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New analytical solutions for nonlinear physical models of the coupled Higgs equation and the Maccari system via rational $\exp(-\varphi(\eta))$-expansion method

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Abstract. In this article, a variety of solitary wave solutions are found for some nonlinear equations. In mathematical physics, we studied two complex systems, the Maccari system and the coupled Higgs field equation. We construct sufficient exact solutions for nonlinear evolution equations. To study travelling wave solutions, we used a fractional complex transform to convert the particular partial differential equation of fractional order into the corresponding partial differential equation and the rational $\exp(-\varphi(\eta))$-expansion method is implemented to find exact solutions of nonlinear equation. We find hyperbolic, trigonometric, rational and exponential function solutions using the above equation. The results of various studies show that the suggested method is very effective and can be used as an alternative for finding exact solutions of nonlinear equations in mathematical physics. A comparative study with the other methods gives validity to the technique and shows that the method provides additional solutions. Graphical representations along with the numerical data reinforce the efficacy of the procedure used. The specified idea is very effective, pragmatic for partial differential equations of fractional order and could be protracted to other physical phenomena.

Keywords. Rational $\exp(-\varphi(\eta))$-expansion method; coupled Higgs equation; Maccari system; travelling wave solutions; fractional calculus; Caputo's fractional derivative.

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1. Introduction

Most of the scientific problems and phenomena arise nonlinearly in various fields of mathematical physics and engineering, such as, solid mechanics, plasma physics, fluid dynamics, solid-state physics, biophysics, Higgs mechanism, etc. The nonlinear equations have a major role in looking into the internal mechanism of nonlinear physical phenomena in travelling wave solutions [1–38]. Due to exact elucidation of nonlinear proceedings, the fractional differential equations [1–8] have gained considerable importance. A fractional differential equation is a generalized form of classical differential equation of integer order. Recently, differential equations of fractional order have been attracting much attention due to their frequent applications in various fields. However, many effective techniques, such as the extended tanh-function method [9], the tanh-function method [10], the exp-function method and the $\exp(-\varphi(\eta))$-expansion method [11–15], the (G'/G)-expansion method [16], the homogeneous balance method [17], the auxiliary equation technique [18], the Jacobi elliptic function method [19], the Weierstrass elliptic function method [20], the modified exp-function method [21], the modified simple equation method [22], the F-expansion method [23], the homotopy perturbation method [24], the homotopy analysis technique [25], (G'/G)-expansion method for nonlinear evolution equations [30–36] and $\exp(-\Phi(\xi))$-expansion method [37, 38], have been used to study exact solutions of nonlinear equations.

The method has not been applied to the coupled Higgs equation and Maccari system in the given literature. The benefit of the suggested method over the other methods is that it delivers new exact solutions with additional free parameters. The travelling wave solutions have great significance to explore the core mechanism of the physical occurrences. When the particular values are substituted by related physical parameters, we gained kink, periodic and solitary wave solutions. The subject matter of the technique is that the exact solutions of partial differential equation can be articulated by a polynomial in $\exp(-\varphi(\eta))\cdot(\varphi'(\eta)) = \exp(-\varphi(\eta)) + \mu\exp(\varphi(\eta)) + \lambda$.

The paper is organized as follows: Section 2 gives the explanation of $\exp(-\varphi(\eta))$-expansion method; §3 gives the application of the method to establish the exact solutions of fractional order to the coupled Higgs equation and the Maccari system, and physical interpretations, §4 presents the results and §5 conclusions. At the end some references are given.

1.1 *Caputo's fractional derivative*

In modelling physical phenomena, using differential equation of fractional order, some drawbacks of Riemann–Liouville derivatives were observed. In this section we set up the notations and recall some significant possessions.

DEFINITION 1

A real function $f(x)$, $x > 0$ is supposed to be in space C_α, $\alpha \in \Re$, if there exists a real number p ($>\alpha$), such that

$$f(x) = x^p f_1(x), \quad \text{where } f_1(x) \in C[0, \infty). \tag{1}$$

DEFINITION 2

A real function $f(x)$, $x > 0$ is supposed to be in space C_α^m, $m \in \mathbb{N} \bigcup \{0\}$, if $f^{(m)} \in C_\alpha$.

DEFINITION 3

Let $f \in C_\alpha$ and $\alpha \geq -1$, then the (left-sided) Riemann–Liouville integral of order μ, $\mu > 0$ is given by

$$I_t^\mu f(x,t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-T)^{\mu-1} f(x,T)\mathrm{d}T, \quad t > 0. \tag{2}$$

DEFINITION 4

The (left-sided) Caputo partial fractional derivative of f with respect to t, $f \in C_{-1}^m$, $m \in \mathbb{N} \bigcup \{0\}$, is defined as

$$D_t^\mu f(x,t) = \frac{\partial^m}{\partial t^m} f(x,t), \quad \mu = m \tag{3}$$

$$D_t^\mu f(x,t) = I_t^{m-\mu} \frac{\partial^m}{\partial t^m} f(x,t), \quad m-1 \leq \mu < m, m \in N. \tag{4}$$

Note that

$$I_t^\mu D_t^\mu f(x,t) = f(x,t) - \sum_{k=0}^{m-1} \frac{\partial^k f}{\partial t^k}(x,0) \frac{t^k}{k!}, \quad m-1 < \mu \leq m, m \in \mathbb{N} \tag{5}$$

$$I_t^\mu t^\nu = \frac{\Gamma(\nu+1)}{\Gamma(\mu+\nu+1)} t^{\mu+\nu}. \tag{6}$$

2. Explanation of rational $\exp(-\varphi(\eta))$-expansion method

Now the $\exp(-\varphi(\eta))$-expansion method will be explained for constructing travelling wave solutions. Consider that the general fractional partial differential equation for $u(x,t)$ is given by

$$\phi(u, D_t^\alpha u, u_x, u_{xx}, D_t^{2\alpha} u, D_t^\alpha u_x, \ldots) = 0, \quad x \in R, t > 0, 0 \leq \alpha \leq 1, \tag{7}$$

where $D_t^\alpha u$, $D_x^\alpha u$, $D_{xx}^\alpha u$ are derivatives with respect to t, x, xx respectively and $u(\eta) = u(x,t)$, ϕ is a polynomial of u and its derivatives. Solving (7) by the above method we have the following steps:

Step 1: Using a transformation for eq. (7), we have

$$u = u(\eta), \quad \eta = x \pm V \frac{t^\alpha}{\Gamma(\alpha+1)}, \tag{8}$$

where V is a constant without zero and $u(\eta)$ is a function of η. Putting eq. (8) in eq. (7) and using the definition of fractional derivative, we get a nonlinear ODE for $u(\eta)$

$$\varphi(u, \pm V u', k u', V^2 u'', k^2 u'', \ldots) = 0. \tag{9}$$

Step 2: Now the solution of eq. (9) can be articulated as follows:

$$u(\eta) = \frac{\sum_{n=0}^M a_n \mathrm{e}^{-\varphi(\eta)}}{\sum_{n=0}^M b_n \mathrm{e}^{-\varphi(\eta)}}, \tag{10}$$

where a_n and b_n ($n = 0, 1, 2, \ldots, M$) are constants such that $a_n \neq 0$, $b_n \neq 0$ and $\varphi(\eta)$ satisfies the equation

$$\varphi'(\eta) = \mu \mathrm{e}^{\varphi(\eta)} + \mathrm{e}^{-\varphi(\eta)} + \lambda, \tag{11}$$

where prime denotes the derivative with respect to η. From eq. (11) we get

Family 1: When $\lambda^2 - 4\mu > 0$, we have

$$\varphi(\eta) = \ln\left\{\frac{1}{2\mu}\left(-\sqrt{(\lambda^2-4\mu)} \times \tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\eta + c_1)\right) - \lambda\right)\right\}.$$

Family 2: When $\lambda^2 - 4\mu < 0$, we have

$$\varphi(\eta) = \ln\left\{\frac{1}{2\mu}\left(\sqrt{(\lambda^2-4\mu)} \times \tan\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\eta + c_1)\right) - \lambda\right)\right\}.$$

Family 3: When $\lambda^2 - 4\mu > 0$, $\mu = 0$ and $\lambda \neq 0$,

$$\varphi(\eta) = -\ln\left\{\frac{\lambda}{\exp(\lambda(\eta+k)) - 1}\right\}.$$

Family 4: When $\lambda^2 - 4\mu = 0$, $\lambda \neq 0$ and $\mu \neq 0$,

$$\varphi(\eta) = \ln\left\{\frac{2(\lambda(\eta+k)+2)}{\lambda^2(\eta+k)}\right\}.$$

Family 5: When $\lambda^2 - 4\mu = 0$, $\lambda = 0$ and $\mu = 0$,

$$\varphi(\eta) = \ln(\eta + k).$$

Step 3: By considering the homogeneous principle, we obtain M. Considering eqs (10), (9) and (11), we have $e^{M\varphi(\eta)}$. We get algebraic equations with a_n, V, λ, μ, after comparing the same powers of $e^{\varphi(\eta)}$ to zero. We put the above values in eq. (10) and with eq. (11), we get some valuable travelling wave solutions of eq. (7).

3. Solution procedure

Consider the coupled Higgs equation given below:

$$D_t^{2\alpha}\rho - \rho_{xx} + |\rho|^2\rho = 2\rho\delta,$$

$$D_t^{2\alpha}\delta + \delta_{xx} = (|\rho|^2)_{xx}, \quad 0 < \alpha \leq 2. \tag{12}$$

Equation (12) describes a system of conserved scalar nucleon interaction with a neutral scalar meson where δ represents a real scalar meson field and ρ a complex scalar nucleon field. Equations (12) are related to some nonlinear models with physical interests. Consider the following transformation:

$$\rho(x,t) = e^{i\omega}u(\eta), \quad \delta(x,t) = v(\eta),$$

where

$$\omega = px + r\frac{t^\alpha}{\Gamma(\alpha+1)}, \quad \eta = x + c\frac{t^\alpha}{\Gamma(\alpha+1)}.$$

The coupled Higgs equation transforms into system of ODEs as

$$(c^2-1)u'' + (p^2 - r^2)u + u^3 = 2uv, \tag{13}$$

$$(c^2+1)v'' = (u^2)''. \tag{14}$$

Integrating eq. (14) then using it into eq. (13) we have

$$(c^4-1)u'' + (c^2+1)(p^2-r^2)u + (c^2-1)u^3 = 0,$$

where double prime denotes the derivative with respect to η. We get $M = 1$ if we follow homogeneous balancing phenomena, and therefore the suggested algorithm has the following trial solution:

$$u(\eta) = \frac{\sum_{n=0}^{1} a_n e^{-n\varphi(\eta)}}{\sum_{n=0}^{1} b_n e^{-n\varphi(\eta)}} = \frac{a_0 + a_1 e^{-\varphi(\eta)}}{b_0 + b_1 e^{-\varphi(\eta)}}. \tag{15}$$

By putting eq. (13) in eqs (12) and (11), we get an algebraic equation in $e^{\varphi(\eta)}$ given below:

$$-\frac{1}{\left(b_0 e^{\varphi(\eta)} + b_1\right)^3} \times \left(C_0 + C_1 e^{\varphi(\eta)} + C_2 e^{2\varphi(\eta)} + C_3 e^{3\varphi(\eta)}\right) = 0. \tag{16}$$

On comparing the coefficients, we obtain

$$[C_0 = 0, C_1 = 0, C_2 = 0, C_3 = 0]. \tag{17}$$

We obtain the following solutions by solving the system of eq. (17).

Solution 1

$$c = 1, \quad p = \pm r, \quad r = r, \quad a_0 = a_0,$$

$$a_1 = a_1, \quad b_0 = b_0, \quad b_1 = b_1.$$

Case 1: When $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, hyperbolic travelling wave solution is attained.

$$\rho_1(x,t) = e^{ir\left(\pm x + \frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{ \frac{a_0 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}{b_0 + b_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}} \right\},$$

$$\delta_1(x,t) = \frac{1}{2}\left\{ \frac{a_0 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}{b_0 + b_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}} \right\},$$

where c_1 is a constant.

Case 2: When $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, trigonometric solution is obtained.

$$\rho_2(x,t) = e^{ir\left(\pm x + \frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \left\{ \frac{a_0 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}{b_0 + b_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}} \right\},$$

$$\delta_2(x,t) = \frac{1}{2}\left\{ \frac{a_0 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}{b_0 + b_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}} \right\},$$

where c_1 is a constant.

Case 3: When $\mu = 0$ and $\lambda \neq 0$, exponential solution is obtained.

$$\rho_3(x,t) = e^{ir\left(\pm x + \frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{ \frac{a_0 + a_1\left[\frac{\lambda}{\exp\left(\lambda\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - 1}\right]^{-1}}{b_0 + b_1\left[\frac{\lambda}{\exp\left(\lambda\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - 1}\right]^{-1}} \right\},$$

$$\delta_3(x,t) = \frac{1}{2}\left\{ \frac{a_0 + a_1\left[\frac{\lambda}{\exp\left(\lambda\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - 1}\right]}{b_0 + b_1\left[\frac{\lambda}{\exp\left(\lambda\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - 1}\right]} \right\},$$

where c_1 is a constant.

Case 4: When $\lambda^2 - 4\mu = 0$, $\lambda \neq 0$ and $\mu \neq 0$, rational function solution is obtained.

$$\rho_4(x,t) = e^{ir\left(\pm x + \frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{ \frac{a_0 + a_1\left[\frac{2\left(\lambda\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right) + 2\right)}{\lambda^2\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)}\right]^{-1}}{b_0 + b_1\left[\frac{2\left(\lambda\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right) + 2\right)}{\lambda^2\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)}\right]^{-1}} \right\},$$

$$\delta_4(x,t) = \frac{1}{2}\left\{ \frac{a_0 + a_1\left[\frac{2\left(\lambda\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right) + 2\right)}{\lambda^2\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)}\right]^{-1}}{b_0 + b_1\left[\frac{2\left(\lambda\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right) + 2\right)}{\lambda^2\left(x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)}\right]^{-1}} \right\},$$

where c_1 is a constant.

Case 5: When $\lambda = 0$ and $\mu = 0$, rational function solution is obtained.

$$\rho_5(x,t) = \mathrm{e}^{ir\left(\pm x+\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{\frac{a_0 + a_1\left[x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right]^{-1}}{b_0 + b_1\left[x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right]^{-1}}\right\},$$

$$\delta_5(x,t) = \frac{1}{2}\left\{\frac{a_0 + a_1\left[x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right]^{-1}}{b_0 + b_1\left[x - \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right]^{-1}}\right\},$$

where c_1 is a constant.

Solution 2

$c = -1, \quad p = \pm r, \quad r = r, \quad a_0 = a_0,$
$a_1 = a_1, \quad b_0 = b_0, \quad b_1 = b_1.$

Case 1: When $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, hyperbolic travelling wave solution is attained.

$$\rho_6(x,t) = \mathrm{e}^{ir\left(\pm x+\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \left\{\frac{a_0 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x + \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}{b_0 + b_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x + \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}\right\},$$

$$\delta_6(x,t) = \frac{1}{2}\left\{\frac{a_0 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x + \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}{b_0 + b_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x + \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}\right\},$$

where c_1 is a constant.

Case 2: When $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, trigonometric solution is obtained.

$$\rho_7(x,t) = \mathrm{e}^{ir\left(\pm x+\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \left\{\frac{a_0 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x + \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}{b_0 + b_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x + \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}\right\},$$

$$\delta_7(x,t) = \frac{1}{2}\left\{\frac{a_0 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x + \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}{b_0 + b_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x + \frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - \lambda\right)\right]^{-1}}\right\},$$

where c_1 is a constant.

Case 3: When $\mu = 0$ and $\lambda \neq 0$, exponential solution is obtained.

$$\rho_8(x,t) = \mathrm{e}^{ir\left(\pm x+\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \left\{\frac{a_0 + a_1\left[\frac{\lambda}{\exp\left(\lambda\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-1}\right]^{-1}}{b_0 + b_1\left[\frac{\lambda}{\exp\left(\lambda\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-1}\right]^{-1}}\right\},$$

$$\delta_8(x,t) = \frac{1}{2}\left\{\frac{a_0 + a_1\left[\frac{\lambda}{\exp\left(\lambda\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-1}\right]}{b_0 + b_1\left[\frac{\lambda}{\exp\left(\lambda\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-1}\right]}\right\},$$

where c_1 is a constant.

Case 4: When $\lambda^2-4\mu=0$, $\lambda\neq 0$ and $\mu\neq 0$, rational function solution is obtained.

$$\rho_9(x,t) = e^{ir\left(\pm x+\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{\frac{a_0 + a_1\left[\frac{2\left(\lambda\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)+2\right)}{\lambda^2\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)}\right]^{-1}}{b_0 + b_1\left[\frac{2\left(\lambda\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)+2\right)}{\lambda^2\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)}\right]^{-1}}\right\},$$

$$\delta_9(x,t) = \frac{1}{2}\left\{\frac{a_0 + a_1\left[\frac{2\left(\lambda\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)+2\right)}{\lambda^2\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)}\right]^{-1}}{b_0 + b_1\left[\frac{2\left(\lambda\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)+2\right)}{\lambda^2\left(x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)}\right]^{-1}}\right\},$$

where c_1 is a constant.

Case 5: When $\lambda = 0$ and $\mu = 0$, rational function solution is obtained.

$$\rho_{10}(x,t) = e^{ir\left(\pm x+\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{\frac{a_0 + a_1\left[x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right]^{-1}}{b_0 + b_1\left[x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right]^{-1}}\right\},$$

$$\delta_{10}(x,t) = \frac{1}{2}\left\{\frac{a_0 + a_1\left[x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right]^{-1}}{b_0 + b_1\left[x+\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right]^{-1}}\right\},$$

where c_1 is a constant.

Solution 3

$$c = c,\quad p = \pm\frac{1}{2}\sqrt{2(c^2-1)(\lambda^2-4\mu)+4r^2},$$
$$r = r,\quad a_0 = \pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1},$$
$$a_1 = \mp\sqrt{2(c^2+1)},\quad b_0 = b_0,\quad b_1 = 0.$$

Case 1: When $\lambda^2-4\mu>0$ and $\mu\neq 0$, hyperbolic travelling wave solution is attained.

$$\rho_{11}(x,t) = e^{ir\left(px+r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{\frac{\pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1}\mp\sqrt{2(c^2+1)}\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x-c\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-\lambda\right)\right]^{-1}}{b_0}\right\},$$

$$\delta_{11}(x,t) = \frac{1}{2}\left\{\frac{\pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1}\mp\sqrt{2(c^2+1)}\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x-c\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-\lambda\right)\right]^{-1}}{b_0}\right\},$$

where c_1 is a constant.

Case 2: When $\lambda^2-4\mu<0$ and $\mu\neq 0$, trigonometric solution is obtained.

$$\rho_{12}(x,t) = e^{i\left(px+r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{\frac{\pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1}\mp\sqrt{2(c^2+1)}\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x-c\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-\lambda\right)\right]^{-1}}{b_0}\right\},$$

$$\delta_{12}(x,t) = \frac{1}{2}\left\{\frac{\pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1}\mp\sqrt{2(c^2+1)}\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x-c\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-\lambda\right)\right]^{-1}}{b_0}\right\},$$

where c_1 is a constant.

Case 3: When $\mu = 0$ and $\lambda \neq 0$, exponential solution is obtained.

$$\rho_{13}(x,t) = e^{i\left(px + r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \left\{ \frac{\pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1} \mp \sqrt{2(c^2+1)}\left[\frac{\lambda}{\exp\left(\lambda\left(x - c\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - 1}\right]^{-1}}{b_0} \right\},$$

$$\delta_{13}(x,t) = \frac{1}{2}\left\{ \frac{\pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1} \mp \sqrt{2(c^2+1)}\left[\frac{\lambda}{\exp\left(\lambda\left(x - c\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)\right) - 1}\right]}{b_0} \right\},$$

where c_1 is a constant.

Case 4: When $\lambda^2 - 4\mu = 0$, $\lambda \neq 0$ and $\mu \neq 0$, rational function solution is obtained.

$$\rho_{14}(x,t) = e^{i\left(px + r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \left\{ \frac{\pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1} \mp \sqrt{2(c^2+1)}\left[\frac{2\left(\lambda\left(x - c\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right) + 2\right)}{\lambda^2\left(x - c\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)}\right]^{-1}}{b_0} \right\},$$

$$\delta_{14}(x,t) = \frac{1}{2}\left\{ \frac{\pm\frac{\lambda b_0}{\sqrt{2}}\sqrt{c^2+1} \mp \sqrt{2(c^2+1)}\left[\frac{2\left(\lambda\left(x - c\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right) + 2\right)}{\lambda^2\left(x - c\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right)}\right]^{-1}}{b_0} \right\},$$

where c_1 is a constant.

Case 5: When $\lambda = 0$ and $\mu = 0$, rational function solution is obtained.

$$\rho_{15}(x,t) = e^{i\left(px + r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{ \frac{\mp\sqrt{2(c^2+1)}\left[x - c\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right]^{-1}}{b_0} \right\},$$

$$\delta_{15}(x,t) = \frac{1}{2}\left\{ \frac{\mp\sqrt{2(c^2+1)}\left[x - c\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right]^{-1}}{b_0} \right\},$$

where c_1 is a constant.

Solution 4

$$c = c, \quad p = \pm\frac{1}{2}\sqrt{2(c^2-1)(\lambda^2-4\mu) + 4r^2},$$

$$r = r, \quad a_0 = \frac{1}{4}\sqrt{-2(c^2+1)}(\lambda^2 - 4\mu)b_1,$$

$$a_1 = 0, \quad b_0 = \frac{1}{2}\lambda b_1, \quad b_1 = b_1.$$

Solution 5

$$c = c, \quad p = \pm\frac{1}{2}\sqrt{2(c^2-1)(\lambda^2-4\mu) + 4r^2},$$

$$r = r, \quad a_0 = -\frac{1}{4}\sqrt{-2(c^2+1)}(\lambda^2 - 4\mu)b_1,$$

$$a_1 = 0, \quad b_0 = \frac{1}{2}\lambda b_1, \quad b_1 = b_1.$$

Solution 6

$$c = c, \quad p = \pm\frac{1}{2}\sqrt{2(c^2-1)(\lambda^2-4\mu) + 4r^2},$$

$$r = r, \quad a_0 = -\frac{\lambda b_0}{\sqrt{-2}}\sqrt{c^2+1},$$

$$a_1 = -\sqrt{-2(c^2+1)}, \quad b_0 = b_0, \quad b_1 = 0.$$

Solution 7

$$c = c, \quad p = \pm\frac{1}{2}\sqrt{2(c^2-1)(\lambda^2-4\mu)+4r^2},$$

$$r = r, \quad a_0 = -\frac{1}{2}\sqrt{-2(c^2+1)}(\lambda b_0 - 2\mu b_1),$$

$$a_1 = \frac{1}{\sqrt{2}}\sqrt{-2(c^2+1)}(\lambda b_1 - 2b_0), \quad b_0 = b_0, \quad b_1 = b_1.$$

Solution 8

$$c = c, \quad p = \pm\frac{1}{2}\sqrt{2(c^2-1)(\lambda^2-4\mu)+4r^2},$$

$$r = r, \quad a_0 = \frac{1}{2}\sqrt{-2(c^2+1)}(\lambda b_0 - 2\mu b_1),$$

$$a_1 = \frac{1}{\sqrt{2}}\sqrt{-2(c^2+1)}(\lambda b_1 - 2b_0), \quad b_0 = b_0, \quad b_1 = b_1.$$

Similarly, we have other solutions from Solutions 4 to Solution 8.

Graphical illustrations of the results are given in figures 1–12.

Consider the Maccari system of equations given below:

$$D_t^\alpha U - U_{xx} = -UV,$$

$$D_t^\alpha V + V_y = -(|U|^2)_x, \quad 0 < \alpha \le 1. \tag{18}$$

$$U(x, y, t) = \mathrm{e}^{i\omega} u(\eta), V(x, y, t) = v(\eta).$$

This system brings nonlinear evolution equations that are frequently used to describe location in a small part of space, and motion of the isolated waves in varied

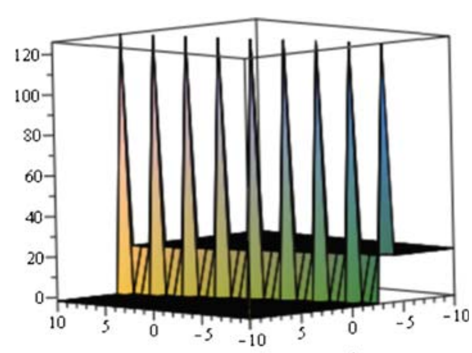

Figure 1. Kink wave solution ρ_1 when $\lambda = 11, \mu = 5, a_0 = 11, a_1 = -0.5, b_0 = 1, b_1 = 1, V = 3, \alpha = 1$.

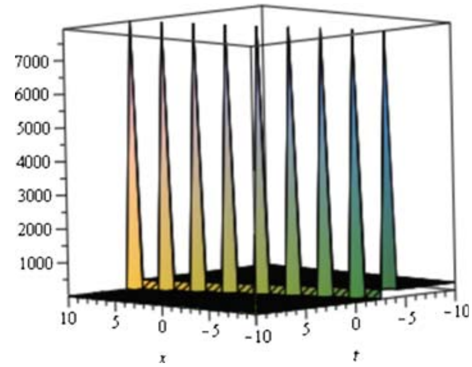

Figure 2. Kink wave solution δ_1 when $\lambda = 11, \mu = 5, a_0 = 11, a_1 = -0.5, b_0 = 1, b_1 = 1, V = 3, \alpha = 1$.

fields, such as nonlinear optics, fluid mechanics, quantum field theory, and plasma physics. Consider the following transformation where

$$\omega = px + qy + r\frac{t^\alpha}{\Gamma(\alpha+1)}, \quad \eta = x + y + c\frac{t^\alpha}{\Gamma(\alpha+1)}.$$

The Maccari system of equation transforms into system of ODEs as

$$u'' + (r + p^2)u = -uv, \tag{19}$$

$$(c+1)v' + (u^2)' = 0. \tag{20}$$

Integrating eq. (20) then using it into eq. (19) we have

$$(c+1)u'' - (c+1)(r+p^2)u - u^3 = 0,$$

where double prime denotes the derivative with respect to η. We get $M = 1$ if we follow homogeneous balancing phenomena, and therefore the suggested algorithm has the following trial solution:

$$u(\eta) = \frac{\sum_{n=0}^{1} a_n \mathrm{e}^{-n\varphi(\eta)}}{\sum_{n=0}^{1} b_n \mathrm{e}^{-n\varphi(\eta)}} = \frac{a_0 + a_1 \mathrm{e}^{-\varphi(\eta)}}{b_0 + b_1 \mathrm{e}^{-\varphi(\eta)}}. \tag{21}$$

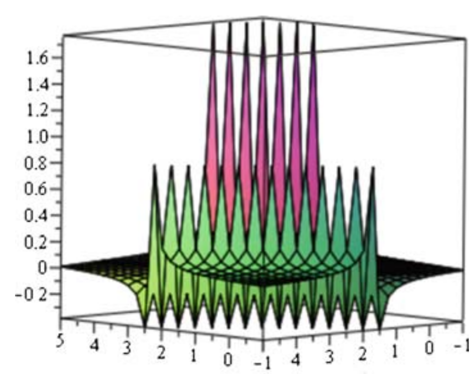

Figure 3. Periodic wave solution ρ_2 when $\lambda = 1.8, \mu = 1, a_0 = 1, a_1 = 1, b_0 = 11, b_1 = 21, V = 1, c = 1, \alpha = 1$.

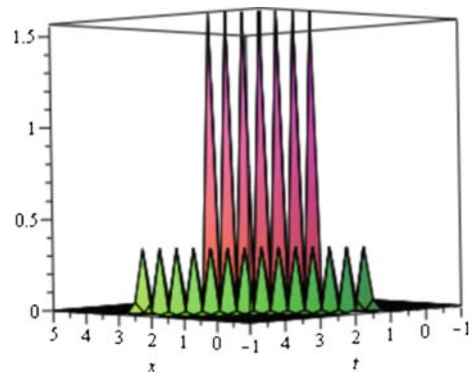

Figure 4. Kink wave solution δ_1 when $\lambda = 1.8$, $\mu = 1$, $a_0 = a_1 = 1$, $b_0 = 11$, $b_1 = 21$, $V = 1$, $c = 1$, $\alpha = 1$.

By putting eq. (19) in eqs (18) and (11), we get an algebraic equation in $e^{\varphi(\eta)}$ given below:

$$-\frac{1}{\left(b_0 e^{\varphi(\eta)} + b_1\right)^3} \times \left(C_0 + C_1 e^{\varphi(\eta)} + C_2 e^{2\varphi(\eta)} + C_3 e^{3\varphi(\eta)}\right) = 0. \quad (22)$$

On comparing the coefficients, we obtain

$$[C_0 = 0, C_1 = 0, C_2 = 0, C_3 = 0]. \quad (23)$$

We obtain the following solutions by solving the system of eq. (23).

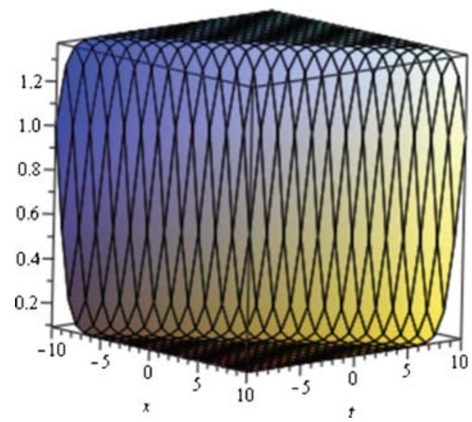

Figure 5. Singular kink wave solution ρ_3 when $\lambda = 1.8$, $a_0 = 1$, $a_1 = 21$, $b_0 = 11$, $b_1 = 21$, $V = 1$, $c = 1$, $\alpha = 1$.

Solution 1

$$c = \frac{1}{2}\frac{a_1^2 - 2b_0^2}{b_0^2}, \quad p = p, \quad q = q,$$

$$r = -\frac{1}{2}\lambda^2 + p^2 + 2\mu, \quad a_0 = \frac{1}{2}\lambda a_1,$$

$$a_1 = a_1, \quad b_0 = b_0, \quad b_1 = 0.$$

Case 1: When $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, hyperbolic travelling wave solution is attained.

$$U_{16}(x,y,t) = e^{i\left(px+qy+r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{\frac{\frac{1}{2}\lambda a_1 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-\lambda\right)\right]^{-1}}{b_0}\right\},$$

$$V_{16}(x,y,t) = \frac{1}{2}\left\{\frac{\frac{1}{2}\lambda a_1 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-\lambda\right)\right]^{-1}}{b_0}\right\},$$

where c_1 is a constant.

Case 2: When $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, trigonometric solution is obtained.

$$U_{17}(x,y,t) = e^{i\left(px+qy+r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \times \left\{\frac{\frac{1}{2}\lambda a_1 + a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-\lambda\right)\right]^{-1}}{b_0}\right\},$$

$$V_{17}(x,y,t)=\frac{1}{2}\left\{\frac{\frac{1}{2}\lambda a_1+a_1\left[\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-\lambda\right)\right]^{-1}}{b_0}\right\},$$

where c_1 is a constant.

Case 3: When $\mu=0$ and $\lambda\neq 0$, exponential solution is obtained.

$$U_{18}(x,y,t)=e^{i\left(px+qy+r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)}\left\{\frac{\frac{1}{2}\lambda a_1+a_1\left[\frac{\lambda}{\exp\left(\lambda\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-1}\right]^{-1}}{b_0}\right\},$$

$$V_{18}(x,y,t)=\frac{1}{2}\left\{\frac{\frac{1}{2}\lambda a_1+a_1\left[\frac{\lambda}{\exp\left(\lambda\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)\right)-1}\right]}{b_0}\right\},$$

where c_1 is a constant.

Case 4: When $\lambda^2-4\mu=0$, $\lambda\neq 0$ and $\mu\neq 0$, rational function solution is obtained.

$$U_{19}(x,y,t)=e^{i\left(px+qy+r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)}\left\{\frac{\frac{1}{2}\lambda a_1+a_1\left[\frac{2\left(\lambda\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)+2\right)}{\lambda^2\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)}\right]^{-1}}{b_0}\right\},$$

$$V_{19}(x,y,t)=\frac{1}{2}\left\{\frac{\frac{1}{2}\lambda a_1+a_1\left[\frac{2\left(\lambda\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)+2\right)}{\lambda^2\left(x+y-\frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)}+c_1\right)}\right]^{-1}}{b_0}\right\},$$

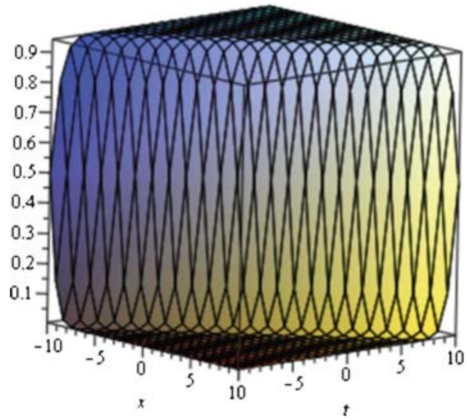

Figure 6. Kink wave solution δ_1 when $\lambda = 1.8, a_0 = 1, a_1 = 21, b_0 = 11, b_1 = 21, V = 1, c = 1, \alpha = 1$.

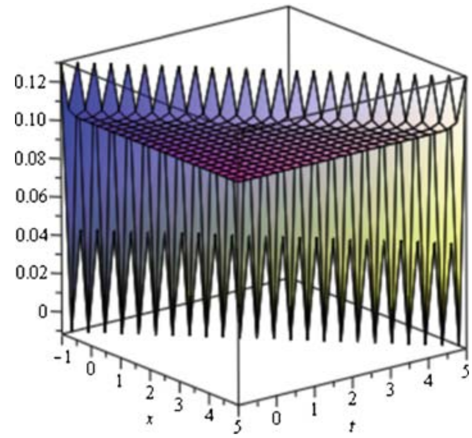

Figure 7. Singular kink wave solution ρ_6 when $\lambda = 8, \mu = 1, a_0 = a_1 = 1, b_0 = 11, b_1 = 21, V = 1, c = 1, \alpha = 1$.

where c_1 is a constant.

Case 5: When $\lambda = 0$ and $\mu = 0$, rational function solution is obtained.

$$U_{20}(x, y, t) = \mathrm{e}^{i\left(px+qy+r\frac{t^\alpha}{\Gamma(\alpha+1)}\right)} \left\{ \frac{a_1\left[x + y - \frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right]^{-1}}{b_0} \right\},$$

$$V_{20}(x,y,t) = \frac{1}{2}\left\{ \frac{a_1\left[x + y - \frac{1}{2}\frac{a_1^2-2b_0^2}{b_0^2}\frac{t^\alpha}{\Gamma(\alpha+1)} + c_1\right]^{-1}}{b_0} \right\},$$

where c_1 is a constant.

Solution 2

$$c = c, \quad p = p, \quad q = q,$$

$$r = -\frac{1}{2}\lambda^2 + p^2 + 2\mu,$$

$$a_0 = \mp\frac{1}{2}\sqrt{2\,(1+c)}\,(\lambda b_0 - 2\mu b_1),$$

$$a_1 = \pm\frac{1}{2}\sqrt{2(1+c)}(\lambda b_1 - 2b_0),$$

$$b_0 = b_0, \quad b_1 = b_1.$$

Solution 3

$$c = -\frac{\lambda^4 b_1^2 - 8\lambda^2\mu b_1^2 + 16\mu^2 b_1^2 - 8a_0^2}{b_1^2(\lambda^2 - 8\lambda^2\mu + 16\mu^2)},$$

$$p = p, \quad q = q, \quad r = -\frac{1}{2}\lambda^2 + p^2 + 2\mu,$$

$$a_0 = a_0, \quad a_1 = 0, \quad b_0 = \frac{1}{2}\lambda b_1, \quad b_1 = b_1.$$

In the same way, we can explore the other results for Solutions 2 and 3.

4. Results and discussion

The proposed technique provides abundant new solitary wave solutions with some free parameters. The travelling wave solutions have extensive significance in interpreting the inner structures of natural phenomena. We have explained different types of solitary wave solutions by setting the physical parameters as special values. However, in this study, we protracted the rational $\exp(-\varphi(\eta))$-expansion method for finding new results of these physical phenomena. Hafez *et al* [26]

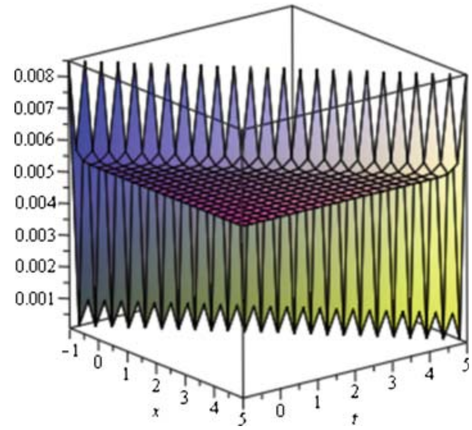

Figure 8. Kink wave solution δ_6 when $\lambda = 1.8$, $\mu = 1$, $a_0 = a_1 = 1$, $b_0 = 11$, $b_1 = 21$, $V = 1$, $c = 1$, $\alpha = 1$.

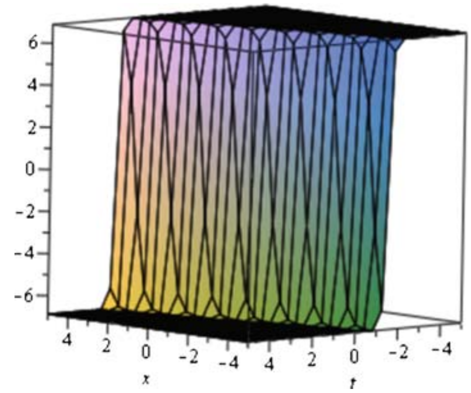

Figure 9. Kink wave solution U_{16} when $\lambda = 5$, $\mu = 1$, $a_0 = 1$, $a_1 = 3$, $V = 3$, $b_1 = 0.5$, $c = 1$, $b_0 = 1$, $\alpha = 1$.

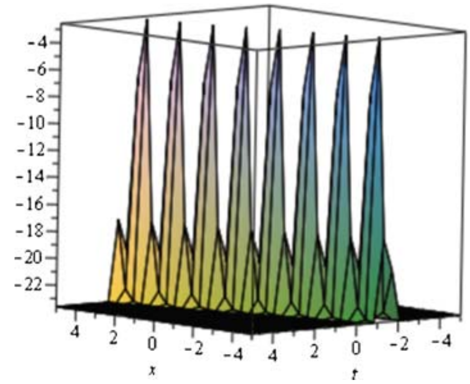

Figure 10. Kink wave solution V_{16} when $\lambda = 5$, $\mu = 1$, $a_0 = 1$, $a_1 = 3$, $V = 3$, $b_1 = 0.5$, $c = 1$, $b_0 = 1$, $\alpha = 1$.

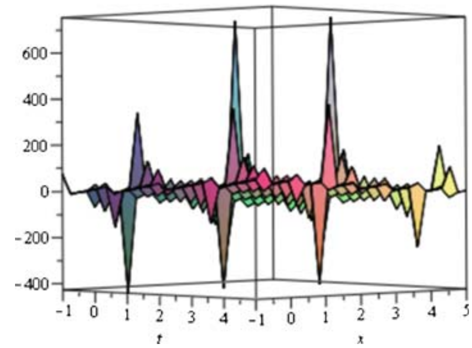

Figure 11. Kink wave solution U_{17} when $\lambda = 5$, $\mu = 11$, $a_0 = 10$, $a_1 = 3$, $V = 10$, $b_1 = 5$, $c = 10$, $b_0 = -1$, $\alpha = 1$.

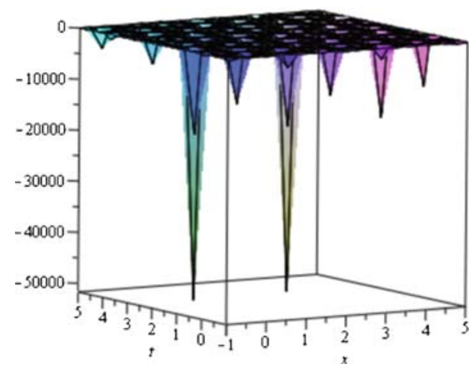

Figure 12. Kink wave solution V_{17} when $\lambda = 5$, $\mu = 11$, $a_0 = 10$, $a_1 = 3$, $V = 10$, $b_1 = 5$, $c = 10$, $b_0 = -1$, $\alpha = 1$.

used $\exp(-\varphi(\eta))$-expansion method to establish solitary wave solutions of coupled Higgs and Maccari system of equations. It is to be noted that for $\alpha = 1$, $b_0 = 1$ and $b_1 = 0$ our solutions $\rho_i(x,t)$, $\delta_i(x,t)$, $i = 1,\ldots,5$ are equivalent to $u_{2i-1}(x,t)$, $v_{2i-1}(x,t)$, $i = 1,\ldots,5$, for $\alpha = 1$, $b_0 = 1$ and $b_1 = 0$, our solutions $\rho_i(x,t)$, $\delta_i(x,t)$, $i = 5,\ldots,10$ are comparable to $u_{2i}(x,t)$, $v_{2i}(x,t)$, $i = 1,\ldots 5$, for $\alpha = 1$ and $b_0 = 1$, solutions $\rho_i(x,t)$, $\delta_i(x,t)$, $i = 11,\ldots,15$ are similar to $u_i(x,t)$, $v_i(x,t)$, $i = 11,\ldots,15$ for $\alpha = 1$ and $b_0 = 1$, solutions $U_i(x,y,t)$, $V_i(x,y,t)$, $i = 16,\ldots,20$ are similar to $u_i(x,y,t)$, $v_i(x,y,t)$, $i = 16,\ldots,20$. Lee and Sakthivel [28] used the Kudryashov method for finding travelling wave solutions of the Maccari system and Higgs equation. $\delta_3(x,t)$, $\rho_8(x,t)$, $\delta_8(x,t)$, $\rho_4(x,t)$ and $\delta_4(x,t)$ obtained in this article are similar to $u_i(x,t)$, $v_i(x,t)$, $i = 1,\ldots,3$, for the Higgs equations. For the Maccari system the solutions $u_{16}(x,y,t)$ and $v_{13}(x,y,t)$ found in [28] are equivalent to $\rho_{16}(x,t)$ and $\delta_{16}(x,t)$. Figures 13–16 show solitary wave solutions for different values of parameters. New solutions could be obtained from the remaining solution sets.

In ref. [29], the generalized Maccari system is condensed to the Maccari system (18). Ahmed *et al* used the Lie symmetry analysis and mapping method to gain solitary type solutions of the generalized Maccari system. In this study, by using appropriate variation of the physical parameters, most of our solutions show solitary wave solutions and solitons. Therefore, in ref. [27], we observe that with the specific selection of

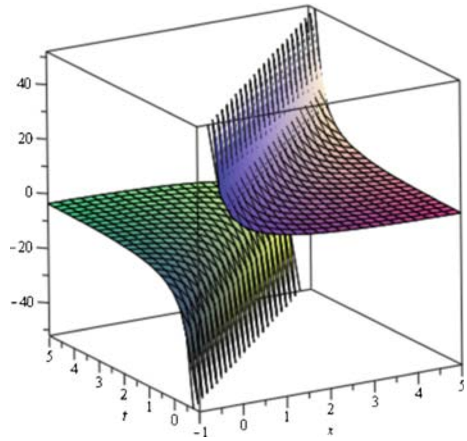

Figure 13. Kink wave solution U_{18} when $\lambda = 0.5$, $a_1 = 13$, $V = 1$, $c = 0.1$, $b_0 = 1$, $\alpha = 1$.

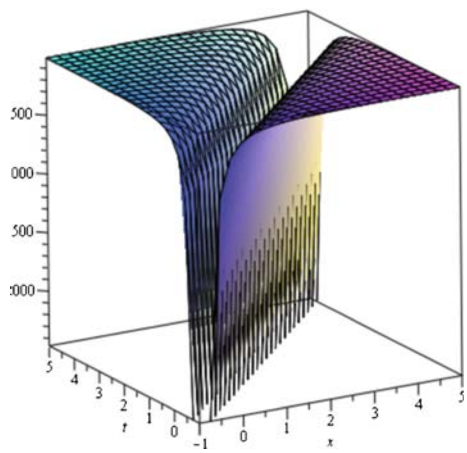

Figure 14. Kink wave solution V_{18} when $\lambda = 0.5$, $a_1 = 13$, $V = 1$, $c = 0.1$, $b_0 = 1$, $\alpha = 1$.

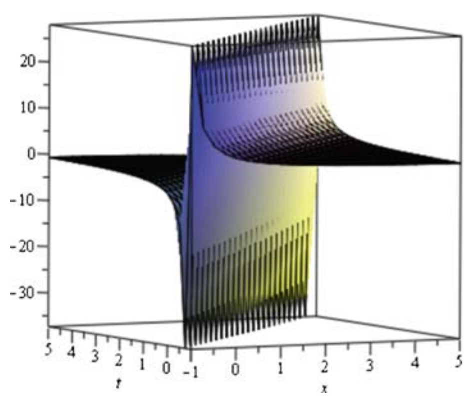

Figure 15. Singular solitary wave solution U_{19} when $\lambda = 7$, $a_1 = 8$, $V = 1$, $c = 1$, $b_0 = 1$, $\alpha = 1$.

particular values of physical parameters, some of our results overlap with the particular solutions obtained by the former techniques. The graphical demonstrations

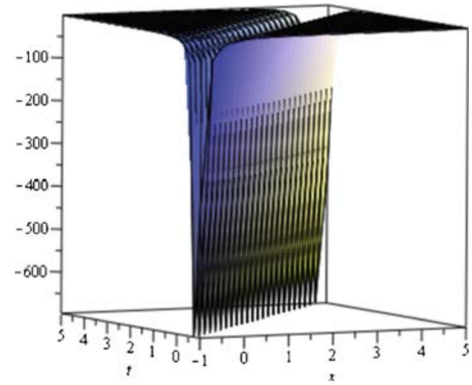

Figure 16. Singular solitary wave solution V_{19} when $\lambda = 7$, $a_1 = 8$, $V = 1$, $c = 1$, $b_0 = 1$, $\alpha = 1$.

show the behaviours of oscillatory particles which gratify the equations mentioned above. Using this modified algorithm called rational $\exp(-\varphi(\eta))$-expansion method, we obtained some new solitary wave solutions. Therefore, the rational $\exp(-\varphi(\eta))$-expansion method provides some new exact solutions which are not found in other literature. All the solutions present in this article are checked using *Maple 2015*, putting back into the original equation.

5. Conclusion

In this work, with the help of a suitable transformation and the rational $\exp(-\varphi(\eta))$-expansion method, we have gained abundant analytical solutions of the Maccari system and coupled Higgs equation. The obtained solutions show that the suggested technique can solve equations of fractional order. It is noticed that the suggested method fully validates the competence and reliability of computational work and may be utilized for other physical problems. The procedure implemented demonstrates that the rational $\exp(-\varphi(\eta))$-expansion method is an improvement and better development in the exactness of the $\exp(-\varphi(\eta))$-expansion method. Accordingly, for solving nonlinear problems, the suggested technique would be expected to perform better, with more exact solutions compared to the traditional $\exp(-\varphi(\eta))$-expansion method.

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