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Stabilization effect of Weibel modes due to inverse bremsstrahlung absorption in laser fusion plasma using Krook collisions model

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Abstract. In this work, the Weibel instability due to inverse bremsstrahlung absorption in laser fusion plasma has been investigated. The stabilization effect due to the coupling of the self-generated magnetic field by Weibel instability with the laser wave field is explicitly showed. The main result obtained in this work is that the inclusion of self-generated magnetic field due to Weibel instability to the inverse bremsstrahlung absorption causes a stabilizing effect of excited Weibel modes. We found a decrease in the spectral range of Weibel unstable modes. This decrease is accompanied by a reduction of two orders in the growth rate of instability or even stabilization of these modes. It has been shown that the previous analyses of the Weibel instability due to inverse bremsstrahlung have overestimated the values of the generated magnetic fields. Therefore, the generation of magnetic fields by the Weibel instability due to inverse bremsstrahlung should not affect the experiences of an inertial confinement fusion.

Keywords. Thermonuclear fusion; inverse bremsstrahlung; Weibel instability; stabilization; Krook collision.

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1. Introduction

Weibel instability [1] is a microconvective instability corresponding to the excitation of electromagnetic modes in plasmas characterized by temperature anisotropy. In a microscopic way, this corresponds to plasma described by an anisotropic distribution function in velocity space. The temperature anisotropy can be generated in plasma by different mechanisms, specifically the heat transport, the expansion of the plasma, and the inverse bremsstrahlung absorption [2–4].

In this work, we aim to investigate the Weibel instability due to inverse bremsstrahlung absorption by taking into account the stabilization effect induced by the coupling of self-generated magnetic field with the laser wave field. This is needed to derive the dispersion relation of low-frequency electromagnetic Weibel modes in plasma heated by a laser pulse. These results highlight new terms in the dispersion relation, due to the coupling between the laser electric field and the resulting magnetic field by the WI. These terms contribute to the instability and the convection of Weibel modes.

The present work is organized as follows: in §2, we present the basic equation used in our theoretical model which is the Fokker–Planck equation (or Vlasov–Landau) [5]. We consider homogeneous plasma in interaction with a high-frequency, low-magnitude laser field. We calculate the distribution function from the anisotropic Fokker–Planck equation. For this we use the method of separation of time-scales and the iterative method. Section 3 is devoted to the Weibel instability analysis. We calculate the perturbed distribution function due to the Weibel instability, by solving the linear part of the Fokker–Planck equation using the inversion of the collisions propagator by the continuous fractions method [6,7]. We establish the dispersion relation of the Weibel modes from the slow-frequency Maxwell equations coupled with the perturbed distribution function. Solving the dispersion relation leads to the calculation of the instability growth rate. Finally, in §4, a brief conclusion summarizing our main results is given.

2. Basic equation

To describe fully ionized plasma where the interactions between particles are dominated by the Coulomb interactions, it is judicious to use the Fokker–Planck equation [5]. For electrons, it is written in the laboratory frame as

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}}-\frac{e}{m_e}(\vec{E}+\vec{v}\times\vec{B})\frac{\partial f}{\partial \vec{v}}=C_{ei}(f,f)+C_{ee}(f), \quad (1)$$

where f is the distribution functions of electrons, m_e is the electron mass, e is the elementary charge, $\vec{E}$ and $\vec{B}$ are respectively the electric and the magnetic fields present in the plasma. $C_{ee}(f)$ and $C_{ei}(f)$ are respectively electron–electron collision and electron–ion collision operators [8]. They can be presented as

$$\vec{E}=\vec{E}_h+\vec{E}_s \quad \text{and} \quad \vec{B}=\vec{B}_h+\vec{B}_s,$$

where $\vec{E}_h$ and $\vec{B}_h$ represent the high-frequency fields associated with the laser wave, $\vec{E}_s$ and $\vec{B}_s$ mean low-frequency fields associated with the disturbance in the plasma. We point out that $\vec{B}_h$ has a small magnitide compared to which $\vec{E}_h$:$\vec{E}_h/\vec{B}_h\sim C$. In addition, the spatiotemporal evolution of the laser wave is of the form:

$$\vec{E}_h=\vec{E}_0\mathrm{Re}[\exp(i\omega_l t)], \quad (2)$$

where $\vec{E}_0$ and ω_l are respectively the wave electric field magnitude and the wave frequency.

In order to solve eq. (1), we consider two time-scales, a low-frequency hydrodynamic time-scale and high-frequency (laser field) one. Therefore, the electronic distribution function f can be written as the sum of a quasistatic distribution function, f_s, which varies slowly in time and a high-frequency distribution function, f_h, following the temporal variation of hig-frequency laser electric field, $\vec{E}_h$. So

$$f(\vec{r},\vec{v},t)=f_s(\vec{r},\vec{v},t)+\mathrm{Re}[f_h(\vec{v})\exp(i\omega t)], \quad (3)$$

Note that the indices 's' and 'h' denote the time-scales (low-frequency) and (high-frequency) respectively, and will be used throughout this work.

The separation of time-scales in eq. (1) leads to the following high-frequency and low-frequency kinetic equations:

$$\frac{\partial f_h}{\partial t}-\frac{e}{m_e}\vec{E}_h\cdot\frac{\partial f_s}{\partial \vec{v}}=\frac{e}{m_e}(\vec{E}_s+\vec{v}\times\vec{B}_s)\cdot\frac{\partial f_h}{\partial \vec{v}}+C_{ei}(f_h), \quad (4)$$

$$\frac{\partial f_s}{\partial t}+\vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}}-\frac{e}{m_e}(\vec{E}_s+\vec{v}\times\vec{B}_s)\cdot\frac{\partial f_s}{\partial \vec{v}}-C_{ei}(f_s)=\left\langle\frac{e}{m_e}\vec{E}_h\cdot\frac{\partial f_h}{\partial \vec{v}}\right\rangle, \quad (5)$$

where ⟨ and ⟩ denote the average over the laser wave cycle time $T=2\pi/\omega_l$: systems constituting the two coupled equations (4) and (5) form the basic equations in, $\vec{B}_s$ and magnetic field, $\vec{E}_s$ our work. Note here that the terms in the electric field reflect the inclusion, in our study, of the low-frequency electromagnetic field. In particular, the first term on the right-hand side of eq. (4) reflects the coupling of quasistatic fields with the laser field. Let us remember here that the quasistatic field is generated in plasma by the mechanism of the Weibel instability. The *ei* operator frequency is modulated here as the Krook type [5]:

$$C_{ei}(f)=-\nu_{ei}(v)[f-f_M], \quad (6)$$

where $\nu_{ei}=\upsilon/v^3$ is the *e–i* collision frequency with $\upsilon=v_t^4/2\lambda_{ei}$, $v_t=\sqrt{T_e/m_e}$ is the thermal velocity of the electrons and λ_{ei} is the *e–i* mean free path. The expression of ν_{ei} is obtained by using the Landau operator [5] by considering that this frequency remains constant for all anisotropies of the distribution function [1,2].

2.1 *Computation of high-frequency distribution function*

First we analyse eq. (4) for typical physical parameters of laser–plasma interactions: electronic temperature, $T_e=1$ keV *e–i* mean free path, $\lambda_{ei}=1$ μm and laser wave length $\lambda_l=1.06$ μm. It appears that the laser frequency w_l is more important than the collisions frequency, ν_{ei}: ($w_l=1.7\times10^{15}$ s^{-1}, $\nu_{ei}=1.3\times10^{13}$ s^{-1}). Consequently, the collisions term, C_{ei}, is proportional to w_l. Also

$$\frac{E_s}{B_s}\sim\frac{\omega}{k}\ll 1,$$

where k and w are respectively the wave number and the pulsation of the Weibel mode. Then it is judicious to consider the following scaling for the high-frequency distribution function:

$$f_h(\vec{v})=f_h^{(0)}(\vec{v})+\vartheta\left(\frac{\nu_{ei}}{\omega_l},\frac{\omega_c}{\omega_l}\right)f_h^{(1)}(\vec{v}), \quad (7)$$

where the indices 0 and 1 correspond to the order of magnitude of the high-frequency distribution function. At the zero order we can neglect the right-hand side of eq. (5) and then we obtain the (hf) distribution function as

$$f_h^{(0)}(\vec{v})=\frac{e}{m_e}\frac{1}{i\omega_l}\vec{E}_h\cdot\frac{\partial f_s}{\partial \vec{v}}. \quad (8)$$

By using this zero-order solution (8) and the expression (7) of $f_h(\vec{v})$, we can iteratively resolve eq. (5). So

$$f_h(\vec{v}) = -\frac{e}{m_e}\frac{i}{\omega_l}E_{hJ}\cdot\frac{\partial f_s}{\partial \vec{v}} - \frac{e^2}{m_e^2}(\vec{E}_s + \vec{v}\times\vec{B}_s)\frac{\partial}{\partial v_i} \times\left[E_{hJ}\cdot\frac{\partial f_s}{\partial v_j}\right] + \frac{e}{m_e}\frac{v_{ei}}{\omega_l^2}E_{hJ}\cdot\frac{\partial f_s}{\partial v_j}, \quad (9)$$

where we have used the Einstein notation [9] corresponding to the summation over the repeat index. Equation (9) represents the high-frequency component of the electronic distribution function which depends on its low-frequency component f_s.

2.2 *Resolution of the low-frequency Fokker–Planck equation*

After some calculation using the expression of f_h, eq. (4) can be presented as

$$\frac{\partial f_s}{\partial t} + \vec{v}.\frac{\partial f_s}{\partial \vec{r}} - \frac{e}{m_e}(\vec{E}_s + \vec{v}\times\vec{B}_s)\cdot\frac{\partial f_s}{\partial \vec{v}} + v_{ei}(v)[f_s] = S_{BI}, \quad (10)$$

where

$$S_{BI} = -\frac{1}{2}\frac{e^2}{m^2\omega_l^2}E_{0l}E_{0j}\frac{\partial}{\partial v_l}\left[v_{ei}(v)\frac{\partial f_s}{\partial v_j}\right] - \frac{1}{2}\frac{e}{m}\frac{e^2}{m^2\omega_l^2}E_{0l}E_{0j}E_{si}\frac{\partial}{\partial v_l}\left[\frac{\partial^2 f_s}{\partial v_i \partial v_j}\right] - \frac{1}{2}\frac{e}{m}\frac{e^2}{m^2\omega_l^2}E_{0l}E_{0j}\frac{\partial}{\partial v_l}\left[(\vec{v}\times\vec{B}_s)_i\frac{\partial^2 f_s}{\partial v_i \partial v_j}\right]. \quad (11)$$

The next step in our analysis is to develop eq. (11) by setting

$$f_s = f_s^{(0)} + f_s^{(1)} \quad (12)$$

with $f^{(1)} \ll f_s^{(0)}$. The distribution function $f_s^{(0)}$ describes the plasma in the presence of high-frequency laser field E_h. However, $f_s^{(1)}$ corresponds to the perturbations associated with quasistatic electromagnetic field E_s and B_s. Using eq. (12), we obtain the zero order of eq. (10) as

$$\frac{\partial f_s^{(0)}}{\partial t} - \frac{1}{2}\left(\frac{e}{m_e\omega_l}\right)^2 E_{0j}E_{0l}\cdot\frac{\partial}{\partial v_l}\left(v_{ei}(v)\frac{\partial}{\partial v_j}f_s^{(0)}\right) = -v_{ei}(v)[f_s^{(0)}]. \quad (13)$$

Subsequently, we suppose that the non-perturbed plasma is homogeneous in the presence of high-frequency laser electric field, $\vec{E}_h$, with a linear polarization on the x direction. In this geometry, the above equation becomes

$$\frac{\partial f_s^{(0)}}{\partial t} - \frac{1}{2}v_0^2\cdot\frac{\partial}{\partial v_x}\left(v_{ei}(v)\frac{\partial}{\partial v_x}f_s^{(0)}\right) = -v_{ei}(v)[f_s^{(0)}], \quad (14)$$

where

$$v_0 = \frac{eE_{hl}}{m_e\omega}$$

is the magnitude of the oscillation velocity of the electrons in the laser electric field, $\vec{E}_h$. The oscillation of the electrons induces an anisotropic distribution function in the direction of $\vec{E}_h$:$f_s^{(0)}(\vec{v},t) = f_s^{(0)}(v, v_x, t)$. With the introduction of the variable, $\mu = v_x/v$, eq. (14) is presented as

$$\frac{\partial f_s^{(0)}}{\partial t} - \frac{v}{2}\frac{v_0^2}{v^2}\left[\mu\frac{1}{v}\frac{\partial^2 f_s^{(0)}}{\partial v^2} + (1-4\mu^2)\frac{\partial f_s^{(0)}}{\partial v}\right] + \frac{3}{v^3}\frac{v_0^2}{v^2}\mu(1-\mu^2)\frac{\partial f_s^{(0)}}{\partial \mu} - \frac{v}{v^2}\frac{v_0^2}{v^2}(1-\mu^2)\frac{\partial^2 f_s^{(0)}}{\partial\mu\partial v} - \frac{v}{2v^3}\frac{v_0^2}{v^2}\frac{\partial^2 f_s^{(0)}}{\partial \mu^2} = -\frac{v}{v^3}[f_s^{(0)}]. \quad (15)$$

Now, we develop the distribution function $f_s^{(0)}(v,\mu)$ on the Legendre polynomials $p_l(\mu)$ [9,10]. Then, eq. (15) can be developed. After some calculations using recurrence relations between Legendre polynomials of several orders, we find the equation of the isotropic distribution function, which corresponds to the projection of eq. (15) on the Legendre polynomial of order 0, P_0. So

$$\frac{\partial f_{s0}^{(0)}}{\partial t} - \frac{v}{6}\frac{v_0^2}{v^2}\left[\frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial f_{s0}^{(0)}}{\partial v}\right)\right] = C_{ee}(f_{s0}^{(0)}). \quad (16)$$

In this equation, the terms proportional to the second anisotropic distribution function $f_{s2}^{(0)}$ are ignored. This is justified by the fact that

$$\frac{f_{s2}^{(0)}}{f_{s0}^{(0)}} \sim \frac{v_0^2}{v_t^2} \ll 1$$

corresponding to the low-magnitude laser wave approximation is largely fulfilled in the laser–plasma interaction experiments. Note that this equation corresponds to that obtained by Langdon in ref. [11].

The equation of the second anisotropic, $f_{s2}^{(0)}$, is calculated under the same approximation by the projection of eq. (16) on Legendre polynomial, p_2. So

$$\frac{\partial f_{s2}^{(0)}}{\partial t} - \frac{v}{3\sqrt{5}}vv_0^2\left[\frac{\partial}{\partial v}\left(\frac{1}{v^4}\frac{\partial f_{s0}^{(0)}}{\partial v}\right)\right] = -\frac{v}{v^3}f_{s2}^{(0)}, \quad (17)$$

where we have neglected the terms proportional to $f_{s4}^{(0)}$ and those proportional to $(\nu_0^2/\nu_t^2) f_{s2}^{(0)}$. Then, the expression of the second anisotropic distribution function obtained in the stationary approximation is

$$\frac{\partial f_{s2}^{(0)}}{\partial t} \approx 0,$$

generally considered in the Weibel instability analysis. So

$$f_{s2}^{(0)} = \frac{1}{3\sqrt{5}} \frac{\nu_0^2}{\nu^2} \nu^6 \left[\frac{\partial}{\partial \nu} \left(\frac{1}{\nu^4} \frac{\partial f_{s0}^{(0)}}{\partial \nu} \right) \right]. \tag{18}$$

By supposing that the isotropic distribution function is a Maxwellian f_M, eq. (18) is presented as

$$f_{s2}^{(0)} = \frac{1}{3\sqrt{5}} \frac{\nu_0^2}{\nu_t^2} \left[3 + \frac{\nu_0^2}{\nu_t^2} \right] f_M. \tag{19}$$

It appears from eq. (15) that the anisotropic distribution function of first order is vanished. By limiting our development at the second order, the non-perturbed distribution function can be written as

$$f_s^{(0)} = f_{s2}^{(0)}(\nu) + p_2(\mu) f_{s2}^{(0)}(\nu). \tag{20}$$

3. Weibel instability analysis

This section is devoted to the analysis of the Weibel instability. For this, we establish the dispersion relation from the Maxwell's equations for Weibel quasistatic electromagnetic field, coupled with the perturbed distribution function. The growth rate of instability is deduced from the dispersion relation.

3.1 *Perturbed distribution function*

The perturbed distribution function $f_s^{(1)}$ is associated with an electromagnetic perturbation due to the Weibel instability. After some algebra, the spatiotemporal evolution of the perturbed distribution function $f_s^{(1)}$, is obtained from the Fokker–Planck equation (5), by considering the scaling equation (12), as

$$\frac{\partial f_s^{(1)}}{\partial t} + \vec{\nu} \cdot \frac{\partial f_s^{(1)}}{\partial \vec{r}} - \frac{e}{m_e} (\vec{E}_s + \vec{\nu} \times \vec{B}_s) \cdot \frac{\partial f_s^{(0)}}{\partial \vec{\nu}} + \nu_{ei}(\nu) f_s^{(1)} = I_{BI}(f_s^{(1)}) + I_{BI}(f_s^{(0)}), \tag{21}$$

where

$$I_{BI}(f_s^{(1)}) = -\frac{1}{2} \nu_0^2 \frac{\partial}{\partial \nu_x} \left(\nu_{ei}(\nu) \frac{\partial}{\partial \nu_x} I_{BI}(f_s^{(1)}) \right)$$

and

$$I_{BI}(f_s^{(0)}) = -\frac{1}{2} \frac{e}{m_e} \nu_0^2 E_{si} \frac{\partial}{\partial \nu_x} \left[\frac{\partial^2 f_{s0}}{\partial \nu_i \partial \nu_x} \right] - \frac{1}{2} \frac{e}{m_e} \nu_0^2 \frac{\partial}{\partial \nu_x} (\vec{\nu} \times \vec{B}_s) \frac{\partial^2 f_{s0}}{\partial \nu_i \partial \nu_x}. \tag{22}$$

It appears that the perturbed distribution function $f_s^{(1)}$ depends on the non-perturbed distribution function $f_s^{(0)}$. Some simplification can be made on eq. (22). In fact the term, $(\partial f_s^{(1)}/\partial t) \sim \omega f_s^{(1)}$ (where ω is the Weibel mode pulsation), can be neglected compared to the collisions term $\sim \nu_{ei}$, that the interested Weibel modes are quasistatics: $\omega \ll \nu_{ei}$. Furthermore, by considering that $(\nu_0^2/\nu_t^2) \ll 1$, corresponding to a low-magnitude laser wave, it appears also that $I_{BI}(f_s^{(1)})$ is small compared to $\nu_{ei} f_s^{(1)}$. Finally, by considering the Faraday law, $E_s = (\omega/k) B_s$, and the condition, $(\omega/k\nu_t) \ll 1$, the electric field term can be neglected compared to the magnetic field term in the expression of $f_s^{(1)}$. With these approximations, eq. (21) becomes

$$\vec{\nu} \cdot \frac{\partial f_s^{(1)}}{\partial \vec{r}} - \frac{e}{m_e} (\vec{E}_s + \vec{\nu} \times \vec{B}_s) \cdot \frac{\partial f_s^{(0)}}{\partial \vec{\nu}} + \nu_{ei}(\nu)[f_s^{(1)}] = I_{BI}(f_s^{(0)}), \tag{23}$$

where

$$I_{BI}(f_s^{(0)}) = -\frac{1}{2} \frac{e}{m_e} \nu_0^2 \frac{\partial}{\partial \nu_x} (\vec{\nu} \times \vec{B}_s) \frac{\partial^2 f_{s0}}{\partial \nu_i \partial \nu_x}. \tag{24}$$

The linear polarization of the laser wave in the x direction allows a positive anisotropy in temperature: $T_X > T_\perp$. This is susceptible to excite perpendicular Weibel modes: $\vec{k} \perp \vec{x}$. For simplification, we consider the geometry: $(\vec{E}_s \| \vec{E}_h \| \vec{x}, B_s \| \vec{y}, \vec{k} \| \vec{z})$. In this geometry, eq. (24) can be presented as

$$ik\nu_z \cdot f_s^{(1)} + \nu_{ei}(\nu) f_s^{(1)} = \frac{eE_s}{m_e} \frac{\partial f_s^{(0)}}{\partial \nu_x} - \frac{eB_s}{m_e} \left(\nu_z \frac{\partial (P_2 f_{s2}^{(0)})}{\partial \nu_X} - \nu_x \frac{\partial (P_2 f_{s2}^{(0)})}{\partial \nu_z} \right) + I_{BI}(f_s^{(0)}), \tag{25}$$

where

$$I_{BI}(f_s^{(0)}) = -\frac{1}{2} \frac{eB_s}{m_e} \nu_0^2 \left(\nu_x \frac{\partial^2 f_s^{(0)}}{\partial \nu_X \partial \nu_z} - \nu_z \frac{\partial^2 f_s^{(0)}}{\partial \nu_x^2} \right). \tag{26}$$

In eq. (26), the anisotropic component has been neglected in terms of E_s. In the right-hand side of eq. (26), the first term is the source term of Weibel instability.

However, the term $I_{BI}(f_s^{(0)})$ corresponds to the coupling between the quasistatic magnetic field, B_s, and the laser wave field ($\sim\nu_0$). This term has been ignored in the previous studies of the Weibel instability [2,3] even though it is comparable to the source term. To resolve eq. (26), first we express the velocity vector in spherical coordinates $v_z = v\cos\theta$, $v_x = v\sin\theta\cos\varphi$, $v_y = v\sin\theta\sin\varphi$, and eq. (26) can be presented as

$$(ikv\cos\theta + \nu_{ei}(v))f_s^{(1)} = S_E + S_B, \tag{27}$$

where

$$S_E = \frac{eE_s}{m_e}\sin\theta\cos\varphi\frac{\partial f_{s0}^{(0)}}{\partial v} \tag{28}$$

$$SB = \left[3\sqrt{5}\frac{e}{m_e}f_{s2}^{(0)} - \frac{1}{2}v_0^2 v\frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial f_{s0}^{(0)}}{\partial v}\right)\right] \times B_s.\cos\theta\sin\theta\cos\varphi. \tag{29}$$

The following step is to develop $f_s^{(1)}(v,\theta,\varphi)$ on the spherical harmonics $y_l^m(\theta,\varphi)$ [9,10]:

$$f_s^{(1)}(v,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{m=l} f_{sl}^{(1)}(v)y_l^m(\theta,\varphi). \tag{30}$$

Equation (25) then becomes

$$(ikv\cos\theta + \nu_{ei}(v))\sum_{l=0}^{\infty}\sum_{m=-l}^{m=l} f_{sl}^{(1)}(v)y_l^m(\theta,\varphi) = S_{E,1}^{\pm1} + S_{B,2}^{\pm1}, \tag{31}$$

$$S_{E,1}^{\pm1} = \pm\sqrt{\frac{2\pi}{3}}\frac{e}{m_e}E_s\frac{\partial f_{s0}^{(0)}}{\partial v}y_1^{\pm1} \tag{32}$$

and

$$S_{B,2}^{\pm1} = \mp\sqrt{\frac{2\pi}{15}}B_s\left[3\sqrt{5}\frac{e}{m_e}f_{s2}^{(0)} - \frac{1}{2}v_0^2 v\frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial f_{s0}^{(0)}}{\partial v}\right)\right]y_2^{\pm1}. \tag{33}$$

After some investigation using the properties of spherical harmonics [9,10], eq. (31) becomes

$$\nu_{ei}(v)f_{sl,m}^{(1)}(v) + ikv\left(\frac{l^2-m^2}{4l^2-1}\right)f_{sl-1,m}^{(1)}(v) + ikv\left(\frac{(l+1)^2-m^2}{4(l+1)^2-1}\right)f_{sl+1,m}^{(1)}(v) = S_{E,l}^m + S_{B,l}^m, \tag{34}$$

where $S_{E,l}^m$ and $S_{B,l}^m$ are respectively the projections of S_E and S_B on the spherical harmonic of order (l,m). Note here that only the components $S_{E,1}^{\pm1}$ and $S_{B,2}^{\pm1}$ are not vanishing.

Equation (32) is the basic equation to calculate $f_s^{(1)}$. It is a recurrence relation between $f_{sl,m}^{(1)}$, $f_{sl-1,m}^{(1)}$, and $f_{sl+1,m}^{(1)}$. Note that for ≥ 3, this equation can be written as

$$\nu_{ei}(v)f_{sl,m}^{(1)}(v) = -ikv\left(\frac{l^2-m^2}{4l^2-1}\right)f_{sl-1,m}^{(1)}(v) - ikv\left(\frac{(l+1)^2-m^2}{4(l+1)^2-1}\right)f_{sl+1,m}^{(1)}(v). \tag{35}$$

System (35) is resolved by using a mathematical method based on the inversion of the collision propagator in spherical harmonics basis using the continuous fractions [6,9]. The following solution is obtained:

$$f_{s3,m}^{(1)} = -ikv^4\left(\frac{9-m^2}{35}\right)F_{3,m}f_{s2,m}^{(1)}, \tag{36}$$

where $F_{3,m}$ means the continuous fraction defined by the recurrence relation

$$F_{l,m}(k,v) = \left[\nu_{ei} + k^2v^8\left(\frac{(l+1)^2-m^2}{4(l+1)^2-1}\right)F_{l+1,m}\right]^{-1}. \tag{37}$$

Note here that eq. (36) is the exact solution of (35). It gives the relation between $f_{s3,m}^{(1)}$ and $f_{s2,m}^{(1)}$ by including the contributions of all anisotropies $f_{sl,m}^{(1)}$ through the continuous fractions $F_{l,m}$.

From eqs (35) and (36) coupled with the continuous fraction (37), we can calculate all anisotropic $f_{sl,m}^{(1)}$ of the perturbed distribution function. In this work, we limit $f_{s1,1}^{(1)}$ and $f_{s1,-1}^{(1)}$ enough for studying the Weibel instability.

The equation of $f_{s1,1}^{(1)}$ is deduced from (35) by putting $l=1$ and $m=1$. So

$$\nu_{ei}f_{s1,1}^{(1)} + \sqrt{\frac{3}{15}}ikv^4 f_{s2,1}^{(1)} = v^3 S_{E,1}^1. \tag{38}$$

The equation of $f_{s2,1}^{(1)}$ is deduced also from (34) by putting $l=2$ and $m=1$. So

$$\nu_{ei}f_{s2,1}^{(1)} + \sqrt{\frac{3}{15}}ikv^4 f_{s1,1}^{(1)} + \sqrt{\frac{8}{35}}ikv^4 f_{s3,1}^{(1)} = v^3 S_{B,2}^1. \tag{39}$$

By substituting (36) in (39) and by using (37), starting from (37), we obtain, after some calculation, the explicit expression of $f_{s1,1}^{(1)}$. So

$$f_{s1,1}^{(1)} = v^3 F_{1,1}S_{E,1}^1 - \frac{i\sqrt{5}}{kv}(1 - vF_{1,1})S_{B,2}^1. \tag{40}$$

By the same method, we compute the explicit expression of $f^{(1)}_{s1,-1}$. So

$$f^{(1)}_{s1,-1} = v^3 F_{1,1} S^{-1}_{E,1} - \frac{i\sqrt{5}}{kv}(1 - \nu F_{1,1}) S^{-1}_{B,2}. \qquad (41)$$

For $f^{(0)}_{s0} = f_M$, by using the expression of $f^{(0)}_{s2}$ (eq. (20)), the component is obtained as

$$f^{(1)}_{s1,\pm1} = \pm 4\sqrt{\frac{2\pi}{3}}\frac{e}{m_e}E_s v_t^2 y^2 F_{1,1} f_M \mp \frac{e}{m_e}\frac{i}{kv_t} \times \sqrt{\frac{\pi}{3}} B_s \frac{v_0^2}{v_t^2}(1-\nu F_{1,1})(3+y)y^{-1/2} f_M. \qquad (42)$$

3.2 *Dispersion relation*

The calculus of dispersion relation starts from the Maxwell equations [8,12] for the quasistatic Weibel modes coupled with the perturbed distribution function $f_s^{(1)}$ through the current density $\vec{J}$. So

$$\vec{\nabla}\Lambda\vec{E}_s = \frac{-\partial\vec{B}_s}{\partial t}, \qquad (43)$$

$$\vec{\nabla}\Lambda\vec{B}_s = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}_s}{\partial t}, \qquad (44)$$

where $\vec{J}$ is defined as

$$\vec{J} = -e\int \vec{v} f_s^{(1)} \overrightarrow{\mathrm{d}v}. \qquad (45)$$

By considering the spatio-temporal dependence of the field $\vec{E}_s$ and $\vec{B}_s$ as a Fourier mode $\sim \exp(izt - i\vec{k}\cdot\vec{r})$, eqs (43) and (44) can be represented as

$$kE_s = \omega B_s, \qquad (46)$$

$$kB_s = -ie\mu_0\sqrt{\frac{2\pi}{3}}\int v^3(f^{(1)}_{s1,1} - f^{(1)}_{s1,-1})\mathrm{d}v. \qquad (47)$$

Then dispersion relation is obtained as

$$\frac{c^2k^2}{\omega_p^2} = \frac{8\sqrt{2}}{3\sqrt{\pi}}\omega v_t^3 \int y^3 F_{1,1}\exp(-y)\mathrm{d}y + \frac{1}{3\sqrt{\pi}}\frac{v_0^2}{v_t^2}\int (1-\nu F_{1,1})(3+y)y^{1/2} \times \exp(-y)\mathrm{d}y. \qquad (48)$$

The growth rate of Weibel instable mode [13,14], γ: $\omega = \omega_r + i\gamma$, is obtained explicitly from this dispersion relation. So

$$\gamma(k) = -\frac{3}{8}\sqrt{\frac{\pi}{2}}\frac{c^2k^2}{\omega_p^2}\frac{1}{v_t^3}\frac{1}{\int \mathrm{d}y\, y^3\, F_{1,1}\exp(-y)} + \frac{1}{4\sqrt{2}}\frac{1}{v_t^3}\frac{v_0^2}{v_t^2} \times \frac{\int \mathrm{d}y\, y^{1/2}(1-2\nu F_{1,1})\left(1-\frac{y}{3}\right)\exp(-y)}{\int \mathrm{d}y\, y^3\, F_{1,1}\exp(-y)}. \qquad (49)$$

We have presented in figures 1 and 2 the growth rate spectra of Weibel instability $\gamma(k)$, as a function of collision parameter $k\lambda_{ei}$ for typical parameters of the laser pulse and plasma. We point out that without the stabilization term, $I_{BI}(f^{(0)}_{s0})$, our results correspond to the results given in ref. [4]. In addition, the comparison of the obtained spectra with previous works shows an overestimaion by two orders in the growth rate of the most unstable Weibel mode.

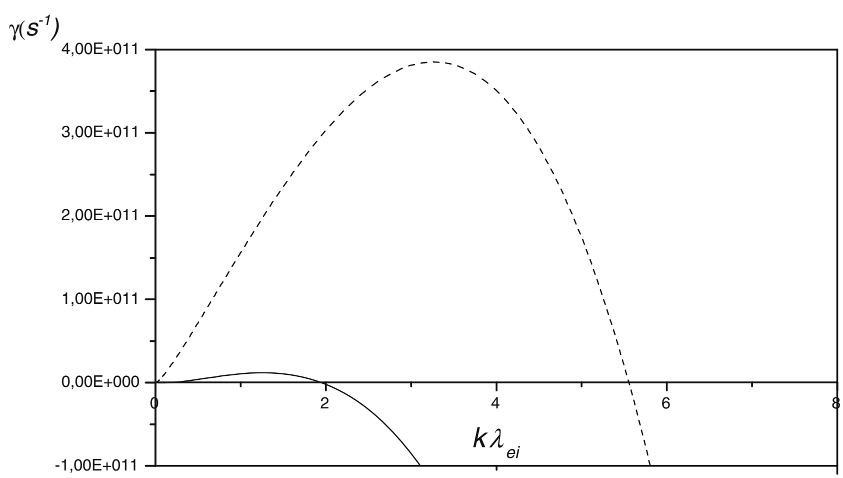

Figure 1. Rate of instability γ from the Krook model C_{ei}, depending on the collision parameter $k\lambda_{ei}$ for the following physical parameters: $n_e = 10^{27}$ m^{-3}, $T_e = 2$ keV, $(v_0/v_t) = 0.3$ and $z = 4$. The dotted line corresponds to the γ values without the term $I_{BI}(f^{(0)}_{s0})$.

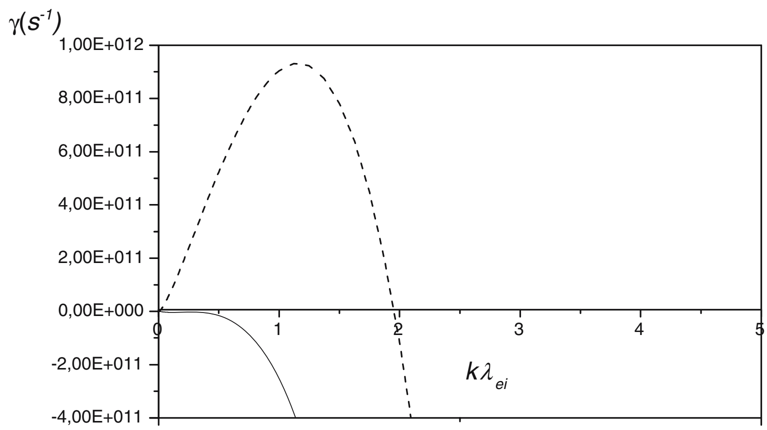

Figure 2. Rate of instability γ from the Krook model C_{ei}, depending on the collision parameter $k\lambda_{ei}$ for the following physical parameters: $n_e = 9 \cdot 10^{27}$ m^{-3}, $T_e = 2$ keV, $(\nu_0/\nu_t) = 0.3$ and $z = 4$. The dotted line corresponds to the γ values without the term $I_{BI}(f_{s0}^{(0)})$.

4. Discussion and conclusion

The Weibel instability is theoretically studied using the Fokker–Planck equation by considering the Krook collisions model. The dispersion relation of the Weibel modes is explicitly established under some justified approximation in the laser-fusion experiments [15,16]. Taking into account the stabilization effect by including the term $I_{BI}(f_s^{(0)})$ led to a significant reduction in the Weibel instability growth rate. Numerical treatment of model equations shows that the growth rate of the most unstable Weibel mode decreases by two orders of magnitude. This decrease in the growth rate magnitude is accompanied by a greater reduction in the spectral range of instability. For high density plasma, the Weibel modes become completely stable. Therefore, the generation of magnetic fields by the Weibel instability due to inverse bremsstrahlung should not affect the experiences of inertial confinement fusion. It is possible to extend this study [17–19], by taking into account the nonlinear effect due to the high-intensity laser pulse.

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