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Magnetized anisotropic dark energy models with constant deceleration parameter

A Y SHAIKH[1,*] and S D KATORE[2]

[1]Department of Mathematics, Indira Gandhi Mahavidyalaya, Ralegaon 445 402, India
[2]PG Department of Mathematics, S.G.B. Amravati University, Amravati 444 602, India
*Corresponding author. E-mail: shaikh_2324ay@yahoo.com

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Abstract. In this paper, we have studied the solutions of plane-symmetric Universe with variable ω in the presence and the absence of magnetic field of energy density ρ_B. A special law of variation for Hubble's parameter proposed by Bermann in *Nuovo Cimento B* **74**, 182 (1983) has been utilized to solve the field equations. Some physical and kinematical properties of the models are also discussed.

Keywords. Dark energy; magnetism.

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1. Introduction

That the Universe is undergoing an accelerating expansion is suggested by current cosmological observations [1–3]. The availability driving this acceleration is assumed to be 'dark energy' whose origin remains a mystery in fashionable cosmology. Present cosmological experimental knowledge suggests that 75% of the Universe is dominated by this dark energy. Dark energy has been conventionally characterized by the equation of state (EoS) parameter $\omega = p/\rho$ that is not essentially constant. The only dark energy candidate is the vacuum energy ($\omega = -1$), that is mathematically corresponding to the constant (Λ). The other conventional alternatives are quintessence ($\omega > -1$) [4], phantom energy ($\omega < -1$) [5] and quintom as evolved and have time-dependent EoS parameter. In addition, there are interacting dark energy models like Chaplygin gas [6,7], holographic models [8–10], brane-world models [11,12] etc. The observational data of Wilkinson microwave anisotropy probe (WMAP) experiment [13] establish the bounds for the value of ω in the range of $-1.11 < \omega < -0.86$. Some other limits obtained from observational results coming from SNe Ia data [14] and combination of SNe Ia data with CMBR anisotropy and galaxy clustering statistics [15] are $-1.67 < \omega < -0.62$ and $-1.33 < \omega < -0.79$, respectively.

LRS Bianchi type-I cosmological models are investigated by Akarsu and Kilinic [16] in the presence of dynamically anisotropic dark energy and perfect fluid. Precise solutions of Einstein's field equations are obtained by using a special law of variation for the mean Hubble parameter. Yadav and Yadav [17] investigated Bianchi type-III dark energy models with constant deceleration parameter. Bianchi type-III magnetized anisotropic dark energy models with constant deceleration parameter are considered by Tade and Sambhe [18]. New dark energy models in anisotropic Bianchi type-I space–time with constant deceleration parameter have been investigated by Pradhan *et al* [19]. Kumar and Singh [20] considered a spatially homogeneous and totally anisotropic Bianchi-I space–time with perfect fluid and anisotropic dark energy, which has dynamical energy density. Katore and Sancheti [21] studied Bianchi type-VI_0 magnetized anisotropic dark energy models with constant deceleration parameter. Spatially homogeneous and anisotropic LRS Bianchi-I cosmological model in the presence of magnetized dark energy has been obtained by Yadav *et al* [22]. Dark energy models with anisotropic fluid in Bianchi type-VI_0 space–time have been discussed by Pradhan *et al* [23]. Priyanka *et al* [24] studied Bianchi type-VI_0 space–time perfect fluid cosmological model satisfying the barotropic equation of state.

Bijan Shah [25] studied the evolution of the Universe filled with dark energy within the scope of a Bianchi type-VI model. The evolution of the Universe filled with dark energy within the scope of a Bianchi type-V model is obtained by Bijan Saha [26]. Mahanta *et al* [27] have obtained dark energy cosmological models in an anisotropic Bianchi type-III space–time in Barber's second self-creation theory of gravitation. Ghate and Sonttake [28] have obtained Bianchi type-IX cosmological models with variable EoS parameter ω in Saez-Ballester theory of gravitation. Recently, Katore and Shaikh [29,30] studied magnetized anisotropic dark energy models for Bianchi type-V space–time and hypersurface homogenous metric with constant deceleration parameter. Very recently, Sahoo and Mishra [31] investigated axially-symmetric cosmological model with anisotropic dark energy.

Motivated by these works, in this paper, we have studied plane-symmetric Universe with variable ω in the presence and the absence of magnetic field of energy density ρ_B together with constant deceleration parameter. Some physical and kinematical properties of the models are also discussed. The outline of the paper is as follows: In §2, the model and field equations are described. The solution of field equations are presented in §3 and §4 concludes the findings.

2. Model and field equations

The Universe is spherically symmetric and the matter distribution is isotropic and solid. However, throughout the first stages of evolution, it is unlikely that it may have had such a smoothened out image. Hence we tend to contemplate plane symmetry that provides a chance for studying irregularity. The plane-parallel metric within

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(\mathrm{d}x^2 + \mathrm{d}y^2) - B^2\mathrm{d}z^2, \tag{1}$$

where A, B are functions of t only.

Panigrahi in association with Sahu [32] have investigated an aeolotropic solid plane-symmetric cosmological micromodel in the presence of massless field in self-creation cosmology. Tilted plane-symmetric cosmological models of perfect fluid with heat conductivity and disordered radiation are investigated by Pawar *et al* [33]. Mahanta and Biswal [34] thought about quark matter coupled to the string cloud and domain walls within the context of Lyra geometry. Recently, Katore and Shaikh [35] have investigated plane-symmetric cosmological model in the presence of cosmic string and bulk consistency in Saez-Ballester scalar–tensor theory of gravitation.

We assume that the Universe is stuffed with aeolotropic fluid for which there is no field whereas the magnetic flux is homeward-bound on coordinate axis. Jacobs [36] studied the impact of a regular, early magnetic flux on Bianchi type-I cosmological model. He found that the early magnetic flux made large growth anisotropies throughout the radiation-dominated part and it had negligible impact throughout the mud-dominated part. To discuss the effects of magnetic flux on the evolution of the Universe, King and Coles [37] used the magnetic perfect fluid energy–momentum tensor. Bianchi type-I cosmological model in the presence of magnetic aeolotropic dark energy is obtained by Sharif and Zubair [38]. Katore *et al* [39] investigated Bianchi type-III cosmological model in the presence of magnetic aeolotropic dark energy. The only generalization of the EoS parameter of the perfect fluid could also be to work out the EoS parameter singly on every spatial axis by protecting the diagonal style of the energy–momentum tensor during a consistent method with the thought-about metric. Here we are dealing with a magnetized anisotropic dark energy fluid whose energy–momentum tensor is

$$T_\mu^\nu = \mathrm{diag}[T_1^1, T_2^2, T_3^3, T_4^4]. \tag{2}$$

Then, one may parametrize it as follows:

$$\begin{aligned} T_\mu^\nu &= \mathrm{diag}[-p_x - \rho_B, -p_y - \rho_B, -p_z + \rho_B, \rho + \rho_B], \\ &= \mathrm{diag}[-\omega_x\rho - \rho_B, -\omega_y\rho - \rho_B, \\ &\qquad -\omega_z\rho + \rho_B, \rho + \rho_B], \\ &= \mathrm{diag}[-\omega\rho - \rho_B, -(\omega+\delta)\rho - \rho_B, \\ &\qquad -(\omega+\gamma)\rho + \rho_B, \rho+\rho_B], \end{aligned} \tag{3}$$

where ρ is the energy density of the fluid, ρ_B is the energy density of the magnetic field, p_x, p_y, p_z are pressures and ω_x, ω_y and ω_z are the directional EoS parameters for the fluid on x-, y- and z-axes respectively, ω is the deviation-free EoS parameter of the fluid. We have parametrized the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameters δ and γ that are the deviations from ω on y- and z-axes respectively. Zeldovich *et al* [40] underlined the importance of magnetic field for a variety of astrophysical phenomena. Magnetic field could have cosmological origin [41]. Therefore, it is interesting to study the influence of magnetism on physical parameters in the presence of dark energy.

The Einstein's field equations, in natural limits ($8\pi G = 1$ and $c = 1$) are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -T_{\mu\nu}, \tag{4}$$

where $g_{\mu\nu} u^{\mu} u^{\nu} = 1$; $u^{\mu} = (1, 0, 0, 0)$ is the four-velocity vector, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar and $T_{\mu\nu}$ is the energy–momentum tensor.

In comoving coordinate system, Einstein's field equations (4), for plane-symmetric metric (1), in case of (3), leads to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -\omega\rho - \rho_B, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -(\omega + \delta)\rho - \rho_B, \tag{6}$$

$$2\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} = -(\omega + \gamma)\rho + \rho_B, \tag{7}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = \rho + \rho_B, \tag{8}$$

where the overhead dot denotes derivative with respect to cosmic time t.

Using eqs (6) and (5), we obtain the skewness parameter on y-axis as null, i.e. $\delta = 0$.

Thus, the system of equations from eqs (5)–(8) may reduce to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -\omega\rho - \rho_B, \tag{9}$$

$$2\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} = -(\omega + \gamma)\rho + \rho_B, \tag{10}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = \rho + \rho_B. \tag{11}$$

3. Solution of the field equations

The field equations (9)–(11) are a system of three equations with six unknown parameters $A, B, \rho, \rho_B, \omega, \gamma$. Thus, we can introduce more conditions either by an assumption corresponding to some physical situation or by an arbitrary mathematical supposition; however, these procedures have some drawbacks. Physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may lead to a non-physical situation. The energy conservation equation $T^{\nu}_{\mu;\nu} = 0$, leads to two equations for the anisotropic fluid and magnetic field. We assumed that the magnetized anisotropic dark energy is minimally interacting, hence the Bianchi identity has been split into two separately additive conserved components; namely, the conservation of the energy–momentum tensor for the anisotropic fluid and for the magnetic field (King and Coles [37]).

$$\dot{\rho} + (1 + \omega)\rho\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \rho\left[\delta\frac{\dot{A}}{A} + \delta\frac{\dot{B}}{B}\right] = 0 \tag{12}$$

and

$$\rho_B = \frac{\beta}{A^4}. \tag{13}$$

The average scale factor of the plane-symmetric metric is given by

$$R = (A^2 B)^{1/3}. \tag{14}$$

The directional Hubble's parameters in the direction of x-, y- and z-axes respectively for the plane-symmetric metric are

$$H_x = \frac{\dot{A}}{A},\ H_y = \frac{\dot{A}}{A},\ H_z = \frac{\dot{B}}{B}. \tag{15}$$

The mean Hubble's parameter H is given by

$$H = \frac{1}{3}(H_x + H_y + H_z). \tag{16}$$

The proper volume V is defined by

$$V = (-g)^{1/2} = (A^2 B). \tag{17}$$

From eqs (14)–(17), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{R}}{R} = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right). \tag{18}$$

In order to solve the field equations, we use a physical condition that the expansion scalar is proportional to shear scalar, i.e.

$$A = B^n, \tag{19}$$

where n is the proportionality constant.

The work of Thorne [42] is the motivation for the consideration of eq. (19). According to Thorne [42], the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the Universe is isotropic within about 30% range approximately [43,44] and red-shift studies place the limit $(\sigma/H) \leq 0.3$, on the ratio of shear σ to Hubble H in the neighbourhood of our galaxy today. The physical significance of this condition for the perfect fluid and barotropic EoS in a more general case has been discussed by Collins *et al* [45]. Sharif and Zubair [46], Yadav and Yadav [47] use this condition to find exact solutions of cosmological models.

The line element (1) is completely characterized by Hubble's parameter H. Therefore, let us consider that mean Hubble's parameter H is related to the average scale factor R by the relation

$$H = k_1 R^{-s}, \tag{20}$$

where k_1 (>0) and s (≥0) are constants. The law of variation of Hubble's parameter yields a constant value of deceleration parameter and such a type of relation has already been considered by Bermann [48] for solving FRW models. Many relativists (Saha and Yadav [49], Singh and Kumar [50–52], Reddy *et al* [53], Adhav *et al* [54], Singh and Bhaghel [55]) have studied flat FRW and Bianchi type models by using the special law of Hubble's parameter that yield constant value of deceleration parameter. With the help of the special law of variation for Hubble's parameter, we obtain exact solutions of the field equations which represent an anisotropic cosmological model with negative constant deceleration parameter.

An important observational quantity is the deceleration parameter q, which is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2}. \tag{21}$$

From eqs (18) and (20), we have

$$\dot{R} = k_1 R^{-n+1}, \tag{22}$$

$$\ddot{R} = -k_1^2 (s-1) R^{-2n+1}. \tag{23}$$

Using eqs (21)–(23), we get constant values for the deceleration parameter for the mean scale factor as

$$q = s - 1 \quad \text{for } s \neq 0, \tag{24}$$

$$q = -1 \quad \text{for } s = 0. \tag{25}$$

The sign of q indicates whether or not the model accelerates or not. The positive sign of q (i.e. $s > 1$) corresponds to plain decelerating models whereas the negative sign of $-1 \leq q < 0$ indicates acceleration $q = 0$ and for $s = 1$ corresponds to growth with constant rate. These observations of SN Ia [1,2] and cosmic microwave background radiation favour fast models, i.e. $q < 0$. However, one does not altogether rule out the decelerating ones that are in keeping with these observations [56].

Using eq. (22), we obtain the law of average scale factor as

$$R = (Dt + c_1)^{1/s} \quad \text{for } s \neq 0, \tag{26}$$

and

$$R = c_2 \mathrm{e}^{k_1 t} \quad \text{for } s = 0, \tag{27}$$

where c_1 and c_2 are constants of integration.

From eq. (27), for $s \neq 0$, it is clear that the condition for accelerating expansion of Universe is $0 < s < 1$.

Case i: *Model for* $s \neq 0$ $(q \neq -1)$

From eqs (16), (18), (19) and (26), we get the following exact expression for the scale function:

$$A(t) = l_2 (Dt + c_1)^{n/r}, \tag{28}$$

$$B(t) = l_1 (Dt + c_1)^{1/r}, \tag{29}$$

where

$$l_1 = c_3^{-3/(2n+1)}, \quad l_2 = l_1^n \quad \text{and} \quad r = \frac{(2n+1)s}{3}.$$

Therefore, model (1) becomes

$$\mathrm{d}s^2 = \mathrm{d}t^2 - l_2^2 (Dt + c_1)^{2n/r} (\mathrm{d}x^2 + \mathrm{d}y^2) - l_1^2 (Dt + c_1)^{2/r} \mathrm{d}z^2. \tag{30}$$

The Universe (30) has no initial singularity, which represents the plane-symmetric magnetized dark energy model.

The expression for kinematical parameters, i.e. the Hubble's parameter H, the scalar expansion Θ, mean anisotropic parameter A_m and shear scalar σ for model (30) are given by

$$H = \frac{(2n+1)D}{3r(Dt + c_1)}, \tag{31}$$

$$\Theta = 3H = \frac{(2n+1)D}{r(Dt + c_1)}, \tag{32}$$

$$A_m = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_i - H}{H} \right)^2 = \frac{2(n-1)^2}{(2n+1)^2}, \tag{33}$$

$$\sigma^2 = \frac{3}{2} A_m H^2 = \frac{(n-1)^2 D^2}{3r^2 (Dt + c_1)^2}. \tag{34}$$

Equations (32) and (34) lead to

$$\frac{\sigma}{\Theta} = \frac{(n-1)}{\sqrt{3}(2n+1)}. \tag{35}$$

From the above results, it can be seen that the spatial volume is zero at $t = -(c_1/D)$, and it increases with the cosmic time. The Hubble's parameter is a decreasing function with increasing cosmic time. It is observed that the expansion scalar and shear scalar decreases with increasing cosmic time. Thus, the rate of expansion of the Universe is slowing down with the passage of time. At early stages, it was expanding with high rate. The nature of the fluid is dissipative at large time. The mean anisotropic parameter is uniform through the whole evolution of the Universe and does not depend on the cosmic time t.

The ratio

$$\frac{\sigma}{\Theta} = \frac{(n-1)}{\sqrt{3}(2n+1)},$$

shows that the Universe is anisotropic throughout the evolution.

Using eq. (28) in eq. (13), we obtain energy density for the magnetic field as

$$\rho_B = \frac{\beta}{l_2^4 (Dt + c_1)^{4n/r}}. \tag{36}$$

It is observed that the nature of energy density of the magnetic field decreases with increasing cosmic time (figure 1). It was large near $t = 0$ and tends to zero at large time.

Using eqs (11), (19), (28), (29) and (36), we obtain energy density for the fluid as

$$\rho = \frac{(n+2)D^2}{r^2 (Dt + c_1)^2} - \frac{\beta}{l_2^4 (Dt + c_1)^{4n/r}}. \tag{37}$$

Due to coupling of the dark energy and the magnetic field, the energy density decayed to be negative at the early stage of evolution. The weak energy condition is violated by the negative energy density and it does not violate any law of physics. The negative energy density indicates that there is vacuum instability in the inertial frames at early stages of evolution. The energy density may be vacuum in future. The energy density of the fluid was small initially compared to energy density of magnetism [57,58] (figure 2).

Using eqs (9), (19), (28), (29), (36) and (37) the equation of state parameter ω is obtained as

$$\omega = \frac{-1}{\rho} \left\{ \frac{D^2 (n^2 - nr + 1 - r + n)}{r^2 (Dt + c_1)^2} + \frac{\beta}{l_2^4 (Dt + c_1)^{4n/r}} \right\}. \tag{38}$$

It is observed that the equation of state ω is time-dependent and it can be a function of red-shift z or scale factor R as well. Figure 3 depicts the variation of EoS parameter vs. cosmic time for the accelerating phase of the Universe, and it is a representative

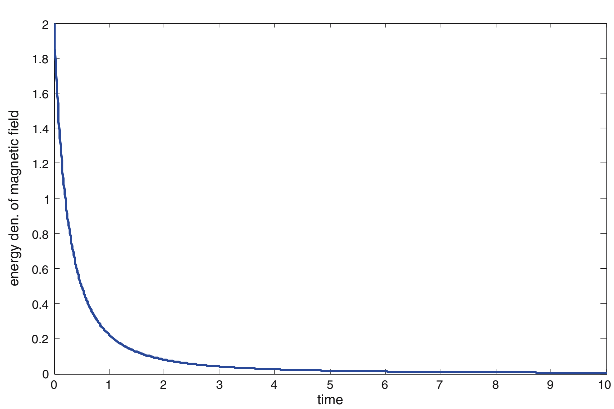

Figure 1. Energy density of the magnetic field vs. time.

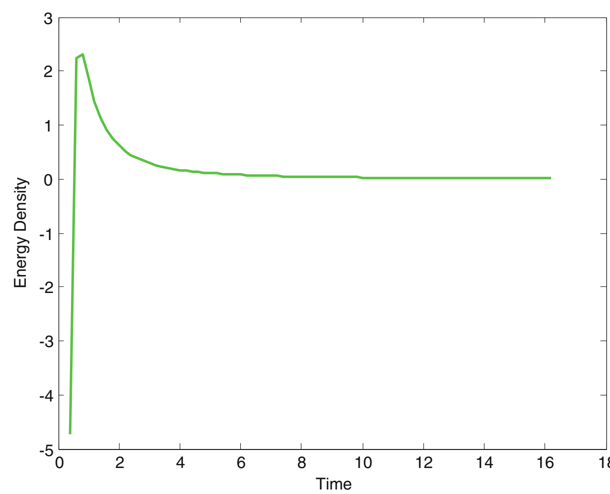

Figure 2. Energy density vs. time.

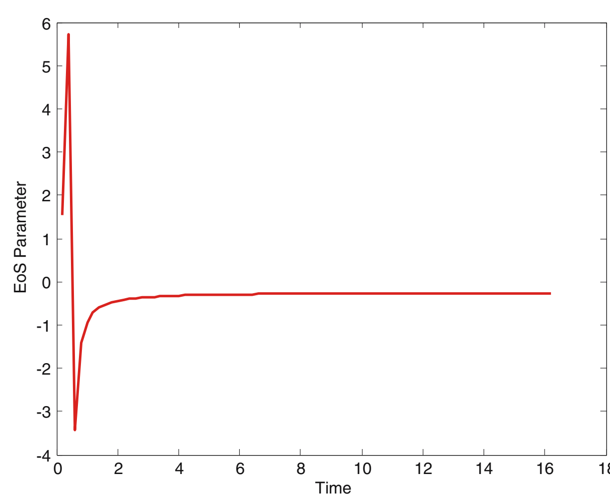

Figure 3. EoS parameter vs. time.

case with appropriate choice of constants of integration and other physical parameters. The time dependence of the EoS parameter allows it to transit from $\omega > -1$ to $\omega < -1$ [59]. Here, as we assume that the equation of state is the function of time, the dark energy affects the CMB at early time. The SN Ia data suggest that $-1.67 < \omega < -0.62$ [14] while the limit imposed on ω by a combination of SN Ia data (with CMB anisotropy) and galaxy clustering statistics is $-1.33 < \omega < -0.79$ [60]. At early times, i.e. $t = 0$ it tends to be positive, i.e. the Universe is dominated by matter. We conclude that at an initial epoch, the EoS parameter ω was positive (i.e. the Universe was matter-dominated) and as time increases it is evolving with negative value (i.e. at the present time). Figure 3 clearly shows that ω evolves within a range, which is in good agreement with SN Ia and CMB observations.

Using eqs (10), (19), (28), (29), (36), (37) and (38), the skewness parameter γ (i.e. deviation from ω along z-axis) is given by

$$\gamma = \frac{1}{\rho}\left\{\frac{(1-2r-2n^2+nr+n)D^2}{r^2\,(Dt+c_1)^2} + \frac{2\beta}{l_2^4(Dt+c_1)^{4n/r}}\right\}. \tag{39}$$

Expression (39) reveals that, $\gamma \to \infty$ as $t \to 0$ and as $t \to \infty$ then $\gamma \to 0$, i.e. at early stages of evolution, the fluid was anisotropic and at large time it isotropizes which is consistent with the expectation which is similar to Akarsu and Kilinc [61].

In the absence of magnetic field, i.e. $\beta \to 0$ the values of Hubble's parameter H, the scalar expansion Θ and shear scalar σ remain as it is and energy density for the magnetic field, energy density for the fluid, the EoS parameter ω and the skewness parameter γ are given by

$$\rho_B = 0,$$

$$\rho = \frac{(n+2)D^2}{r^2\,(Dt+c_1)^2},$$

$$\omega = \frac{-1}{\rho}\left\{\frac{D^2(n^2-nr+1-r+n)}{r^2\,(Dt+c_1)^2}\right\},$$

$$\gamma = \frac{1}{\rho}\left\{\frac{(1-2r-2n^2+nr+n)D^2}{r^2\,(Dt+c_1)^2}\right\}.$$

Case ii: *When* $s = 0\ (q = -1)$

From eqs (18), (19) and (27), we get the following exact expressions for the scale function:

$$A(t) = L_2 e^{nk_2 t}, \tag{40}$$

$$B(t) = L_1 e^{k_2 t}, \tag{41}$$

where

$$L_1 = \left(\frac{c_2}{c_4}\right)^{3/(2n+1)},$$

$$L_2 = L_1^n \quad \text{and} \quad k_2 = \frac{3k_1}{2n+1}.$$

Therefore, model (1) becomes

$$ds^2 = dt^2 - L_2^2 e^{2nk_2 t}(dx^2+dy^2) - L_1^2 e^{2k_2 t} dz^2. \tag{42}$$

The Universe (42) is free from singularity which represents the plane-symmetric magnetized dark energy Universe.

The expression for kinematical parameters, i.e. the Hubble's parameter H, the scalar expansion Θ, mean anisotropic parameter A_m and shear scalar σ for model (42) are given by

$$H = \frac{(2n+1)k_2}{3}, \tag{43}$$

$$\Theta = 3H = (2n+1)k_2, \tag{44}$$

$$A_m = \frac{2(n-1)^2}{(2n+1)^2}, \tag{45}$$

$$\sigma^2 = \frac{(n-1)^2 k_2^2}{3}. \tag{46}$$

Equations (44) and (46) give

$$\frac{\sigma}{\Theta} = \frac{(n-1)}{\sqrt{3}(2n+1)}. \tag{47}$$

From eq. (43), $(dH/dt) = 0$ implies the greatest value of the Hubble's parameter and the fastest rate expansion of the Universe. Thus, Universe (42) may represent the inflationary era in the early Universe. It is observed that throughout the evolution of the Universe, the expansion scalar is constant. The ratio

$$\frac{\sigma}{\Theta} = \frac{(n-1)}{\sqrt{3}(2n+1)},$$

shows that Universe is anisotropic throughout the evolution.

Using eq. (40) in eq. (13), we obtain energy density for the magnetic field as (figure 4)

$$\rho_B = \frac{\beta}{L_2^4 e^{4nk_2 t}}. \tag{48}$$

From eq. (48), it is observed that the energy density of the magnetic field is always positive and a decreasing function of time. The interesting point is that energy density of the magnetic field in our model is defined at $t = 0$ and we do not have any singularity.

Using eqs (11), (40), (41) and (48), we obtain the energy density for the fluid as

$$\rho = n(n+2)k_2^2 - \frac{\beta}{L_2^4 e^{4nk_2 t}}. \tag{49}$$

It is noted that the proper energy density could be a decreasing function of time and it approaches a tiny low value worth at this epoch. This behaviour is clearly represented in figure 5 as a representative case with acceptable alternative of constants of integration and alternative physical parameters exploitation moderately well-known things. Here it is found that the model begins with Big-Bang having infinite density and as time increases (for finite time) the energy density ρ tends to finite value. Hence after some finite value the models approach steady state.

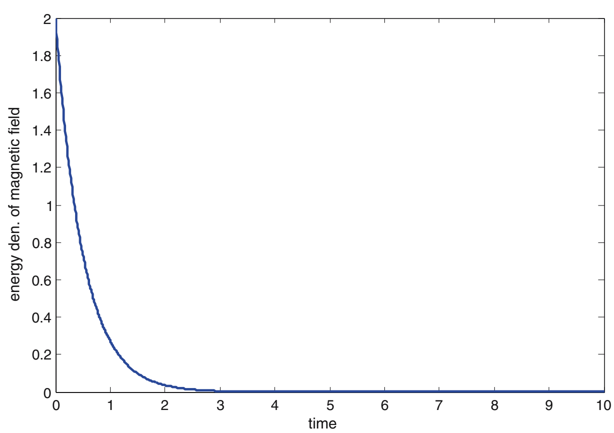

Figure 4. Energy density of the magnetic field vs. time.

Using eqs (9), (40), (41), (48) and (49), the EoS parameter ω is obtained as

$$\omega = \frac{-1}{\rho}\left\{(n^2+n+1)k_2^2 + \frac{\beta}{L_2^4 e^{4nk_2 t}}\right\}. \tag{50}$$

Figure 6 depicts the variation of EoS parameter ω vs. cosmic time as a representative case with appropriate choice of constants of integration and other physical parameters. It is shown that the growth of ω takes place with negative sign. Figure 6 clearly shows that ω evolves with negative and its range is in good agreement with large-scale structure data [13].

Using eqs (10), (40), (41), (48), (49) and (50), the skewness parameter γ (i.e. deviation from ω along the z-axis) is given by

$$\gamma = \frac{-1}{\rho}\left\{(1-2n^2+n)k_2^2 + \frac{2\beta}{L_2^4 e^{4nk_2 t}}\right\}. \tag{51}$$

In the absence of magnetic field, i.e $\beta \to 0$, the values of Hubble's parameter H, the scalar expansion Θ and shear scalar σ remain as they are and the energy density for the magnetic field, energy density for the fluid, the EoS parameter ω and the skewness parameter γ are given by

$$\rho_B = 0,$$

$$\rho = n(n+2)k_2^2,$$

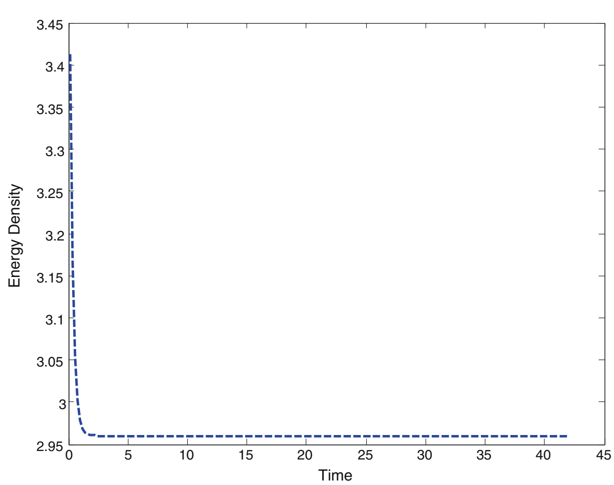

Figure 5. Energy density vs. time.

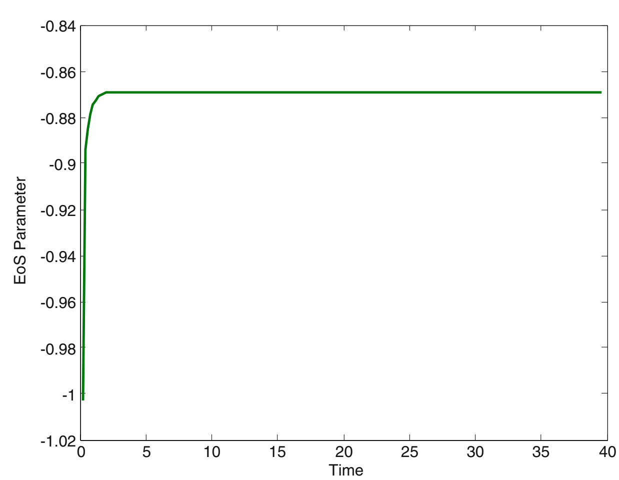

Figure 6. EoS parameter vs. time.

$$\omega = \frac{-1}{\rho}\{(n^2+n+1)k_2^2\},$$

$$\gamma = \frac{-1}{\rho}\{(1-2n^2+n)k_2^2\}.$$

4. Conclusions

(i) We have investigated magnetized anisotropic dark energy for plane-symmetric space–time in general relativity. For that purpose we have considered the special law of Hubble's parameter and analysed two models: (1) for $s \neq 0$ and (2) for $s = 0$.

(ii) We consider that the energy–momentum tensor consists of an anisotropic fluid with anisotropic EoS $p = \omega\rho$ and a uniform magnetic field of energy density ρ_B.

(iii) The law of variation for Hubble's parameter defined in (20) for plane-symmetric space–time model gives two types of cosmologies where the EoS parameter ω is a function of time: first one (for $s \neq 0$) shows the solution for positive value of deceleration parameter indicating the power-law expansion of the Universe, whereas second one (for $s = 0$) shows the solution for negative value of deceleration parameter, indicating the exponential expansion of the Universe.

(iv) In both cases we also discuss the parameters in the absence of magnetic field.

(v) It is observed that in both cases, EoS parameter ω is a variable function of time which has been supported by recent observations (Knop *et al* [14], Tegmark *et al* [15]). The EoS parameter of DE evolves within the range predicted by the observations.

(vi) In the derived model, the EoS parameter ω is evolving with negative sign which may be attributed to the current accelerated expansion of the Universe. Hence, from the theoretical perspective, the present model can be a viable model to explain the late-time acceleration of the Universe which resembles the investigations of Saha and Yadav [49].

(vii) We observed that, the model is free from Big-Bang singularity and the values of Hubble's parameter, expansion scalar and shear scalar are constants at the initial epoch and decrease with time approaching zero as $t \to \infty$.

(viii) The values of Hubble's parameter, expansion scalar and mean anisotropic parameter of the expansion and the shear scalar remain the same in the presence and the absence of magnetic field while the component of magnetic field reduces energy density of the anisotropic fluid.

(ix) It is interesting to note that our results resemble the results obtained by Yadav *et al* [22] and Sahoo and Mishra [31] in the absence of magnetic field.

This study will throw some light on the structure formation of the Universe, which has astrophysical significance.

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