Pramana – J. Phys. (2016) 87: 42
DOI 10.1007/s12043-016-1253-3

A mini review on CP-violating minimal supersymmetric Standard Model Higgs

AMIT CHAKRABORTY[1,*] and DILIP KUMAR GHOSH[2]

[1]Department of Theoretical Physics, Tata Institute of Fundamental Research,
1, Homi Bhabha Road, Mumbai 400 005, India
[2]Department of Theoretical Physics, Indian Association for the Cultivation of Science,
2A & 2B, Raja S.C. Mullick Road, Jadavpur, Kolkata 700 032, India
*Corresponding author. E-mail: amit@theory.tifr.res.in

Published online 24 August 2016

Abstract. We discuss the present status of the Higgs sector of the CP-violating minimal supersymmetric Standard Model (CPVMSSM). In the Standard Model (SM) of particle physics, the only source of CP violation is the complex phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix. By now we all know that this single phase is not large enough to explain the observed baryon asymmetry of our Universe. Hence, one require additional sources of CP violation. The MSSM with several complex phases is one such scenario. The tree-level CP invariance of the MSSM Higgs potential is broken at one-loop level in the presence of complex phases in the MSSM Lagrangian. The presence of these additional phases modifies Higgs masses, mixings and couplings significantly. These additional phases have non-trivial impact on several low-energy observables; like the electric dipole moments (EDMs) of atoms and molecules, the CP asymmetry in rare b-decays etc. We first present a brief outline of the CPVMSSM Higgs sector, and then discuss the current limits/bounds obtained from the measurements of several low-energy observables. We also comment on the current bounds coming from the high-energy collider experiments, specially the Large Electron Positron (LEP) Collider and the ongoing Large Hadron Collider (LHC) at the CERN.

Keywords. Higgs boson; supersymmetry; CP violation; Large Hadron Collider.

PACS Nos 14.80.Bn; 14.80.Ec

1. Introduction

In physics, symmetry plays an important role towards our understanding of the laws of nature. It essentially simplifies the description of the physical phenomena. CP is a discrete symmetry of nature denoted as the product of two symmetry operations: charge conjugation (C) and parity (P). Charge conjugation transforms a particle into the corresponding antiparticle. For example, if we apply C to an electron with electric charge -1, we will obtain a positron with electric charge $+1$. On the other hand, parity transforms everything into its mirror image, i.e., left-handed system becomes right-handed and vice versa. The electromagnetic and strong interactions are symmetric under both C and P, and so they are also symmetric under the combined CP transformation. However, the weak interaction is not invariant under the C and P operation. In 1957, Chien-Shiung Wu and collaborators performed an experiment with Cobalt-60 and confirmed that weak interaction does not conserve parity [1]. The realization that both C- and P-symmetry are violated, leads to the idea of a new symmetry, called CP symmetry. The philosophy behind defining this new symmetry is that C and P may not be the right symmetry of nature, rather CP is actually the more fundamental symmetry. In fact, according to the CPT theorem, any Lorentz-invariant local quantum field theory with a Hermitian Hamiltonian must be symmetric under a combined operation of C, P and T, where T represents the time-reversal operator. So, any violation of the CP symmetry would automatically imply violation of the T symmetry.

In 1964, CP violation was first observed experimentally by Cronin *et al* in the rare decay of neutral K-meson [2]. The two CP eigenstates of the K-meson, namely K_S^0 and K_L^0 [2a], are the linear combinations of the electrically neutral K-meson K^0 (containing a down quark and a strange antiquark) and its antiparticle $\bar{K}^0$ (containing a strange quark and a down antiquark) states, i.e.,

$$K_S^0 = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K^0}\rangle)$$

$$K_L^0 = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K^0}\rangle). \qquad (1)$$

From eq. (1) it is clear that the CP eigenvalues of K_S^0 and K_L^0 are $+1$ and -1 respectively. Thus, in order to preserve the CP symmetry, the state K_S^0 should always decay to $\pi^+\pi^-$ or $\pi^0\pi^0$ while K_L^0 should not decay to a pair of πs [2b]. But, in the experiment, Cronin *et al* observed the decay of K_L^0 to a pair of pions (one in 500 decays) which is only possible if CP symmetry is violated. Even though, the observed effect was very small, still it provided an important information that laws of physics are not the same for matter and antimatter [2c]. Similarly, a neutral B-meson either can decay directly, or may oscillate into its antiparticle before decaying. The difference in decay paths leads to a non-zero phase, which leads to a CP asymmetry. The LHCb experiment at the CERN has observed such CP asymmetries in the $B_0 \to K^+\pi^-$ and $B_s^0 \to K^-\pi^+$ decays with an integrated luminosity of 1.0 fb^{-1} at 7 TeV centre-of-mass energy [3]. Similarly, CP violation effects are also observed in both the charged and neutral K-mesons and B-mesons as well as in the neutral D-mesons at the Belle and BaBar experiments and also at the LHCb experiment [3–9]. For more updates we refer the Particle Data Group [10].

Before we proceed, let us ask ourselves why CP violation is so important to us. According to the Big-Bang theory, equal amounts of matter and antimatter were initially produced. As the Universe cools, the annihilation of matter and antimatter starts producing photons and nothing else. The relic of this primordial annihilation is the 2.7 K cosmic microwave background (CMB) radiation that fills the entire Universe. However, our Universe at present is composed of matter particles (i.e., protons, neutrons, electrons etc.) only. So one can immediately ask why our present Universe is completely dominated by matter or, where did those antimatter particles go! In 1967, Andrei Sakharov first proposed a solution to this puzzle. Among three boundary conditions, as proposed by Sakharov, CP violation is a necessary condition for baryogenesis that can explain how a Universe initially containing equal amounts of matter and antimatter finally evolved into a matter-dominated Universe [11].

CP violation requires the presence of couplings which, in general, are complex. One can always express the complex coupling in terms of its absolute value multiplied by the exponential of some angle or phase. For example, for an arbitrary complex coupling $y = |y| \times e^{i\theta_y}$ where θ_y is the associated phase. In the Standard Model (SM) [12–14], the only source of CP violation is the phase in the 3×3 complex unitary quark mixing matrix, known as Cabibbo–Kobayashi–Maskawa (CKM) matrix [15] which was introduced by Kobayashi and Maskawa [15a]. Applying the unitarity condition on the matrix and performing phase redefinitions, one finds four independent quantities, namely three real angles and one phase. This non-vanishing phase of the CKM matrix is the only source of CP violation within the SM. All parameters of the CKM matrix can be described as the sides and angles of a triangle, known as 'unitarity triangle', and the height of this triangle is the measure of CP-violation in the theory. Interestingly, the single phase of the CKM matrix can perfectly explain all the experimental observations obtained till date. However, the amount of baryon asymmetry predicted by this phase is a few orders of magnitude smaller compared to what we have observed in nature [16–21]. So, we need additional sources of CP violation. Many extensions of the SM have been proposed, and most of them contain additional sources of CP violation and thus can explain the observed baryon asymmetry. But, how do we measure the CPV effects in an experiment? We generally search for these effects in a variety of processes, e.g., rare decay of mesons, oscillations between flavour eigenstates of mesons, measurement of electric dipole moments of electron, neutron and nuclei etc. In fact, we precisely measure various observables and asymmetries and compare our findings with the SM predictions. Any sign of discrepancy will directly lead to a potential signature of physics beyond the Standard Model (BSM).

Supersymmetry (SUSY) [22–24] has been one of the most popular choices for formulating BSM physics. Supersymmetric extensions of the SM which provide the most natural solution to the gauge hierarchy problem by cancelling large quadratically divergent contributions to the Higgs mass introducing scalar partners of the SM particles, provides stable dark matter candidates, explains neutrino masses etc. The simplest minimal realization, known as the minimal supersymmetric Standard Model (MSSM), has been the prime interest of several theoretical and experimental studies.

So far, no SUSY particle has been found experimentally which in turn put strong bounds on the sparticle masses [25,26].

Majority of collider analyses of the MSSM assume that all the SUSY parameters are real (i.e., a CP-conserving scenario). However, here we shall allow terms in the Lagrangian which can, in general, be complex, thus posses CP-violating phases. The most general MSSM Lagrangian contains three Yukawa matrices, three triscalar coupling matrices and five scalar mass matrices. Now, each of the Yukawa and triscalar matrices are, in general, 3×3, and thus each has nine real and nine imaginary elements. On the other hand, each of the scalar mass matrix is a 3×3 Hermitian matrix, and thus has six real and three imaginary elements. Altogether, we have 84 real elements and 69 phases. However, not all of these parameters are physical. Some of these can be rotated away by redefinition of fields, leaving 69 real and 41 CP-violating phases [27]. These additional phases of the MSSM may thus be considered as new sources of CP violation. Thus, in the presence of these new phases, one can expect to obtain large contributions to electric dipole moments (EDM) of fundamental particles (see §4 for details). However non-observation of permanent EDMs puts strong constraints on these new CP-violating phases.

The MSSM Higgs potential is CP invariant at the tree level. However, loop contributions involving these extra CP-violating phases associated with different SUSY parameters, for example phase of Higgsino mass parameter μ (ϕ_μ), trilinear couplings $A_{t,b,\tau}$ ($\phi_{A_{t,b,\tau}}$), gaugino mass parameters $M_{1,2,3}$ ($\phi_{M_{1,2,3}}$) etc., can break the tree-level CP invariance of the MSSM Higgs potential. Thus, unlike the CP-conserving MSSM, the physical Higgs eigenstates now become an admixture of CP-even and CP-odd states. Moreover, the presence of these CP-violating phases, and thereby mixing between the CP eigenstates, can substantially modify the Higgs boson masses and couplings, leading to a variety of interesting signatures at the colliders.

The rest of the article is organized as follows: in §2 we introduce the minimal supersymmetric Standard Model (MSSM) and its particle contents. In §3 we discuss the CP-violating MSSM, in particular CP-violating MSSM Higgs sector. Section 4 describes different observables which can be used to probe CP-violating effects in the MSSM, while Higgs searches at the colliders along with its implications on the CPVMSSM parameter space are discussed in §5. Finally we conclude in §6.

2. Minimal supersymmetric Standard Model

In this section we introduce the minimal supersymmetric Standard Model (MSSM). Here, the word 'minimal' refers to the minimal choice of additional particles added to the SM to build the supersymmetric field theory. In the MSSM, every SM matter field is replaced by a chiral superfield, and every vector field is replaced by a vector superfield, which actually doubles the SM particle content. The MSSM particle spectrum and their transformation properties under SM gauge group ($G_{\rm SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$) can be symbolically written as [23,28]

$$\begin{aligned} Q_i &\equiv \begin{pmatrix} u_i & \tilde{u}_i \\ d_i & \tilde{d}_i \end{pmatrix}_L \sim (3, 2, 1/3) \\ \bar{U}_i &= U_i^c \equiv (u_i^\dagger \quad \tilde{u}_i^*) \sim (3, 1, -4/3) \\ \bar{D}_i &= D_i^c \equiv (d_i^\dagger \quad \tilde{d}_i^*) \sim (3, 1, 2/3) \\ L_i &\equiv \begin{pmatrix} \nu_{e_i} & \tilde{\nu}_{e_i} \\ e_i & \tilde{e}_i \end{pmatrix}_L \sim (1, 2, -1) \\ \bar{E}_i &= E_i^c \equiv (e_i^\dagger \quad \tilde{e}_i^*) \sim (1, 1, 2). \end{aligned}$$

Here, i stands for the three generations ($i = 1, 2, 3$) for quarks and leptons, while the superscript c reminds ourselves that the corresponding chiral superfield is made up of conjugates of the SM fields. The numbers in the parenthesis describe the colour, isospin and hypercharge assignments respectively of the particles/fields present in the supermultiplets. The scalar partners of the quark and lepton are called as 'squark' and 'slepton' respectively, and altogether we call them 'sfermions'. We denote the superpartners using 'tilde' on their SM counterpart. The SM fermions (quarks and leptons) and their scalar superpartners sfermions (squarks and sleptons) define the chiral supermultiplet. As the left-handed and right-handed fermions transform differently under the SM gauge group, they belong to different chiral supermultiplets. In the MSSM, we require two Higgs doublets to give masses to the up-type quarks and down-type quarks and the leptons. The superpartner of the Higgses are called 'Higgsinos' [28a]. The two Higgs fields in the MSSM are:

$$\begin{aligned} H_1 &\equiv \begin{pmatrix} H_1^0 & \tilde{H}_1^0 \\ H_1^- & \tilde{H}_1^- \end{pmatrix}_L \sim (1, 2, -1), \\ H_2 &\equiv \begin{pmatrix} H_2^+ & \tilde{H}_2^0 \\ H_2^- & \tilde{H}_2^+ \end{pmatrix}_L \sim (1, 2, 1). \end{aligned} \tag{2}$$

Moreover in the MSSM, corresponding to each SM gauge field we introduce a vector superfield which transforms in the adjoint representation of the SM gauge group. Each vector superfield contains the SM gauge boson and its fermionic superpartner, called 'gauginos'. The three vector superfields of the MSSM are as follows:

$$V_s^a: \quad (G_\mu^a \quad \tilde{g}^a) \sim (8, 1, 0),$$
$$V_W^I: \quad (W_\mu^I \quad \tilde{W}^I) \sim (1, 3, 0),$$
$$V_Y: \quad (B_\mu^0 \quad \tilde{B}^0) \sim (1, 1, 0). \tag{3}$$

In the above, G_μ^a and $\tilde{g}^a$ are the gluon fields and their superpartner fields called 'gluinos' respectively, where a runs from 1 to 8. The Ws and Bs with $I = 1, 2, 3$ are the $SU(2)$ and $U(1)$ gauge fields respectively, while their respective superpartners $\tilde{W}$ and $\tilde{B}$ are called 'winos' and 'binos' respectively. The chiral and gauge supermultiplets, as shown in tables 1 and 2, make up the particle content of the MSSM.

If supersymmetry is an exact symmetry of nature, then one would expect to have the same mass for the particles and their corresponding superpartners. For example, there will be selectrons $\tilde{e}_L$ and $\tilde{e}_R$ with masses exactly equal to the mass of the electron, or there would be massless gluino and photino along with the massless gluon and photon. These particles would have been then already discovered in the past high-energy experiments. However, none of the superpartners of the SM particles have been discovered till date, and therefore SUSY must be broken. The mass eigenstates and mixing patterns of the particles and sparticles crucially depend on the nature of the SUSY breaking [28b]. The SUSY breaking terms, that we introduce in the SUSY Lagrangian, result in the mass splitting between the ordinary particles and their superpartners.

Once we break the SUSY and the $SU(2)_L \times U(1)_Y$ EW symmetry, particles and the corresponding sparticles become massive. The left squarks (sleptons) mix with the right squarks (sleptons), intergeneration mixings are also allowed. The EW gauginos mix with Higgsinos to form charged and neutral mass eigenstates known as charginos ($\tilde{\chi}_i^\pm, i = 1, 2$) and neutralinos

Table 1. Chiral supermultiplets of the MSSM [23].

	Spin		SM gauge group		
Supermultiplet	Spin-0	Spin-1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q_i	$(\tilde{u}_L \quad \tilde{d}_L)_i$	$(u_L \quad d_L)_i$	3	2	1/3
U_i	$\tilde{u}_{Ri}^*$	$u_{Ri}^\dagger$	$\bar{3}$	1	−4/3
D_i	$\tilde{d}_{Ri}^*$	$d_{Ri}^\dagger$	$\bar{3}$	1	2/3
L_i	$(\tilde{\nu}_{\ell L} \quad \tilde{\ell}_L)_i$	$(\nu_{\ell L} \quad \ell_L)_i$	1	2	1/3
E_i	$\tilde{e}_{Ri}^*$	$e_{Ri}^\dagger$	1	1	2
H_2	$(H_2^+ \quad H_2^0)$	$(\tilde{H}_2^+ \quad \tilde{H}_2^0)$	1	2	1
H_1	$(H_1^0 \quad H_1^-)$	$(\tilde{H}_1^0 \quad \tilde{H}_1^-)$	1	2	−1

Table 2. Gauge supermultiplets of the MSSM [23].

	Spin		SM gauge group		
Supermultiplet	Spin-1	Spin-1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
V_s^a	G^a	$\tilde{g}^a$	8	1	0
V_W^I	$W^\pm,\ W^3$	$\tilde{W}^\pm,\ \tilde{W}^3$	1	3	0
V_Y	B^0	$\tilde{B}^0$	1	1	0

Table 3. The neutralinos, charginos and Higgs bosons in the MSSM after electroweak symmetry breaking.

Name	Spin	Gauge eigenstates	Mass eigenstates
Higgs	0	$H_2^+, H_2^0, H_1^0, H_1^-$	$h, H, A, H^\pm$
Neutralinos	1/2	$\tilde{W}^3, \tilde{B}^0, \tilde{H}_2^0, \tilde{H}_1^0$	$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$
Charginos	1/2	$\tilde{W}_{1,2}, \tilde{H}_2^+, \tilde{H}_1^-$	$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$

($\tilde{\chi}_i^0$, $i = 1$–4). In addition to these, there are five Higgs bosons: two CP-even neutral h, H, one CP-odd neutral A and two charged $H^{\pm}$ bosons. These are summarized in table 3.

3. CP-violating MSSM Higgs sector

In the MSSM, the Higgs potential is CP-invariant at the tree-level. However, this CP-invariance of the MSSM Higgs potential can be broken spontaneously through radiative corrections [29,30]. One can also take another avenue by assuming some of the parameters/couplings present in the MSSM Lagrangian complex. One-loop effects can now introduce CP violation in the MSSM Higgs sector explicitly through these complex couplings. For example, the trilinear couplings (A_f, $f = u, d, c, s, t, b$) and the gaugino mass parameters ($M_{1,2,3}$) can acquire phases, and thus break the tree-level CP invariance. To distinguish the spontaneous and explicit CP violation, one generally defines various observables which are invariant under the weak basis transformations but change sign under CP transformations (for details, see refs [31–36]).

In the MSSM, the presence of several complex parameters in the superpotential and the soft SUSY breaking Lagrangian [36a] introduce an explicit CP violation:

$$\begin{aligned} W_{\text{MSSM}} &= \mu(H_1 \cdot H_2) - f_{ij}^e (H_1 \cdot L_i)\bar{E}_j \\ &\quad - f_{ij}^d (H_1 \cdot Q_i)\bar{D}_j - f_{ij}^u (Q_i \cdot H_2)\bar{U}_j \\ \mathcal{L}_{\text{SOFT}} &= \tilde{q}_{iL}^* (\mathcal{M}_{\tilde{q}}^2)_{ij} \tilde{q}_{jL} + \tilde{u}_{iR}^* (\mathcal{M}_{\tilde{u}}^2)_{ij} \tilde{u}_{jR} \\ &\quad + \tilde{d}_{iR}^* (\mathcal{M}_{\tilde{d}}^2)_{ij} \tilde{d}_{jR} + \tilde{l}_{iL}^* (\mathcal{M}_{\tilde{l}}^2)_{ij} \tilde{l}_{jL} \\ &\quad + \tilde{e}_{iR}^* (\mathcal{M}_{\tilde{e}}^2)_{ij} \tilde{e}_{jR} + [\tilde{\Phi}_1 \cdot \tilde{l}_{iL} (f^e A^e)_{ij} \tilde{e}_{jR}^* \\ &\quad + \tilde{\Phi}_1 \cdot \tilde{q}_{iL} (f^d A^d)_{ij} \tilde{d}_{jR}^* \\ &\quad + \tilde{q}_{iL} \cdot \Phi_2 (f^u A^u)_{ij} \tilde{u}_{jR}^* + \text{h.c.}] + m_1^2 |\tilde{\Phi}_1|^2 \\ &\quad + m_2^2 |\Phi_2|^2 + (m_{12}^2 \tilde{\Phi}_1 \cdot \Phi_2 + \text{h.c.}) \\ &\quad + \frac{1}{2}(M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}^I \tilde{W}^I + M_3 \tilde{g}^a \tilde{g}^a), \end{aligned} \tag{4}$$

where $\tilde{\Phi}_1$ is the scalar part of the Higgs chiral superfield H_1 and Φ_2 is the scalar part of the Higgs chiral superfield H_2. The soft bilinear Higgs-mixing mass parameter is denoted as $m_{12}^2 \equiv B\mu$. The MSSM parameters which, in principle, can have CP-odd phases are:

(1) The Higgsino mass parameter μ associated with the bilinear mixing of the two Higgs chiral superfields.
(2) The soft SUSY breaking gaugino masses $M_{1,2,3}$ corresponding to the gauginos of the gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ respectively.
(3) The soft bilinear Higgs-mixing mass parameter $m_{12}^2 = B\mu$.
(4) The soft SUSY breaking trilinear couplings A_f of the Higgs bosons to the scalar fermions f.

The Higgs potential of the CP-violating MSSM (CPVMSSM) is described by the two Higgs doublets $\tilde{\Phi}_1$ and Φ_2 with opposite hypercharges ($Y_{\tilde{\Phi}_1} = -1$, $Y_{\Phi_2} = 1$) in order to give masses to the up-type and down-type fermions and, at the same time, to cancel the chiral anomalies induced by the fermionic partners of the two Higgs fields. The MSSM Higgs potential (tree-level) in terms of the two Higgs fields $\Phi_1 \equiv -i\tau_2\tilde{\Phi}_1^*$ and Φ_2 can be written as

$$\begin{aligned} \mathcal{L}_V &= \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) + m_{12}^2(\Phi_1^\dagger\Phi_2) \\ &\quad + m_{12}^{*2}(\Phi_2^\dagger\Phi_1) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ &\quad + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ &\quad + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1), \end{aligned} \tag{5}$$

with

$$\begin{aligned} &\mu_1^2 = -m_1^2 - |\mu|^2, \quad \mu_2^2 = -m_2^2 - |\mu|^2, \\ &m_{12} = B\mu, \quad \lambda_1 = \lambda_2 = -\frac{1}{8}(g^2 + g_Y^2), \\ &\lambda_3 = -\frac{1}{4}(g^2 - g_Y^2), \quad \lambda_4 = \frac{1}{2}g^2. \end{aligned} \tag{6}$$

The complex parameter m_{12}^2 in eq. (5) as well as the real parameters m_1^2 and m_2^2 in eq. (6) are associated with the soft SUSY-breaking Higgs masses. Furthermore, g and g_Y are the usual $SU(2)_L$ and $U(1)_Y$ gauge couplings respectively.

To determine the ground state of the CPVMSSM Higgs potential (5), we express the two Higgs doublets in the following form:

$$\begin{aligned} \Phi_1 &= \begin{pmatrix} \phi_1^+ \\ (1/\sqrt{2})(v_1 + \phi_1^0 + i\zeta_1^0) \end{pmatrix}, \\ \Phi_2 &= \mathrm{e}^{i\xi} \begin{pmatrix} \phi_2^+ \\ (1/\sqrt{2})(v_2 + \phi_2^0 + i\zeta_2^0) \end{pmatrix}, \end{aligned} \tag{7}$$

where v_1 and v_2 are the vacuum expectation values (VEV) of the Higgs doublets, ξ is their relative phase and ϕ_i^0 ($i = 1, 2$) and ζ_i^0 ($i = 1, 2$) are the scalar and pseudoscalar components of the two Higgs doublets respectively. If m_{12} is real at the tree-level, vanishing of the CP-odd tadpole contributions imply $\xi = 0$. However, a non-zero ξ can always be generated at the one-loop level, but using judicious choice of $\Im m(m_{12})$

to cancel the tadpole graphs, we can always keep ξ zero in all orders of perturbation theory [37,38]. The minimization of the Higgs potential requires the vanishing of the following tadpole parameters:

$$T_{\phi_1^0} \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial \phi_1^0} \right\rangle = v_1 \left[\mu_1^2 + \Re e(m_{12}^2 \mathrm{e}^{i\xi}) \tan\beta - \frac{1}{2} M_Z^2 \cos 2\beta \right], \quad (8)$$

$$T_{\phi_2^0} \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial \phi_2^0} \right\rangle = v_2 \left[\mu_2^2 + \Re e(m_{12}^2 \mathrm{e}^{i\xi}) \cot\beta + \frac{1}{2} M_Z^2 \cos 2\beta \right], \quad (9)$$

$$T_{\zeta_1^0} \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial \zeta_1^0} \right\rangle = v_2 \Im m(m_{12}^2 \mathrm{e}^{i\xi}), \quad (10)$$

$$T_{\zeta_2^0} \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial \zeta_2^0} \right\rangle = -v_1 \Im m(m_{12}^2 \mathrm{e}^{i\xi}), \quad (11)$$

where $\tan\beta = v_2/v_1$ and $M_Z^2 = (g^2 + g_Y^2)v^2/4$ is the Z-boson mass squared with $v^2 = v_1^2 + v_2^2$. To conserve the electric charge of the physical ground state, we further require the vanishing of the variation of the potential with respect to the charged fields ϕ_1^+ and ϕ_2^+. One can perform an orthogonal rotation in the (ζ_1^0, ζ_2^0) plane,

$$\begin{pmatrix} \zeta_1^0 \\ \zeta_2^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \quad (12)$$

such that the G^0 field becomes the would-be Goldstone boson to be eaten up by the longitudinal component of the Z-boson. In this weak basis, the tree-level CP-odd scalar mass squared becomes

$$M_A^2 = \frac{\Re e(m_{12}^2 \mathrm{e}^{i\xi})}{\sin\beta\cos\beta}. \quad (13)$$

In the presence of CP-violating phases, the neutral Higgs boson mass square matrix is given by

$$\mathcal{L}_{\text{mass}} = \left(\zeta_1^0 \; \zeta_2^0 | \phi_1^0 \; \phi_2^0 \right) \left(\begin{array}{c|c} \mathcal{M}_P^2 & \mathcal{M}_{SP}^2 \\ \hline [\mathcal{M}_{SP}^2]^T & \mathcal{M}_S^2 \end{array} \right) \begin{pmatrix} \zeta_1^0 \\ \zeta_2^0 \\ \hline \phi_1^0 \\ \phi_2^0 \end{pmatrix}. \quad (14)$$

The 4×4 mass square matrix can be divided into four 2×2 blocks with $\mathcal{M}_P^2$, $\mathcal{M}_S^2$ and $\mathcal{M}_{SP}^2$ as the independent components. One should note that, the quantity $\mathcal{M}_{SP}^2$ is identically equal to zero in the CP-conserving MSSM. However, in the CP-violating MSSM, one-loop effects can generate such a term as given below [39]:

$$\mathcal{M}_{SP}^2 \approx \mathcal{O}\left(\frac{M_t^4 \, |\mu| |A_t|}{v^2 32\pi^2 M_{\text{SUSY}}^2} \right) \sin\Phi_{\text{CP}} \times \left[6, \frac{|A_t|^2}{M_{\text{SUSY}}^2}, \frac{|\mu|^2}{\tan\beta M_{\text{SUSY}}^2}, \frac{\sin 2\Phi_{\text{CP}} |A_t||\mu|}{\sin\Phi_{\text{CP}} M_{\text{SUSY}}^2} \right], \quad (15)$$

where $\Phi_{\text{CP}} = \arg(A_t\mu)$, $v = 246$ GeV and the mass scale M_{SUSY} is defined as

$$M_{\text{SUSY}}^2 = \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{2}, \quad (16)$$

with $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ being the stop masses. One can easily estimate the degree of CP violation in the Higgs sector by looking at the possible dominant contributions. For example, sizable scalar–pseudoscalar mixing is possible for a large CP-violating phase Φ_{CP}, $|\mu|$ and $|A_t| > M_{\text{SUSY}}$. Now, as the massless Goldstone boson G^0 does not mix with the other neutral states, the 4×4 mass matrix can be reduced to a 3×3 Higgs mass-squared matrix $\mathcal{M}_{ij}^2$, in the basis (A, ϕ_1^0, ϕ_2^0), where A is the appropriate eigenstate of $\mathcal{M}_P^2$. The 3×3 symmetric matrix $\mathcal{M}_{ij}^2$ can now be diagonalized by an orthogonal matrix $\mathcal{O}$, i.e., $M_i^2 \delta_{ij} = O_{ik} \mathcal{M}_{kl}^2 O_{jl}$, where M_i^2 corresponds to the diagonal mass eigenstates. The physical mass eigenstates h_1, h_2 and h_3 ($M_{h_1} < M_{h_2} < M_{h_3}$) are therefore mixtures of CP-odd A and CP-even ϕ_1^0 and ϕ_2^0 components. Moreover, as A is no longer a physical state, the charged Higgs boson mass $M_{H^\pm}$ is a more appropriate parameter for describing CPVMSSM Higgs sector in place of M_A often used in the CP-conserving MSSM.

3.1 *Higgs couplings to gauge bosons and fermions*

As we have already discussed, the physical Higgs bosons ($h_{1,2,3}$) of the CPVMSSM do not have any well-defined CP properties due to the admixture of CP-even and CP-odd mass eigenstates. Hence, the coupling of these Higgses h_i ($i = 1, 2, 3$) to the SM weak gauge bosons $V = W^\pm, Z$ will obviously be different from those in the CP-conserving MSSM [39]:

$$\mathcal{L}_{hVV} = g M_W \sum_{i=1}^{3} g_{h_i VV} \times \left(h_i W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} h_i Z_\mu Z^\mu \right),$$

$$\mathcal{L}_{hhZ} = \frac{g}{2\cos\theta_W}\sum_{i,j=1}^{3} g_{h_ih_jZ}(h_i \overleftrightarrow{\partial}_\mu h_j)Z^\mu + \text{h.c.},$$

$$\mathcal{L}_{hH^\mp W^\pm} = \frac{g}{2\cos\theta_W}\sum_{i=1}^{3} g_{h_iH^+W^-} \times (h_i \overleftrightarrow{\partial}_\mu H^+)W^{-\mu} + \text{h.c.}, \qquad (17)$$

where

$$g_{h_iVV} = \mathcal{O}_{1i}\cos\beta + \mathcal{O}_{2i}\sin\beta$$

$$g_{h_ih_jZ} = \mathcal{O}_{3i}(\cos\beta\mathcal{O}_{2j} - \sin\beta\mathcal{O}_{1j}) - (i \leftrightarrow j)$$

$$g_{h_iH^+W^-} = \mathcal{O}_{2i}\cos\beta - \mathcal{O}_{1i}\sin\beta + i\mathcal{O}_{3i}. \qquad (18)$$

These couplings obey the following sum rules:

$$\sum_{i=1}^{3} g^2_{h_iVV} = 1, \quad g^2_{h_iVV} + |g_{h_iH^+W^-}|^2 = 1,$$

$$g_{h_kVV} = \epsilon_{ijk}g_{h_ih_jZ} \qquad (19)$$

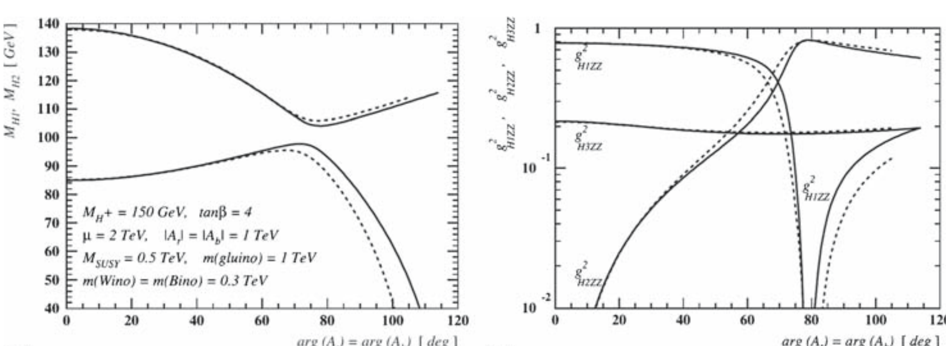
from which one can see that if two of the g_{h_iZZ} are known, then the whole set of couplings of the neutral Higgs boson to the gauge bosons are determined. It is interesting to see from eq. (19) that in the case of large scalar–pseudoscalar mixing, the suppressed h_1VV coupling means an enhanced $h_1H^+W^-$ coupling.

The Lagrangian involving the interactions of the neutral Higgs boson with the quarks and charged leptons is given by

$$\mathcal{L}_{h\bar{f}f} = -\sum_{f=u,d,l}\frac{gm_f}{2M_W}\sum_{i=1}^{3} h_i\,\bar{f} \times \left(g^S_{h_i\bar{f}f} + ig^P_{h_i\bar{f}f}\gamma_5\right) f. \qquad (20)$$

At the tree-level, the coupling constants are given by: $(g^S, g^P) = (O_{1i}/\cos\beta, -O_{3i}\tan\beta)$ and $(g^S, g^P) = (O_{2i}/\sin\beta, -O_{3i}\cot\beta)$ for $f = (l, d)$ and $f = u$ respectively. However, in the case of third-generation quarks, finite threshold corrections induced by the exchanges of gluinos, charginos and neutralinos are significant and one should include all these corrections in order to have a meaningful perturbative expansion. The detailed discussion of the CP-violating effects on all the loop-induced couplings are discussed in ref. [40] and the references therein.

We have already mentioned that the possibility of having non-zero CPV phases corresponding to the soft SUSY breaking parameters lead to significant modification in the Higgs boson masses and its couplings with the fermions and gauge bosons. Now, it is well known that the radiative correction to the lightest Higgs boson h_1 strongly depends on the stop mixing parameter $|X_t| \equiv |A_t - \mu\cot\beta|$. The one-loop corrected Higgs boson mass increases as $|X_t|$ increases, reaching to its maximum value when $|X_t|/M_{\rm SUSY} \approx 2.45$. As $|X_t|$ further increases, the one-loop corrections to the light Higgs (h_1)-boson mass decreases and it may become negative which eventually makes M_{h_1} very small, pushing it to an experimentally disallowed region. In the CPVMSSM, $|X_t|$ can be increased by varying only the phase of A_t but keeping $|A_t|$ and $|\mu|$ fixed. This feature is depicted in figure 1a where we show the variation of two light Higgs boson masses M_{h_1} and M_{h_2} as a function of the phase of A_t for two different values of $\arg(M_3) \equiv \arg(M_{\tilde{g}})$ namely 0 (solid line) and $\pi/2$ (dashed lines) respectively [41]. In figure 1b we display the variation of the square of the h_iZZ coupling, $g^2_{h_iZZ}$, as a function of $\arg(A_t)$ for the same set of parameters as in figure 1a. With the increase of the phase of A_t, the scalar–pseudoscalar mixing enhances which eventually leads to large modifications in the Higgs boson couplings to the SM gauge bosons. Moreover, it can be seen from figure 1b that at $\arg(A_t) \approx 80°$, the light Higgs coupling to a pair of Z-boson, $g^2_{h_1ZZ}$, gets highly suppressed [40a] which in turn implies that



(a) (b)

Figure 1. Variation of (**a**) M_{h_1} and M_{h_2} and (**b**) $g^2_{h_iZZ}$ as a function of $\arg(A_t)$ for a representative MSSM parameter. Solid lines and dashed lines correspond to $\arg(m_{\tilde{g}}) = 0$ and $\pi/2$ respectively (figure taken from ref. [41], where h is given as H).

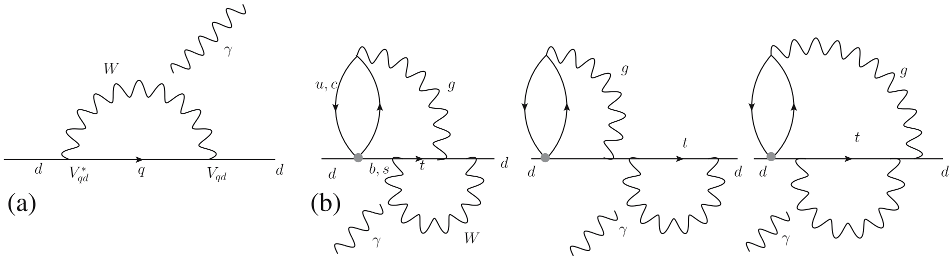


Figure 2. The diagram for the EDM of d-quark at (**a**) one-loop and (**b**) three-loop level in the SM (figure taken from ref. [80]).

the LEP-2 cannot detect the light Higgs via $e^+e^- \to Z^* \to Zh_1$. On the other hand, for the same value of the phase, h_2 and h_3 have enhanced coupling to the Z-boson [41]. An avalanche of phenomenological studies in the context of CP-violating MSSM Higgs exists in the literature, for example [42–72]. Moreover, as we have already discussed, the presence of these additional CP-violating phases would lead to non-zero values of electric dipole moments (EDM), which we have not discovered in any experiment yet. So, EDMs put strong constraints on these new CP-violating phases [36,73–77]. In the next section, we shall briefly discuss about various EDMs and their impact on the SUSY parameter space.

4. Observables to probe CP-violating effects

The time-reversal symmetry of nature states that all the physical interactions should behave the same if the direction of flow of time is reversed. The invariance under time reversal infers that fundamental particles having intrinsic spin cannot posses permanent electric dipole moments (EDMs). But we know that CP is not a good symmetry of nature and we do observe matter–antimatter asymmetry in nature. Now, if we assume the conservation of charge–parity–time reversal (CPT) symmetry, then any signature of CP violation would imply violation of the T-symmetry, thereby non-zero values of EDMs. To measure EDMs of the fundamental particles like leptons and quarks, paramagnetic and diamagnetic atoms and molecules are used. In the SM, EDMs are negligibly small (e.g. $d_{\rm e} \sim 10^{-41}$ e-cm [78]), as they appear in three loops for the quarks while they appear in four loops for leptons [79–81]. In figure 2, we show a sample diagram for calculating the d-quark EDM in the SM at the 1-loop level (figure 2a) and 3-loop level (figure 2b). Note that, the SM limit on EDMs is much smaller than the present upper bounds from the experiments and also compared to future proposed sensitivity of those experiments. In table 4, we summarize the present most stringent bounds on the EDMs of neutron, thallium, mercury and electron.

In the MSSM, all the new particles put significant contributions via loop corrections and also via new CP-violating effective interaction terms present in the theory to the EDMs of the fundamental particles. For example, EDM of the electron receives significant 1-loop corrections from the chargino and neutralino exchange diagrams. Similarly, 1-loop corrections coming from the gluino, chargino, neutralino exchange diagrams and 2-loop contributions coming from the gluino–quark–squark diagrams can enhance the EDM of neutron significantly (see figure 3). In addition, there exists non-negligible 2-loop Barr–Zee-type of contributions via pseudoscalar Higgs and also via SM gauge boson mediation diagrams (see figure 4) (for detailed description of several possible one- and two-loop SUSY corrections, please refer refs [76,87–92] and references therein). Non-observation of permanent EDMs of elementary particles put strong bounds on the CP-violating phases. For a detailed description of

Table 4. Summary table for the current experimental limits on $d_{\rm n}$, $d_{\rm Tl}$, $d_{\rm Hg}$ and $d_{\rm e}$.

System	Present limit on absolute value
$\lvert d_{\rm n}\rvert$	3.3×10^{-26} e-cm (95% CL) [82,83]
$\lvert d_{\rm Tl}\rvert$	9.0×10^{-25} e-cm (90% CL) [84]
$\lvert d_{\rm Hg}\rvert$	3.1×10^{-29} e-cm (95% CL) [85]
$\lvert d_{\rm e}\rvert$	8.7×10^{-29} e-cm (90% CL) [86]

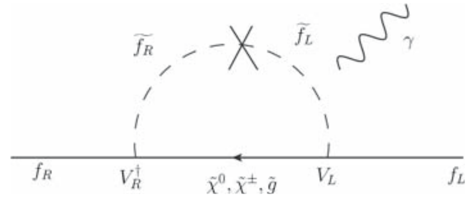


Figure 3. SUSY contributions to the EDM of fermions. Note that, Gluino ($\tilde{g}$) will contribute only for quark EDM (figure taken from ref. [78]).

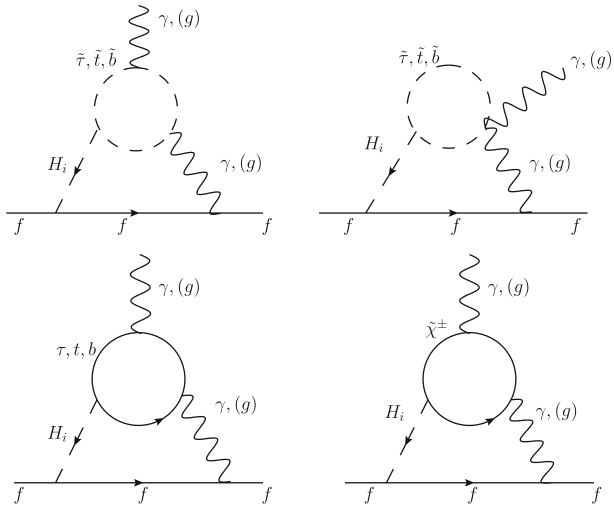


Figure 4. The Barr–Zee diagrams, where H_i denotes all the three neutral Higgs bosons with CP-violating Higgs boson mixing (figure taken from ref. [78]).

dependence of all these phases on different EDMs, please refer ref. [93].

The null results from EDM measurements pose serious constraints on the CP-violating MSSM parameter space. For example, if we assume superpartner masses around electroweak scale, say 100 GeV, then an $\mathcal{O}(1)$ CP-violating phase would violate the experimental constraints by a few orders of magnitude [94–96]. Thus, if we want to satisfy all the EDM constraints with light SUSY particles, then the obvious choice would be to make the phases very small. This is the well-known SUSY CP problem. Several prescriptions have been proposed to evade the EDM constraints, as summarized below:

(1) Heavy superpartners: If we assume that the squarks and sleptons of the first two generations are much heavier than the third-generation squarks and sleptons, then SUSY contributions to the EDMs can be very small [97–99]. However, in order to cancel the large quadratically divergent radiative corrections to the Higgs boson mass, the third-generation sfermions are preferably kept under the TeV scale. These relatively light third-generation sparticles can, in principle, induce significantly large loop contributions to various EDMs (see figures 3 and 4).

(2) Phases are small: A trivial choice to suppress the EDMs is to assume an approximate CP symmetry of the soft SUSY breaking sector, and choose all the additional CPV phases very small $\mathcal{O}$ (10^{-2}–10^{-3}) [100].

(3) Accidental cancellations: Another possibility is to allow partial or complete cancellation between several loop contributions to the EDMs in the presence of different CP-violating couplings [101–107]. As the superparticle mass spectrum of MSSM is completely unknown and also there exist several additional sources of CP violation, in principle, one cannot exclude such a possibility. However, a thorough analysis of the MSSM parameter space reveals that such a 'cancellation scenario' requires significant amount of tuning ($\sim\mathcal{O}(10^{-2})$) of the model parameters [108,109].

With the expected improvements in the EDM measurements, it is going to be one of the most important tests for signatures of BSM physics, and most importantly for supersymmetric theories where there are plenty of additional sources of CP violation. Interested readers can refer refs [78,110,111] for a complete review on the search for the signatures of new physics focussing on several EDM measurements.

Apart from the EDM constraints, the CP-violating asymmetry in $b \to s\gamma$ decay ($A_{\rm CP}$), SUSY contributions to the $B_0 - \overline{B_0}$ mixing are equally sensitive to the CP-violating phases [112,113]. For the B_d system, the experimental and the SM results for the mass differences are $\Delta M_{B_d}^{\rm Exp} = 0.510 \pm 0.004$ ps^{-1} [114–116] and $\Delta M_{B_d}^{\rm SM} = 0.502 \pm 0.006$ ps^{-1} [117], respectively. On the other hand, for the B_s system they are, $\Delta M_{B_s}^{\rm Exp} = 17.768 \pm 0.024$ ps^{-1} [115,118] and $\Delta M_{B_s}^{\rm SM} = 17.3 \pm 2.6$ ps^{-1} [119] respectively. One can calculate the SUSY contributions to the mass differences $\Delta M_{B_d}^{\rm SUSY}$ and $\Delta M_{B_s}^{\rm SUSY}$ respectively, subtracting the SM prediction from the experimentally measured value, i.e. $\Delta M_{B_d}^{\rm SUSY} = \Delta M_{B_d}^{\rm Exp} - \Delta M_{B_d}^{\rm SM}$ [120,121]. However, note that the theoretical uncertainty associated with ΔM_{B_s} is dominated by the experimental uncertainties, while for ΔM_{B_d} both theoretical and experimental error bars are relatively small. Besides, a non-zero value of $A_{\rm CP}$ also imposes strong constraints on the CP-violating phases, as it can receive significant amount of higher-order loop corrections in the CP-violating MSSM framework, thereby violating the stringent limit on $A_{\rm CP}$ measurement [114].

Another very useful method to study the CP-violating effects is to construct T-odd observables using triple products of the momenta/spin vectors of the particles involved in the production and/or decay processes [122–125]. These observables allow us to measure both the magnitude and sign of these CPV phases. Now we discuss the implication of these observables by considering a particular SUSY process; for example, associated squark–gluino production at the LHC [126]. The tree-level production process proceeds via the s-channel light quark exchange and t-channel squark/gluino exchange diagrams. The squark then subsequently decays to a quark and heavier neutralino, followed by 3-body decay of neutralino to a pair of leptons and the LSP. This process is very sensitive to the phases ϕ_1 and ϕ_μ, as expected. Now, to understand the interplay of these phases, one can define a scalar triple product, such as

$$\mathcal{T} = \vec{p}_q \cdot (\vec{p}_{\ell^+} \times \vec{p}_{\ell^-}),$$

using the three momentum vectors $\vec{p}_q$, $\vec{p}_{\ell^+}$, $\vec{p}_{\ell^-}$ of the final-state quark and leptons. Note that, this triple product observable is odd under a naive time-reversal symmetry [126a], and so can be considered as a CP-odd observable, as T-violation implies CP-violation with CPT being conserved [126b]. Note that the triple product correlations are tree-level effects and so one can expect large observable effects coming from these spin correlations. One can now define a quantity, called the T-asymmetry, as

$$\mathcal{A}_\mathcal{T} = \frac{N_{\mathcal{T}_+} - N_{\mathcal{T}_-}}{N_{\mathcal{T}_+} + N_{\mathcal{T}_-}},$$

where $N_{\mathcal{T}_+}$ ($N_{\mathcal{T}_-}$) denotes the number of events for which $\mathcal{T}$ is positive (negative). The asymmetry can be visualized as the difference between the number of events where $\vec{p}_q$ lies above or below the plane constructed by $\vec{p}_{\ell^+}$ and $\vec{p}_{\ell^-}$ vectors, normalized by the total number of events. So, if there are no CP-violating phases, there will be no CP-asymmetry and so $\vec{p}_q$ will always lie on the plane. Several studies have been performed to understand the potential of observing CP-violating effects in several production and decay processes at the LHC and future linear colliders using these triple product observables. For example, in the context of LHC triple product asymmetries have been studied via decays of neutralinos [125,126,128], charginos, stops and sbottoms and staus [68,129–134]. A more precise measurement is expected at the ILC and several studies have been performed using two-body and three-body decays of charginos and neutralinos [124,125,135–151].

5. CP-violating MSSM Higgs search at the colliders

5.1 *LEP Higgs data on CP-violating MSSM*

The four LEP Collaborations (ALEPH, DELPHI, OPAL and L3), searched for the Higgs bosons in e^+e^- collision data with centre-of-mass energies from 91 GeV to 209 GeV. The Higgs searches at the LEP were primarily focussed on the Higgsstrahlung process [152], i.e.,

$$e^+e^- \to \begin{cases} Z^* \to Z + h_i \to \ell^+\ell^- + b\bar{b}, \;\; i = 1, 2, 3, \\ h_1 + h_2 \to 4b, \\ h_1 + h_2 \to 3h_1 \to 6b, \\ Z + h_2 \to Z + 2h_1 \to (\ell^+\ell^-) + 4b. \end{cases}$$

The CP-violating effects on the neutral Higgs boson masses are significant when the product $\Im m(\mu A_t)/M_{\text{SUSY}}$ is relatively large (see eq. (15)). This observation leads to the introduction of a new CP-violating benchmark scenario, called the 'CPX scenario' [62], where the CP-violation effects are maximum. This CPX scenario may be summarized with the following parameter choices:

$$\begin{aligned} &\tilde{M}_Q = \tilde{M}_t = \tilde{M}_b = M_{\text{SUSY}}, \quad \mu = M_{\text{SUSY}}, \\ &|A_t| = |A_b| = M_{\text{SUSY}}, \quad \arg(A_t) = 90^\circ, \\ &|m_{\tilde{g}}| = 1 \text{ TeV}, \quad \arg(M_3) = 90^\circ. \end{aligned} \tag{21}$$

Without any loss of generality, the μ parameter is considered to be real, while the CP-odd angles $\arg(A_t)$ and $\arg(M_3)$ are chosen to their maximal CP-violating values. As we have already discussed, the three neutral Higgs mass eigenstates $h_i (i = 1, 2, 3)$ do not have any well-defined CP properties, rather they are mixed CP eigenstates. At the tree-level, CP-violating MSSM Higgs sector is mainly described by two parameters $M_{H^\pm}$ and $\tan\beta$. However, significant contributions may come from M_2, μ, A_f and M_3. Furthermore, the CP-violation effects also strongly depend on the precise measurement of the top quark mass. Figure 5 shows the exclusion limits for the CPX benchmark scenario from LEP-2 searches. In figure 5a, we see that for large values of m_{h_2}, the lightest Higgs h_1 is almost CP-even and thus the limit on m_{h_1} stands at 114 GeV which is the same as the SM Higgs mass bound. However, a large CP admixture may lead to relatively small values of m_{h_2}, and also allow lower values of m_{h_1} which are not excluded by LEP data (see figure 5b). In other words, LEP data do not exclude the possibility of having relatively light Higgs bosons (h_1) with mass ≈40–50 GeV [153–155], with h_1ZZ, h_1WW and $h_1t\bar{t}$ couplings being substantially reduced [41,62]. It is to be noted that the exclusion regions vary considerably with the top quark mass and also with the MSSM model parameters. Moreover, due to the suppression in the standard couplings, none of the canonical search channels for h_1 at the Tevatron and LHC are expected to be viable [154,156–158], which implies that there is a 'blind spot' or 'hole' in the parameter space permitted

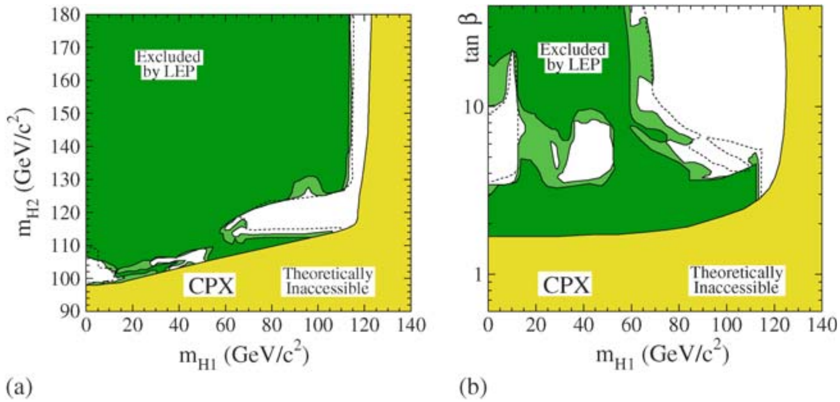


Figure 5. The 95% CL (light-green) and the 99.7% CL (dark-green) exclusion limits are shown in the M_{h_1}–M_{h_2} (**a**) and $M_{h_1}-\tan\beta$ (**b**) planes. Here we consider the *CPX* scenario with $m_t = 174.3$ GeV. The regions where unphysical results are obtained are denoted as theoretically inaccessible regions, yellow in the figure. The dashed lines indicate the boundaries of the excluded regions at the 95% CL on the basis of Monte Carlo simulations with no signal (figure taken from refs [152,153]).

by all the experimental results till date. This apparently inaccessible region is known as 'LEP-hole' region. There exist several studies in the literature which aim to propose search strategy to probe this inaccessible 'blind spot' region at the LHC [69,70,72,159,160].

5.2 *LHC Higgs data on CP-violating MSSM*

A SM Higgs-like particle with mass close to 125 GeV has been discovered by the ATLAS [161] and CMS [162] Collaborations at the LHC. The possibility to have a 125 GeV SM-like Higgs signal in the context of CP-violating MSSM satisfying the current LHC data as well as other relevant experimental constraints has been discussed thoroughly in refs [163–168]. As we have already discussed, the mass of the Higgs bosons crucially depends on various CPV phases, especially the radiative corrections to the Higgs boson masses are strongly dependent on the stop mixing parameter $|X_t| \equiv |A_t - \mu\cot\beta|$. Now, if we assume μ to be real, then the complex nature of A_t changes X_t, and so one can expect significant variations of the Higgs mass. In figure 6b we illustrate such an effect for the lightest Higgs boson mass, where the red, blue and brown curves correspond to $\phi_3 = 0$, $\phi_3 = \pi/2$ and $\phi_3 = \phi_{A_b} = \phi_{A_t}$, respectively for certain fixed values of MSSM parameters. The horizontal solid curve denotes the value of M_{h_1} in the CP-conserving MSSM, with the shaded region being the 1σ uncertainty band around the observed Higgs boson mass

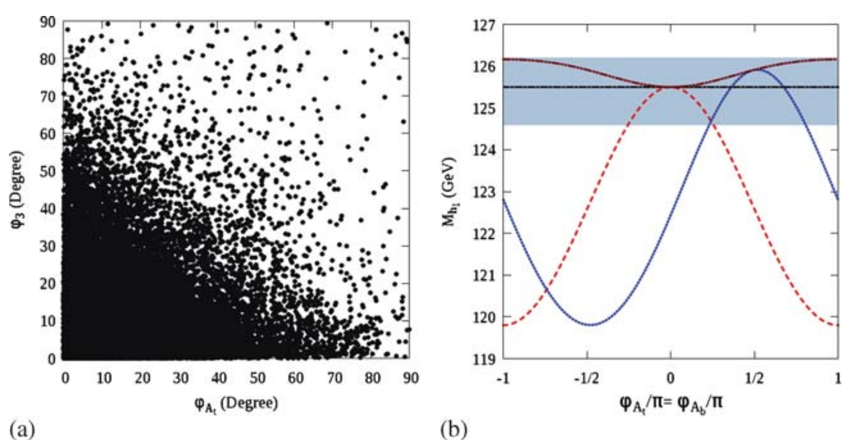


Figure 6. (**a**) Correlation of ϕ_{A_t} with ϕ_3. The black points satisfy all the constraints including EDM constraints. (**b**) Variation of Higgs boson mass with phase of A_t. Here the red, blue and brown curves correspond to $\phi_3 = 0$, $\phi_3 = \pi/2$ and $\phi_{A_t} = \phi_{A_b} = \phi_3$ respectively. The horizontal solid curve shows the M_{h_1} value in the CP-conserving MSSM, while the shaded region corresponds to the 1σ range of the observed Higgs boson mass 125.3 ± 0.4 (stat.) $\pm$ 0.5 (syst.) GeV by the CMS Collaboration [162] (figure taken from ref. [163]).

125.3 ± 0.4 (stat.) $\pm$ 0.5 (syst.) GeV by the CMS Collaboration. Chakraborty *et al* [163] studied the role of CPV phases in some of the soft SUSY-breaking parameters on h_1 and on the decay rates for the processes $gg \to h_1 \to \gamma\gamma$, $gg \to h_1 \to ZZ^* \to 4l$, $gg \to h_1 \to WW^* \to l\nu l\nu$, $pp \to Vh_1 \to Vb\bar{b}$ and $pp \to Vh_1 \to V\tau^+\tau^-$, with $V \equiv W^\pm, Z$ at the LHC and also considered the impact of low-energy flavour constraints and EDM constraints. They showed that small $\tan\beta$, large A_t and light stop are sufficient to obtain a 125 GeV Higgs boson in CP-violating MSSM satisfying the current lower limits on different SUSY particles. In figure 6a, a correlation plot in the $\phi_{A_t}-\phi_3$ plane is shown for all the points which satisfy all the constraints considered in the analysis, and from the figure it is clear that relatively large CP-violating phases (say 50–60°) are allowed after satisfying all the present constraints [163].

A well-dedicated global analysis involving all the Higgs couplings and the available current LHC results has been performed with and without the current EDM constraints [164–166]. These global fits reveal that present data do not exclude the possibility of having non-zero pseudoscalar Higgs couplings. It is to be remembered that the presence of non-zero pseudoscalar Higgs couplings is a characteristic signature of CP-violating MSSM. Considering the current EDM bounds as well as LHC Higgs data, refs [164–166] report that the pseudoscalar part of the Higgs Yukawa couplings can be approximately 10^{-2} [165], with some fixed values of the model parameters. For more details, we would like the readers to refer to refs [164–166] and the references therein. Recently, it has been shown that, even though the Higgs CP property measurements at the LHC mostly rely on coupling of the Higgs with the massive vector bosons, the gluon fusion process might be sensitive enough at the 14 TeV run to study the CP properties of the Higgs boson [169].

A geometric approach was proposed to study the interplay of additional CP-violating phases of MSSM satisfying all the current EDM constraints [170]. The basic theme of this prescription was to optimize any given observable allowing non-zero values of the CP-violating phases and then find optimal EDM-free directions in this multidimensional parameter space. Recently, this approach has been extended to study the possible values of CP-violating effects in the light of updated constraints on Higgs physics, flavour physics, dark matter relic density constraints, dark matter direct detection constraints and EDM constraints [167]. Working in the maximal CP-violating minimal flavour violating (MCPMFV) framework with six independent CP-violating phases (ϕ_1, ϕ_2, ϕ_3, ϕ_{A_t}, ϕ_{A_b} and ϕ_{A_b}) and four most updated EDM constraints, namely EDMs of thallium, mercury, neutron and electron, they found that EDM constraints do not force all the six CPV phases to be small. In fact, the phases might

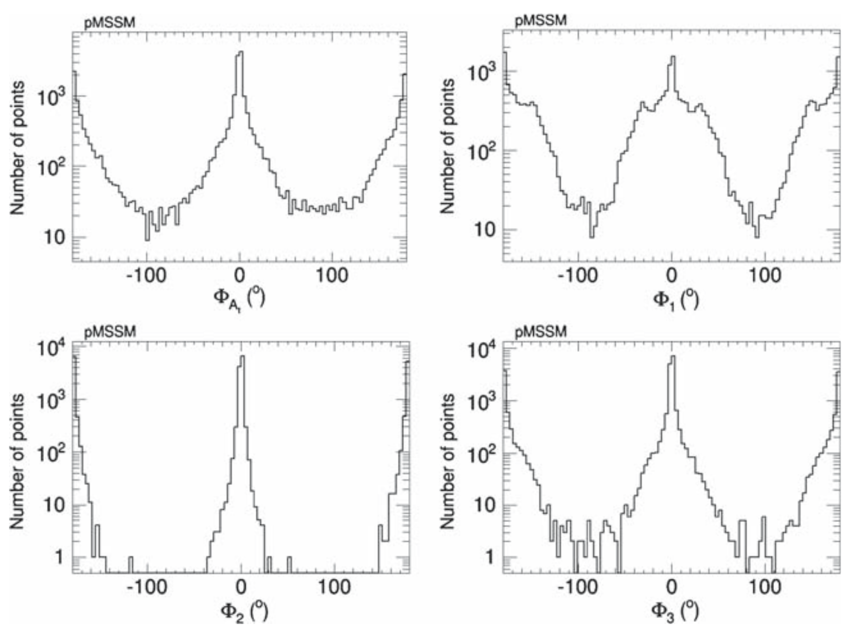


Figure 7. CP-violating phases (ϕ_1, ϕ_2, ϕ_3, ϕ_{A_t}) in the pMSSM scenario generated using the geometric approach and satisfying all the experimental constraints including the EDMs. Remaining phases have also similar behaviour (figure taken from ref. [167]).

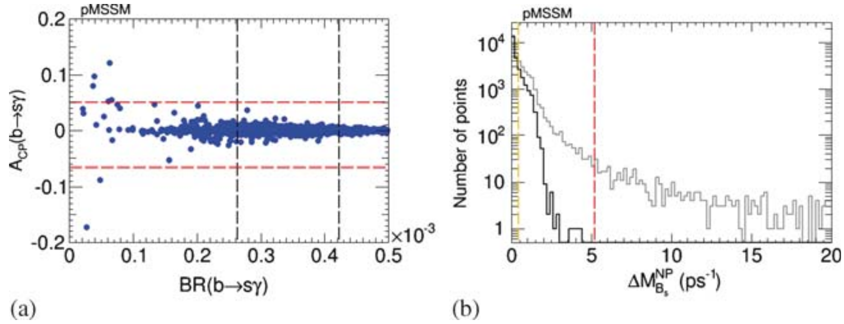


Figure 8. (**a**) Scatter plot in the BR($b \to s\gamma$)–A_{CP} plane. The vertical black dashed lines represent the allowed range of the BR($b \to s\gamma$) and the horizontal red dashed lines represent experimental limits on A_{CP}. (**b**) The amount of new physics contribution in the B_s mass mixing ($\Delta M^{\mathrm{NP}}_{B_s}$). The grey lines represent the full dataset without applying the EDM constraints, while the black histogram is with EDM constraints (figure taken from ref. [167]).

be large enough to find observable impact on quantities like CP asymmetry in $b \to s\gamma$ decay (A_{CP}), new physics contribution on B_s mixing (ΔM_s) etc. In figure 7, the number distribution of the CP-violating phases ϕ_1, ϕ_2, ϕ_3 and ϕ_{A_t} obtained after the global fit analysis are shown. From the distributions one can see that large values of ϕ_{A_t}, ϕ_1, ϕ_3 are perfectly allowed. However, ϕ_2 is very restricted, and only small deviation from the CP-conserving value is allowed.

Arbey *et al* [167] also studied the possibility of new physics signatures in B_s meson mass mixing and the CP asymmetry (A_{CP}) in the $b \to s\gamma$ decay. Figure 8a displays a scatter plot in the BR($b{\to}s\gamma$)–A_{CP} plane. The vertical black dashed lines represent the allowed range of BR($b \to s\gamma$) and the horizontal red dashed lines represent experimental limits on A_{CP}. In figure 8b, we show the distribution of the amount of new physics (NP) contribution in the B_s mass mixing ($\Delta M^{\mathrm{NP}}_{B_s}$). The grey lines represent the full dataset without applying the EDM constraints, while the black histogram is for the same dataset but now with EDM constraints. The study shows that with improved A_{CP} measurement and with precise theoretical calculations of the SM contributions to B_s meson mixing, one can make precise predictions of the additional CP-violating phases present in the model. Furthermore, it has been shown that one cannot identify the second lightest Higgs boson as the observed 125 GeV Higgs boson as it is not possible to satisfy all the Higgs signal strength constraints. But, it is possible to match the heaviest Higgs boson (h_3) with the observed SM-like Higgs boson. However, both of these possibilities are ruled out by the bounds on the heavy Higgs and charged Higgs masses/cross-sections obtained from the direct searches at the LHC.

Before we end this section, we would like to make some comments on the relation of these additional CP-violating phases to the electroweak baryogenesis and the present LHC data. As we have already mentioned, the origin of the baryon asymmetry of the Universe is one of the most important open questions in cosmology and particle physics. Baryogenesis at the electroweak phase transition (EWPT) is an attractive prescription to explain the baryon asymmetry of the Universe. For this mechanism to work, extensions of the SM is required to incorporate additional sources of CP-violation other than the CKM phase. However, the discovery of a SM-like Higgs boson at the LHC with mass around 125 GeV along with non-observation of permanent electric dipole moments in atoms and molecules severely constrains some of these simplified versions of the SM extensions. Detailed discussion on these issues is beyond the scope of this article, for interested readers we refer refs [171–173] where both earlier formulations of electroweak baryogenesis as well as recent developments incorporating current LHC data in the context of CP-violating MSSM have been considered.

6. Conclusion

In the SM of particle physics, the only source of CP violation is the CKM phase. However, this single phase is not sufficient enough to explain the observed baryon asymmetry of the Universe. The minimal supersymmetric extension of the SM introduces many new sources of CP violation, and thus helps to explain the baryon asymmetry. In this review, we discuss the effect of this new CP-violating (CPV) phases in the Higgs sector of the minimal supersymmetric Standard Model (MSSM). The Higgs potential of the CP-violating MSSM (CPVMSSM) is CP invariant at the tree-level.

However, tree-level CP invariance can be broken spontaneously through radiative corrections or, explicitly by making the couplings present in the MSSM Lagrangian complex. Here we consider the explicit CP violation in the MSSM Higgs sector. The parameters which could be complex and thus posses CPV phases are, for example, the Higgsino mass parameter (μ), the soft SUSY breaking gaugino masses (M_1, M_2, M_3), soft trilinear couplings (A_f, $f = u, d, c, s, t, b$) etc.

The presence of non-zero CPV phases lead to significant modification in the Higgs boson masses and its couplings with fermions and gauge bosons. We discuss how the phase of A_t (ϕ_{A_t}) changes the Higgs boson masses and mixing, and also its coupling with gauge bosons. In fact, for large values of ϕ_{A_t} the scalar–pseudoscalar mixing becomes large enough to suppress the light Higgs boson coupling with gauge bosons. Due to this suppressed H_1ZZ coupling, the limit on the light Higgs boson mass from LEP-2 search is highly relaxed. In §5.1, we discuss in detail the impact of the LEP data on the CPVMSSM Higgs sector. We then proceed to analyse the possibility of having the observed SM-like Higgs boson with mass around 125 GeV in §5.2. We find that non-zero CPV phases along with relatively small $\tan\beta$ (<15) and large A_t helps to obtain a 125 GeV Higgs boson in the CPVMSSM. In fact, some of the new CPV phases can be as large as 50–60° after satisfying all the present constraints. Results obtained from dedicated global fit analysis incorporating all the available constraints also reveal the fact that these new CPV phases need not be small, and they can be large enough to have observable effects on quantities like CP asymmetry in $b \to s\gamma$ ($A_{\rm CP}$), new physics contribution to B_s mixing etc. Unambiguous predictions of these additional CP-violating phases can be made with improved $A_{\rm CP}$ measurement and with more precise theoretical calculations of the SM contributions to $B_{s,d}$ meson mixing. Furthermore, the presence of these additional phases also leads to large loop contributions to various EDMs, say electron, neutron, muon, mercury EDMs. The SM predictions for these EDMs are negligibly small, in fact beyond the reach of near-future experiments. However, the large SUSY contributions make it possible to find some signature of these new phases. So far, we have not observed any signal of these EDMs in an experiment, which in turn has already placed strong bounds on the CPV phases. With the expected improvements in the EDM measurements, it is going to be one of the most important tests for signatures of BSM physics, and most importantly for supersymmetric theories where there are plenty of additional sources of CP violation.

References

[1] C S Wu, E Ambler, R W Hayward, D D Hoppes and R P Hudson, *Phys. Rev.* **105**, 1413 (1957)

[2] J H Christenson, J W Cronin, V L Fitch and R Turlay, *Phys. Rev. Lett.* **13**, 138 (1964)

[2a] The subscript S and L stand for short and long and denote the fact that $K^0_{\rm S}$ has very short life-time ($\tau \sim 10^{-11}$ sec.) while $K^0_{\rm L}$ stays longer ($\tau \sim 10^{-8}$ sec.).

[2b] The intrinsic parity of the pions are -1, while the charge conjugation operation on a particle–antiparticle system gives $+1$, thus to conserve parity (and so CP) of $K^0_{\rm S}$ it should decay to two pions.

[2c] Fitch and Cronin were awarded the Nobel Prize in 1980 for this discovery.

[3] LHCb Collaboration: R Aaij *et al*, *Phys. Rev. Lett.* **110(22)**, 221601 (2013), arXiv:1304.6173 [hep-ex]

[4] Belle Collaboration: C C Wang *et al*, *Phys. Rev. Lett.* **94**, 121801 (2005), hep-ex/0408003

[5] BaBar Collaboration: B Aubert *et al*, hep-ex/0408099

[6] Belle Collaboration: K Abe *et al*, hep-ex/0507045

[7] BaBar Collaboration: B Aubert *et al*, *Phys. Rev. Lett.* **93**, 131801 (2004), hep-ex/0407057

[8] Belle Collaboration: A Poluektov *et al*, *Phys. Rev. D* **81**, 112002 (2010), DOI: 10.1103/PhysRevD.81.112002, arXiv:1003.3360 [hep-ex]

[9] LHCb Collaboration: R Aaij *et al*, *Phys. Rev. Lett.* **110(10)**, 101802 (2013), arXiv:1211.1230 [hep-ex]

[10] Particle Data Group Collaboration: K A Olive *et al*, *Chin. Phys. C* **38**, 090001 (2014), DOI: 10.1088/1674-1137/38/9/090001

[11] A D Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* **5**, 32 (1967); *JETP Lett.* **5**, 24 (1967); *Sov. Phys. Usp.* **34**, 392 (1991); *Usp. Fiz. Nauk* **161**, 61 (1991)

[12] S L Glashow, *Nucl. Phys.* **22**, 579 (1961)

[13] A Salam, *Conf. Proc. C* **680519**, 367 (1968)

[14] S Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967)

[15] M Kobayashi and T Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973)

[15a] Kobayashi and Maskawa were awarded the Nobel prize in 2008 for the 'discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature'.

[16] G R Farrar and M E Shaposhnikov, *Phys. Rev. D* **50**, 774 (1994), hep-ph/9305275

[17] G R Farrar and M E Shaposhnikov, *Phys. Rev. Lett.* **70**, 2833 (1993); Erratum, *ibid*. **71**, 210 (1993), hep-ph/9305274

[18] M B Gavela, M Lozano, J Orloff and O Pene, *Nucl. Phys. B* **430**, 345 (1994), hep-ph/9406288

[19] M B Gavela, P Hernandez, J Orloff, O Pene and C Quimbay, *Nucl. Phys. B* **430**, 382 (1994), hep-ph/9406289

[20] P Huet and E Sather, *Phys. Rev. D* **51**, 379 (1995), hep-ph/9404302

[21] M Dine and A Kusenko, *Rev. Mod. Phys.* **76**, 1 (2003), hep-ph/0303065

[22] For reviews on Supersymmetry, see, J Wess and J Bagger, *Supersymmetry and supergravity*, 2nd edn (Princeton University Press, Princeton, 1991)
M Drees, P Roy and R M Godbole, *Theory and phenomenology of sparticles* (World Scientific, Singapore, 2005)
H E Haber and G Kane, *Phys. Rep.* **117**, 75 (1985)
H P Nilles, *Phys. Rep.* **110**, 1 (1984)
[23] S P Martin, hep-ph/9709356
[24] A Djouadi, *Phys. Rep.* **459**, 1 (2008)
[25] CMS SUSY: combined summary plots
[26] ATLAS SUSY: combined summary plots
[27] S Dimopoulos and D W Sutter, *Nucl. Phys. B* **452**, 496 (1995), DOI: 10.1016/0550-3213(95)00421-N, hep-ph/9504415
[28] S K Vempati, arXiv:1201.0334 [hep-ph]
[28a] The Higgsinos are fermionic particles, and so they contribute to SM chiral anomalies. The presence of two Higgs doublets with opposite hypercharge ($Y_{H_1} = 1$ and $Y_{H_2} = -1$) ensures that the Higgsinos mutually cancel their anomaly contributions.
[28b] Details of SUSY breaking mechanisms is beyond the scope of this article. Interested readers can refer [23,24].
[29] T D Lee, *Phys. Rev. D* **8**, 1226 (1973)
[30] T D Lee, *Phys. Rep.* **9**, 143 (1974)
[31] C Jarlskog, *Phys. Rev. Lett.* **55**, 1039 (1985)
[32] L Lavoura and J P Silva, *Phys. Rev. D* **50**, 4619 (1994), hep-ph/9404276
[33] F J Botella and J P Silva, *Phys. Rev. D* **51**, 3870 (1995), hep-ph/9411288
[34] J F Gunion and H E Haber, *Phys. Rev. D* **72**, 095002 (2005), hep-ph/0506227
[35] G C Branco, M N Rebelo and J I Silva-Marcos, *Phys. Lett. B* **614**, 187 (2005), hep-ph/0502118
[36] K A Olive, M Pospelov, A Ritz and Y Santoso, *Phys. Rev. D* **72**, 075001 (2005), hep-ph/0506106
[36a] The term 'soft' means that these additional terms which we introduce to break the SUSY explicitly, contains only mass terms and couplings with positive mass dimension. In other words, these terms do not reintroduce the quadratically divergent terms in the radiative corrections to the scalar masses to all orders in perturbation theory (for details, see refs [23,24]).
[37] A Pilaftsis, *Phys. Rev. D* **58**, 096010 (1998), hep-ph/9803297
[38] A Pilaftsis, *Phys. Lett. B* **435**, 88 (1998), hep-ph/9805373
[39] A Pilaftsis and C E M Wagner, *Nucl. Phys. B* **553**, 3 (1999), hep-ph/9902371
[40] J S Lee, A Pilaftsis, M S Carena, S Y Choi, M Drees, J R Ellis and C E M Wagner, *Comput. Phys. Commun.* **156**, 283 (2004), hep-ph/0307377
[40a] For this value of $\arg(A_t)$ and other MSSM parameters, h_1 becomes almost CP-odd.
[41] M Carena, J R Ellis, A Pilaftsis and C E M Wagner, *Nucl. Phys. B* **586**, 92 (2000)
[42] A Dedes and S Moretti, *Nucl. Phys. B* **576**, 29 (2000)
[43] A Dedes and S Moretti, *Phys. Rev. Lett.* **84**, 22 (2000)
[44] S Hesselbach, S Moretti, S Munir and P Poulose, *J. Phys. Conf. Ser.* **335**, 012020 (2011)
[45] S Hesselbach, S Moretti, S Munir and P Poulose, *AIP Conf. Proc.* **1200**, 498 (2010)
[46] S Moretti, S Munir and P Poulose, *Phys. Lett. B* **649**, 206 (2007)
[47] S Hesselbach, S Moretti, S Munir and P Poulose, *Eur. Phys. J. C* **54**, 129 (2008)
[48] S Hesselbach, S Moretti, S Munir and P Poulose, *Phys. Rev. D* **82**, 074004 (2010)
[49] O Kittel, hep-ph/0504183
[50] I Hinchliffe and N Kersting, *Phys. Rev. D* **63**, 015003 (2001)
[51] S W Ham, S G Jo, S K Oh and D Son, arXiv:0711.3951 [hep-ph]
[52] T Gajdosik, R M Godbole and S Kraml, *J. High Energy Phys.* **0409**, 051 (2004)
[53] T-F Feng, X-Q Li and J Maalampi, *Phys. Rev. D* **73**, 035011 (2006)
[54] J R Ellis, J S Lee and A Pilaftsis, *Phys. Rev. D* **70**, 075010 (2004)
[55] S Baek and P Ko, *Phys. Rev. Lett.* **83**, 488 (1999)
[56] W Altmannshofer, A J Buras and P Paradisi, *Phys. Lett. B* **669**, 239 (2008)
[57] D A Demir, A Masiero and O Vives, *Phys. Rev. D* **61**, 075009 (2000)
[58] D A Demir, *Phys. Rev. D* **60**, 055006 (1999)
[59] S Y Choi and J S Lee, *Phys. Rev. D* **61**, 115002 (2000)
[60] S Y Choi, K Hagiwara and J S Lee, *Phys. Lett. B* **529**, 212 (2002)
[61] G L Kane and L-T Wang, *Phys. Lett. B* **488**, 383 (2000)
[62] M S Carena, J R Ellis, A Pilaftsis and C E M Wagner, *Phys. Lett. B* **495**, 155 (2000)
[63] J R Ellis, J S Lee and A Pilaftsis, *Phys. Rev. D* **72**, 095006 (2005)
[64] R M Godbole, S Kraml, S D Rindani and R K Singh, *Phys. Rev. D* **74**, 095006 (2006); Erratum, *ibid. D* **74**, 119901 (2006)
[65] A G Akeroyd and A Arhrib, *Phys. Rev. D* **64**, 095018 (2001)
[66] S Y Choi, K Hagiwara and J S Lee, *Phys. Rev. D* **64**, 032004 (2001)
[67] S Y Choi, M Drees, J S Lee and J Song, *Eur. Phys. J. C* **25**, 307 (2002)
[68] F Deppisch and O Kittel, *J. High Energy Phys.* **0909**, 110 (2009); Erratum, *ibid.* **1003**, 091 (2010)
[69] D K Ghosh, R M Godbole and D P Roy, *Phys. Lett. B* **628**, 131 (2005)
[70] D K Ghosh and S Moretti, *Eur. Phys. J. C* **42**, 341 (2005)
[71] A Arhrib, D K Ghosh and O C W Kong, *Phys. Lett. B* **537**, 217 (2002)
[72] B Bhattacherjee, A Chakraborty, D Kumar Ghosh and S Raychaudhuri, *Phys. Rev. D* **86**, 075012 (2012), arXiv:1204.3369 [hep-ph]
[73] J Ellis, S Ferrara and D V Nanopoulos, *Phys. Lett. B* **114**, 231 (1982)
W Buchmüller and D Wyler, *Phys. Lett. B* **121**, 321 (1983)
J Polchinski and M Wise, *Phys. Lett. B* **125**, 393 (1983)
F del Aguila, M Gavela, J Grifols and A Méndez, *Phys. Lett. B* **126**, 71 (1983)
D V Nanopoulos and M Srednicki, *Phys. Lett. B* **128**, 61 (1983)
M Dugan, B Grinstein and L Hall, *Nucl. Phys. B* **255**, 413 (1985)
T Falk, K A Olive and M Srednicki, *Phys. Lett. B* **354**, 99 (1995)

S Pokorski, J Rosiek and C A Savoy, *Nucl. Phys. B* **570**, 81 (2000)
E Accomando, R Arnowitt and B Dutta, *Phys. Rev. D* **61**, 075010 (2000)
[74] P Nath, *Phys. Rev. Lett.* **66**, 2565 (1991)
Y Kizukuri and N Oshimo, *Phys. Rev. D* **46**, 3025 (1992)
[75] J Dai, H Dykstra, R G Leigh, S Paban and D A Dicus, *Phys. Lett. B* **237**, 216 (1990); Erratum, *B* **242**, 547 (E) (1990)
[76] D Chang, W-Y Keung and A Pilaftsis, *Phys. Rev. Lett.* **82**, 900 (1999); Erratum, *ibid.* **83**, 3972 (1999), hep-ph/9811202
[77] A Pilaftsis, *Nucl. Phys. B* **644**, 263 (2002), hep-ph/0207277
[78] T Fukuyama, *Int. J. Mod. Phys. A* **27**, 1230015 (2012), arXiv:1201.4252 [hep-ph]
[79] E P Shabalin, *Sov. J. Nucl. Phys.* **28**, 75 (1978); *Yad. Fiz.* **28**, 151 (1978)
[80] A Czarnecki and B Krause, *Phys. Rev. Lett.* **78**, 4339 (1997), hep-ph/9704355
[81] C A Baker, D D Doyle, P Geltenbort, K Green, M G D van der Grinten, P G Harris, P Iaydjiev, S N Ivanov *et al*, *Phys. Rev. Lett.* **98**, 149102 (2007), arXiv:0704.1354 [hep-ex]
[82] A P Serebrov, E A Kolomenskiy, A N Pirozhkov, I A Krasnoshekova, A V Vasiliev, A O Polyushkin, M S Lasakov, A K Fomin *et al*, arXiv:1310.5588 [nucl-ex]
[83] C A Baker, D D Doyle, P Geltenbort, K Green, M G D van der Grinten, P G Harris, P Iaydjiev, S N Ivanov *et al*, *Phys. Rev. Lett.* **97**, 131801 (2006)
[84] B C Regan, E D Commins, C J Schmidt and D DeMille, *Phys. Rev. Lett.* **88**, 071805 (2002)
[85] W C Griffith, M D Swallows, T H Loftus, M V Romalis, B R Heckel and E N Fortson, *Phys. Rev. Lett.* **102**, 101601 (2009)
[86] ACME Collaboration: J Baron *et al*, arXiv:1310.7534 [physics.atom-ph]
[87] A Pilaftsis, *Phys. Lett. B* **471**, 174 (1999), hep-ph/9909485
[88] D Chang, W F Chang and W Y Keung, *Phys. Rev. D* **66**, 116008 (2002), hep-ph/0205084
[89] S Weinberg, *Phys. Rev. Lett.* **63**, 2333 (1989)
[90] D A Dicus, *Phys. Rev. D* **41**, 999 (1990)
[91] D A Demir, O Lebedev, K A Olive, M Pospelov and A Ritz, *Nucl. Phys. B* **680**, 339 (2004), hep-ph/0311314
[92] Y Li, S Profumo and M Ramsey-Musolf, *Phys. Rev. D* **78**, 075009 (2008), arXiv:0806.2693 [hep-ph]
[93] Y Li, S Profumo and M Ramsey-Musolf, *J. High Energy Phys.* **1008**, 062 (2010), arXiv:1006.1440 [hep-ph]
[94] J R Ellis, S Ferrara and D V Nanopoulos, *Phys. Lett. B* **114**, 231 (1982)
[95] W Buchmuller and D Wyler, *Phys. Lett. B* **121**, 321 (1983)
[96] J Polchinski and M B Wise, *Phys. Lett. B* **125**, 393 (1983)
[97] P Nath, *Phys. Rev. Lett.* **66**, 2565 (1991)
[98] Y Kizukuri and N Oshimo, *Phys. Rev. D* **46**, 3025 (1992)
[99] A G Cohen, D B Kaplan and A E Nelson, *Phys. Lett. B* **388**, 588 (1996), hep-ph/9607394
[100] M Dine, E Kramer, Y Nir and Y Shadmi, *Phys. Rev. D* **63**, 116005 (2001), hep-ph/0101092
[101] T Ibrahim and P Nath, *Phys. Lett. B* **418**, 98 (1998), hep-ph/9707409
[102] T Ibrahim and P Nath, *Phys. Rev. D* **57**, 478 (1998); Erratum, *ibid. D* **58**, 019901 (1998); Erratum, *ibid. D* **60**, 079903 (1999); Erratum, *ibid. D* **60**, 119901 (1999), hep-ph/9708456
[103] T Ibrahim and P Nath, *Phys. Rev. D* **58**, 111301 (1998); Erratum, *ibid. D* **60**, 099902 (1999), hep-ph/9807501
[104] M Brhlik, G J Good and G L Kane, *Phys. Rev. D* **59**, 115004 (1999), hep-ph/9810457
[105] T Ibrahim and P Nath, *Phys. Rev. D* **63**, 035009 (2001), hep-ph/0008237
[106] T Falk and K A Olive, *Phys. Lett. B* **375**, 196 (1996), hep-ph/9602299
[107] J R Ellis, J S Lee and A Pilaftsis, *J. High Energy Phys.* **0810**, 049 (2008), arXiv:0808.1819 [hep-ph]
[108] V D Barger, T Falk, T Han, J Jiang, T Li and T Plehn, *Phys. Rev. D* **64**, 056007 (2001), hep-ph/0101106
[109] S Abel, S Khalil and O Lebedev, *Nucl. Phys. B* **606**, 151 (2001), hep-ph/0103320
[110] J S M Ginges and V V Flambaum, *Phys. Rep.* **397**, 63 (2004), physics/0309054
[111] M Pospelov and A Ritz, *Ann. Phys.* **318**, 119 (2005), hep-ph/0504231
[112] A J Buras, P H Chankowski, J Rosiek and L Slawianowska, *Nucl. Phys. B* **659**, 3 (2003)
[113] P Ball and R Fleischer, *Eur. Phys. J. C* **48**, 413 (2006)
[114] Particle Data Group: J Beringer *et al*, *Phys. Rev. D* **86**, 010001 (2012)
[115] G Isidori and F Teubert, *Eur. Phys. J. Plus* **129**, 40 (2014)
[116] Heavy Flavor Averaging Group Collaboration: Y Amhis *et al*, arXiv:1412.7515 [hep-ex]
[117] UTfit Collaboration: M Bona *et al*, *J. High Energy Phys.* **0610**, 081 (2006)
[118] LHCb Collaboration: R Aaij *et al*, *New J. Phys.* **15**, 053021 (2013)
[119] A Lenz and U Nierste, arXiv:1102.4274 [hep-ph]
[120] G Isidori and P Paradisi, *Phys. Lett. B* **639**, 499 (2006)
[121] N Fornengo, S Scopel and A Bottino, *Phys. Rev. D* **83**, 015001 (2011)
[122] G A Moortgat-Pick, H Fraas, A Bartl and W Majerotto, *Eur. Phys. J. C* **9**, 521 (1999); Erratum, *ibid. C* **9**, 549 (1999), hep-ph/9903220
[123] S Y Choi, H S Song and W Y Song, *Phys. Rev. D* **61**, 075004 (2000), hep-ph/9907474
[124] A Bartl, T Kernreiter and W Porod, *Phys. Lett. B* **538**, 59 (2002), hep-ph/0202198
[125] A Bartl, H Fraas, T Kernreiter and O Kittel, *Eur. Phys. J. C* **33**, 433 (2004), hep-ph/0306304
[126] G Moortgat-Pick, K Rolbiecki, J Tattersall and P Wienemann, *J. High Energy Phys.* **1001**, 004 (2010), arXiv:0908.2631 [hep-ph]
[126a] By naive, we mean reversal of the momentum and spin of the particles without interchanging the initial- and final-state particles which can be expected in an exact time-reversal operation [127].
[126b] Here we assume that phases coming from the loops, Breit–Wigner propagators and final-state interactions are small.
[127] D Atwood, S Bar-Shalom, G Eilam and A Soni, *Phys. Rep.* **347**, 1 (2001), hep-ph/0006032
[128] P Langacker, G Paz, L T Wang and I Yavin, *J. High Energy Phys.* **0707**, 055 (2007), hep-ph/0702068 [hep-ph]
[129] A Bartl, E Christova, K Hohenwarter-Sodek and T Kernreiter, *Phys. Rev. D* **70**, 095007 (2004), hep-ph/0409060
[130] J Ellis, F Moortgat, G Moortgat-Pick, J M Smillie and J Tattersall, *Eur. Phys. J. C* **60**, 633 (2009), arXiv:0809.1607 [hep-ph]

[131] G Moortgat-Pick, K Rolbiecki and J Tattersall, *Phys. Rev. D* **83**, 115012 (2011), arXiv:1008.2206 [hep-ph]
[132] A Bartl, E Christova, K Hohenwarter-Sodek and T Kernreiter, *J. High Energy Phys.* **0611**, 076 (2006), hep-ph/0610234
[133] F F Deppisch and O Kittel, *J. High Energy Phys.* **1006**, 067 (2010), arXiv:1003.5186 [hep-ph]
[134] H Dreiner, O Kittel, S Kulkarni and A Marold, *Phys. Rev. D* **83**, 095012 (2011), arXiv:1011.2449 [hep-ph]
[135] A Bartl, H Fraas, S Hesselbach, K Hohenwarter-Sodek and G A Moortgat-Pick, *J. High Energy Phys.* **0408**, 038 (2004), hep-ph/0406190
[136] A Bartl, H Fraas, O Kittel and W Majerotto, *Phys. Lett. B* **598**, 76 (2004), hep-ph/0406309
[137] O Kittel, A Bartl, H Fraas and W Majerotto, *Phys. Rev. D* **70**, 115005 (2004), hep-ph/0410054
[138] H K Dreiner, O Kittel and A Marold, *Phys. Rev. D* **82**, 116005 (2010), arXiv:1001.4714 [hep-ph]
[139] A Bartl, H Fraas, S Hesselbach, K Hohenwarter-Sodek, T Kernreiter and G Moortgat-Pick, *Eur. Phys. J. C* **51**, 149 (2007), hep-ph/0608065
[140] S Y Choi, A Djouadi, M Guchait, J Kalinowski, H S Song and P M Zerwas, *Eur. Phys. J. C* **14**, 535 (2000), hep-ph/0002033
[141] S Y Choi, B C Chung, J Kalinowski, Y G Kim and K Rolbiecki, *Eur. Phys. J. C* **46**, 511 (2006), hep-ph/0504122
[142] J A Aguilar-Saavedra, *Phys. Lett. B* **596**, 247 (2004), hep-ph/0403243
[143] A Bartl, K Hohenwarter-Sodek, T Kernreiter, O Kittel and M Terwort, *J. High Energy Phys.* **0907**, 054 (2009), arXiv:0905.1782 [hep-ph]
[144] J A Aguilar-Saavedra, *Nucl. Phys. B* **717**, 119 (2005), hep-ph/0410068
[145] A Bartl, K Hohenwarter-Sodek, T Kernreiter, O Kittel and M Terwort, *Nucl. Phys. B* **802**, 77 (2008), arXiv:0802.3592 [hep-ph]
[146] A Bartl, H Fraas, O Kittel and W Majerotto, *Phys. Rev. D* **69**, 035007 (2004), hep-ph/0308141
[147] A Bartl, T Kernreiter and O Kittel, *Phys. Lett. B* **578**, 341 (2004), hep-ph/0309340
[148] S Y Choi, M Drees, B Gaissmaier and J Song, *Phys. Rev. D* **69**, 035008 (2004), hep-ph/0310284
[149] A Bartl, H Fraas, O Kittel and W Majerotto, *Eur. Phys. J. C* **36**, 233 (2004), hep-ph/0402016
[150] S Y Choi and Y G Kim, *Phys. Rev. D* **69**, 015011 (2004), hep-ph/0311037
[151] J A Aguilar-Saavedra, *Nucl. Phys. B* **697**, 207 (2004), hep-ph/0404104
[152] A Sopczak for the LEP Higgs Working Group, presented at SUSY-2005, arXiv:hep-ph/0602136
[153] LEP Higgs Working Group, LHWG Note 2005-01, presented at the Lepton Photon Symposium (2005)
[154] M Carena, J R Ellis, S Mrenna, A Pilaftsis and C E M Wagner, *Nucl. Phys. B* **659**, 145 (2003), arXiv:hep-ph/0211467
[155] OPAL Collaboration: G Abbiendi *et al*, *Eur. Phys. J. C* **37**, 49 (2004)
[156] V Buescher and K Jakobs, *Int. J. Mod. Phys. A* **20**, 2523 (2005), hep-ph/0504099
[157] M Schumacher, hep-ph/0410112
[158] E Accomando, A G Akeroyd, E Akhmetzyanova, J Albert, A Alves, N Amapane, M Aoki, G Azuelos *et al*, hep-ph/0608079
[159] V Buescher and K Jakobs, *Int. J. Mod. Phys. A* **20**, 2523 (2005), arXiv:hep-ph/0504099
[160] P Bandyopadhyay, A Datta, A Datta and B Mukhopadhyaya, *Phys. Rev. D* **78**, 015017 (2008)
A C Fowler and G Weiglein, *J. High Energy Phys.* **01**, 108 (2010)
P Draper, T Liu and C E M Wagner, *Phys. Rev. D* **81**, 015014 (2010)
P Bandyopadhyay, *J. High Energy Phys.* **08**, 016 (2011)
P Bandyopadhyay and K Huitu, arXiv:1106.5108 [hep-ph]
[161] ATLAS Collaboration: G Aad *et al*, *Phys. Lett. B* **716**, 1 (2012)
[162] CMS Collaboration: S Chatrchyan *et al*, *Phys. Lett. B* **716**, 30 (2012)
[163] A Chakraborty, B Das, J L Diaz-Cruz, D K Ghosh, S Moretti and P Poulose, *Phys. Rev. D* **90(5)**, 055005 (2014), arXiv:1301.2745 [hep-ph]
[164] K Cheung, J S Lee and P Y Tseng, *J. High Energy Phys.* **1305**, 134 (2013), arXiv:1302.3794 [hep-ph]
[165] K Cheung, J S Lee, E Senaha and P Y Tseng, *J. High Energy Phys.* **1406**, 149 (2014), arXiv:1403.4775 [hep-ph]
[166] K Cheung, J S Lee and P Y Tseng, arXiv:1407.8236 [hep-ph]
[167] A Arbey, J Ellis, R M Godbole and F Mahmoudi, arXiv:1410.4824 [hep-ph]
[168] L Bian, T Liu and J Shu, arXiv:1411.6695 [hep-ph]
[169] M J Dolan, P Harris, M Jankowiak and M Spannowsky, arXiv:1406.3322 [hep-ph]
[170] J Ellis, J S Lee and A Pilaftsis, *J. High Energy Phys.* **1010**, 049 (2010), arXiv:1006.3087 [hep-ph]
[171] M Carena, J M Moreno, M Quiros, M Seco and C E M Wagner, *Nucl. Phys. B* **599**, 158 (2001), DOI:10.1016/S0550-3213(01)00032-3, hep-ph/0011055
S J Huber, P John and M G Schmidt, *Eur. Phys. J. C* **20**, 695 (2001), DOI:10.1007/PL00022989, hep-ph/0101249
K Funakubo, S Otsuki and F Toyoda, *Prog. Theor. Phys.* **102**, 389 (1999), DOI:10.1143/PTP.102.389, hep-ph/9903276
[172] K Funakubo, S Tao and F Toyoda, *Prog. Theor. Phys.* **109**, 415 (2003), DOI:10.1143/PTP.109.415, hep-ph/0211238
M Carena, G Nardini, M Quiros and C E M Wagner, *Nucl. Phys. B* **812**, 243 (2009), DOI:10.1016/j.nuclphysb.2008.12.014, arXiv:0809.3760 [hep-ph]
V Cirigliano, Y Li, S Profumo and M J Ramsey-Musolf, *J. High Energy Phys.* **1001**, 002 (2010), DOI:10.1007/JHEP01(2010)002, arXiv:0910.4589 [hep-ph]
[173] T Cohen, D E Morrissey and A Pierce, *Phys. Rev. D* **86**, 013009 (2012), DOI:10.1103/PhysRevD.86.013009, arXiv:1203.2924 [hep-ph]
D Curtin, P Jaiswal and P Meade, *J. High Energy Phys.* **1208**, 005 (2012), DOI:10.1007/JHEP08(2012)005, arXiv:1203.2932 [hep-ph]
M Carena, G Nardini, M Quiros and C E M Wagner, *J. High Energy Phys.* **1302**, 001 (2013), DOI:10.1007/JHEP02(2013)001, arXiv:1207.6330 [hep-ph]
K Fuyuto, J Hisano and E Senaha, arXiv:1510.04485 [hep-ph]