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Relativistic quantum correlations in bipartite fermionic states

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Abstract. The influences of relative motion, the size of the wave packet and the average momentum of the particles on different types of correlations present in bipartite quantum states are investigated. In particular, the dynamics of the quantum mutual information, the classical correlation and the quantum discord on the spin correlations of entangled fermions are studied. In the limit of small average momentum, regardless of the size of the wave packet and the rapidity, the classical and the quantum correlations are equally weighted. On the other hand, in the limit of large average momentum, the only correlations that exist in the system are the quantum correlations. For every value of the average momentum, the quantum correlations maximize at an optimal size of the wave packet. It is shown that after reaching a minimum value, the revival of quantum discord occurs with increasing rapidity.

Keywords. Quantum correlations; special relativity; quantum information.

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1. Introduction

Relativistic quantum information theory [1,2] has been the focus of research for the last few years. Many researchers have contributed significantly in growing different aspects of the field. The focus of most of the early work has been the dynamics of entanglement between modes of quantum fields observed by accelerating observers [3–7], using wave packets when the detectors are in relative motion [8] and the capacity of quantum channel in relative motion [9]. Other studies that explore the influence of relativity on different important issues include, the Unruh effect [10–17], black holes [18,19] and special relativity [20–22]. Relativistic quantum information plays a key role in situations involving extremely long distances and its most useful applications are in entanglement-enhanced communication, quantum teleportation [23,24], quantum clock synchronization [25], reference frame alignment [26–32], quantum cryptography [33] and dense coding [34].

Entanglement is one of the fundamental features of quantum physics and its credit of being a unique property of quantum systems has been extensively utilized in constructing different protocols in many quantum information scenarios. Recent developments in relativistic quantum information brought to fore the relative behaviour of entanglement. It is an observer-dependent quantity which degrades with acceleration from the perspective of accelerated observer. On the other hand, correlations other than entanglement exist in quantum system whose benefits in quantum information have already been explored. Among such correlations, one is known as quantum discord, and its detail is given in the next section. It has already been used as a resource for certain quantum computation models [35], encoding of information onto a quantum state [36] and quantum state merging [37,38]. It has been pointed out that the localized Gaussian modes have different correlation properties compared to global Unruh modes. The quantum discord and the geometric discord for free modes of fermionic fields degrade to a nonvanishing value [39,40] in the limit of infinite acceleration in noninertial frames. However, for localized Gaussian modes, the quantum discord in noninertial frames asymptotes to zero [41] in the limit of infinite acceleration.

In this work we investigate the effect of relative motion on the spin correlations of entangled fermions. In particular, we examine the dynamics of quantum mutual information, classical correlations and quantum correlations quantified by quantum discord. The influence of the relative motion of the detectors, the size of the wave packet and the average momentum of the particle are the focus of this paper. Our results are peculiar in the sense that the dynamics of the correlations show equivalence in some domains of varying parameters while in other domains they drastically change their behaviour in such a way that quantum correlation becomes dominant. Specifically, there is always an optimal size of the wave packet at which quantum correlations exactly balance the total correlations leaving no space for the existence of classical correlations.

The paper is organized as follows: In the first part of §2, we present a brief review of the main concepts regarding the measures of different correlations. The rest of the section explains the model and the necessary mathematical machinery. Section 3 consists of a detailed discussion of our results. In §4 we summarize the paper and present the conclusion of the work.

2. Measurements of correlations and the system

As the main focus of our study is to investigate the behaviour of quantum discord, presenting its main formulas will make the paper self-contained. Introduced independently for bipartite states in refs [42–44], it is a more general measure of quantum correlations than the entanglement, in the sense that it captures correlations which do not come in the domain of the measures of entanglement. For a bipartite state defined by a density matrix ρ^{ab} it is, mathematically, given as the difference of total correlations and classical correlations. The measure of total correlations, also known as quantum mutual information, for such a state is given by

$$\mathcal{I}(\rho^{ab}) = \mathcal{S}(\rho^a) + \mathcal{S}(\rho^a) - \mathcal{S}(\rho^{ab}), \tag{1}$$

where $\mathcal{S}(\rho) = -\mathrm{Tr}[\rho \log_2 \rho]$ stands for the von Neumann entropy. The measure of classical correlations is given as

$$\mathcal{C}_b(\rho^{ab}) = S(\rho^b) - \min_{\{\Pi_k^a\}} \sum_k p_k S(\rho_k^b), \tag{2}$$

where Π_k^a $(k = 1, 2)$ represent the set of projectors onto the Hilbert space of subsystem a. The state $\rho_k^b = \mathrm{Tr}_a((\Pi_k^a \otimes I^b)\rho^{ab}(\Pi_k^a \otimes I^b))/p_k$ is the postmeasurement state of subsystem b after obtaining the outcome k on subsystem a with probability $p_k = \mathrm{Tr}((\Pi_k^a \otimes I^b)\rho^{\mathrm{AB}} \times (\Pi_k^a \otimes I^b))$ and I^b is the identity operator acting on the Hilbert space of subsystem b of the composite system. It is important to mention that the measure of classical correlations depends on which marginal state the projective measurement is made. This asymmetric behaviour ($\mathcal{C}_a(\rho^{ab}) \neq \mathcal{C}_b(\rho^{ab})$) of the measure of classical correlations accordingly make quantum discord an asymmetric quantifier for quantum correlations. Nevertheless, regardless of the asymmetry arising from the projective measurement on one or the other subsystem, quantum discord is always positive, $\mathcal{D}_a(\rho^{ab}), \mathcal{D}_b(\rho^{ab}) \geq 0$. Moreover, if ρ^{ab} is a classical quantum state, then $\mathcal{D}_a(\rho^{ab}) = \mathcal{D}_b(\rho^{ab}) = 0$. As we shall be dealing with two-dimensional marginal states, the general relation for the projectors onto the space of subsystem a can be written as

$$\Pi_k^a = \frac{1}{2}(\sigma^0 \pm \mathbf{n} \cdot \sigma), \tag{3}$$

where $\pm$ corresponds to the two values of k and $\sigma = \sigma(\sigma^1, \sigma^2, \sigma^3)$ with σ^j $(j = 0, 1, 2, 3)$, respectively, represent the 2×2 identity matrix and the three Pauli spin matrices. The vector $\mathbf{n} = n(x_1, x_2, x_3)$ is a unit vector on Bloch sphere with its three Cartesian components denoted by x_i. Using the definition and eqs (1) and (2), the quantum discord becomes

$$\begin{aligned}\mathcal{D}_a(\rho) &= \mathcal{I}(\rho) - \mathcal{C}_b(\rho)\\ &= \mathcal{S}(\rho^a) - \mathcal{S}(\rho^{ab}) + \min_{\{\Pi_k^a\}} \mathcal{S}(\rho^b|\{\Pi_k^a\}).\end{aligned} \tag{4}$$

Analytical evaluation of quantum discord for all possible quantum states is not possible. The main hurdle in doing so comes in the complex minimization procedure of the quantum conditional entropy $\mathcal{S}(\rho^b|\Pi_\pm^a)$ over all the possible von Neumann measurements.

We consider two identical spin-1/2 particles (A, B) with zero total spin angular momentum as our system and study the dynamics of quantum correlations in the relativistic set-up measured by two moving observers equiped with detectors sensitive only to the spin degrees of freedom. We assume that all the motions are along x-axis and the state of particle A is observed through the left detector whereas the right one focusses on the state of particle B. The normalized state for the composite system can be written as [45]

$$|\psi\rangle = \sum_{s_{\mathrm{A}}, s_{\mathrm{B}}} \int \mathrm{d}\mathbf{p}_{\mathrm{A}} \mathrm{d}\mathbf{p}_{\mathrm{B}} \psi_{s_{\mathrm{A}}, s_{\mathrm{B}}}(\mathbf{p}_{\mathrm{A}}, \mathbf{p}_{\mathrm{B}})\, |s_{\mathrm{A}}, \mathbf{p}_{\mathrm{A}}\rangle\, |s_{\mathrm{B}}, \mathbf{p}_{\mathrm{B}}\rangle, \tag{5}$$

with

$$\sum_{s_A, s_B} \int d\mathbf{p}_A d\mathbf{p}_B \left|\psi_{s_A, s_B}(\mathbf{p}_A, \mathbf{p}_B)\right|^2 = 1 \tag{6}$$

and

$$\langle s'_\chi, \mathbf{p}'_\chi | s_\chi, \mathbf{p}_\chi \rangle = \delta_{s'_\chi, s_\chi} \delta(\mathbf{p}'_\chi, \mathbf{p}_\chi), \tag{7}$$

where $\chi = (\mathrm{A}, \mathrm{B})$. We assume that the two-particle system is initially prepared in the following singlet state:

$$|\psi(\mathbf{p}_A, \mathbf{p}_B)\rangle = \frac{1}{\sqrt{2}} \left[\psi_{1/2}(\mathbf{p}_A) \otimes \psi_{-1/2}(\mathbf{p}_B) - \psi_{-1/2}(\mathbf{p}_A) \otimes \psi_{1/2}(\mathbf{p}_B)\right], \tag{8}$$

with

$$\psi_{1/2}(\mathbf{p}_\chi) = \begin{pmatrix} f_{\mathbf{k}_\chi}(\mathbf{p}_\chi) \\ 0 \end{pmatrix} \text{ and } \psi_{-1/2}(\mathbf{p}_\chi) = \begin{pmatrix} 0 \\ f_{\mathbf{k}_\chi}(\mathbf{p}_\chi) \end{pmatrix}. \tag{9}$$

In eq. (9), $f_{\mathbf{k}_\chi}(\mathbf{p}_\chi)$ are Gaussian in nature and are explicitly given by $f_{\mathbf{k}_\chi}(\mathbf{p}_\chi) = \pi^{-3/4} w^{-3/2} \exp[-(\mathbf{p}_\chi - \mathbf{k}_\chi)^2/2w^2]$, with w being a positive real parameter. The parameters w and $\mathbf{k}$ represent the momentum dispersion and average momentum of the particles. As mentioned already, we shall focus our analysis on the situation in which both the particles are moving in the opposite direction from the origin at the same rate along x-axis of the laboratory frame with wave vectors $k_A = -k_B = (|k|, 0, 0)$. Moreover, we consider that the velocities of the two detectors are, respectively, given by $\mathbf{v}_{d_A} = (\mathbf{v}_{d_A}, 0, 0)$ and $\mathbf{v}_{d_B} = (v_{d_B}, 0, 0)$. It is well known that in the proper frames of the detectors, the observed wavefunction $|\varphi\rangle$ is a transformed wavefunction, which is obtained according to the following unitary transformation of $|\psi\rangle$ [2,45]:

$$|\varphi\rangle = U(\Lambda_{d_A}) \otimes U(\Lambda_{d_B}) |\psi\rangle, \tag{10}$$

with

$$U(\Lambda_{d_\chi})|s_\chi, p_\chi\rangle = \sqrt{[(\Lambda_{d_\chi} p_\chi)^0/p^0]} \times \sum_{s'_\chi} D'_{s_\chi, s'_\chi}(\Lambda_{d_\chi}, p_\chi)|s'_\chi, p_\chi\rangle, \tag{11}$$

where $D(\Lambda_{d_\chi}, p_\chi)$ is known as the Wigner rotation. In matrix form it can be written as

$$D(\Lambda_{d_\chi}, p_\chi) = \frac{(p^0_\chi + m)\sigma^0 \cosh(\alpha_{d_\chi}/2) + (p^x_\chi \sigma^0 + i\epsilon^{xij} p^i_\chi \sigma^j) \sinh(\alpha_{d_\chi}/2)}{[(p^0_\chi + m)((\Lambda_{d_\chi} p_\chi)^0 + m)]^{1/2}}. \tag{12}$$

In eq. (12), the new parameter α_{d_X} stands for rapidity, which is given by $\alpha_{d_X} = -\tanh^{-1} v_{d_X}$. For the boost along x direction, we have

$$\Lambda_{d_X} = \begin{bmatrix} \cosh(\alpha_{d_X}) & \sinh(\alpha_{d_X}) & 0 & 0 \\ \sinh(\alpha_{d_X}) & \cosh(\alpha_{d_X}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

Through back substitution of eq. (13) into eq. (12) and then into eq. (11), the result of which along with eq. (5) substituted into eq. (10) gives the transformed wavefunction, which can further be reduced to the following form by the use of eq. (9):

$$|\varphi(\mathbf{p}_A, \mathbf{p}_B)\rangle = \frac{1}{\sqrt{2}} \left[\varphi_{1/2}(\mathbf{p}_A) \otimes \varphi_{-1/2}(\mathbf{p}_B) - \varphi_{-1/2}(\mathbf{p}_A) \otimes \varphi_{1/2}(\mathbf{p}_B)\right], \tag{14}$$

where

$$\varphi_{1/2}(\mathbf{p}_\chi) = \begin{pmatrix} a_1(\mathbf{p}_\chi) \\ a_2(\mathbf{p}_\chi) \end{pmatrix} \text{ and } \varphi_{-1/2}(\mathbf{p}_\chi) = \begin{pmatrix} -a_2(\mathbf{p}_\chi) \\ a_1^*(\mathbf{p}_\chi) \end{pmatrix}, \tag{15}$$

with

$$\begin{aligned} a_1(\mathbf{p}_\chi) &= k_\chi f_{\mathbf{k}_\chi}(\mathbf{q}_\chi)[c_\chi(q^0_\chi + m) + s_\chi(q^x_\chi + iq^y_\chi)], \\ a_2(\mathbf{p}_\chi) &= k_\chi f_{\mathbf{k}_\chi}(\mathbf{q}_\chi) s_\chi q^z_\chi, \\ k_\chi &= [(q^0_\chi/p^0_\chi)/((q^0_\chi + m)(p^0_\chi + m))]^{1/2}. \end{aligned} \tag{16}$$

In the preceding equations the new parameters are defined as $c_\chi = \cosh(\alpha_{d_\chi}/2)$, $s_\chi = \sinh(\alpha_{d_\chi}/2)$, $q_\chi = \Lambda^{-1}_{d_\chi}[p_\chi]$. In deriving eq. (14), we have made the change of variable $p_\chi \to \Lambda^{-1}_{d_\chi} p_\chi$ using the fact that $d\mathbf{p}_\chi/p^0_\chi$ is a relativistic invariant. Equation (14) gives the final wavefunction in the proper frames of the detectors at which they act to produce measured values. As the detectors are sensitive only to the spin degrees of freedom of the total wavefunction, the part of the wavefunction representing momenta degrees of freedom needs to be discarded. But, in general, it is impossible to investigate the spin of a particle independently by tracing over its momentum degree of freedom [46,47]. Nevertheless, under particular considerations in which both the detectors are boosted with the same absolute rapidity and having the same size of the wave packet and average momentum of the particles, the partial trace can be implemented [48]. We work in this limit and construct the density matrix from the total transformed wavefunction and then partial tracing over the momenta degrees of freedom to obtain the reduce density matrix only in the spin degrees of freedom as follows [8]:

$$\varrho = \int \mathrm{d}\mathbf{p}_{\mathrm{A}}\mathrm{d}\mathbf{p}_{\mathrm{B}}\varphi(\mathbf{p}_{\mathrm{A}},\mathbf{p}_{\mathrm{B}})\varphi^{\dagger}(\mathbf{p}_{\mathrm{A}},\mathbf{p}_{\mathrm{B}}) = \frac{1}{2}[\varrho_1^{\mathrm{A}}\otimes\varrho_1^{\mathrm{B}} - \varrho_2^{\mathrm{A}}\otimes\varrho_2^{\mathrm{B}} - \varrho_3^{\mathrm{A}}\otimes\varrho_3^{\mathrm{B}} + \varrho_4^{\mathrm{A}}\otimes\varrho_4^{\mathrm{B}}], \quad (17)$$

where

$$\varrho_1^{\mathrm{A}} = \begin{bmatrix} 1-V(\alpha_{d_{\mathrm{A}}}) & 0 \\ 0 & V(\alpha_{d_{\mathrm{A}}}) \end{bmatrix},$$

$$\varrho_2^{\mathrm{A}} = \begin{bmatrix} 0 & 1-3V(\alpha_{d_{\mathrm{A}}}) \\ -V(\alpha_{d_{\mathrm{A}}}) & 0 \end{bmatrix},$$

$$\varrho_3^{\mathrm{B}} = \begin{bmatrix} 0 & 1-3V(\alpha_{d_{\mathrm{B}}}) \\ -V(\alpha_{d_{\mathrm{B}}}) & 0 \end{bmatrix},$$

$$\varrho_4^{\mathrm{B}} = \begin{bmatrix} 1-V(\alpha_{d_{\mathrm{B}}}) & 0 \\ 0 & V(\alpha_{d_{\mathrm{B}}}) \end{bmatrix},$$

$$\varrho_4^{\mathrm{A}} = Adj[\varrho_1^{\mathrm{A}}], \quad \varrho_1^{\mathrm{B}} = Adj[\varrho_4^{\mathrm{B}}],$$

$$\varrho_3^{\mathrm{A}} = [\varrho_2^{\mathrm{A}}]^T \quad \text{and} \quad \varrho_2^{\mathrm{B}} = [\varrho_3^{\mathrm{B}}]^T, \quad (18)$$

and

$$V(\alpha_{d_\chi}) = \sinh^2(\alpha_{d_\chi}/2)\int \mathrm{d}\mathbf{q}_\chi \frac{|f_{\mathbf{k}_\chi}(\mathbf{q}_\chi)|^2 q_\chi^{z^2}}{(q_\chi^0+m)(p_\chi^0+m)}, \quad (19)$$

where we have used $\mathrm{d}\mathbf{p}_\chi/p_\chi^0 = \mathrm{d}\mathbf{q}_\chi/q_\chi^0$.

3. Results and discussion

With all the formulas being known, we are now in a position to derive the required results for investigating the dynamics of correlations under the influence of relative motion. Using the defining equation, the analytical result for the total correlation $\mathcal{I}(\varrho^{\mathrm{AB}})$ can easily be derived, which can be written as follows:

$$\mathcal{I}(\varrho^{\mathrm{AB}}) = 2 + \eta\log[\eta] + 2[\mu-2\eta]\log[\mu-2\eta] + [1-2\mu+3\eta]\log[1-2\mu+3\eta]. \quad (20)$$

To derive the analytical results for classical correlations and quantum discord, we need to minimize the conditional entropy over all the von Neumann measurements. Through the normalization condition for the Bloch vector, we can write the magnitude of one of its components, say x_2, in terms of the magnitudes of the other two components. With this substitution, the final result is only x_1-dependent, which can straightforwardly be checked for evaluating the minimum. The analytical results for classical correlation $\mathcal{C}(\varrho^{\mathrm{AB}})$ and the quantum discord $\mathcal{D}^{\mathrm{A}}(\varrho^{\mathrm{AB}})$ are then obtained as follows:

$$\mathcal{C}(\varrho^{\mathrm{AB}}) = 1 + (1-\mu+\eta)\log[1-\mu+\eta] - (\mu-\eta)\log[\mu-\eta], \quad (21)$$

$$\mathcal{D}^{\mathrm{A}}(\varrho^{\mathrm{AB}}) = 1 + \eta\log[\eta] + 2[\mu-2\eta]\log[\mu-2\eta] + [1-2\mu+3\eta]\log[1-2\mu+3\eta] - (1-\mu+\eta)\log[1-\mu+\eta] - (\mu-\eta)\log[\mu-\eta]. \quad (22)$$

In the preceding equations, we have substituted $\mu = V(\alpha_{d_{\mathrm{A}}}) + V(\alpha_{d_{\mathrm{B}}})$ and $\eta = 2V(\alpha_{d_{\mathrm{A}}})V(\alpha_{d_{\mathrm{B}}})$. The above results for measuring correlations are general, and we can adjust them for specific cases. We focus our attention on the case where both detectors are boosted in the opposite directions and the same absolute rapidity $\alpha_{d_{\mathrm{A}}} = -\alpha_{d_{\mathrm{B}}} = -|\alpha|$, so that $v_{d_{\mathrm{A}}} = -v_{d_{\mathrm{A}}} = \tanh|\alpha|$. Although we managed to obtain the analytical results, there is still an issue in the form of integral of eq. (19), which cannot always be analytically solved. So, we shall confine ourselves to investigate the dynamics of correlations by solving the said integral numerically. In order to do that, we rewrite eq. (19) in cylindrical coordinates with Q_X as the symmetry axis

$$V(\alpha) = \frac{\sinh^2(\alpha/2)}{\sqrt{\pi}W^3}\int_{-\infty}^{\infty}\mathrm{d}Q_x \times \int_0^{\infty}\mathrm{d}Q_r \frac{Q_r^3\exp[-(Q_x-K)^2+Q_r^2]/W^2}{(Q_0+1)(Q_0\cosh(\alpha)-Q_x\sinh(\alpha)+1)}. \quad (23)$$

In eq. (23), we have defined the normalized nondimensional variables $Q_r = q_r/m$, $Q_x = q_x/m$, $W = w/m$, $\mathbf{K} = \mathbf{k}_{\mathrm{A}}/m = \mathbf{k}_{\mathrm{B}}/m$ and $Q_0 = \sqrt{Q_r^2+Q_x^2+1}$. We can now simulate the results for different correlations by solving this integral numerically and can investigate the relativistic effects on these correlations.

The effect of rapidity on the dynamics of the three types of correlations for average momentum being small is shown in figure 1. Here, figure 1a represents the behaviour of quantum mutual information, figure 1b shows the dynamics of classical correlations and figure 1c shows the behaviour of quantum discord. Each curve in these figures corresponds to a different value of the dispersion parameter W (values specified in the caption of the figure) with a fixed average momentum parameter K, set to 0.01. As can be seen from the figures, the qualitative behaviour of all the three types

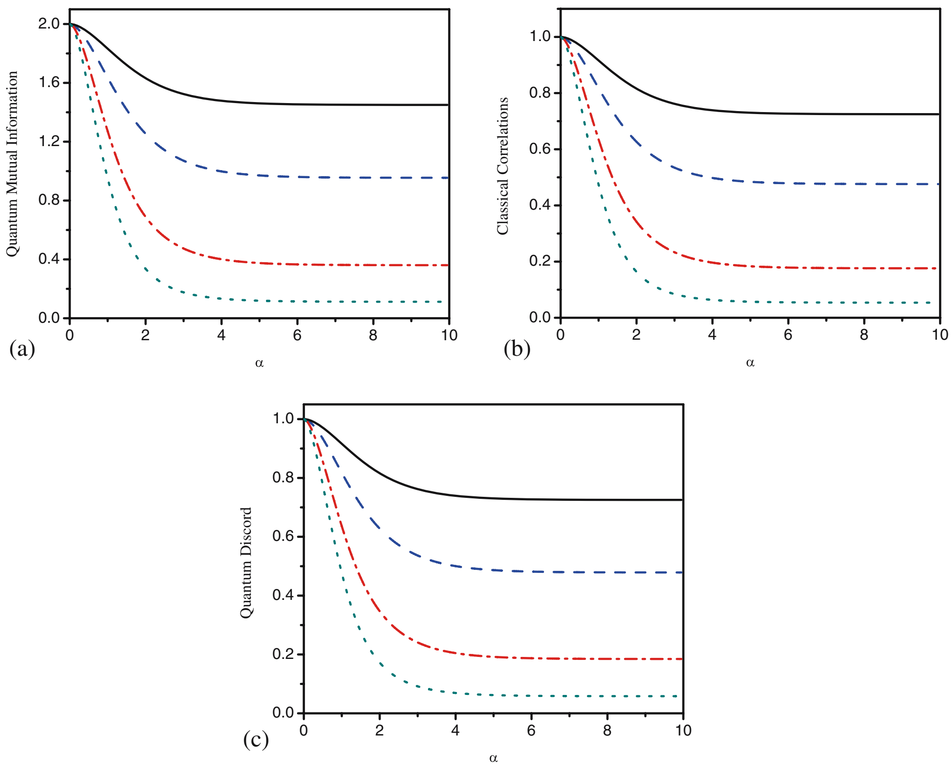

Figure 1. The three types of correlations, ((**a**) quantum mutual information, (**b**) classical correlations and (**c**) quantum discord) are plotted against rapidity for $K = 0.01$. In each figure the values of W for the curves are as follows: $W = 0.5$ (black curve), $W = 1$ (blue curve), $W = 3$ (red curve) and $W = 30$ (cyan curve).

of correlations against rapidity for every corresponding value of W is identical. For every value of W, they initially decrease monotonically with increasing rapidity, reaching a minimum and then become constant. It is important to note that the loss in every type of correlations with increasing rapidity is small for small degree of dispersion in momentum. Similarly, for the whole range of rapidity and for every value of W, the total correlation that exists in the system is evenly distributed between the classical and the quantum correlations. From the perspective of quantum information theory, this means that neither of the two correlations is more beneficial for quantum information processing purposes than the other.

The dynamics of the different types of correlations against rapidity for comparatively large average momentum are shown in figure 2. Again, figures 2a, 2b and 2c, respectively, represent the total correlations, the classical correlations and the quantum discord. Similar to the case of figure 1, each curve in these figures corresponds to a particular value of W with the average momentum K frozen to a fixed value of 100. The value of W for each curve is given in the caption of the figure. In this case, the total correlations is no more evenly distributed between the classical and the quantum correlations, rather its behaviour is two-fold against rapidity. In the range consisting of small values of rapidity, the total correlation is evenly distributed between the classical and the quantum correlations. As can be seen, all the three types of correlations are monotonically decreasing functions of rapidity. However, each of them reaches a different minimum for every value of W. This makes the dynamics of these correlations very interesting in the range of values of rapidity beyond the one corresponding to each minimum. For all values of W, the classical correlations are frozen at zero, and the total correlation as well the quantum correlation start growing, reaching a maximum and becomes stable with rapidity. It is important to note that the rate of revival of the quantum correlations is fast and more for the intermediate range of values of the dispersion parameter W. The revival of quantum correlations with increasing rapidity is a remarkable feat from the quantum information point of view, which shows its utility and dominance over classical correlations.

To further investigate the dynamics of these correlations, we plot each of them in figure 3 against the width W of the wave packet for large, however fixed, rapidity at different values of the average momentum K. In all these figures, the black curve represents classical correlations, the red curve shows quantum discord and the blue curve represents quantum mutual information.

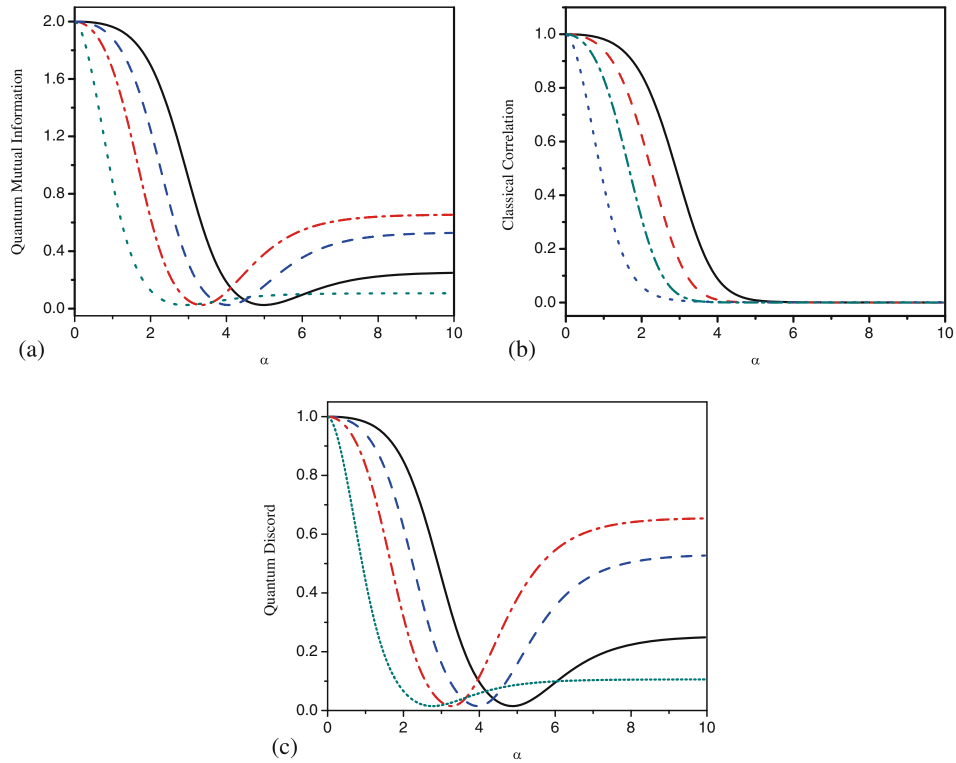

Figure 2. The three types of correlations ((**a**) quantum mutual information, (**b**) classical correlations and (**c**) quantum discord) are plotted against rapidity for $K = 100$. In each figure $W = 5$ is represented by black curve, $W = 10$ is represented by blue curve, $W = 20$ is represented by red curve and $W = 100$ is represented by cyan curve.

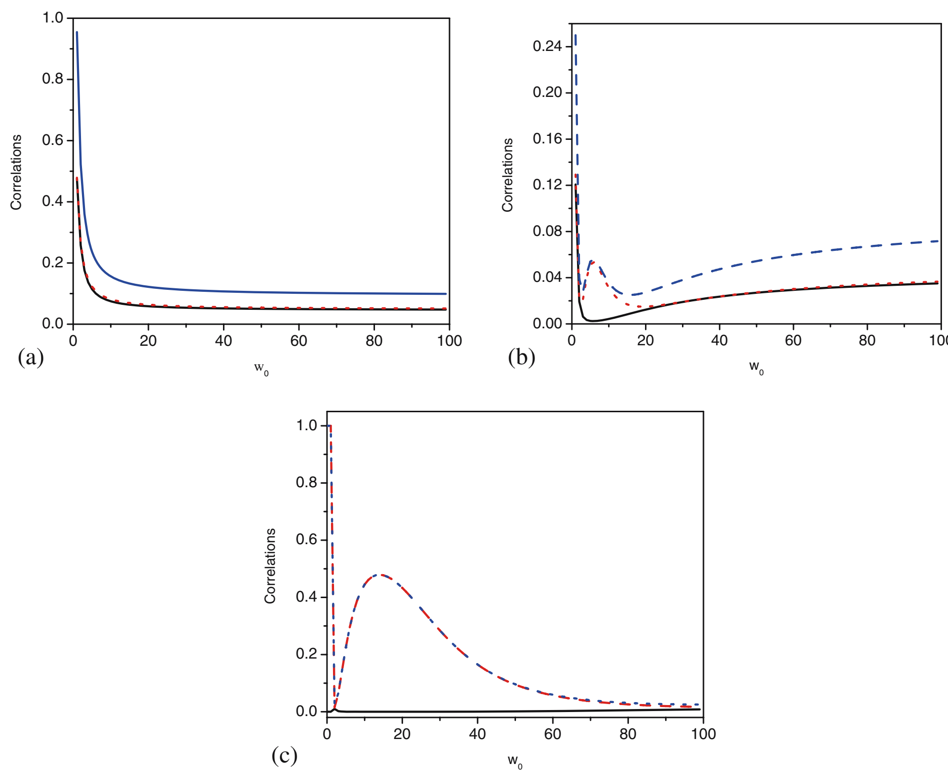

Figure 3. The three types of correlations are plotted against the size of wave packet W with rapidity $\alpha = 300$. In each of these figures, the black curve represents classical correlations, red is for quantum discord and the blue stands for quantum mutual information. The value of K in each figure is: (**a**) $K = 0.01$, (**b**) $K = 10$ and (**c**) $K = 100$.

It can be seen that for small values of K (figure 3a), there is no difference between the classical and the quantum correlations, they fall sharply together to a minimum, however, become stable at a nonzero value in the limit of large dispersion. For large values of K, there is always a range of values of W in which the quantum correlation beats their classical counterpart and remains high (figure 3b). It is important to note that the width of the range of W in which the quantum correlations remain dominant extends with the increasing value of K (figure 3c). To further comment, for large values of K, the total correlations in the system exist in the form of quantum correlations with no classical one and the optimal size of the wave packet containing maximum quantum correlations varies according to the value of K. However, as the size of the wave packet grows, the correlations die out completely.

4. Conclusions

In summary, the effects of relativity, the size of the wave packet and the average momentum of the particles on the dynamics of different correlations present in the spin degree of freedom of a bipartite fermionic system are investigated. It is shown that regardless of the size of the wave packet, qualitatively there is no difference in the behaviour of all the three types of correlations in the limit of small average momentum. In this limit, the total correlations are evenly distributed between the classical and the quantum correlations. However, in the limit of large average momentum, the quantum correlations are not only stronger than the classical ones but also regrow with increasing rapidity after achieving a minimum value. It is shown that for large momentum and high rapidity, the only type of correlation that exists in the system is the quantum correlation. We show that for every value of the average momentum, there is an optimal size of the wave packet that possesses more quantum correlations. From the perspective of quantum information processing, the presence of high degree quantum correlations at large momentum and high rapidity, and in particular its revival, are motivating features that need further exploration in other quantum systems under similar conditions.

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