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Naturalness problem: Off the beaten track

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Abstract. Assuming that there is some symmetry which keeps the SM Higgs boson at the electroweak scale, we discuss the feasibility of some minimalistic expansions of the SM.

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1. The problem

July 4, 2012 is the first time that a fundamental scalar has been discovered [1,2]. All its properties, including the decay widths into different channels and the quantum numbers like spin and parity, indicate that this is indeed the Higgs boson [3], or the celebrated 'incomplete scalar multiplet' predicted by Peter Higgs fifty years ago. The question that remains is whether the scalar sector is as minimalistic as this, or whether this is just the tip of the iceberg, the first member of a big family of fundamental scalars.

It is good that we have not seen any fundamental scalar before 2012, because such scalars are known to carry their own problems. Electron self-energy has a superficial degree of divergence $D = 1$, and so one would expect it to be linearly divergent, proportional to the momentum cut-off Λ if we use the cut-off regularization. However, in the limit of vanishing electron mass, the action has an extra symmetry, called the chiral symmetry: $\psi \to \gamma_5\psi$. This symmetry must be there even for the quantum corrections, because they too arise from the higher-order terms of the action. If the self-energy is proportional to Λ, it should be nonzero at the limit $m \to 0$; thus, such terms cannot be present, and the self-energy is actually logarithmically divergent. In other words, chiral symmetry protects the electron mass. Similarly, gauge symmetry protects the mass of gauge bosons; without that, the vacuum polarization diagram could have been quadratically divergent, proportional to Λ^2. So even if Λ is high, the quantum corrections are never that large; for $\Lambda = 10^{19}$ GeV, electron self-energy gets a correction proportional to $\ln(\Lambda/m) \approx 51$.

There is no symmetry that protects the scalar mass. The mass term, $m^2\Phi^\dagger\Phi$ does not break any symmetry that is respected by the potential $\lambda(\Phi^\dagger\Phi)^2$. So the superficial degree of divergence of the scalar self-energy, $D = 2$, is its actual degree of divergence; the scalar self-energy is quadratically divergent. The only reason why the Higgs mass is not at the Planck scale, 10^{19} GeV, but at the electroweak scale, is that there must be a tremendous cancellation between the huge quantum correction and the bare mass, leaving a tiny nonzero contribution. This is the fine-tuning or naturalness problem [3a].

There is another way to look at the fine-tuning problem. Suppose the bare mass squared of the unrenormalized theory is $m^2_{\rm bare}$. You add a counterterm and get the renormalized mass squared, m^2. Now change $m^2_{\rm bare}$ to $m^2_{\rm bare} + \delta m^2_{\rm bare}$, with $\delta m^2_{\rm bare}/m^2_{\rm bare} \sim \mathcal{O}(1)$. This will change m^2 to $m^2 + \delta m^2$. If $\delta m^2/m^2 \gg 1$, the theory is fine-tuned.

When we talk about the degree of divergence, it is always better to use the cut-off regularization, which clearly brings out how badly divergent an amplitude is. In dimensional regularization, all divergences are lumped into $1/\epsilon$, which is good if we want to subtract the divergent part and work with the finite part, but not so helpful if we want to know which divergence is worse, scalar self-energy or electron self-energy. As we are talking about the fine-tuning problem, we shall use only the cut-off regularization (figure 1).

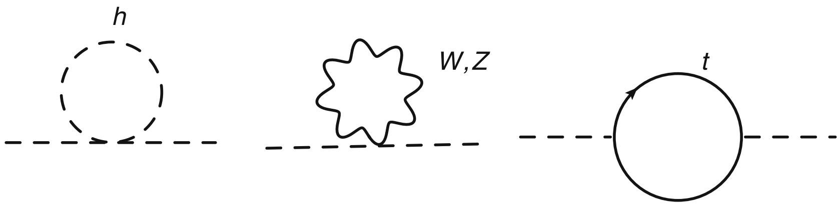

Figure 1. The quadratically divergent self-energy diagrams in unitary gauge. In any other gauge, the Goldstone loops will also be there.

There are several ways to address this issue. One might bring in more degrees of freedom, invoking a perfect symmetry between bosons and fermions, so that the divergence coming from a bosonic loop is exactly cancelled by that coming from a fermionic loop (supersymmetry). One might lower the Planck scale somehow so that the fine-tuning problem loses its severity (extra dimensions). The Higgs might not be a fundamental scalar after all but just a composite object made out of fermions (technicolour or top-condensate), or it might be the (pseudo)-Goldstone boson of a higher symmetry group which keeps its mass small (little Higgs models). There can always be the anthropic principle; bizarre that it may sound, it is perhaps no more bizarre than to ask for an almost complete cancellation between two huge and uncorrelated terms.

Recently, it was pointed out in refs [4,5] that the Higgs boson quartic coupling, λ, has such a value for $m_h \approx 125$ GeV that it might remain perturbative all the way up to the Planck scale; neither does it blow up and hit the Landau pole, nor does it become negative and make the electroweak vacuum unstable [5a]. Thus, there seems to be a distinct possibility that a desert lies between the electroweak scale, parametrized by v ($\approx$246 GeV), the vacuum expectation value (VEV) of the Higgs field, and the Planck scale $M_{\rm Pl}$ ($\sim 10^{19}$ GeV); the fine-tuning problem indeed exists.

2. The bottom-up approach

The conventional solutions of the fine-tuning problem have been well explored. Unfortunately, none seems to work perfectly. Even at the LHC, we have not seen any new particles. Supersymmetry, for example, gives a perfect solution for the fine-tuning problem if and only if the particles are degenerate with superparticles. Even if the superparticles are at the TeV scale, one must entertain fine-tuning at a fraction of a percent level [5b].

All these solutions are more or less based on some underlying symmetry of the theory. All these are top-down approaches, one first invokes the symmetry and then explores the consequences. Suppose we look at the problem from the other end. That the Higgs mass is at 125 GeV and not at the Planck scale means there is some symmetry that keeps the scalar light. We do not know what the symmetry is but we know at least one of its consequences: the scalar masses must be protected. Let us try to develop this idea logically and see where we reach.

The scalar potential of the Standard Model is given by

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \tag{1}$$

with μ^2, $\lambda > 0$. This ensures the stability of the potential (a well-defined vacuum state) as well as spontaneous symmetry breaking. Let us use the standard convention of setting the VEV of Φ as $\langle \Phi \rangle = v/\sqrt{2}$, so that $v = 246$ GeV.

At one-loop, the Higgs self-energy receives a quadratically divergent correction

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(6\lambda + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2\right) \equiv \frac{\Lambda^2}{16\pi^2} f_h\,, \tag{2}$$

where g_1 and g_2 are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, and $g_t = \sqrt{2}m_t/v$ is the top quark Yukawa coupling. One can safely treat all other fermions as massless. For our future reference, the quantity inside the parentheses is denoted by f_h, and the quadratically divergent correction for any scalar s, δm_s^2, is generically denoted by f_s times $\Lambda^2/16\pi^2$.

The self-energy diagrams are formally divergent, so what regularization should one use to make sense of these amplitudes? We advocate the cut-off regularization; this is not Lorentz invariant but this nicely separates the quadratic ($\sim\Lambda^2$) and the logarithmic ($\sim\ln(\Lambda^2/m^2)$) divergences. The logarithmic divergences are much softer, and anyway they also exist for fermions and gauge bosons. The quadratically divergent terms are responsible for the naturalness problem, so if there is some unknown symmetry that makes the sum of all the quadratic divergences zero, we are through. This is known as the Veltman condition [6].

Thus, the Veltman condition, in its original form, is written as

$$f_h = 0 \Longrightarrow 2\lambda + \frac{1}{4}g_1^2 + \frac{3}{4}g_2^2 - 2g_t^2 = 0. \quad (3)$$

In terms of the masses, this reads as

$$m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2 = 0. \quad (4)$$

This needs $m_h = 316$ GeV, and so in the Standard Model, the Veltman condition is far from being satisfied.

There are a number of points worth discussing.

(1) We have not talked about any fundamental theory or mechanism that keeps the Higgs light, so is the idea at all appealing? Not if you know very definitely that one of the few beyond Standard Model ideas that have been proposed must be the truth. Unfortunately, we do not know this, we are groping in the dark, and this is perhaps the best that a cartographer in a newly discovered continent can do. That is why this is a venture off the beaten track: 'the goal is to first find the minimal extension of the Standard Model that solves the fine-tuning problem, and then try to embed the solution in some concrete, well-motivated model.' In a sense, supersymmetry implements the Veltman condition, but there some underlying dynamics of the theory ensures that the condition is satisfied, at least in the limit of exact supersymmetry.

(2) The Veltman condition is not satisfied in cut-off regularization; does dimensional regularization help? As we have noted, dimensional regularization does not separate between quadratic and logarithmic divergences, and dumps everything as the coefficient of the ubiquitous $1/\epsilon$. We get a slightly different correction with dimensional regularization [7]:

$$\delta m_h^2 \propto \frac{1}{\epsilon}\left(6\lambda + \frac{1}{4}g_1^2 + \frac{3}{4}g_2^2 - 6g_t^2\right). \quad (5)$$

So even this does not help much. As our goal is to cancel the strongest divergence, from now on we shall use only the cut-off regularization.

(3) Is a strict implementation of eq. (3) necessary? There are at least two reasons why we do not expect this. First, the higher loop effects are there; what has been shown is only the one-loop result. Compared to the leading order, the higher loops are suppressed by further powers of $\log(\Lambda^2/m^2)/16\pi^2$. So their contributions have to be subleading, but definitely at the level of a few per cent. Second, we can always accommodate some cancellation between the bare Higgs mass and the radiative corrections. If $\delta m_h^2 \sim m_h^2$, there is no fine-tuning, and so this is perfectly acceptable. If $\delta m_h^2 \sim 10 m_h^2$, there is a cancellation of one in 10, or a tuning of 10%, which is nothing to worry about. Without any fine-tuning, i.e. $|\delta m_h^2| \leq m_h^2$, one finds

$$|m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2| \leq \frac{16\pi^2}{3}\frac{v^2}{\Lambda^2}m_h^2. \quad (6)$$

This inequality is clearly not satisfied in the SM for $v^2/\Lambda^2 \leq 0.1$ or $\Lambda \geq 760$ GeV, and the onset of NP at such a low scale is almost ruled out by the LHC. If we allow a fine-tuning of 1 in N, the scale goes up by a factor of $\sqrt{N}$. But that is what we expect: if there is some NP at a few TeV, no one bothers about the fine-tuning problem.

(4) One might also expect the Veltman condition to be satisfied for every energy scale till the theory remains valid. This upper range might be the Planck scale, the GUT scale, or some intermediate scale where the theory ceases to be valid any more. In other words, the stability condition in the Standard Model reads

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(2\lambda + \frac{1}{4}g_1^2 + \frac{3}{4}g_2^2 - 2g_t^2\right) \sim \frac{v^2}{\Lambda^2}, \quad (7)$$

where $t = \log(Q^2/\mu^2)$. If we use only the one-loop β-functions, this becomes [8]

$$\begin{aligned} 288\lambda^2 &+ 144g_t^2\lambda - 180g_t^4 - 36\lambda(g_1^2 + 3g_2^2) \\ &+ 25g_1^4 - 15g_2^4 + 9g_1^2 g_2^2 \\ &+ g_t^2(192g_3^2 + 34g_1^2 + 54g_2^2) \sim \frac{v^2}{\Lambda^2}. \quad (8) \end{aligned}$$

Again, this is far from being satisfied. Of course, two-loop calculations exist. It was shown in [7,9] that in any generic Yukawa theory, if the Higgs mass correction is to remain zero at all scales ($f_h = 0$, $f_h' \equiv \mathrm{d}f_h/\mathrm{d}t = 0$) using one-loop β-functions, it is precisely the same condition as quadratic divergences at two-loop to vanish [9a]. Recently, it was pointed out in [10] that the correction to the Higgs mass changes sign at a very high scale, the exact value of which again depends on the top quark mass, leading to the conjecture [11] that the sign reversal may trigger electroweak symmetry breaking. However, Jones [12] has disputed the claims of [10].

It is clear that one needs more bosonic degrees of freedom to satisfy the Veltman condition. One can try to have more gauge bosons; however, more scalars are needed anyway to give them gauge invariant masses. So let us just try to enhance the scalar sector, adding scalars that couple to the Standard Model doublet Φ. In fact, all multiplets do that, because $\mathcal{S}^\dagger\mathcal{S}\Phi^\dagger\Phi$ is always a gauge singlet, no matter how $\mathcal{S}$ transforms. There are other non-trivial constraints though. The first is the stability of the scalar potential: there cannot be any direction in the field space where the potential becomes unbounded from below, or breaks $U(1)_{\rm em}$ [12a]. The second one is that the new scalars must also have their own Veltman conditions satisfied if they should remain at the electroweak scale. In fact, if they are very heavy, one can always integrate them out at the electroweak scale and the Higgs fine-tuning problem will be back. Thus, we have three necessary conditions:

(1) There must be new scalars.
(2) The scalar potential must be stable.
(3) The new scalars must also couple to fermions [12b].

In the rest of the article, we shall concentrate on three different models, each formulated with a completely different motivation:

(1) Models with one or more singlet scalar: This is by far the minimal extension of the Standard Model. If the singlet does not mix with the doublet, it can also act as a WIMP dark matter. With enough singlets, the electroweak phase transition can be made first-order. However, singlets do not couple to chiral fermions, and so we need vectorial fermions to address their own fine-tuning.
(2) Two-Higgs doublet models (2HDM): There are four different types of 2HDM that do not allow any FCNC, and there are more if controllable FCNCs are allowed. Except for the type-I 2HDM, both the doublets couple to ordinary fermions, and so we do not need any extra fermions.
(3) Scalar triplet models: Complex triplets are interesting in the context of type-II see-saw mechanism to generate neutrino masses. Triplets do not couple to ordinary fermions, but they do couple to lepton doublets through lepton number violating interactions. With only one complex triplet, the ρ-parameter forces the triplet VEV to be small, but that is more than welcome for generating the neutrino masses. One might also introduce a complex as well as a real triplet and keep the triplet VEV large through a custodial symmetry.

3. The minimal extension: One real scalar singlet

The singlet extension of the scalar sector is the most economical option. Moreover, it does not run afoul of the ρ-parameter, and can also be a very good cold dark matter candidate if it does not mix with the doublet Higgs. The role of one (or more) extra singlet(s) has been extensively considered in [8,13–16]. Here, we shall mostly follow ref. [15]. We shall also talk only about the one-loop analysis; that is the easiest to follow.

Let us first consider the SM augmented with one real singlet scalar S. The potential is

$$V(\Phi, S) = V_{\rm SM} + V_{\rm singlet} = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 - M^2S^2 + \tilde{\lambda}S^4 + aS^2(\Phi^\dagger\Phi). \quad (9)$$

We shall take both μ^2, $M^2 > 0$ to start with. The remnant of Φ after spontaneous symmetry breaking is the Higgs boson h. There might be a cubic term cS^3 in the potential, but that would not affect the subsequent analysis. The linear terms in S giving rise to tadpole diagrams are assumed to cancel out and they will remain so even after the quantum corrections. This happens if we take the tadpole potential to be

$$V_{\rm tadpole} = \alpha_1 S + \alpha_2\Phi^\dagger\Phi S \quad (10)$$

with $\alpha_1 + \frac{1}{2}\alpha_2 v^2 = 0$. There might also be a discrete symmetry, like $S \to -S$, preventing odd terms. For N singlets with an $O(N)$ symmetry, the potential looks like

$$V(\Phi, S_i) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 - M^2\sum_i S_i^2 + \tilde{\lambda}\left(\sum_i S_i^2\right)^2 + a(\Phi^\dagger\Phi)\sum_i S_i^2. \quad (11)$$

With one extra singlet S, as in eq. (9), the VC is modified to

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(6\lambda + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2 + a\right) = \frac{\Lambda^2}{16\pi^2} f_h. \quad (12)$$

If there are N number of identical singlets (i.e., an $O(N)$ symmetric singlet sector), the last term is replaced by Na.

For $N = 1$, we find that $a = 4.17$, which is quite large even if not nonperturbative (nonperturbativity sets in when the coupling is at about $4\pi \approx 12.56$). More singlets bring down the value to $4.17/N$. This is to be taken as an indicative value only, as there is no reason why this value would be absolutely stable if one takes higher-order corrections (for an estimate, see [14], where it can be seen that such corrections bring a marginal change). To be consistent, we shall use only one-loop renormalization group (RG) equations to calculate the evolution of the couplings.

The singlet VC, independent of whether the singlet develops a VEV or not, reads as

$$\delta m_S^2 = \frac{\Lambda^2}{16\pi^2}[(8+4N)\tilde{\lambda} + 4a]. \tag{13}$$

So, only with singlet, we need a large (and definitely nonperturbative) and negative quartic coupling, and the potential develops a minimum unbounded from below in the direction $|\Phi| =$ constant and $|S| \to \infty$. Thus, this solution is clearly unacceptable.

While one needs some negative contribution to eq. (13), one notes that this cannot come from chiral fermions of the SM as they do not couple to S. Thus, one is led to introduce vector fermions – either singlets or doublets under $SU(2)$. This introduces further terms in the potential:

$$\mathcal{L}_{\rm VF} = -m_F \bar{F} F - \zeta_F \bar{F} F S, \tag{14}$$

and the mass of F is $m_F + \zeta_F \langle S \rangle$. Note that a symmetry like $S \to -S$ implies $F \to i\gamma_5 F$ and hence forbids the bare mass term, unless the symmetry is explicitly broken. We shall not pursue this possibility further as we show explicitly that a nonzero VEV to the singlet is disfavoured. So the vector fermions must get their masses from the bare mass term. For this case, i.e., $M^2 < 0$ and $\langle S \rangle = 0$, there is no mass term from the Yukawa couplings. Direct searches at the LHC put a lower limit of the order of 500 GeV on the mass of vector quarks.

For simplicity, we assume a complete generation of vector fermions (N, E), (U, D) with all Yukawa couplings ζ_i to be the same (the heavy neutrino N is not to be confused with N, the number of singlets). One can, in principle, consider only one such fermion in the spectrum, or only the lepton or quark doublet. The VC for S now reads as

$$\delta m_S^2 = \frac{\Lambda^2}{16\pi^2}[(8+4N)\tilde{\lambda} + 4a - 4Z^2], \tag{15}$$

where

$$Z^2 = \sum_i N_c \zeta_i^2 = \zeta_E^2 + \zeta_N^2 + 3(\zeta_U^2 + \zeta_D^2) = 8\zeta^2, \tag{16}$$

N_c being the colour of the corresponding fermions. While this does not guarantee a degenerate generation, that remains a distinct possibility, and thus one can avoid the strong constraints coming from oblique S and T parameters – because of the vectorial nature and degeneracy.

If there is only one singlet and $M^2 > 0$, the minimization conditions are

$$-\mu^2 + \lambda v^2 + a v'^2 = 0, \quad -M^2 + 2\tilde{\lambda} v'^2 + \frac{1}{2} a v^2 = 0, \tag{17}$$

where $\langle S \rangle = v'$. The mass term can be written as

$$(h\ S)\,\mathcal{M}\begin{pmatrix} h \\ S \end{pmatrix} = (h\ \ S)\begin{pmatrix} \lambda v^2 & a v v' \\ a v v' & 4\tilde{\lambda} v'^2 \end{pmatrix}\begin{pmatrix} h \\ S \end{pmatrix}. \tag{18}$$

The condition for both masses to be real is

$$4\lambda\tilde{\lambda} \ge a^2. \tag{19}$$

For $\lambda \approx 0.13$ and $a \approx 4$, this makes $\tilde{\lambda}$ very large (≥ 33) and clearly nonperturbative. While this by itself may still be acceptable, the fact that all the scalar couplings hit their respective Landau poles almost right at the electroweak scale rules this option out (figure 2).

One might wonder whether the situation improves in the large-N limit. However, if the original scalar sector has an $O(N)$ symmetry, a spontaneous symmetry breaking will result in $N - 1$ Goldstone bosons, which couple to the doublet Higgs h, and therefore will give a very large invisible decay width of h. This is again unacceptable from the measurement of branching fractions of the Higgs at the LHC.

Thus, we are forced to take $M^2 < 0$, so that the singlet does not develop a VEV and there is no singlet–doublet mixing. This is true even if there are more than one such singlets. The mass-squared of the singlet is given by

$$m_S^2 = 2M^2 + a v^2. \tag{20}$$

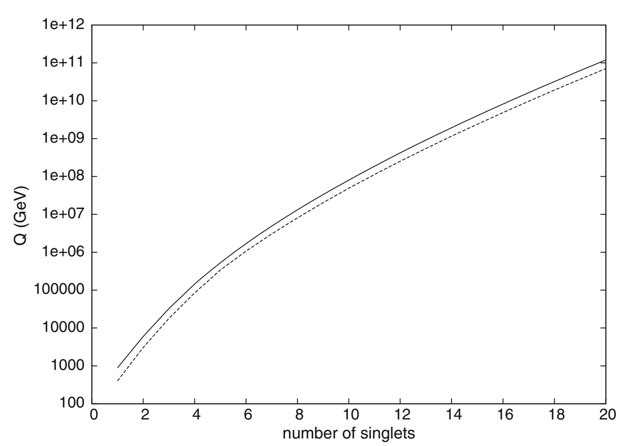

Figure 2. The energy scales where the scalar quartic couplings hit the Landau poles (assuming one-loop RG equations to be still valid) (upper curve) and at least one of the scalar couplings ceases to be perturbative ($\leq 4\pi$) (lower curve).

So, for $N = 1$, the lowest possible mass for the singlet is about 500 GeV, and goes down as $1/\sqrt{N}$. This means no change in the decay pattern of the 125 GeV scalar from the SM Higgs, and no $h \to SS$ invisible decay unless N is so large that $m_S < m_h/2$ (this happens for $N \geq 66$).

Unfortunately, the singlet scalar model ceases to be valid much before the Planck scale. This can be understood from the relevant one-loop β-functions:

$$\begin{aligned}
16\pi^2\beta_\lambda &= 12\lambda^2 + 6g_t^2\lambda + Na^2 - \frac{3}{2}\lambda(g_1^2 + 3g_2^2) \\
&\quad - 3g_t^4 + \frac{3}{16}(g_1^4 + 2g_1^2g_2^2 + 3g_2^4), \\
16\pi^2\beta_{\tilde{\lambda}} &= (32 + 4N)\tilde{\lambda}^2 + a^2 + 4\tilde{\lambda}Z^2 - \sum_i N_c\zeta_i^4, \\
16\pi^2\beta_a &= \Big[6\lambda + 12\tilde{\lambda} + 4a + 6g_t^2 \\
&\quad + 4Z^2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\Big]a, \qquad (21)
\end{aligned}$$

where $\beta_h \equiv dh/dt$ and $t \equiv \ln(Q^2/\mu^2)$. The scalar quartics are coupled and hence all of them hit the Landau pole almost simultaneously; we may take this as the range of validity of the model.

There are cosmological constraints on S, coming from the spin-independent DM-nucleon scattering cross-section. This depends on a, the coupling of S with Higgs, because the process is Higgs-mediated. Compatibility with XENON100 demands a low value of a (so, there may be more than one singlets, but the model can easily be tweaked to accommodate that); compatibility with Dama/I needs larger values of a and hence smaller number of singlets. For details, we refer the reader to ref. [15].

4. Two-Higgs doublet models

The two-Higgs doublet models (2HDM) [17] are one of the most widely investigated scenarios that go beyond the Standard Model. Any 2HDM consists of five physical scalars: two CP-even neutral h and H, one CP-odd neutral A, and two charged bosons $H^\pm$. The CP quantum numbers are, of course, assigned with the assumption that the scalar potential is CP-conserving and hence the mass eigenstates are also CP eigenstates. However, a generic 2HDM suffers from large flavour-changing neutral currents (FCNC); to prevent this, one invokes the Glashow–Weinberg–Paschos theorem [18,19]. The theorem states that there will be no tree-level FCNC if all right-handed fermions of a given electric charge couple to only one of the doublets. This can be achieved in 2HDMs by introducing discrete symmetries for fermions or scalars.

Let us denote the two doublets by Φ_1 and Φ_2, and invoke an Z_2 symmetry $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$. There are four types of 2HDM, depending on the transformation of the fermions under this Z_2, for which there will be no tree-level FCNC. They are: (i) type-I, for which all fermions couple with Φ_2 and none with Φ_1; (ii) type-II, for which up-type quarks couple to Φ_2 and down-type quarks and charged leptons couple to Φ_1 (this is the type that is embedded in the minimal supersymmetric SM (MSSM) and hence has received the most attention); (iii) type-Y (sometimes called type-III or flipped), for which up-type quarks and charged leptons couple to Φ_2 and down-type quarks couple to Φ_1, and (iv) type-X (sometimes called type-IV or lepton-specific), for which all charged leptons couple to Φ_1 and all quarks couple to Φ_2.

We do not need any extra fermions for 2HDM; apart from type-I, both the doublets couple to fermions. However, the extra constraint regarding the Veltman conditions implies that $\tan\beta = v_2/v_1$ is no longer a free parameter; it will be determined by the scalar quartic couplings. Consequences of applying the VC to 2HDMs were discussed in ref. [20] and then later discussed in more detail in ref. [21]. We shall base our discussion on a recent paper by us [22].

4.1 *2HDM in brief*

We shall follow the notations and conventions of ref. [17]. There are two scalar doublets, Φ_1 and Φ_2, with hypercharge $+1$. The lower components, which are electrically neutral, get nonzero VEV:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2\end{pmatrix}, \qquad (22)$$

with $\tan\beta = v_2/v_1$ and $m_W = \frac{1}{2}g_2\sqrt{v_1^2 + v_2^2}$. The CP-conserving scalar potential can be written as

$$\begin{aligned}
V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) \\
&\quad + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 \\
&\quad + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\
&\quad + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&\quad + \frac{1}{2}\lambda_5[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2], \qquad (23)
\end{aligned}$$

where m_{12}^2 softly breaks the Z_2 symmetry. The two CP-even neutral states ρ_1 and ρ_2, which are components of Φ_1 and Φ_2 respectively, are not mass eigenstates.

The corresponding mass matrix can be diagonalized through a rotation by an angle α, and the mass eigenstates are

$$h = \rho_2 \cos\alpha - \rho_1 \sin\alpha,$$
$$H = \rho_2 \sin\alpha + \rho_1 \cos\alpha, \tag{24}$$

where $h\,(H)$ is the lighter (heavier) eigenstate. Note that if $\alpha - \beta = \pi/2(0)$, $h(H)$ will be the SM Higgs boson, with a VEV of $v = \sqrt{v_1^2 + v_2^2}$. For example, the $hhVV^*$ $(HHVV^*)$ coupling is just the SM coupling times $\sin^2(\alpha-\beta)$ $(\cos^2(\alpha-\beta))$, where V is any weak gauge boson. The CP-odd scalar A does not couple to gauge bosons.

Before we proceed any further, let us note that one should formulate the VCs for h and H. However, if we demand the quadratic divergences for both h and H to vanish, we might as well formulate them for ρ_1 and ρ_2. This is what we shall do in our subsequent discussion, and perform the entire analysis in terms of the couplings and not the masses. While the propagators are ill-defined in the Φ_1–Φ_2 basis, this does not affect our analysis as long as we focus on purely the divergent terms.

The most generic Yukawa interactions for these four models can be written as [17]

$$\mathcal{L}_Y = -\sum_{j=1}^{2}[Y_j^d \bar{Q}_L d_R \Phi_j + Y_j^u \bar{Q}_L u_R \tilde{\Phi}_j + Y_j^e \bar{L}_L l_R \Phi_j + \text{h.c.}], \tag{25}$$

where $\tilde{\Phi}_j = i\tau_2\Phi_j^*$, Q_L, L_L, d_R, u_R and l_R are generic doublet quarks, doublet leptons, singlet down-type and singlet up-type quarks, and singlet charged leptons respectively. Y_j^d, Y_j^u, Y_j^e are 3×3 complex matrices, containing Yukawa couplings for the down, up, and leptonic sectors respectively. In our analysis, we shall consider only top, bottom, and τ Yukawa couplings to be nonzero.

4.1.1 *Stability conditions.* The requirement that the scalar potential always remains bounded from below leads to the following stability conditions:

$$\lambda_1, \lambda_2 \geq 0, \quad \lambda_3 \geq -\sqrt{\lambda_1\lambda_2},$$
$$\lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1\lambda_2}. \tag{26}$$

Thus, λ_3, λ_4, and λ_5 can potentially be negative. There can be charge-breaking or CP-breaking stable points of the potential; however, if the normal minimum is deeper, such stable points can at best be saddle points. The last condition shows that $\lambda_5 = 0$ leads to the most stable configuration for a given set of the other quartic couplings.

4.2 *Veltman conditions*

If the Yukawa couplings are neglected, the VCs for ρ_1 and ρ_2 are the same for all 2HDMs. The self-energy corrections are [22a]

$$\delta' m_{\rho_1}^2 = \frac{\Lambda^2}{16\pi^2}\left[\left(\frac{9}{4}g_2^2 + \frac{3}{4}g_1^2\right) + 2\lambda_3 + 3\lambda_1 + \lambda_4\right] \equiv \frac{\Lambda^2}{16\pi^2} f'_{\rho_1}, \tag{27}$$

$$\delta' m_{\rho_2}^2 = \frac{\Lambda^2}{16\pi^2}\left[\left(\frac{9}{4}g_2^2 + \frac{3}{4}g_1^2\right) + 2\lambda_3 + 3\lambda_2 + \lambda_4\right] \equiv \frac{\Lambda^2}{16\pi^2} f'_{\rho_2}. \tag{28}$$

Even if we neglect the gauge couplings, no solutions are consistent with eq. (26), except the trivial solution $\lambda_i = 0$. Note that there is no term proportional to λ_5; the quadratically divergent contributions cancel out.

With the introduction of the Yukawa couplings (only for t, b, and τ), the corrections turn out to be as follows:

(1) Type I:

$$f_{\rho_1} = f'_{\rho_1},$$
$$f_{\rho_2} = f'_{\rho_2} - 3\,(Y_2^b)^2 - 3\,(Y_2^t)^2 - (Y_2^\tau)^2. \tag{29}$$

(2) Type II:

$$f_{\rho_1} = f'_{\rho_1} - 3\,(Y_1^b)^2 - (Y_1^\tau)^2,$$
$$f_{\rho_2} = f'_{\rho_2} - 3\,(Y_2^t)^2. \tag{30}$$

(3) Lepton-specific:

$$f_{\rho_1} = f'_{\rho_1} - (Y_1^\tau)^2,$$
$$f_{\rho_2} = f'_{\rho_2} - 3\,(Y_2^b)^2 - 3\,(Y_2^t)^2. \tag{31}$$

(4) Flipped:

$$f_{\rho_1} = f'_{\rho_1} - 3\,(Y_1^b)^2,$$
$$f_{\rho_2} = f'_{\rho_2} - 3\,(Y_2^t)^2 - (Y_2^\tau)^2. \tag{32}$$

Thus, the complete one-loop quadratically divergent corrections are

$$\delta m^2_{\rho_{1(2)}} = \frac{\Lambda^2}{16\pi^2} f_{\rho_{1(2)}}, \tag{33}$$

and the strict enforcement of the VCs require $f_{\rho_1} = 0$, $f_{\rho_2} = 0$. In addition, if they are to hold at all energy scales, we also need $\mathrm{d}f_{\rho_1}/\mathrm{d}(\ln q^2) = 0$, $\mathrm{d}f_{\rho_2}/\mathrm{d}(\ln q^2) = 0$. We shall not show the RG equations here; they can be found in [17] or in [22].

4.3 $\tan\beta$ *as a function of quartics*

We are not going into any detailed discussion here, but let us just focus on the major outcomes. Note that 2HDMs are unlike the singlet extension; the 125 GeV Higgs is potentially a mixture of ρ_1 and ρ_2. However, this scalar behaves exactly like the SM Higgs, within the margin of error, and so $\cos(\alpha - \beta) \approx 0$. There are more constraints: the parameters should be so adjusted as to produce a light CP-even state at about 125 GeV, and the charged Higgs should not be lighter than 300 GeV for type-II and flipped models. This bound is independent of the precise value of $\tan\beta$ as long as $\tan\beta > 1$ and comes from the rate of the radiative decay $b \to s\gamma$. There is no such bound on the charged Higgs in the lepton-specific model.

Let us note here that the Yukawas for f_{ρ_1} and f_{ρ_2} should be of the same order, given all other quartic and gauge couplings. This forces large values of $\tan\beta$. For type-II and flipped, the lowest possible value of $\tan\beta$ is about 31.5 amd 42.5 respectively and increases almost linearly with λ_1. There is of course a band in the allowed values of $\tan\beta$, but that is very narrow, and depends on how strictly we impose the Veltman conditions. In the strictest limit, the width goes to zero. The lowest value of $\tan\beta$ comes from the fact that stability prevents $\lambda_1 < 0$. For lepton-specific models, the lowest value of $\tan\beta$ is about 140. Thus, the Yukawa couplings are so large that the potential quickly becomes unstable, at about a scale of 1 TeV (remember that the Yukawa couplings drive the scalar quartic couplings towards negative).

This, in some sense, is the most interesting result: if you demand a solution to the naturalness problem, you are forced to have large values of $\tan\beta$. Needless to say, this will affect the search strategies.

There is still a large parameter space consistent with the stability of potential as well as the direct and indirect constraints where both the Veltman conditions are satisfied. Unlike the singlet case, the theory does not blow up before the Planck scale for most of this parameter space. Again, this is true only for type-II and flipped models.

What about the stability of f_{ρ_1} and f_{ρ_2}? Everything would have been perfect if the Veltman conditions were absolutely stable with the scale variation. Unfortunately, this is not so. But this may be due to the simplistic approach of keeping only the one-loop terms; at higher orders, we might expect a scale independence.

5. Complex scalar triplet

Let us now concentrate on the last variant, the triplet extension [23]. The SM is augmented with a complex triplet, whose potential can be found in refs [24,25].

Such complex triplet models are interesting in the context of neutrino mass generation through the see-saw mechanism [26] with a lepton number (L) violating interaction, and also the type-II leptogenesis scenario [27]. As the complex triplet can couple to left-handed leptons to generate Majorana masses for the neutrinos through $\Delta L = 2$ terms, again there is no need to introduce any additional fermions in the model.

The VEV of the triplet is, of course, restricted from the ρ-parameter to be at most of a few GeV [28]. However, it is more than enough to generate the neutrino masses if the corresponding Yukawa couplings are of order unity. The mixing between the triplet and doublet states is proportional to the triplet VEV, which, being tiny, makes the mixing small too [29]. Thus, the 125 GeV scalar is almost a pure doublet, which is completely consistent with its production cross-section and decay branching ratios.

The complex triplet X has a weak hypercharge $Y = 2$. The VEVs are

$$\langle\phi^0\rangle = \frac{v_1}{\sqrt{2}}, \qquad \langle X^0\rangle = v_2. \tag{34}$$

We can express the triplet in a bidoublet notation:

$$X = \begin{pmatrix} X^+/\sqrt{2} & X^{++} \\ X^0 & -X^+/\sqrt{2} \end{pmatrix}, \tag{35}$$

and the generic form of the $\Delta L = 2$ terms is

$$V_{\Delta L=2} = -i f_{ab} L_a^T C^{-1} \tau_2 X L_b + \text{h.c.}, \tag{36}$$

where C is the charge conjugation operator and $L = (\nu \;\; \ell)^T$ is the left-handed lepton doublet. If there is no leptonic flavour-changing neutral current, we can take the Yukawa coupling f_{ab} to be diagonal. Life is further simplified by $f_{ab} = f\delta_{ab}$; this does not reproduce the neutrino data but is enough for the analysis.

The scalar potential can be written as [25]

$$V = V_2 + V_3 + V_4, \tag{37}$$

where the individual terms are

$$V_2 = -\mu_1^2(\Phi^\dagger\Phi) + \mu_2^2(X^\dagger X),$$
$$V_3 = a_0(\Phi\Phi X^\dagger) + \text{h.c.},$$
$$V_4 = \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2(X^\dagger X)^2 + \lambda_3(\Phi^\dagger\Phi)(X^\dagger X) + \lambda_4(\Phi^\dagger\tau_i\Phi)(X^\dagger t_i X) + \lambda_5|X^T\tilde{C}X|^2\,, \tag{38}$$

with

$$\tilde{C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{39}$$

and τ_is and t_is ($i = 1$–3) are the 2×2 and 3×3 Pauli matrices respectively, with $t_1 = \delta_{i,i+1} + \delta_{i,i-1}$, $t_2 = -i(\delta_{i,i+1} - \delta_{i,i-1})$, and $t_3 = \text{diag}(1, 0, -1)$. Note that the triplet has a 'right-sign' mass term, which ensures that the triplet VEV will arise only through the trilinear and quartic interactions, and can remain small without necessarily keeping the triplet light and hence jeopardizing the experimental constraints [29a]. Without the trilinear term, there is a global $O(2)$ symmetry in the neutral scalar sector, so that there will be a physical Goldstone boson in the spectrum if both neutral fields acquire VEV. Further ramifications of the trilinear term can be found in [25].

5.1 *Veltman conditions*

With the triplet, the VC for the SM Higgs is modified to

$$f_h = 6\lambda_1 + 3\lambda_3 + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2. \tag{40}$$

With $m_h = 125$ GeV, $m_W = 80.41$ GeV, $m_Z = 91.19$ GeV, and $m_t = 174$ GeV, this fixes $\lambda_3 \approx 1.39$. This is large but still within the perturbative limit of 4π. With N identical triplets, $\lambda_3 \approx 1.39/N$.

The stability conditions of the scalar potential read as

$$\lambda_1, \lambda_2 \geq 0, \qquad \lambda_2 + 2\lambda_5 \geq 0,$$
$$\lambda_3 \pm \lambda_4 \geq -2\sqrt{\lambda_1\lambda_2} \tag{41}$$

plus some other conditions that are not independent of these. Note that λ_4 and λ_5 can be negative. The VC for the triplet, which couples to the leptons through $\Delta L = 2$ interaction, reads as

$$f_X = 4\lambda_2 + \lambda_3 + 2\lambda_5 + \frac{1}{2}g_1^2 + g_2^2 - 3f^2\,. \tag{42}$$

Without the Yukawa term, δm_X^2 can never be made to vanish, even with possible negative values of λ_5, due to the stability conditions. There is no contribution proportional to λ_4 in eq. (42); the quadratically divergent contributions cancel out. Also, even in the limit $\lambda_1, \lambda_5 \to 0$, the large value of λ_3 necessitates correspondingly large value of the Yukawa coupling f and hence an extremely tiny tripet VEV v_2, completely consistent with the ρ-parameter bound, as well as to the identification of the 125 GeV resonance as the almost-pure SM doublet. The $3f^2$ term in eq. (42) appears because of universal leptonic Yukawa couplings. For normal (inverted) hierarchy, we expect $3f^2 \approx f_{\text{normal}}^2(2f_{\text{inverted}}^2)$.

The RG equations can be found in [23] (figure 3).

5.2 *A new intermediate scale is necessary*

Just like the singlet extension, the triplet model cannot be extended to the Planck scale. Sooner or later, the scalar couplings blow up. The blowing up can be postponed up to about 10^7 GeV if we start with a very

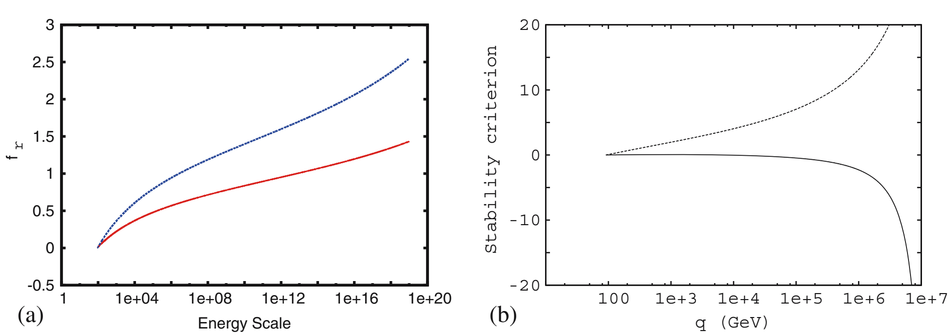

Figure 3. Stability of the Veltman conditions or 2HDM type-II (**a**) and complex triplet model (**b**). For figure 3a, the lower red line is for f_{ρ_1} and the upper blue line is for f_{ρ_2}, drawn with $\lambda_1 = 0$, $\lambda_2 = \lambda_3 = 0.25$, $\lambda_4 = -0.01$, $\lambda_5 = 0$, and $\tan\beta = 26.5$ [22]. For figure 3b, the lower line is for f_X and the upper line is for f_h, drawn with $\lambda_2 = 0.01$, $\lambda_4 = \lambda_5 = 0$ [23].

small value of λ_2 and a negative value of λ_5; the other parameters are not that relevant. In a see-saw mechanism, this scale may have some physical interpretation as well. But that is beyond the scope of this article; the onus of model building is not on us, we shall just supply the necessary guidelines.

6. Naturalness in an effective theory

So far we have discussed models where the new physics appears below the cut-off scale Λ. The other side of the coin is to assume a mass gap between the electroweak scale v and Λ and to take all new particles to be heavier than Λ. Thus, below Λ only virtual effects of new particles are observable. The correct framework is an effective field theory (EFT) with higher dimensional operators (suppressed by suitable powers of Λ) whose coefficients parametrize the new physics effects [30]. The effective Lagrangian is of the form

$$\mathcal{L}_{\rm eff} = \sum_{n=1}^{\infty} \frac{1}{\Lambda^n} \sum_i C_i \mathcal{O}_i, \tag{43}$$

where the new physics effects are contained in the Wilson coefficients C_i. These effective operators can also give rise to quadratic divergences. Suppose we have an effective operator containing two Higgs fields, two bosons of type X and two bosons of type Y. This is suppressed by $1/\Lambda^2$, but there are two bosonic loops on the Higgs line, and so the divergence is $(\Lambda^2)^2 = \Lambda^4$. Overall, the effect of this operator is quadratically divergent.

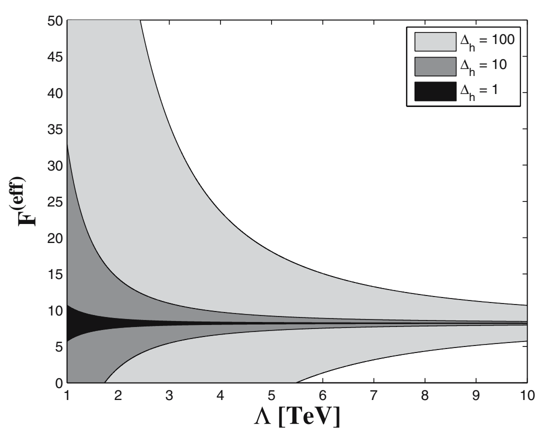

Figure 4. Regions in the $F^{\rm eff}$–Λ plane where naturalness can be restored with no fine-tuning ($\Delta_h = \delta m_h^2/m_h^2 = 1$, in black) and with fine-tuning at the level of 10% (dark gray) and 1% (light gray), corresponding to $\Delta_h = 10$ and 100, respectively (taken from [30]).

The one-loop quadratic corrections to δm_h^2 can be written as

$$\delta m_h^2 = -\frac{\Lambda^2}{16\pi^2} F^{\rm eff}, \tag{44}$$

where $F^{\rm eff}$ comes from the effective operators. Adding the SM contributions, one gets [30]

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2} |F^{\rm eff} - 8.2|. \tag{45}$$

Thus, if $\Lambda^2 \gg m_h^2$, there must be a very fine cancellation between $F^{\rm eff}$ and the SM contributions. This is shown in figure 4.

7. Epilogue

Such bottom-up approaches, of course, do not tell us anything about the underlying dynamics. There may be a fine-tuning with which we have to live unless we want to make the nature more baroque. Scalar extensions are only the simplest solutions; there can be gauge extensions too, or even combined fermionic and bosonic extensions. How do we know whether nature chooses such extensions to get rid of fine-tuning? The only way is to map out the parameter space of the model; find the new particles, understand their properties, measure the interactions. This is analogous to the question: if the theory of supersymmetry were not known but one found the sparticles and measured their spins, masses, and couplings, could one derive the fundamental theory from the data?

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