Pramana – J. Phys. (2016) 87: 70
DOI 10.1007/s12043-016-1275-x

Nonlinear propagation of weakly relativistic ion-acoustic waves in electron–positron–ion plasma

M G HAFEZ[1,3,*], M R TALUKDER[2] and M HOSSAIN ALI[3]

[1]Department of Mathematics, Chittagong University of Engineering and Technology, Chittagong-4349, Bangladesh
[2]Department of Applied Physics and Electronic Engineering, University of Rajshahi, Rajshahi-6205, Bangladesh
[3]Department of Applied Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh
*Corresponding author. E-mail: golam_hafez@yahoo.com

MS received 8 June 2015; revised 27 December 2015; accepted 25 January 2016; published online 6 October 2016

Abstract. This work presents theoretical and numerical discussion on the dynamics of ion-acoustic solitary wave for weakly relativistic regime in unmagnetized plasma comprising non-extensive electrons, Boltzmann positrons and relativistic ions. In order to analyse the nonlinear propagation phenomena, the Korteweg–de Vries (KdV) equation is derived using the well-known reductive perturbation method. The integration of the derived equation is carried out using the ansatz method and the generalized Riccati equation mapping method. The influence of plasma parameters on the amplitude and width of the soliton and the electrostatic nonlinear propagation of weakly relativistic ion-acoustic solitary waves are described. The obtained results of the nonlinear low-frequency waves in such plasmas may be helpful to understand various phenomena in astrophysical compact object and space physics.

Keywords. Electron–positron–ion plasma; weakly relativistic ion-acoustic waves; the ansatz method; generalized Riccati equation mapping method.

PACS Nos 52.35.Fp; 52.35.Sb; 02.30.Jr

1. Introduction

The electron–positron–ion (epi) plasmas are observed in the evolution of early Universe [1], in active galactic nuclei [2], in the centre of the Milky Way galaxy [3], in pulsar magnetospheres [4], in the polar regions of neutron stars [5], etc. The structures and properties of such plasmas are still unrevealed. So, the study of nonlinear low-frequency waves in such plasmas should be helpful for the better understanding of various physical phenomena in astrophysical, space and laboratory plasmas. Already many researchers [6–16] have investigated the basic properties of linear and nonlinear waves in epi plasmas by considering different plasma models. But, most of the studies are concerned with the epi plasmas for non-relativistic regimes. However, the particle velocities in some of the plasmas may become high and approach the speed of light. Many works [17–22] have shown that the relativistic effects may significantly modify the wave dynamics in the plasmas. Thus, the relativistic epi plasmas have drawn the attention of researchers due to their remarkable applications and potentiality in investigating various types of collective processes in astrophysical, space as well as laboratory plasmas. The relativistic plasmas are formed not only during the evolution of early Universe, but also in the inner region of accretion disc in the vicinity of black holes [23], plasma sheath boundary layer of Earth's magnetosphere [24], laser–plasma interaction [25] and space plasma phenomena [26]. However, it is apparent that only a few works have been carried out to investigate the nonlinear propagation of ion-acoustic solitary waves (IASWs) for three-component plasma system, when the ion speed is comparable with the speed of light. For instance, Gill *et al* [17,18] have investigated the effect of weakly relativistic ion-acoustic soliton in the epi plasmas. Han *et al* [19] have studied the ion-acoustic solitary waves and their interaction in weakly relativistic two-dimensional thermal plasmas. Shah *et al* [20] have studied the electrostatic compressive and rarefactive shocks and soliton relativistic plasmas occurring in polar regions of the pulsar. Therefore, the researchers are being motivated to study the basic properties of relativistic ion-acoustic waves recently.

Different types of ion-acoustic, dust-acoustic or electron-acoustic waves have been studied [27–31] by considering one or two components to be non-extensive [32] by ignoring relativistic effect. Very recently, Hafez *et al* [21] have studied the nonlinear propagation of ion-acoustic solitons (IAS) consisting of relativistic thermal hot ions and non-extensive distributed electrons and positrons. Thus, the study of linear and nonlinear properties in magnetized or unmagnetized non-extensive plasmas is one of the most important topic for understanding dynamic behaviour of astrophysical and space plasmas, where the ion speed is comparable to the speed of light. It is motivated to analyze the properties of small but finite amplitude weakly relativistic IASWs in unmagnetized plasma system consisting of relativistic ions, non-extensive electrons and Boltzmann-distributed positrons. The ion continuity and momentum equation in the plasmas are obtained from the conservation law. The relativistic Lorentz factor is included in the equation of motion only. The inclusion of relativistic effect in the equation of motion is already justified by many researchers [17,33,34]. The plasma model may be viewed as an effect of the interactions of relativistic ions coming from the relativistic outflows or extragalactic jets with interstellar clouds having superthermal or subthermal

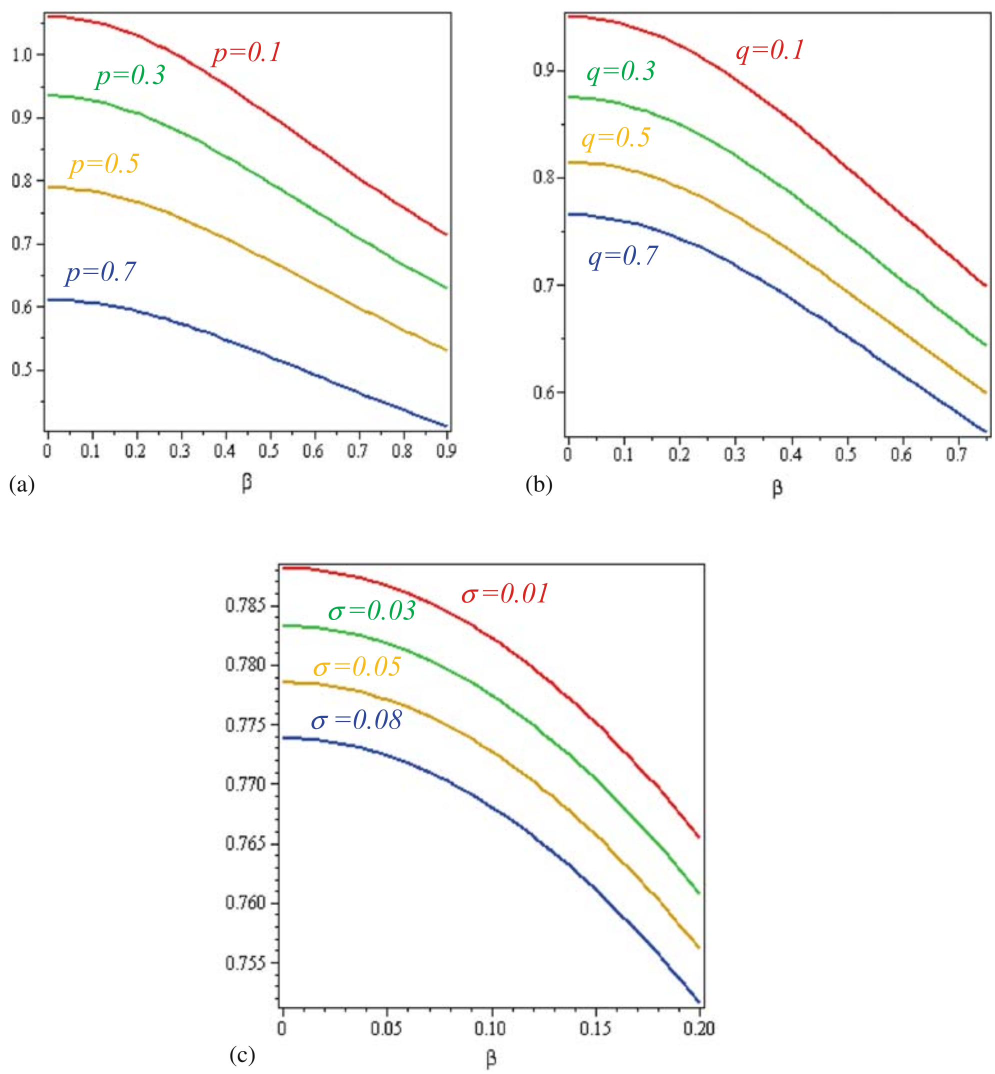

Figure 1. Effects of phase velocity $(v_0 - u_{i0})$ corresponding to the relativistic factor β (**a**) for different values of p when $\sigma = 0.01$ and $q = 0.6$, (**b**) for different values of q when $\sigma = 0.01$ and $p = 0.4$ and **(c)** for different values of σ when $p = 0.5$ and $q = 0.6$.

electrons and isothermal positrons. Therefore, the aim of this work is to investigate the influence of plasma parameters, especially electrons non-extensivity parameter, unperturbed positron-to-electron number density ratio, electron-to-positron temperature ratio and the relativistic streaming factor on the nonlinear propagation of IASWs in plasmas through the two mathematical techniques [35–37].

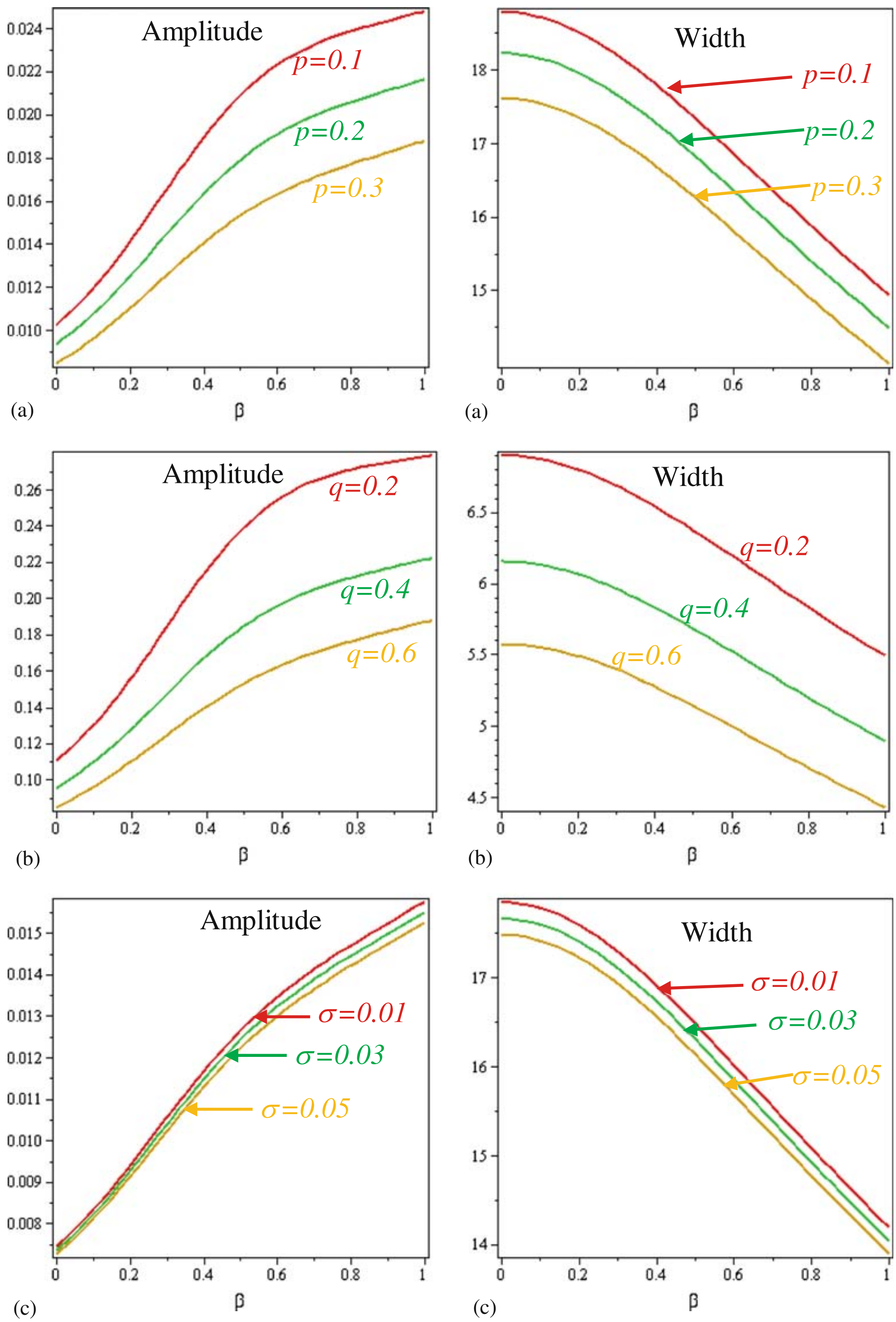

Figure 2. Variation of amplitude and width of the soliton with β (**a**) for different values of p when $\sigma = 0.01$, $V = 0.0075$ and $q = 0.6$, (**b**) for different values of q when $\sigma = 0.01$, $V = 0.0075$ and $p = 0.3$ and (**c**) for different values of σ when $p = 0.5$, $V = 0.0075$ and $q = 0.4$.

2. Mathematical model equations

Let us consider a fully ionized unmagnetized epi plasma system consisting of relativistic ions, Boltzmann-distributed positrons and hot non-extensive electrons. The charge neutrality equilibrium condition is $n_{e0} = n_{p0} + n_{i0}$, where n_{e0}, n_{p0} and n_{i0} are the unperturbed electron, positron and ion densities respectively. The positron number density in the electrostatic potential perturbation was obtained from the Boltzmann distribution as $n_p = p\mathrm{e}^{-\sigma\Phi}$, where $\sigma = T_e/T_p$ is the ratio between the electron and positron temperatures and $p = n_{p0}/n_{e0}$ is the fractional concentration of positrons with regards to electron at equilibrium state for the case of slow ion time-scale in which positrons are in local thermodynamic equilibrium. The electron number density is assumed to obey non-extensive electron distribution function and it can be defined as [38]

$$n_e = [1 + (q-1)\Phi]^{(q+1)/2(q-1)}, \quad q > -1,$$

where q stands for the strength of non-extensivity. It is notable that $q < 1$ indicates the case of superthermality, $q > 1$ indicates the subthermal case, whereas $q \to 1$ reduces to its Maxwell–Boltzmann counterpart. Therefore, the basic normalized equations can be written for the dynamics of electrostatic excitations, where the phase velocity is much higher than the relativistic ion thermal velocity but much smaller than the positron and electron thermal velocities, as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial(\gamma u_i)}{\partial t} + u_i\frac{\partial(\gamma u_i)}{\partial x} = -\frac{\partial\Phi}{\partial x}, \tag{2}$$

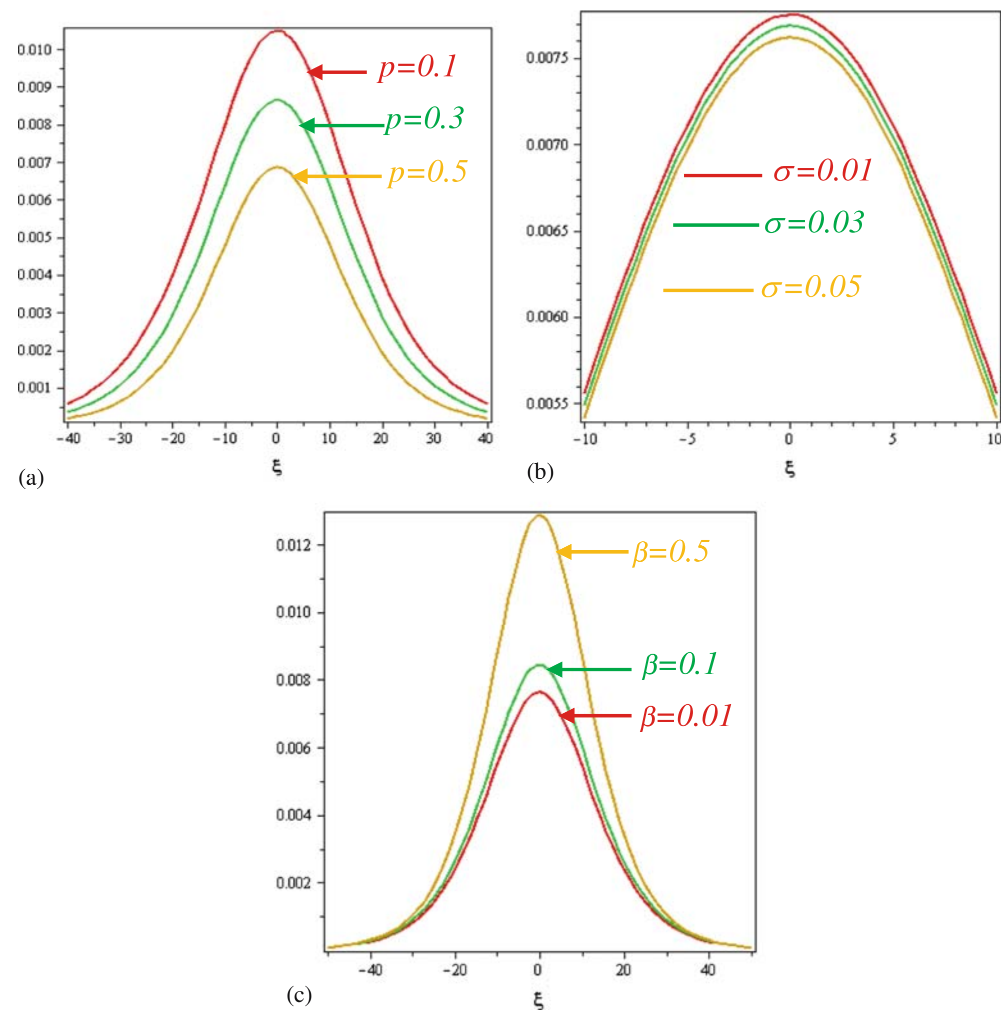

Figure 3. Effects of electrostatic IASWs (**a**) for different values of p when $\sigma = 0.01$, $V = 0.0075$, $\beta = 0.1$, $u_{i0} = 0.9$ and $q = 0.6$, (**b**) for different values of σ when $p = 0.4$, $V = 0.0075$, $\beta = 0.1$, $u_{i0} = 0.9$ and $q = 0.6$ and (**c**) for different values of increasing stream velocity, that is, β when $p = 0.4$, $V = 0.0075$, $\sigma = 0.03$ and $q = 0.6$.

$$\frac{\partial^2 \Phi}{\partial x^2} = [1 + (q-1)\Phi]^{(q+1)/2(q-1)} - p\mathrm{e}^{-\sigma\Phi} - (1-p)n_i, \tag{3}$$

where n_i denotes the ion number density normalized by the unperturbed ion density, u_i denotes the ion fluid velocity normalized by ion-acoustic speed $C_s = (T_e/m_i)^{1/2}$, m_i is the mass of the positive ions and Φ denotes the electrostatic potential normalized by (T_e/e), where e is the electron charge. The space and time variables are normalized by

$$\lambda_{De} = (T_e/4\pi n_{eo} e^2)^{1/2}$$

and

$$\omega_{pi}^{-1} = (m_i/4\pi n_{eo} e^2)^{1/2}$$

respectively and

$$\gamma = (1 - u_i^2/c^2)^{-1/2} \approx 1 + \frac{u_i^2}{2c^2}$$

for weakly relativistic effect. In the non-relativistic limit $\gamma = 1$, we obtain the system of equations in analysing non-relativistic (epi) unmagnetized plasma.

3. Formation of the KdV equation

This section presents the formation of the KdV equation using the well-known reductive perturbation technique [39].

To examine the small but finite-amplitude weakly relativistic IAWs in epi plasmas, the starched variables can be written as

$$\eta = \varepsilon^{1/2}(x - v_0 t), \quad \tau = \varepsilon^{3/2} t, \quad 0 < \varepsilon < 1, \tag{4}$$

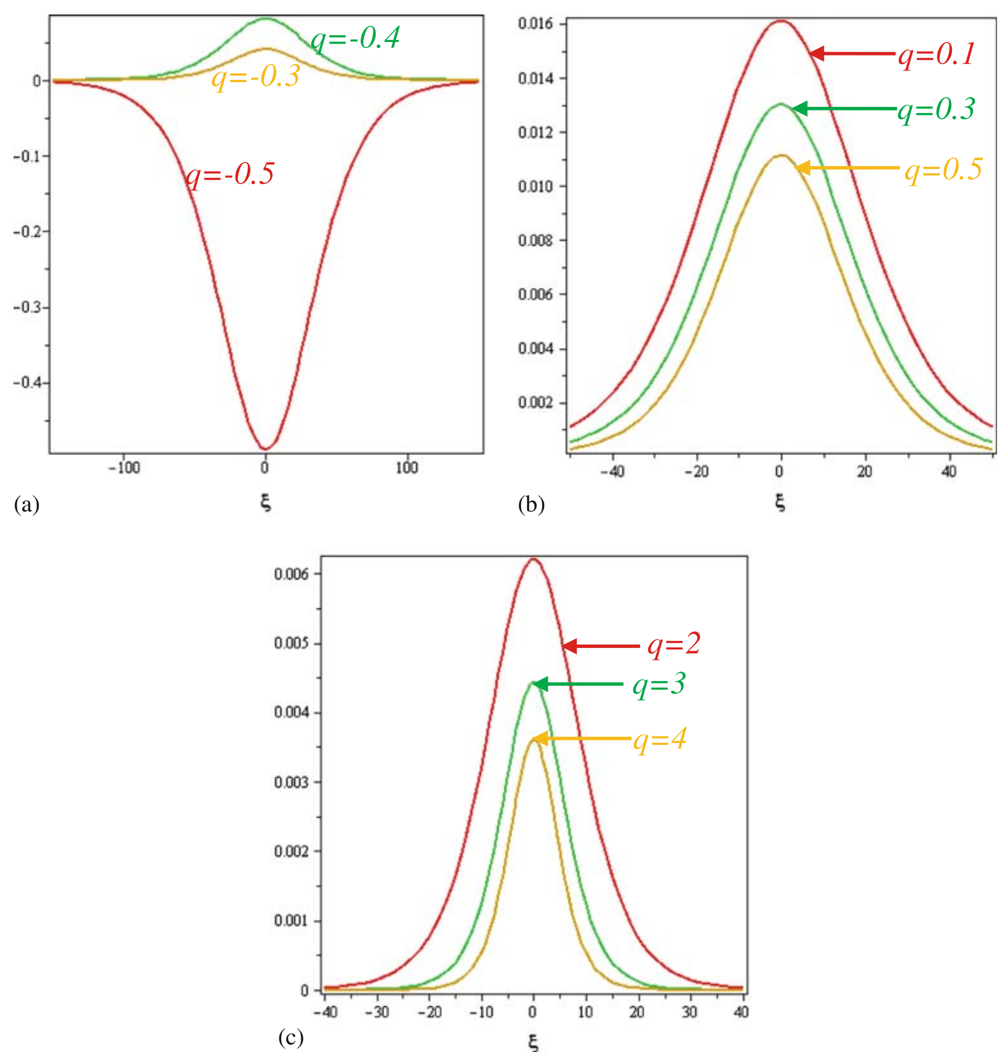

Figure 4. Effect of electrostatic IASWs for different values of q, that is (**a**) $q < 0$ (superthermal case), (**b**) $0 < q < 1$ (superthermal case) and (**c**) for $q > 1$ (subthermal case) when $\sigma = 0.05$, $V = 0.0075$, $\beta = 0.1$, $u_{i0} = 0.9$ and $p = 0.1$.

where v_0 is the unknown linear phase velocity to be determined later.

Using the starched coordinates, eqs (1)–(3) can be reduced as

$$\varepsilon\frac{\partial n_i}{\partial \tau} - v_0\frac{\partial n_i}{\partial \eta} + \frac{\partial}{\partial \eta}[n_i u_i] = 0, \tag{5}$$

$$\varepsilon\frac{\partial(\gamma u_i)}{\partial \tau} - v_0\frac{\partial(\gamma u_i)}{\partial \eta} + u_i\frac{\partial(\gamma u_i)}{\partial \eta} = -\frac{\partial \Phi}{\partial \eta}, \tag{6}$$

$$\varepsilon\frac{\partial^2 \Phi}{\partial \eta^2} = [1+(q-1)\Phi]^{(q+1)/2(q-1)} - p\mathrm{e}^{-\sigma\Phi} - (1-p)n_i. \tag{7}$$

We now expand the quantities n_i, u_i and Φ at the equilibrium state by balancing the nonlinear and the dispersive terms as follows:

$$\left.\begin{aligned} n_i &= 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \cdots \\ u_i &= u_{i0} + \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} + \cdots \\ \Phi &= \varepsilon\,\Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \cdots \end{aligned}\right\}. \tag{8}$$

By substituting (8) into eqs (5)–(7), and developing equations in different orders of ε, the lowest order in ε gives

$$n_i^{(1)} = \frac{1}{v_0 - u_{i0}} u_i^{(1)}, \tag{9}$$

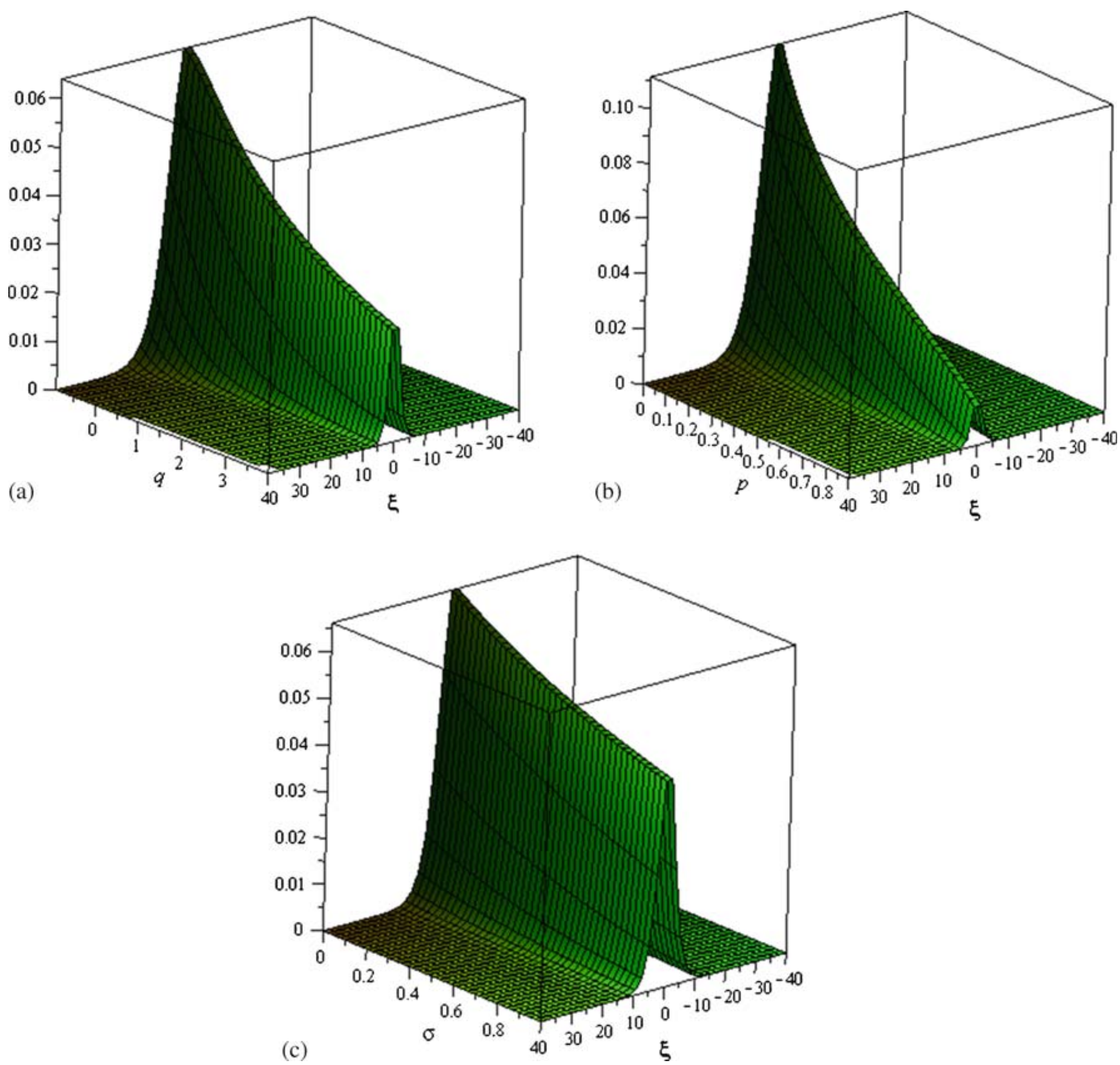

Figure 5. Weakly relativistic effect of electrostatic IASWs with respect to (**a**) q and reference frame $\xi = \eta - V\tau$ when $\sigma = 1$, $\beta = 0.1$, $u_{i0} = 0.9$, $V = 0.3$ and $q = 0.6$, (**b**) p and ξ when $\sigma = 1$, $\beta = 0.1$, $u_{i0} = 0.9$, $V = 0.3$ and $p = 0.5$ and (**c**) σ and ξ when $p = 0.5$, $\beta = 0.1$, $u_{i0} = 0.9$, $V = 0.3$ and $q = 0.6$.

$$u_i^{(1)} = \frac{1}{(v_0 - u_{i0})\gamma_{1i}} \Phi^{(1)}, \tag{10}$$

$$n_i^{(1)} = \left[\frac{q+1+2\sigma p}{2(1-p)}\right] \Phi^{(1)}, \tag{11}$$

where $\gamma_{1i} = 1+1.5\beta^2$ and $\beta = u_{i0}/c$. Solving eqs (9)–(11), the phase velocity can be written in the following form:

$$v_0 = u_{i0} + \left\{\frac{2(1-p)}{\gamma_{1i}(q+1+2p\sigma)}\right\}^{1/2}. \tag{12}$$

The next order of ε gives the following equations:

$$\frac{\partial n_i^{(1)}}{\partial \tau} - (v_0 - u_{i0})\frac{\partial n_i^{(2)}}{\partial \eta} + \frac{\partial u_i^{(2)}}{\partial \eta} + \frac{\partial}{\partial \eta}[n_i^{(1)} u_i^{(1)}] = 0, \tag{13}$$

$$\gamma_{1i}\frac{\partial u_i^{(1)}}{\partial \tau} - \gamma_{1i}(v_0 - u_{i0})\frac{\partial u_i^{(2)}}{\partial \eta} + \left(\gamma_{1i} - 2\gamma_{2i}\frac{v_0 - u_{i0}}{u_{i0}}\right) u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \eta} = -\frac{\partial \Phi^{(2)}}{\partial \eta}, \tag{14}$$

$$\frac{1}{(1-p)}\frac{\partial^2 \Phi^{(1)}}{\partial \eta^2} = \frac{(q+1+2p\sigma)}{2(1-p)}\Phi^{(2)} + \frac{(q+1)(3-q)-4p\sigma^2}{8(1-p)}(\Phi^{(1)})^2 - n_i^{(2)}, \tag{15}$$

where $\gamma_{2i} = 1.5\beta^2$. If we eliminate $n_i^{(2)}$, $u_i^{(2)}$ and $\Phi^{(2)}$ from (13)–(15), we obtain the following nonlinear evolution equation:

$$\frac{\partial \Phi^{(1)}}{\partial \tau} + L\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \eta} + M\frac{\partial^3 \Phi^{(1)}}{\partial \eta^3} = 0. \tag{16}$$

This is the well-known KdV equation. The nonlinear coefficient L and dispersive coefficients M are given by

$$L = \left[\frac{1}{\gamma_{1i}^2(v_0 - u_{i0})}\left(3\gamma_{1i} - 2\gamma_{2i}\frac{v_0 - u_{i0}}{u_{i0}}\right) + (v_0 - u_{i0})\frac{(q-3)(q+1)+4p\sigma^2}{4(q+1+2p\sigma)}\right],$$

$$M = \frac{(v_0 - u_{i0})}{q+1+2p\sigma}. \tag{17}$$

4. Analytical solutions to the KdV equation

This section discusses analytical solutions to the KdV equation using the ansatz method [35,36] and the generalized Riccati equation mapping method [37].

The nonlinear evolution eq. (16) reveals one-dimensional travelling wave solution by assuming $\Phi^{(1)}(\eta, \tau) = \Phi^{(1)}(\xi)$, $\xi = \eta - V\tau$, where V is the constant speed of the moving frame. Thus, eq. (16) can be reduced to a nonlinear ordinary differential equation (ODE) as

$$-V\frac{\mathrm{d}\Phi^{(1)}}{\mathrm{d}\xi} + \frac{L}{2}\frac{\mathrm{d}(\Phi^{(1)})^2}{\mathrm{d}\xi} + M\frac{\mathrm{d}^3\Phi^{(1)}}{\mathrm{d}\xi^3} = 0. \tag{18}$$

Integrating eq. (18) once with respect to ξ and setting the integrating constant to be zero for the equation of time homogeneity, we obtain

$$-V\Phi^{(1)} + \frac{L}{2}(\Phi^{(1)})^2 + M\frac{\mathrm{d}^2\Phi^{(1)}}{\mathrm{d}\xi^2} = 0. \tag{19}$$

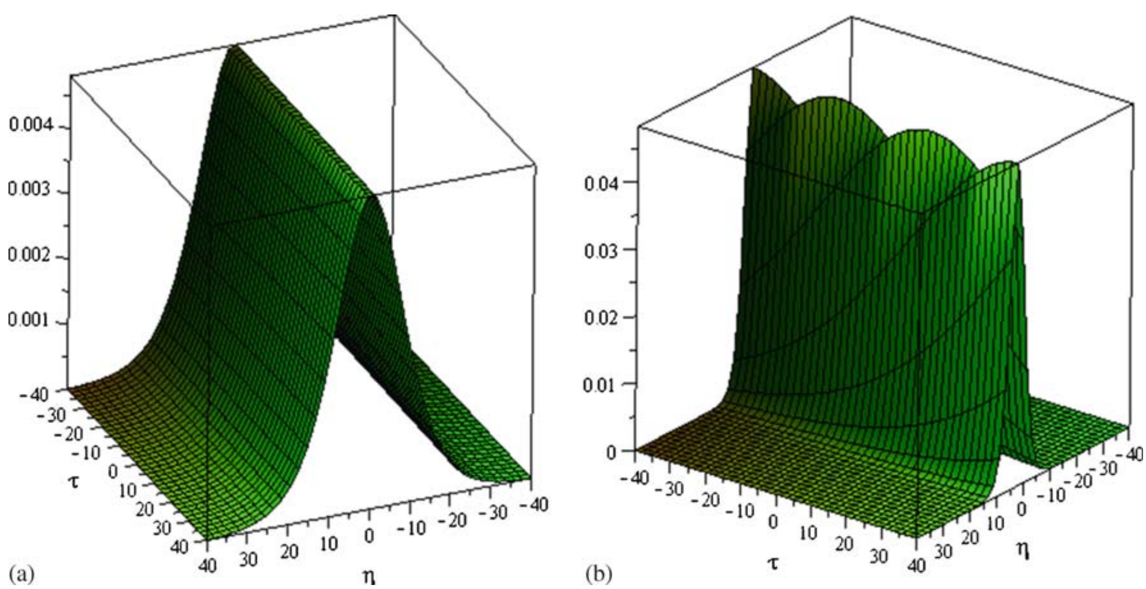

Figure 6. (**a**) Surface plot of solution (24) when $\sigma = 1$, $V = 0.0075$, $\beta = 0.1$, $q = 0.6$ and $p = 0.5$ and (**b**) surface plot of solution (30) when $\sigma = 1$, $V = 0.03$, $\beta = 0.1$, $q = 0.6$ and $p = 0.5$.

To investigate the bright solitons, the solitary wave ansatz [35,36] is considered in the following form:

$$\Phi^{(1)}(\xi) = A\cosh^q(B\xi), \tag{20}$$

where A and B respectively denote the amplitude and inverse width of the soliton. The first and second derivatives of $\Phi^{(1)}(\xi)$ with respect to ξ are given below:

$$\frac{\mathrm{d}\Phi^{(1)}(\xi)}{\mathrm{d}\xi} = ABq\cosh^{q-1}(B\xi)\,\sinh(B\xi)$$

$$\begin{aligned}\frac{\mathrm{d}^2\Phi^{(1)}(\xi)}{\mathrm{d}\xi^2} &= AB^2q(q-1)\cosh^q(B\xi)\\ &\quad - AB^2q(q-1)\cosh^{q-2}(B\xi)\\ &\quad + AB^2q\cosh^q(B\xi).\end{aligned} \tag{21}$$

Substituting the values of (21) along with (20) into eq. (19), we obtain

$$\begin{aligned}&-VA\cosh^q(B\xi) + \frac{L}{2}A^2\cosh^{2q}(B\xi)\\ &\quad + M[AB^2q(q-1)\cosh^q(B\xi) - AB^2q(q-1)\\ &\quad \times\cosh^{q-2}(B\xi) + AB^2q\cosh^q(B\xi)] = 0.\end{aligned} \tag{22}$$

By balancing the exponents of each pair of cosh that appeared in eq. (22) we get $q = -2$. Then collecting the coefficients of equal powers in cosh from eq. (22) and setting them to zeros, we get a system of algebraic equations, which are omitted for convenience. Solving the obtained system of equation, we get the following values for A and B:

$$B = \sqrt{\frac{V}{4M}}, \quad A = \frac{12B^2}{L}. \tag{23}$$

Therefore, the stationary solitary wave solution to eq. (16) according to the ansatz method can be written in the following form:

$$\Phi_1^{(1)}(\eta,\tau) = \frac{3V}{L}\mathrm{sech}^2\left(\sqrt{\frac{V}{4M}}(\eta - V\tau)\right). \tag{24}$$

Again, the travelling wave solutions of eq. (16) according to the generalized Riccati equation mapping method [37] can be written as

$$\Phi^{(1)} = \sum_{i=0}^{2} A_i(\psi(\xi))^i, \quad A_2 \neq 0, \tag{25}$$

where $\psi(\xi)$ satisfies the following nonlinear Riccati equation:

$$\psi'(\xi) = \alpha + \beta\psi(\xi) + \gamma\,\psi^2(\xi), \tag{26}$$

where α, β and γ are the real constants and prime denotes the differentiation with respect to ξ. It is noted that eq. (26) has provided 27 solutions [37]. By substituting eq. (25) into eq. (19) with the help of eq. (26) and setting the coefficients of $(\psi(\xi))^i$, $i = 0, 1, 2, 3, 4$ to zero, one can easily derive the nonlinear algebraic equations that are not mentioned here for convenience. Solving the resulting algebraic equations, we get the following solutions for V, A_0, A_1 and A_2:

$$\begin{aligned}&V = M(-4\gamma\alpha + \beta^2), \quad A_0 = -\frac{12\alpha\beta M}{L},\\ &A_1 = -\frac{12M\beta\gamma}{L}, \quad A_2 = -\frac{12M\gamma^2}{L}.\end{aligned} \tag{27}$$

If we set $\alpha = 0$ and $\beta\gamma \neq 0$, the KdV eq. (16) has the following soliton like new analytical solutions:

$$\begin{aligned}\Phi_2^{(1)}(\eta,\tau) &= \frac{12M\beta^2}{L}\left[\frac{d}{d+\cosh(\beta\xi)-\sinh(\beta\xi)}\right]\\ &\quad - \frac{12M\beta^2}{L}\left[\frac{d}{d+\cosh(\beta\xi)-\sinh(\beta\xi)}\right]^2,\end{aligned} \tag{28}$$

$$\begin{aligned}\Phi_3^{(1)}(\eta,\tau) &= \frac{12M\beta^2}{L}\left[\frac{\cosh(\beta\xi)+\sinh(\beta\xi)}{d+\cosh(\beta\xi)+\sinh(\beta\xi)}\right]\\ &\quad - \frac{12M\beta^2}{L}\left[\frac{\cosh(\beta\xi)+\sinh(\beta\xi)}{d+\cosh(\beta\xi)+\sinh(\beta\xi)}\right]^2,\end{aligned} \tag{29}$$

where d is an arbitrary constant and $\xi = \eta - M\beta^2\tau$. If we set $d = 1$ and $\beta = \sqrt{V/4M}$, then we have

$$\Phi_2^{(1)}(\eta,\tau) = \frac{3V}{L}\left\{\frac{\cosh(\sqrt{V/4M}(\eta-(V/4)\tau)) - \sinh(\sqrt{V/4M}(\eta-(V/4)\tau))}{[1+\cosh(\sqrt{V/4M}(\eta-(V/4)\tau)) - \sinh(\sqrt{V/4M}(\eta-(V/4)\tau))]^2}\right\}. \tag{30}$$

The remaining solutions are ignored for simplicity. As $V > 0$, eqs (24) and (30) clearly indicate that the small but finite IASWs, that is, positive soliton exists if $L > 0$, negative soliton exists if $L < 0$ and no soliton can exist around $L = 0$. One can easily verify the stability of the KdV equation by imposing appropriate boundary conditions for localized perturbations, namely

$$\varphi^{(1)} \to 0, \quad \frac{d\varphi^{(1)}}{d\xi} \to 0, \quad \frac{d^2\varphi^{(1)}}{d\xi^2} \to 0$$
$$\text{at } \xi \to \pm\infty \tag{31}$$

to investigate the asymptotic behaviour and linearize it with regards to $\varphi^{(1)}$. As $V > 0$, it is mentioned that there will be a stable soliton if $M > 0$, else there will be an oscillatory soliton.

5. Numerical results and discussion

The amplitude, width, phase velocity and electrostatic oscillations of the weakly relativistic IASWs will be discussed according to the solutions of eqs (24) and (30) presented in §3.

The variation of phase velocity $(v_0 - u_{i0})$ with relativistic factor β at different values of$p$, q and σ are shown in figures 1a, 1b and 1c, respectively, when other physical parameters are fixed. Figure 2 displays the variation of amplitude and width of solitons with β at different values of p, q and σ for fixed values of the remaining parameters. Figure 3 exhibits the effect of weakly relativistic IASWs according to the solution of eq. (24) for different values of p, q and β respectively when the other parameters are fixed. It is observed from figure 3 that the electrostatic potential has a significant effect on the solitary waves and represents the bell-type solitary wave or bright soliton so that the width as well as the amplitude of the solitary wave decrease with the increase of p and q respectively, while the peak amplitude increases together with increasing streaming velocity u_{i0}, that is β. Figure 4 depicts the effects of weakly relativistic IASWs according to the solution of eq. (24) for different values of q with constant values of $p = 0.5$, $V = 0.0075$ and $q = 0.4$. Figure 4 shows that the peak amplitude as well as width of the soliton decrease with the increase of q and represents the topological bell-shaped solitary wave for the case of superthermality, whereas the peak amplitude of the soliton decrease with the increase of q and represent the narrow-shaped solitary wave for the case of subthermality. The effects of q, p and σ on weakly relativistic IASWs according to the solutions of eq. (30) with fixed values of the remaining parameters are displayed in figure 5. The results show that the amplitude and width of the IASWs are considerably influenced by q, p and σ. As q and σ increase, the amplitude of the soliton decreases but its width becomes slightly narrow while the amplitude decreases but width remains unchanged with the increase of fractional positrons to electron concentration. Furthermore, the cupson-shaped ion-acoustic solitons with existence of the relative number of relativistic ions may arise in the plasmas. This effect may be attributed to the accumulation of electrons, significantly more than the ions, as well as to the depletion of positrons in the epi plasmas. Finally, figure 6 shows the difference between the solutions of eqs (24) and (30) and represents the bell- and cupson-shaped solitary waves, respectively.

6. Concluding remarks

This work has proposed an unmagnetized three-component plasma model consisting of relativistic ions, non-extensive electrons and Boltzmann positrons. Using the reductive perturbation approach, the basic set of equations is reduced to the KdV equation for the lowest-order perturbation. The integration of this equation is carried out using the ansatz method and the generalized Riccati equation mapping method. The generalized Riccati equation mapping method provides new solutions of the nonlinear evolution equation. It is found that the electrostatic potential distribution of the solitary waves becomes bell and cupson types. The results also reveal that the superthermal electrons are responsible for positive as well negative bright solitons, while subthermal electrons are responsible for only positive bright solitons. The present work would be useful to study the non-extensive effects in interstellar and space plasmas. Furthermore, laboratory experiment can be carried out to explore special new features of weakly relativistic IASWs propagating in epi plasmas with relativistic ions, non-extensive electrons and Boltzmann positrons.

References

[1] K A Holcomb and T Tajima, *Phys. Rev. D* **40**, 3809 (1989)

[2] H R Miller and P J Witta, *Active galactic nuclei* (Springer-Verlag, Berlin, 1987)

[3] M L Burns, A K Harding and R Ramaty, *Positron–electron pairs in astrophysics* (American Institute of Physics, New York, 1983)

[4] F C Michel, *Rev. Mod. Phys.* **54**, 1 (1982)

[5] F C Michel, *Theory of neutron star magnetosphere* (Chicago University Press, Chicago, 1991)

[6] P K Shukla, A A Mamun and L Stenflo, *Phys. Scr.* **68**, 295 (2003)

[7] P K Shukla, J T Mendonca and R Bingham, *Phys. Scr. T* **113**, 133 (2004)

[8] S I Popel, S V Vladimirov and P K Shukla, *Phys. Scr.* **2**, 716 (1995)

[9] A Lavagno and D Pigato, *Euro. Phys. J. A* **47**, 52 (2011)
[10] A Kourakis, Esfandyari-Khalejahi, M Mehdipoor and P K Shukla, *Phys. Plasmas* **13**, 052117 (2006)
[11] E I El-Awady, S A El-Tantawy, W M Moslem and P K Shukla, *Phys. Lett. A* **374**, 3216 (2010)
[12] K Roy, A P Misra and P Chattergee, *Phys. Plasmas* **15**, 032310 (2008)
[13] A Shah and R Saeed, *Phys. Lett. A* **373**, 4164 (2009)
[14] M Ferdousi, S Yasmin, S Ashraf and A A Mamun, *Astrophys. Space Sci.* **352**, 579 (2014)
[15] A A Mamun and P K Shukla, *Phys. Lett. A* **374**, 472 (2010)
[16] A A Mamun and P K Shukla, *Phys. Plasmas* **17**, 10 (2010)
[17] T S Gill *et al*, *Phys. Lett. A* **361**, 364 (2007)
[18] T S Gill, A S Bains and N S Saini, *Can. J. Phys.* **87**, 861 (2009)
[19] J Han, S Du and W Duan, *Phys. Plasmas* **15**, 112104 (2008)
[20] A Shah, Q Haque and S Mahmood, *Astrophys. Space Sci.* **335**, 529 (2011)
[21] M G Hafez, M R Talukder and R Sakthivel, *Indian J. Phys.* (2015), doi: 10.1007/s12648-015-0782-9
[22] M G Hafez and M R Talukder, *Astrophys. Space Sci.* **359(1)**, 27 (2015)
[23] M J Rees, *Nature* **229**, 312 (1971)
[24] J I Vette, *Summary of particle population in the magnetosphere* (Reidel, Dordrecht, 1970)
[25] M Marklund and P K Shukla, *Rev. Mod. Phys.* **78**, 591 (2006)
[26] C Grabbe, *J. Geophys. Res.* **94**, 17299 (1989)
[27] S Ashraf, S Yasmin, M Asaduzzaman and A A Mamun, *Astrophys. Space Sci.* **344**, 145 (2013)
[28] S Ashraf, S Yasmin, M Asaduzzaman and A A Mamun, *Plasma Phys. Rep.* **40**, 306 (2014)
[29] S Sultana, I Kourakis, N S Saini and M A Hellberg, *Phys. Plasmas* **17**, 032310 (2010)
[30] H Alinejad, *Astrophys. Space Sci.* **345**, 85 (2013)
[31] S Mahmood and N Akhtar, *Euro. Phys. J. D* **49**, 217 (2008)
[32] C Tsallis, *J. Stat. Phys.* **52**, 479 (1988)
[33] B Ghosh, S Chandra and S N Paul, *Pramana – J. Phys.* **78**, 779 (2012)
[34] S Chandra and B Ghosh, *Ind. J. Pure Appl. Phys.* **51(9)**, 627 (2013)
[35] H Triki and A M Wazwaz, *Phys. Lett. A* **373**, 2162 (2009)
[36] H Triki, M Mirzazadeh, A H Bhrawy, P Razboravva and A Biswas, *Rom. J. Phys.* **60(1–2)**, 72 (2015)
[37] S Zhu, *Chaos, Solitons and Fractals* **37**, 1335 (2008)
[38] R Silva, A Plastino and A Lima, *Phys. Lett. A* **249**, 401 (1998)
[39] M Mehdipoor and A Neirameh, *Astrophys. Space Sci.* **337**, 269 (2012)