m_i}\frac{k^2}{\rho_0}\left(\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)\right] \times [1 - i\omega\tau] + \omega^2 mN_0\right\}u_{z1} = 0. \tag{24}$$

3. A continuously stratified plasma layer

In this section we shall consider the case of incompressible continuously stratified plasma layer of thickness h units confined between two rigid boundaries, in which the density is given by

$$\rho_0(z) = \rho_0(0)\exp(z/L_D)$$

and

$$N_0(z) = N_0(0)\exp(z/L_D),$$

where $\rho_0(0)$, $N_0(0)$ and L_D are constants. Then eq. (24) takes the form

$$\left\{(\omega^2 - \omega_q^2)(1 - i\omega\tau) + \frac{mN_0(0)}{\rho_0(0)}\omega^2\right\}\frac{\mathrm{d}^2 u_{z1}}{\mathrm{d}z^2}$$
$$+ \frac{1}{L_D}\left\{(\omega^2 - \omega_q^2)(1 - i\omega\tau) + \frac{mN_0(0)}{\rho_0(0)}\omega^2\right\}\frac{\mathrm{d}u_{z1}}{\mathrm{d}z}$$
$$- k^2\left\{(1 - i\omega\tau)\left[\omega^2 - \omega_q^2 + \frac{g}{L_D}\right] + \frac{mN_0(0)}{\rho_0(0)}\omega^2\right\}u_{z1} = 0, \tag{25}$$

where

$$\omega_q^2 = \frac{\hbar^2 k^2}{4L_D^2 m_e m_i}$$

represents the quantum effects.

Now, if we choose u_{z1} in the form

$$u_{z1} = \sin\left(\frac{n\pi}{h}z\right)\exp(\lambda z),$$

then the solution of eq. (25) is in the form

$$\left\{(\omega^2 - \omega_q^2)(1 - i\omega\tau) + \frac{mN_0(0)}{\rho_0(0)}\omega^2\right\} \times \left\{\frac{1}{4L_D^2} + \frac{n^2\pi^2}{h^2} + k^2\right\} + \left(\frac{k^2 g}{L_D}\right)(1 - i\omega\tau) = 0, \quad (26)$$

where

$$\lambda = -\frac{1}{2L_D}.$$

Now if we define the dimensionless quantities

$$\omega^{*^2} = \frac{\omega^2}{\omega_{pe}^2}, \quad \omega_q^{*^2} = \frac{\hbar^2}{4L_D^4 m_e m_i \omega_{pe}^2}, \quad \tau^* = \tau\omega_{pe},$$
$$h^{*^2} = \frac{h^2}{L_D^2}, \quad k^{*^2} = k^2 L_D^2, \quad g^* = \frac{g}{\omega_{pe}^2 L_D}, \quad (27)$$

then eq. (26) may be written in the form

$$\left\{(\omega^{*2} - k^{*^2}\omega_q^{*2})(1 - i\omega^*\tau^*) + \frac{mN_0(0)}{\rho_0(0)}\omega^{*^2}\right\} \times \{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}\} + 4g^*k^{*^2}h^{*^2}\{1 - i\omega^*\tau^*\} = 0. \quad (28)$$

Now, where $\omega^* = \omega_r^* + i\omega_i^*$ and substituting in eq. (28) reducing two equations (real and imaginary). From imaginary equation we have $\omega_r^* = 0$, and then substituting in the real one the dispersion relation may be given as

$$\gamma^3 + \{1 + \alpha_0\}f^*\gamma^2 + \left\{k^{*^2}\omega_q^{*^2} - \frac{4g^*k^{*^2}h^{*^2}}{h^{*2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}\right\}\gamma + \left\{k^{*^2}\omega_q^{*^2} - \frac{4g^*k^{*^2}h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}\right\}f^* = 0, \quad (29)$$

where $\alpha_0 = mN_0(0)/\rho_0(0)$ denotes the mass concentration of the suspended particles and we introduce the relaxation frequency parameter $f^* = 1/\tau^*$ of the suspended particles which represents the collisions between suspended dust particles and plasma, and $\omega_i^* = \gamma$ (is the imaginary part of ω^*).

In the absence of suspended particles ($f^* = 0$, $\alpha_0 = 0$), eq. (29) becomes

$$\gamma^2 = \frac{4g^*k^{*^2}h^{*^2}}{h^{*2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}} - k^{*^2}\omega_q^{*^2}. \quad (30)$$

From this equation it is clear that $\partial\gamma^2/\partial\omega_q^{*^2} = -k^{*^2}$ which implies that the quantum term has stability role (see refs [16–25]).

In the absence of quantum effects ($\omega_q^* = 0$) and the mass concentration of the suspended particles ($\alpha_0 = 0$), the dispersion relation (29) becomes

$$\gamma^3 + f^*\gamma^2 - \frac{4g^*k^{*^2}h^{*2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}\gamma - \frac{4g^*k^{*^2}h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}f^* = 0. \quad (31)$$

The stability of the system represented by eq. (31) can be easily discussed without calculating the roots of this equation using Routh–Hurwitz (RH) criterion, where the coefficients are real. We can construct the RH tabulation of eq. (31), which are given below:

γ^3	1	$-\frac{4g^*k^{*^2}h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}$
γ^2	f^*	$-\frac{4g^*k^{*^2}h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}f^*$,
γ^1	b_1	0
γ^0	c_1	0

(32)

$$b_1 = \frac{4g^*k^{*^2}h^{*^2}\{f^* - 1\}}{\{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}\}f^*},$$
$$c_1 = -\frac{4g^*k^{*^2}h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}f^*. \quad (33)$$

From the above table, as there are two sign changes in the first column (i.e. in the first column of the RH table there are two sign changes, and as there are two roots in the right half of the s-plane (unstable roots)) the system is unstable, implying that the relaxation frequency of the suspended dust particles has destabilizing effect on the considered system.

The role of the mass concentration of the suspended particles alone comes from eq. (29). If the quantum term equals zero ($\omega_q^* = 0$) and the relaxation frequency of the suspended particles tends to infinity ($f^* \to \infty$), then the dispersion relation (29) becomes

$$\gamma^2 = \frac{4g^* k^{*^2} h^{*^2}}{\{1+\alpha_0\}\left\{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}\right\}}. \tag{34}$$

From eq. (34) one can see that the mass concentration of the suspended particles has stabilizing effects on the considered system and this role increases as α_0 increases as $\alpha_0 > 1$.

Also from eq. (34) we find that $\gamma^2_{k=0} = 0$, while $\gamma^2_{k\to\infty} = g^*/1+\alpha_0$ implying that the mass concentration of the suspended particles has no critical strength to suppress the instability completely.

In the general case, the stability condition can also be obtained without calculating the roots of eq. (29) using RH criterion, where the coefficients of eq. (29) are real.

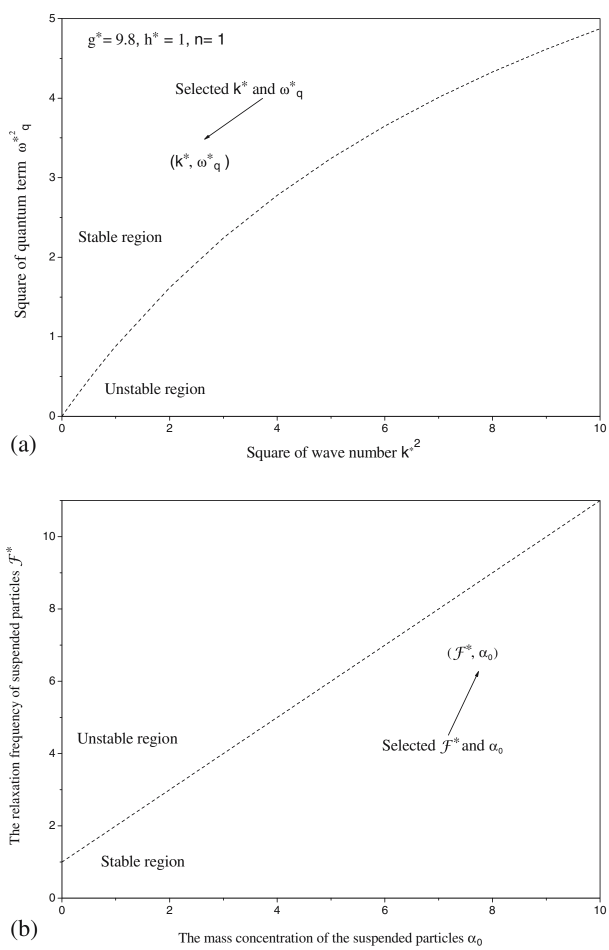

Figure 2. The stability condition of eq. (29). (**a**) The square normalized growth rate (γ^2) plotted against the square normalized wave number k^{*^2} at $n = 1$, $h^* = 1$ and $g^* = 9.8$ and (**b**) the square normalized growth rate (γ^2) plotted against the mass concentration of the suspended particles (α_0).

We construct the RH tabulation of eq. (29), which are given as

$$\begin{array}{l|cc}
\gamma^3 & 1 & k^{*^2}\omega_q^{*^2} - \dfrac{4g^* k^{*^2} h^{*^2}}{h^{*2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}} \\
\gamma^2 & \{1+\alpha_0\} f^* & k^{*^2}\omega_q^{*^2} - \dfrac{4g^* k^{*^2} h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}} f^* \\
\gamma^1 & b_2 & 0 \\
\gamma^0 & c_2 & 0
\end{array} \tag{35}$$

where

$$b_2 = \frac{k^{*^2}\left\{\omega_q^{*^2} - \dfrac{4g^* h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}\right\}\{1+\alpha_0 - f^*\}}{\{1+\alpha_0\} f^*}, \tag{36}$$

$$c_2 = k^{*^2} f^* \left\{\omega_q^{*^2} - \frac{4g^* h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}\right\}. \tag{37}$$

Under the conditions

$$\omega_q^{*^2} > \frac{4g^* h^{*^2}}{h^{*^2} + 4n^2\pi^2 + 4h^{*^2}k^{*^2}}$$

and

$$1 + \alpha_0 > f^*$$

there are no changes in sign in the first column (the first column has the same sign (+ sign)). Now, as all the coefficients in the first column of the RH table are

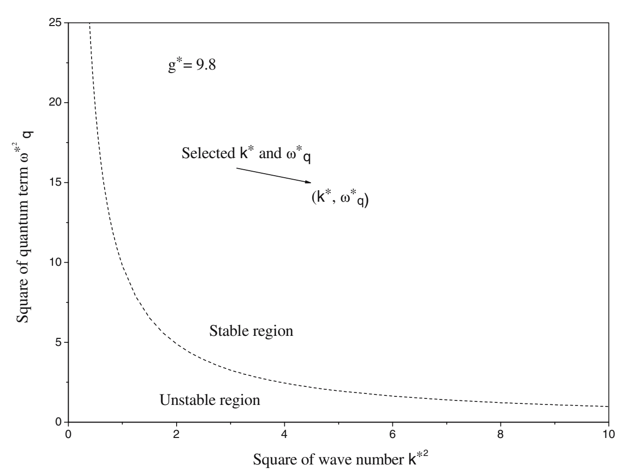

Figure 3. The stability condition of eq. (38), where the square normalized growth rate (γ^2) plotted against the square normalized wave number k^{*^2} at $g^* = 9.8$.

positive and there is no sign change, then the system is stable (see figures 2a and 3b).

Now at $k^* \gg 1$ we find $(\frac{1}{2k^*}) + 1 \cong 1$ and if $n = 0$, then eq. (29) may be written as

$$\gamma^3 + \{1 + \alpha_0\}\gamma^2 f^* + \{k^{*^2}\omega_q^{*^2} - g^*\}\gamma + \{k^{*^2}\omega_q^{*^2} - g^*\}f^* = 0. \quad (38)$$

Again, applying RH criterion on eq. (38) we have

$$\begin{array}{c|cc} \gamma^3 & 1 & k^{*^2}\omega_q^{*^2} - g^* \\ \gamma^2 & \{1 + \alpha_0\}f^* & \{k^{*^2}\omega_q^{*^2} - g^*\}f^* \\ \gamma^1 & b_3 & 0 \\ \gamma^0 & c_3 & 0 \end{array}, \quad (39)$$

$$b_3 = \frac{\{k^{*^2}\omega_q^{*^2} - g^*\}\{1 + \alpha_0 - f^*\}}{\{1 + \alpha_0\}f^*}, \quad (40)$$

$$c_3 = f^*\{\omega_q^{*^2} k^{*^2} - g^*\}. \quad (41)$$

Again, under the condition $\omega_q^{*^2} > g^*/k^{*^2}$ and $1+\alpha_0 > f^*$ there are no changes in sign in the first column and then the system is stable (see figures 2b and 3).

4. Conclusion

In the present paper, the RTI of incompressible quantum plasma is investigated after incorporating the effects of suspended dust particles. The basic QHD set of equations have been modified owing to the influence of suspended dust particles and linearized equations are obtained. The dispersion relation is obtained by applying appropriate boundary conditions and Fourier transform on the perturbed variables. The role of suspended dust particles has been examined in terms of f^*, the relaxation frequency of the suspended particles and α_0 the mass concentration of the suspended particles. It is found that both quantum term and mass concentration of the suspended particles have stabilizing influence on the growth rate of RTI, while the relaxation frequency of the suspended particles has destabilizing effect on the growth rate of linear RTI. In the presence of relaxation frequency of the suspended particles with quantum term or the mass concentration of the suspended dust particles, the system yields a stable quantum diffraction term and relaxation frequency of the suspended dust particles.

Finally, the stabilizing effect that happens in the presence of quantum corrections (quantum pressure gradient) and the mass concentration of the suspended particles can cause absorption of a part of the kinetic energy of the waves, leading to damping in the frequency of the waves. The present results can be used for understanding dispersion properties of electron plasma oscillations at the dusty quantum plasmas.

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