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# Photodetachment of $H^-$ ion in crossed gradient electric and magnetic fields

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**Abstract.** We study the photodetachment of $H^-$ ion in crossed gradient electric and magnetic fields and put forward an analytical formula for calculating the photodetachment cross-section. Compared to the photodetachment of $H^-$ ion in a gradient electric field, the Hamiltonian of the detached electron has three degrees of freedom, which makes the dynamical behaviour of the detached electron complex. Photodetachment cross-section for various external fields and the laser polarization are calculated and displayed. A comparison with the photodetachment cross-section in crossed uniform electric and magnetic fields or in a single gradient electric field has been made. The agreement of our results with the above two special cases suggests the correctness of our calculation. Our study may have some potential applications in the photodetachment microscopy experiment or in ion detection.

**Keywords.** Photodetachment; gradient electric field; magnetic field; closed orbit theory.

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## 1. Introduction

Early researchers showed that the photodetachment cross-section of $H^-$ ion in the presence of external fields displays a 'ripple' structure, which can be considered as a smooth background term plus many sinusoidal oscillations. A fairly complete calculation of these oscillations under various environments has been carried out by many researchers at the quantum and the semiclassical levels [1–7]. Among these previous studies, closed orbit theory has rapidly become one of the most useful and intuitive semiclassical method as it makes a very direct connection between the photodetachment cross-section and the closed orbit of the corresponding classical system [7,8]. Many researchers have used this theory to study the photodetachment of $H^-$ ion in different external electric and magnetic fields [9–13]. In these early studies, the external fields are homogeneous. As to the photodetachment of negative ions in inhomogeneous external fields, the studies are relatively few. However, there are some necessities to conduct such studies. For example, in the ion trap experiment, the electric field in the Paul trap consists of a large-gradient electric field plus a small oscillating field [14]. Under the condition of zero-order approximation, the inhomogeneous external field in the Paul trap can be treated as a gradient electric field. In 1999, Yang *et al* first studied the photodetachment of $H^-$ ion in a gradient electric field using the semiclassical closed orbit theory [15]. Later, Wu and his coworkers used the traditional quantum-mechanical approach to calculate the photodetachment cross-section of the same system [16]. The agreement of their result with Yang *et al*'s semiclassical result gives an eloquent proof of the validity of the closed orbit theory. Very recently, our group has studied the photodetachment of $H^-$ ion in parallel gradient electric and magnetic fields [17]. In these studies, the detached electron's Hamiltonian has a cylindrical symmetry and the detached electron's movement is relatively simple. As to the photodetachment of $H^-$ ion in crossed electric and magnetic fields, the cylindrical symmetry in the Hamiltonian has been broken. The Hamiltonian of the detached electron has three degrees of freedom. Therefore, the dynamical behaviour of the corresponding systems will become complex. In early 1993, Peters and Delos have calculated the photodetachment cross-section of $H^-$ ion in crossed homogeneous electric and magnetic fields [10]. For the photodetachment of $H^-$ ion in crossed inhomogeneous electric and magnetic fields, no report has yet been put forward. In this research, we calculate the photodetachment cross-section of $H^-$ ion in crossed gradient electric and magnetic fields using the semiclassical closed orbit theory. Our result suggests that the photodetachment cross-section of this system depends not only on the external gradient electric field and magnetic field strength, but also related to the laser polarization. Atomic units (abbreviated as a.u.) are used throughout this paper unless indicated otherwise.

## 2. The Hamiltonian and classical motion of the detached electron

The schematic plot of $H^-$ ion in crossed gradient electric and magnetic fields is shown in figure 1. The hydrogen atom is localized at the origin. The interaction potential $V_b(r)$ between the active electron and hydrogen atom is a spherically symmetric, short-ranged potential. The uniform magnetic field $H_0$ is along the $+z$ axis and the gradient electric field is along the $x$-axis, which is given by [15]

$$F = F_0 + \alpha x, \tag{1}$$

where $F_0$ is the strength of a 'background' electric field, $\alpha$ is the electric field gradient along the $x$-axis, $F_0$ and $\alpha$ are positive constants.

The detached electron's Hamiltonian is given by

$$H = \frac{1}{2}\left[P + \frac{1}{c}\mathbf{A}\right]^2 + F_0 x + \frac{1}{2}\alpha x^2 + V_b(r), \tag{2}$$

where $\mathbf{A}$ is the vector potential, which is given by $\mathbf{A} = H_0 x \hat{j}$. After the electron is photodetached from hydrogen atom, the short-ranged potential $V_b(r)$ can be neglected. Then the above Hamiltonian can be further simplified as

$$H = \frac{1}{2}P_x^2 + \frac{1}{2}P_y^2 + \frac{1}{2}P_z^2 + \frac{1}{2}\omega_l^2 x^2 + (\omega_B p_y + F_0)x, \tag{3}$$

where $\omega_B = H_0/c$ is the cyclotron frequency of the electron and $\omega_l = \sqrt{\omega_B^2 + \alpha}$.

By solving the classical Hamiltonian motion equations:

$$\begin{cases} \dot{x} = \partial H/\partial P_x = P_x, \\ \dot{y} = \partial H/\partial P_y = P_y + \omega_B x, \\ \dot{z} = \partial H/\partial P_z = P_z, \\ \dot{P}_x = -\partial H/\partial x = -\omega_l^2 x - (\omega_B P_y + F_0), \\ \dot{P}_y = -\partial H/\partial y = 0, \\ \dot{P}_z = -\partial H/\partial z = 0, \end{cases} \tag{4}$$

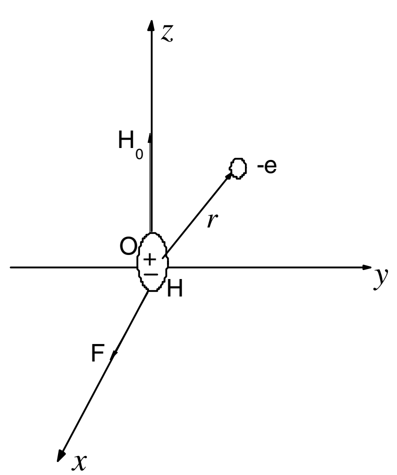



**Figure 1.** The schematic plot of $H^-$ ion in crossed gradient electric and magnetic fields. The magnetic field $H_0$ is along the $+z$-axis and the gradient electric field $F$ is along the $x$-axis.

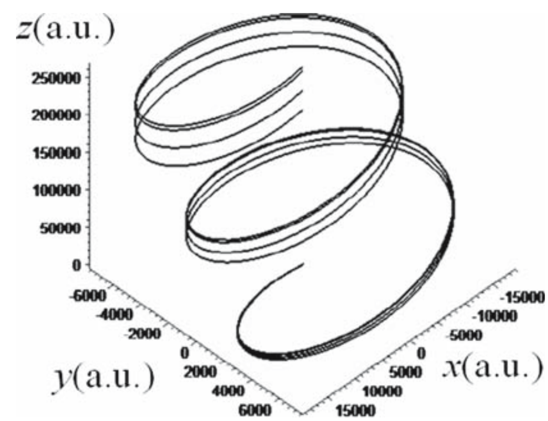



**Figure 2.** A family of trajectories representing electrons propagating away from the H atom in crossed gradient electric and magnetic fields.

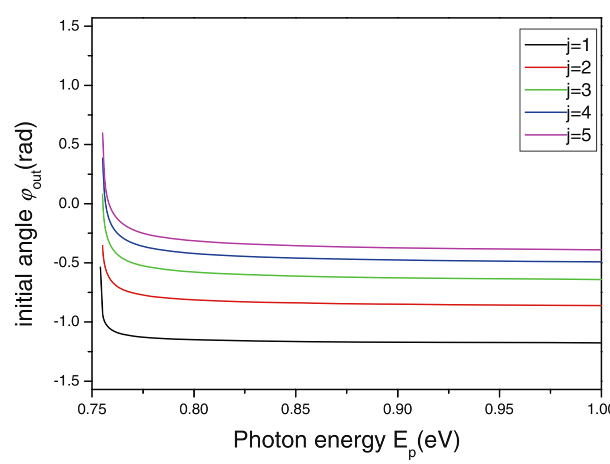



**Figure 3.** Variation of the detached electron's initial angle $\phi_{out}$ with the photon energy for different values of $j$. The electric field gradient $\alpha = 10^{-12}$ a.u., uniform electric field strength $F_0 = 18$ V/cm and magnetic field strength $H_0 = 0.6$ T.

with the initial conditions: $x_0 = 0, y_0 = 0, z_0 = 0, P_{x0} = k\sin\theta_{\text{out}}\cos\varphi_{\text{out}}$, $P_{y0} = k\sin\theta_{\text{out}}\sin\varphi_{\text{out}}$, $P_{z0} = k\cos\theta_{\text{out}}$, where $k = \sqrt{2E}$ is the momentum of the detached electron, $\theta_{\text{out}}$ is the outgoing angle of the detached electron relative to the $z$-axis and $\varphi_{\text{out}}$ is the azimuth angle. We obtain the motion equations for the detached electron as follows:

$$\begin{cases} x(t) = A_0[\sin(\omega_l t + \varphi_0) - \sin(\varphi_0)], & \text{(5a)} \\ y(t) = -(\omega_B/\omega_l)A_0[\cos(\omega_l t + \varphi_0) - \cos(\varphi_0)] \\ \qquad + [P_{y0} - A_0\omega_B\sin(\varphi_0)]t, & \text{(5b)} \\ z(t) = P_{z0}t, & \text{(5c)} \end{cases}$$

where $A_0$ and $\varphi_0$ are two integration constants:

$$\begin{cases} A_0\sin\varphi_0 = \dfrac{P_{y0}\omega_B + F_0}{\omega_l^2}, & \text{(6a)} \\ A_0\cos\varphi_0 = \dfrac{P_{x0}}{\omega_l}. & \text{(6b)} \end{cases}$$

From these equations, we find that the motion of the detached electron can be divided into two separate motions: a uniform linear motion along the $z$-axis and a trochoid motion in the $x$–$y$ plane, which can be described as a circular motion about a centre superposed upon a translational motion down the $y$-axis. Some of the trajectories of the detached electron are shown in figure 2. From this figure, we find that as the trajectories leave the origin, they diverge from each other. Due to the influence of the external electric and magnetic fields, they cross over each other at the caustic, and then, after being turned back by the fields, move close to the atom. The trajectories continue until they pass through a focus, where they repeat. This process repeats itself.

According to the closed orbit theory, each classical orbit of the detached electron that starts from the

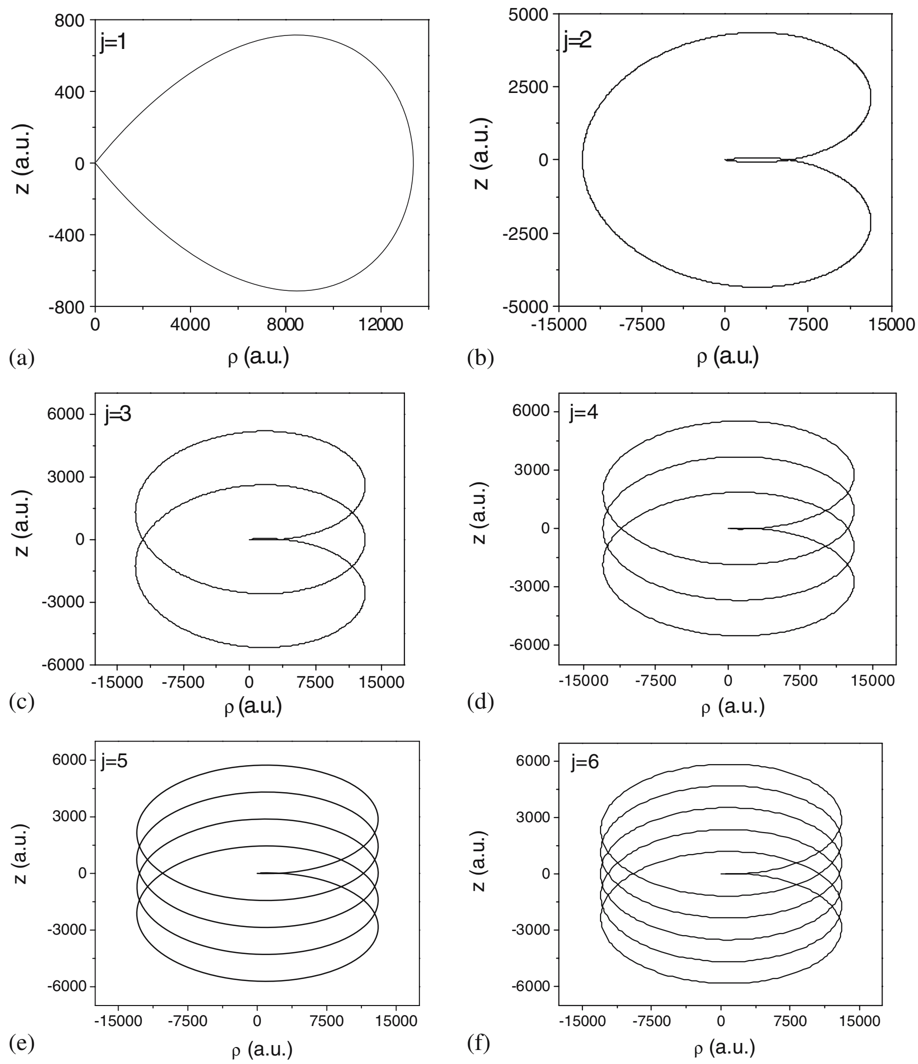



**Figure 4.** Some typical closed orbits of the detached electron in crossed gradient electric and magnetic fields. The electric field gradient $\alpha = 10^{-10}$ a.u., uniform electric field strength $F_0 = 18$ V/cm and magnetic field strength $H_0 = 0.6$ T. The photon energy $E_{\text{p}} = 1.0$ eV.

origin and subsequently returns back to it will produce an oscillation in the photodetachment cross-section. As the motion along the $z$-axis is a uniform linear motion, if the initial momentum of the detached electron along the $z$-axis is not zero, $P_{z0} \neq 0$, the electron will not return to the origin any more. Thus, for the returning orbits, $P_{z0}$ must be equal to zero, which suggests that the outgoing angle $\theta_{\text{out}}$ of the detached electron relative to the $z$-axis is: $\theta_{\text{out}} = \pi/2$. All the closed orbits of the detached electron lie in the $x$–$y$ plane. For returning orbits: $x(t_{\text{ret}}) = 0$, $y(t_{\text{ret}}) = 0$. From eq. (5a), we get the possible return times of the detached electron along the $x$-axis: $t_{\text{ret}} = -2\varphi_0 + (2j-1)\pi$ ($j$ =1,2,3,...). By substituting $t_{\text{ret}}$ into eq. (5b), we can get the condition relating the integration constants $A_0$ and $\varphi_0$:

$$2\omega_B A_0 \cos\varphi_0 + (P_{y0} - \omega_B A_0 \sin\varphi_0) \times[-2\varphi_0 + (2j-1)\pi] = 0. \tag{7}$$

Combining eq. (7) with eqs (6a) and (6b), we can obtain the initial azimuth angle $\varphi_{\text{out}}$ of a closed orbit. The variation of the initial angle $\varphi_{\text{out}}$ of the detached electron with the photon energy for different values of $j$ is given in figure 3. Once the initial angle $\varphi_{\text{out}}$ is given, the initial conditions for that closed orbit are known. Some of the closed orbits are plotted in figure 4.

The classical action of the $j$th closed orbit can be calculated as

$$\begin{aligned} S_j &= \int p\,\mathrm{d}q = \int_0^{t_{\text{ret}}} P_x \mathrm{d}x + \int_0^{t_{\text{ret}}} P_y \mathrm{d}y \\ &= \frac{A_0^2\omega_l^2}{2}\left\{t_{\text{ret}} + \frac{1}{2\omega_l}[\sin(2\omega_l t_{\text{ret}} + 2\varphi_0) - \sin 2\varphi_0]\right\} \\ &\quad + k^2 \sin^2\theta_{\text{out}}^j \sin^2\varphi_{\text{out}}^j t_{\text{ret}} + A\omega_B k \sin\theta_{\text{out}}^j \sin\varphi_{\text{out}}^j \\ &\quad \times \left[\frac{2\cos\varphi_0}{\omega_l} - \sin\varphi_0 t_{\text{ret}}\right]. \end{aligned} \tag{8}$$

## 3. Photodetachment cross-section

As the photodetachment process of our system is similar to the photodetachment of $H^-$ ion in crossed uniform electric and magnetic fields [10], we can use the same procedure to derive the photodetachment cross-section. For simplicity, we briefly summarize the results. According to the closed orbit theory, the photodetachment cross-section is a sum of two terms [10]:

$$\sigma(E) = \sigma_0(E) + \sigma^{\text{osc}}(E). \tag{9}$$

$\sigma_0(E) = \frac{16\sqrt{2}B^2\pi^2E^{3/2}}{3c(E_b+E)^3}$ is the background cross-section, where $E$ is the energy of the detached electron, $E_{\text{b}} = 0.754$ eV is the binding energy of $H^-$ ion and $B = 0.31552$ is the normalization constant. The second part is the oscillating term, which is related to the gradient electric and magnetic fields. It has been shown by Peters and Delos that $\sigma^{\text{osc}}(E)$ can be expressed as [10]

$$\sigma^{\text{osc}}(E) = -\sum_j C_j(E)\sin\left[S_j(E) - \mu_j\cdot\frac{\pi}{2}\right], \tag{10}$$

where the summation includes all the closed orbits of the detached electron. $S_j(E)$ is the classical action of the closed orbit, which is given by eq. (8). $\mu_j$ is the Maslov index characterizing the geometrical properties

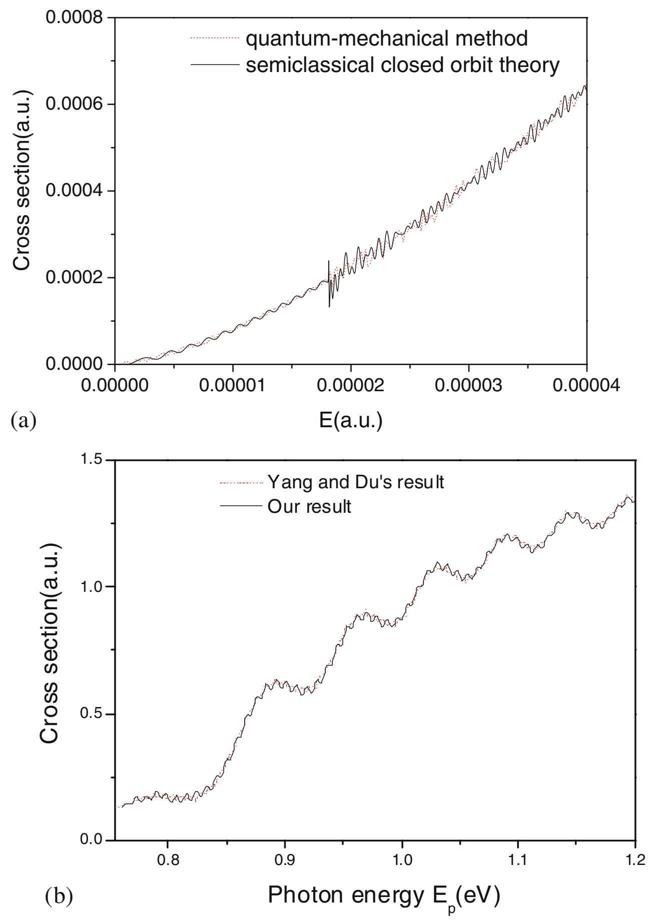



**Figure 5.** Two special cases for the photodetachment cross-section of $H^-$ ion in crossed gradient electric and magnetic fields for the $x$-polarized laser light. (**a**) $\alpha = 0.0$ a.u., the uniform electric field strength $F_0 = 18$ V/cm and the magnetic field strength $H_0 = 0.6$ T. (**b**) The magnetic field strength $H_0 = 0.0$ T, the uniform electric field strength $F_0 = 500$ kV/cm, the electric field gradient $\alpha = 10^{-7}$ a.u.

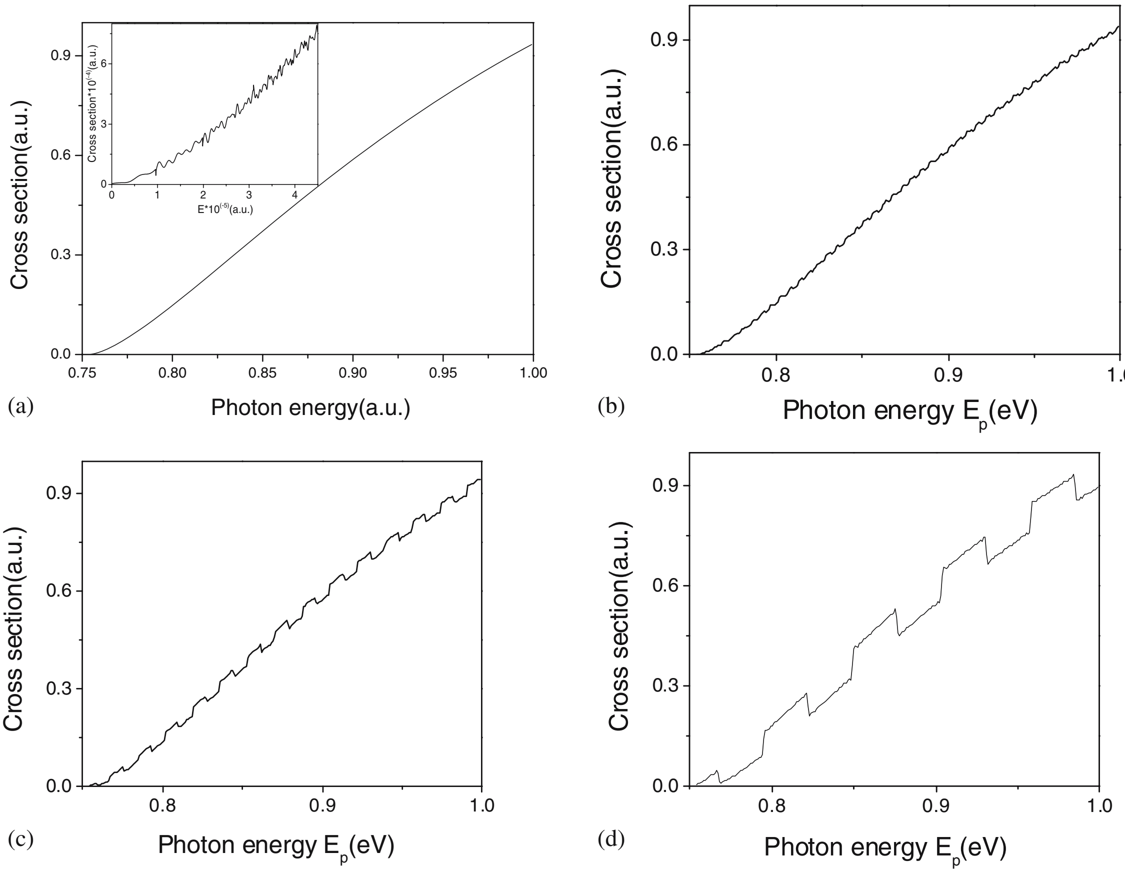



**Figure 6.** Photodetachment cross-section of $H^-$ ion in crossed gradient electric and magnetic fields for $x$-polarized laser light. Uniform electric field strength $F_0 = 18$ V/cm and the magnetic field strength $H_0 = 0.6$ T. The electric field gradient is as follows: (**a**) $\alpha = 10^{-12}$ a.u., (**b**) $\alpha = 10^{-8}$ a.u., (**c**) $\alpha = 10^{-7}$ a.u. and (**d**) $\alpha = 10^{-6}$ a.u.

of the $j$th closed orbit. $C_j(E)$ is the oscillating amplitude, which is given by

$$C_j(E) = \frac{64B^2\pi^3 E}{cE_\mathrm{p}^3} A_j \cdot [\chi(\theta_\mathrm{out}^j, \varphi_\mathrm{out}^j)\chi^*(\theta_\mathrm{ret}^j, \varphi_\mathrm{ret}^j)], \tag{11}$$

where $E_\mathrm{p} = E + E_\mathrm{b}$ is the photon energy. $A_j$ is the amplitude factor, which represents the divergence of adjacent trajectories.

$$A_j = \frac{1}{r_\mathrm{out}} \left| \frac{J_j(t=0)}{J_j(t_\mathrm{ret})} \right|^{1/2},$$

where $r_\mathrm{out}$ is the initial spherical radius. On the surface of this sphere, the electron wave is emitted. $J_j(t)$ is the classical-density Jacobian, which is given by

$$J_j(t) = \frac{\partial(x, y, z)}{\partial(t, \theta_\mathrm{out}, \varphi_\mathrm{out})}. \tag{12}$$

By substituting the classical motion equations (eqs (5a)–(5c)) into the Jacobian, after a lengthy derivation, we obtain the Jacobians at $t = t_0$ and $t = t_\mathrm{ret}$:

$$J_j(t=0) = kr_\mathrm{out}^2 \sin(\theta_\mathrm{out}^j), \tag{13}$$

$$\begin{aligned} J_j(t_\mathrm{ret}) = {} & A\cos(\omega_l t_\mathrm{ret} + \varphi_0)\omega_l k t_\mathrm{ret} \\ & \times \Bigg\{ -\frac{\omega_B}{\omega_l^3}[-\omega_l \cos(\omega_l t_\mathrm{ret}) k \sin\varphi_\mathrm{out}^j \\ & + \omega_l k \sin\varphi_\mathrm{out}^j - \sin(\omega_l t_\mathrm{ret})\omega_B k \cos\varphi_\mathrm{out}^j] \\ & - \left[ \frac{\omega_B^2}{\omega_l^2} k \cos\varphi_\mathrm{out}^j - k\cos\varphi_\mathrm{out}^j \right] t_\mathrm{ret} \Bigg\} \\ & - (A\omega_B \sin(\omega_l t_\mathrm{ret} + \varphi_0) \\ & - A\omega_B \sin\varphi_0 + k\sin\varphi_\mathrm{out}^j) k t_\mathrm{ret} \\ & \times \frac{1}{\omega_l^2}[\cos(\omega_l t_\mathrm{ret})\omega_B k \cos\varphi_\mathrm{out}^j \\ & - \omega_B k \cos\varphi_\mathrm{out}^j \\ & - \omega_l \sin(\omega_l t_\mathrm{ret}) k \sin\varphi_\mathrm{out}^j]. \end{aligned} \tag{14}$$

After substituting the above Jacobians into $A_j$, the amplitude factor can be obtained.

$\chi(\theta, \varphi)$ in eq. (11) is the angular distribution of the outgoing waves, which is related to the laser polarization. Assume that the laser light is linearly polarized

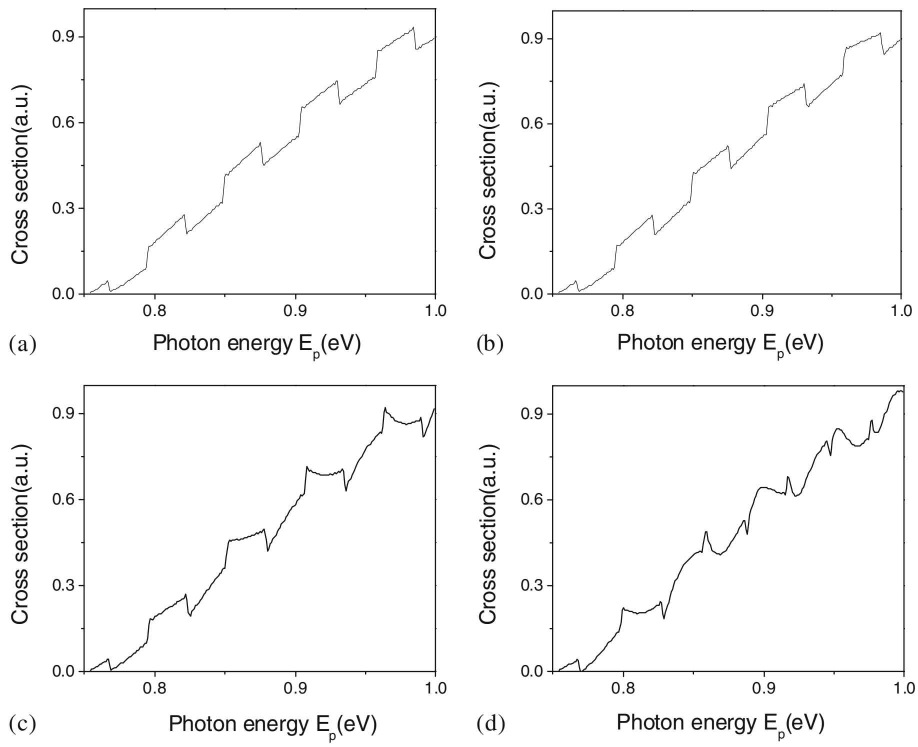



**Figure 7.** Variation of the photodetachment cross-section with the magnetic field strength. The laser light is polarized along the $x$-axis. Uniform electric field strength $F_0 = 18$ V/cm and electric field gradient $\alpha = 10^{-6}$ a.u. The magnetic field strength is: (**a**) $H_0 = 1.0$ T, (**b**) $H_0 = 20.0$ T, (**c**) $H_0 = 50.0$ T and (**d**) $H_0 = 100.0$ T.

along the $x$- or $y$-axis, then the angular factors are [10]

$$\chi_x(\theta_{\rm out}, \varphi_{\rm out}) = \frac{1}{\sqrt{4\pi}} \sin\theta_{\rm out} \cos\varphi_{\rm out}, \tag{15}$$

$$\chi_y(\theta_{\rm out}, \varphi_{\rm out}) = \frac{1}{\sqrt{4\pi}} \sin\theta_{\rm out} \sin\varphi_{\rm out}. \tag{16}$$

The angles $\{\theta^j_{\rm out}, \varphi^j_{\rm out}\}$ and $\{\theta^j_{\rm ret}, \varphi^j_{\rm ret}\}$ refer to the outgoing and returning directions of the $j$th closed orbit. As all the closed orbits lie in the $x$–$y$ plane, the geometry of the returning orbits requires that $\theta_{\rm out} = \theta_{\rm ret} = \pi/2$ and $\varphi_{\rm out} = -\varphi_{\rm ret}$. Therefore, the angular factor in eqs (15) and (16) for $x$ or $y$ linear polarized laser light can be written as

$$\chi_x(\theta_{\rm out}, \varphi_{\rm out})\chi^*_x(\theta_{\rm ret}, \varphi_{\rm ret}) = \frac{1}{4\pi}\cos^2\varphi_{\rm out}, \tag{17}$$

$$\chi_y(\theta_{\rm out}, \varphi_{\rm out})\chi^*_y(\theta_{\rm ret}, \varphi_{\rm ret}) = -\frac{1}{4\pi}\sin^2\varphi_{\rm out}. \tag{18}$$

Correspondingly, the photodetachment cross-sections for different linear polarized laser lights can be written as

$$\sigma_x(E) = \sigma_0(E) - \sum_j \frac{16B^2\pi^2E}{cE^3_{\rm p}} A_j \cos^2\varphi_{\rm out} \times \sin\left[S_j(E) - \mu_j \cdot \frac{\pi}{2}\right], \tag{19}$$

$$\sigma_y(E) = \sigma_0(E) + \sum_j \frac{16B^2\pi^2E}{cE^3_{\rm p}} A_j \sin^2\varphi_{\rm out} \times \sin\left[S_j(E) - \mu_j \cdot \frac{\pi}{2}\right]. \tag{20}$$

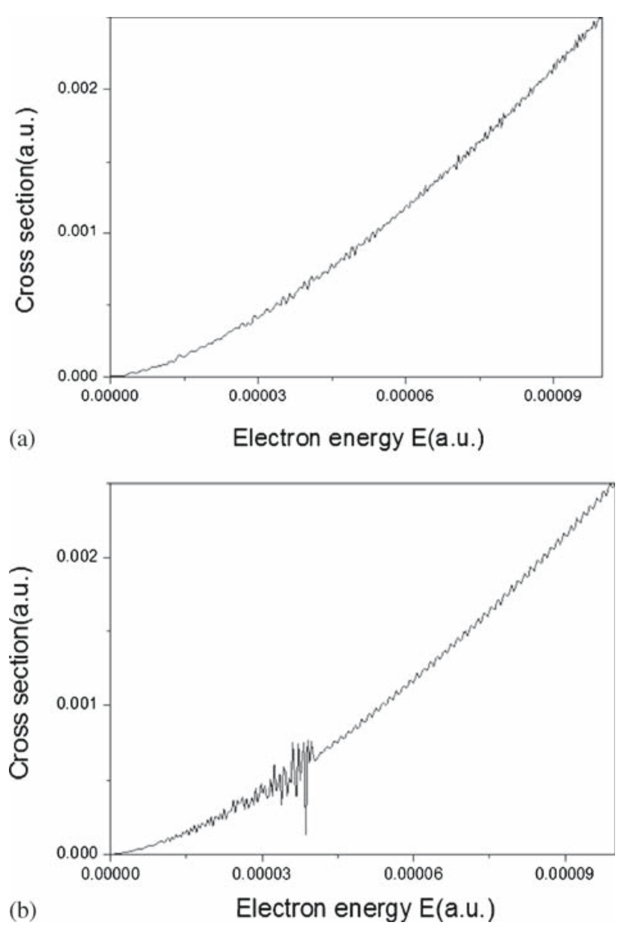
## 4. Results and discussions

Suppose the laser light is polarized along the $x$-axis. Using eq. (19), we calculate the photodetachment cross-section of $H^-$ ion in crossed gradient electric and magnetic fields. Firstly, in order to verify the correctness of our formula, we consider two special cases. One is the case without the gradient electric field, $\alpha = 0.0$ a.u. Then our system reduces to the photodetachment of $H^-$ ion in crossed homogeneous electric and magnetic fields [10]. We keep the magnetic field $H_0 = 0.6$ T and the uniform electric field $F_0 = 18$ V/cm. The energy of the detached electron is varied from 0.0 to $4.0 \times 10^{-5}$ a.u. The result is given in figure 5a. We find that the cross-section is a smooth, rising function superposed upon many oscillations. The solid line represents our result, while the dotted line is the result calculated using the method given by Peters and Delos [10]. These two results agree well with each other, suggesting the correctness of our formula. Another case is the one without the magnetic field, $H_0 = 0.0$ T. Then our system reduces to the photodetachment of $H^-$ ion in a gradient electric field. We choose the background electric field $F_0 = 500$ kV/cm and the electric field gradient $\alpha = 10^{-7}$ a.u. The result is given in figure 5b. We find that the cross-section displays a saw-tooth structure, which approaches the one calculated using the formula given by Yang and Du in [18].

In the following, we keep the magnetic field $H_0 = 0.6$ T and the background electric field $F_0 = 18$ V/cm. Then we show how the photodetachment cross-section varies with the electric field gradient $\alpha$. The result is given in figure 6. Figure 6a shows that the photodetachment cross-section with the electric field gradient is relatively small, $\alpha = 10^{-12}$ a.u. Under this condition, the oscillation in the cross-section is very small and is nearly invisible. However, above the threshold, oscillatory structure still exists. In the inset, we limit the energy of the detached electron varying from 0.0 to $4.5 \times 10^{-5}$ a.u., and we can clearly see the oscillatory structures appearing in the cross-section, which are caused by the interference effect between the returning electron waves with the outgoing source waves. Figure 6b shows the photodetachment cross-section with the electric field gradient $\alpha = 10^{-8}$ a.u. Oscillatory structures are clearly visible in the cross-section. As we increase the electric field gradient, the cross-section exhibits a regular staircase structure and the oscillatory amplitude increases, as we can see from figures 6b–6d.

Next, we calculate the photodetachment cross-section of $H^-$ ion in a constant gradient electric field but with different magnetic field strength. We fix the gradient electric field $F_0 = 18$ V/cm and $\alpha = 10^{-6}$ a.u.; then we change the magnetic field strength $H_0$ from 1.0 to 100 T. Figure 7a shows the cross-section when $H_0 = 1.0$ T. Under this condition, the photodetachment cross-section still exhibits a regular staircase structure. As we increase the strength of the magnetic field, the regular staircase structure becomes irregular. Firstly, the irregular structure only appears in the high-energy region, see figure 7b, $H_0 = 20.0$ T. When $H_0 = 50.0$ T (figure 7c), the region where the irregular structure appears in the cross-section becomes enlarged. As the strength of the magnetic field is very large, the regular staircase structures in the cross-section nearly disappear, as we can see clearly from figure 7d.

Finally, we calculate the photodetachment cross-section of $H^-$ ion in the crossed gradient electric and magnetic fields for different laser light polarization. In our calculation, we choose $F_0 = 18$ V/cm, $\alpha = 10^{-12}$ a.u., $H_0 = 0.6$ T. The energy of the detached electron is varied from 0.0 to $1.0 \times 10^{-4}$ a.u. The result is shown in figure 8. Figure 8a shows the cross-section when the laser light is polarized along the $x$-axis, while figure 8b shows the cross-section when the laser light is polarized along the $y$-axis. From these two figures, we find that the oscillating amplitude for the $y$-polarized laser light is larger than the case for the $x$-polarized laser light. Especially as the detached electron's energy is close to $E = 4.5 \times 10^{-5}$ a.u., there is a large oscillation in the photodetachment cross-section for the $y$-polarized laser light. The reason can be interpreted as follows: The coefficient in the photodetachment cross-section for the $y$-polarized laser light (eq. (20)) is proportional to $\sin^2 \varphi_{out}$. When the energy of the detached electron is close to $4.5 \times 10^{-5}$ a.u., the azimuth angle $\varphi_{out}$ is maximum for that orbit. The trajectory of the orbit leaves and returns primarily oriented in the




(a) (b)

**Figure 8.** Photodetachment cross-section of $H^-$ ion in crossed gradient electric and magnetic fields for different polarized laser light. (**a**) $x$-polarized laser light, (**b**) $y$-polarized laser light. The uniform electric field strength $F_0 = 18$ V/cm and the magnetic field strength $H_0 = 0.6$ T. The electric field gradient $\alpha = 10^{-12}$ a.u.

$y$-direction, and so the interference between the returning wave and the outgoing wave is strong, which makes the amplitude of the oscillation at a maximum at that energy. However, for the $x$-polarized laser light, the opposite happens. The amplitude factor in the photodetachment cross-section (eq. (19)) is proportional to $\cos^2 \varphi_{\text{out}}$. As $\varphi_{\text{out}}$ is maximum, this factor is minimum, which makes the amplitude of the oscillation becomes relatively small. Figure 8 suggests that the photodetachment cross-section of $H^-$ ion in the crossed gradient electric and magnetic fields is not only related to the strength of the external field, but also to the polarization of the laser light.

## 5. Conclusions

In summary, the photodetachment of $H^-$ ion in crossed gradient electric and magnetic fields has been studied in the framework of the closed orbit theory. We put forward an analytical formula for calculating the photodetachment cross-section. Our results show that oscillatory structures appear in the cross-section, which are caused by the interference between the returning electron waves travelling along the closed orbit with the outgoing source waves. Besides, the oscillatory structure in the cross-section depends sensitively on the external field strengths and the laser polarization. Our study may have some possible applications in the photodetachment microscopy experiment or in ion detection. For example, in the photodetachment microscopy experiment, our result may be used in determining the binding energy of the negative ion. Once the detachment electron spectrum is measured after applying a laser light and an external field on the negative ion, one may compare the spectra with the result calculated by using the analytical formula given in this work to determine the binding energy of the detached electron. Using this method, the electron affinities in the negative ion can be measured with an accuracy much higher than any current *ab-initio* method for multielectron systems [19–23].

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