Pramana – J. Phys. (2016) 87: 37
DOI 10.1007/s12043-016-1246-2

Why supersymmetry? Physics beyond the standard model

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Published online 23 August 2016

Abstract. The Naturalness Principle as a requirement that the heavy mass scales decouple from the physics of light mass scales is reviewed. In quantum field theories containing *elementary* scalar fields, such as the Standard Model of electroweak interactions containing the Higgs particle, mass of the scalar field is not a natural parameter as it receives large radiative corrections. How supersymmetry solves this Naturalness Problem is outlined. There are also other contexts where the presence of elementary scalar fields generically spoils the high–low mass scales decoupling in the quantum theory. As an example of this, the non-decoupling of possible Planck scale violation of Lorentz invariance due to quantum gravity effects from the physics at low scales in theories with elementary scalar fields such as the Higgs field is described. Here again supersymmetry provides a mechanism for ensuring that the decoupling of heavy–light mass scales is maintained.

Keywords. Naturalness problem; hierarchy problem; decoupling; supersymmetry; Standard Model; Higgs particle; violation of Lorentz invariance.

PACS Nos 11.30.Pb; 12.15.−y; 12.60.−i; 12.10.−g; 04.60.−m

1. Introduction

Till recently, all elementary particles that were known to exist in Nature were only spin-half fermions and spin-one gauge particles. With the discovery of Higgs particle at the LHC in 2012, we now have the first elementary spin-zero particle. An elementary scalar field, such as the Higgs field, introduces a completely new feature in quantum field theories containing such a field. This new feature is a generic non-decoupling of the heavy mass scales from the physics of low mass scales.

A quantum field theoretic description for physical processes with a characteristic smaller mass scale m_L should not depend sensitively on the physics of larger mass scales m_H. This decoupling requirement is a reasonable expectation so that whatever low mass scale quantum theory we have, can describe the physics at that scale reliably. The only possible allowed dependence of the physics at low mass scale m_L on the higher mass scale m_H is in the form of its inverse powers and at the worst, a milder dependence through logarithms of the high scale, but, as shall be discussed in detail in the following, those with positive powers of this scale are not acceptable at all. Another name for this requirement is ‘Naturalness Principle’.

A quantum field theory containing only spin-half fermions and gauge fields exhibits precisely this decoupling. Such theories are called natural theories and the masses and the gauge couplings are natural parameters. Examples of such theories are: Quantum electrodynamics (QED) and quantum chromodynamics (QCD).

The notion of Naturalness emerged in the late 1970s from the work of Wilson, Gildener and Weinberg and ’t Hooft [1,2]. A concise formulation is provided by ’t Hooft’s Doctrine of Naturalness [2]: *A parameter $\alpha(\mu)$ at any energy scale μ in the description of physical reality can be small*, *if and only if, there is an enhanced symmetry in the limit* $\alpha(\mu) \to 0$. This implies a rule of thumb: quantum corrections to the parameter α (masses and couplings) should be proportional to a positive power of that parameter itself: $(\Delta\alpha)_{\text{quantum}} \sim \alpha^n$, $n \geq 1$. This would be ensured by the associated approximate symmetry. Further, this property implies that the enhanced symmetry holds even at the quantum level as $\alpha \to 0$.

Let us look at QED in some detail. The theory describes the interaction of fermions λ of charge q_f

such as an electron with electromagnetic radiation through a Lagrangian density:

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\lambda}\left[i\gamma^{\mu}\left(\partial_{\mu} - ieq_f A_{\mu}\right) - m_{\rm e}\right]\lambda. \tag{1}$$

Various parameters here, the electromagnetic coupling e and electron mass $m_{\rm e}$, can be naturally small. Limit $m_{\rm e} \rightarrow 0$ leads to an enhanced symmetry, the chiral symmetry: separate conservation of the number of left- and right-handed electrons. It is for this reason that one-loop correction to the electron mass in this theory is given by logarithmically divergent diagrams and is proportional to the electron mass itself.

$$(\Delta m_{\rm e})_{1\ \rm loop} \sim e^2 m_{\rm e} \ln \Lambda. \tag{2}$$

In the limit $m_{\rm e} \rightarrow 0$, this correction disappears reflecting the fact that chiral symmetry is preserved by such perturbative quantum corrections in this limit. Also, $e \rightarrow 0$ results in enhanced symmetry: there is no interaction and hence particle number of each type is conserved. Again, quantum corrections to the coupling e are logarithmically divergent and are, in the lowest order, proportional to e^3:

$$(\Delta e)_{1\ \rm loop} \sim e^3 \ln \Lambda. \tag{3}$$

It is for this reason that atomic physics described by the interactions of electrons and photons is not disturbed by the fact there are other heavier charged fermions such as the muon, tau-lepton, top quarks and others in Nature: $m_{\mu} \sim 200 m_{\rm e}$, $m_{\tau} \sim 3500 m_{\rm e}$, ..., $m_{\rm top} \sim 3.4 \times 10^{11} m_{\rm e}$. Effects of heavier fermions are decoupled from the physics of electrons and photons; showing up at best through logarithmic dependence on them.

As against this, theories with *elementary* scalar fields have completely different behaviour. Elementary scalar fields spoil heavy–light decoupling: quantum field theories with scalar fields are *not natural.* As an example, consider a Yukawa theory of an elementary scalar field ϕ of low mass m_L coupled to a heavy fermion λ of mass m_H:

$$\mathcal{L} = \frac{1}{2}\,\partial^{\mu}\phi\partial_{\mu}\phi - \frac{1}{2}\,m_L^2\phi^2 + \bar{\lambda}\left(i\gamma^{\mu}\partial_{\mu} - m_H\right)\lambda + y\bar{\lambda}\lambda\phi; \qquad m_H \gg m_L. \tag{4}$$

In this theory, the light scalar mass m_L is not a natural parameter. Smallness of m_L cannot be protected by any approximate symmetry against perturbative quantum corrections involving heavy fermions in the loops. In fact, such corrections to m_L^2 appear with quadratic divergences:

$$\Delta m_L^2 \sim -y^2 \int {\rm d}^4 k \frac{k^2 + m_H^2}{(k^2 - m_H^2)^2} \sim -y^2 m_H^2 \ln(m_H^2/\mu^2), \tag{5}$$

where we have used dimensional regularization and minimal subtraction in the last step. Notice that this correction is proportional to m_H^2 and not to m_L^2. Therefore, there is no decoupling of the heavy mass scale from the light mass scale theory. Even in the limit $m_L \rightarrow 0$, this correction does not go away.

2. Naturalness of electroweak theory

Standard Model (SM) of particle physics has an elementary scalar particle, the Higgs particle. Discussion above then implies that its mass is not protected by any symmetry against large radiative corrections.

Tree-level masses for the Higgs particle, gauge particles $W^{\pm}$, Z^0 and the fermions in SM are given by: $m_{\rm Higgs} = \sqrt{\lambda}v$, $m_W = gv/2$, $m_Z = gv/(2\cos\theta_W)$, $m_f = Y_f v/\sqrt{2}$, where v is the vacuum expectation value of the scalar field, λ is the quartic scalar coupling, g is the gauge coupling, Y_f is the Yukawa coupling of the fermions f to the scalar field and θ_W is the weak mixing angle.

Note that the limit $v \rightarrow 0$ does enhance *classical* symmetry: (i) all particles being massless in this limit, we have scale invariance of the classical theory; (ii) the weak gauge bosons are massless resulting in restored $SU(2)$ gauge symmetry and additionally (iii) there is chiral symmetry due to zero masses of the fermions. Yet, v is not a natural parameter. Consequently, masses of the Higgs particle, weak gauge bosons and fermions are not natural. This is due to the Coleman–Weinberg mechanism of radiative breaking of symmetry: quantum fluctuations generate a non-zero quantum vacuum expectation value for the scalar field even when classically v is zero, breaking all these symmetries. There is no enhancement of symmetry at the *quantum* level in the limit where classical vacuum expectation value $v \rightarrow 0$.

In the SM, one-loop radiative corrections to the Higgs mass come from the diagrams of the type where fermions f, gauge fields ($W_{\pm}$, Z) and Higgs field H go around the loop (see figure 1). The diagrams contribute correction to the Higgs mass as

$$\Delta m_{\rm Higgs}^2 = \alpha\Lambda^2$$

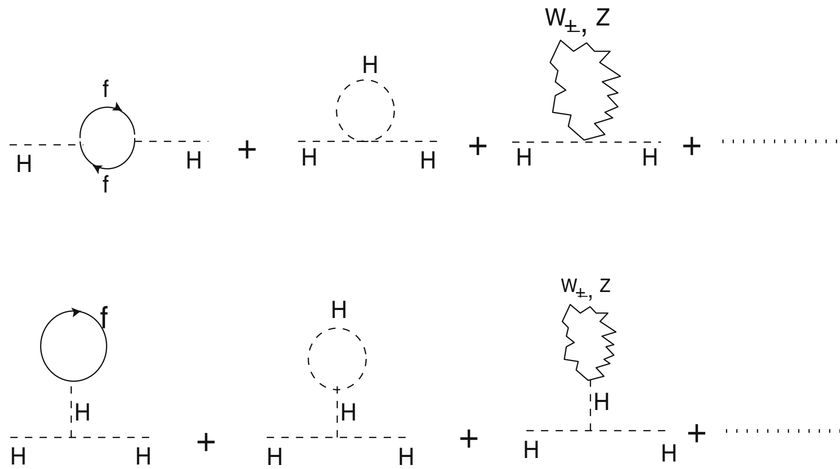

Figure 1. Diagrams showing fermions f, Higgs field H and gauge fields ($W_\pm$, Z) in the loop.

with

$$\alpha = \frac{1}{16\pi^2}(A\lambda + Bg^2 - CY_f^2),$$

where A, B, C are numerical constants respectively associated with the diagrams with scalar fields, gauge fields and the fermions in the loops. Use dimensional regularization (with minimal subtraction) to write this correction as

$$\Delta m^2_{\text{Higgs}} \sim \frac{1}{16\pi^2}\left[A\lambda m^2_{\text{Higgs}} \ln\left(\frac{m^2_{\text{Higgs}}}{\mu^2}\right) + Bg^2 m^2_{\text{gauge}} \ln\left(\frac{m^2_{\text{gauge}}}{\mu^2}\right) - CY_f^2 m_f^2 \ln\left(\frac{m_f^2}{\mu^2}\right)\right].$$

Largest mass particle (top quark) in the loops gives the dominant correction:

$$\Delta m^2_{\text{Higgs}} \sim -\alpha m^2_{\text{top}} \ln(m^2_{\text{top}}/\mu^2). \tag{6}$$

Thus, the radiative correction to scalar mass is generically controlled by the highest mass in the loops. This is to be contrasted with QED where correction to the square of the electron mass is proportional not to the square of any other mass but only to square of the electron mass itself: $\Delta m_e^2 \sim \alpha m_e^2$.

Now from eq. (6), for top mass $m_{\text{top}} = 175$ GeV and the coupling factor $\alpha \sim 1/100$, the radiative contribution $\Delta m^2_{\text{Higgs}}$ is still small for $m_{\text{Higgs}} = 125$ GeV. But if there were a much heavier particle in Nature, such as in a Grand Unified Theory (containing the QCD and the electroweak model), the radiative corrections to Higgs mass would be controlled by this heavy scale and hence very large.

Naturalness breakdown scale of the SM: As we have seen above, one-loop correction to Higgs boson mass due to quantum fluctuations of a size characterized by the scale Λ may be written as: $\Delta m^2_{\text{Higgs}} = \alpha\Lambda^2$. Square of the vacuum expectation value of the scalar field, and also the masses of vector bosons $W^\pm$ and Z^0 and fermions would obtain similar corrections. For coupling $\alpha \sim (100)^{-1}$ and $m_{\text{Higgs}} \sim 100$ GeV, if we require that these radiative corrections to this mass do not exceed its value, $\Delta m^2_{\text{Higgs}} \sim m^2_{\text{Higgs}}$, we have

$$\Lambda_N^2 = \frac{\Delta m^2_{\text{Higgs}}}{\alpha} \sim \frac{(100\ \text{GeV})^2}{(100)^{-1}} = (1000\ \text{GeV})^2 \equiv (1\ \text{TeV})^2. \tag{7}$$

This leads to an estimate of the naturalness breakdown scale for the electroweak theory as: $\Lambda_N \sim 1$ TeV.

3. Large logarithmically divergent corrections in GUT

Not only are the quadratically divergent graphs with heavy mass fields going around the loops responsible for destabilizing the lower mass scale, but there are also some log divergent graphs which contribute to this phenomenon [3]. These graphs appear generically in any grand unified theory (GUT) of the QCD and electroweak $SU(2) \times U(1)$ model.

Consider a gauge theory based on a gauge group G which is spontaneously broken at two stages:

$$G \xrightarrow{F} G_1 \xrightarrow{f} G_2; \quad F \gg f.$$

This is achieved through vacuum expectation values of two scalar fields: $\langle\Phi\rangle_{\text{vac}} = F \sim M_1$ and $\langle\phi\rangle_{\text{vac}} = f \sim M_2$. It was in this context of GUTs that the Naturalness Problem was noticed in its earliest versions by

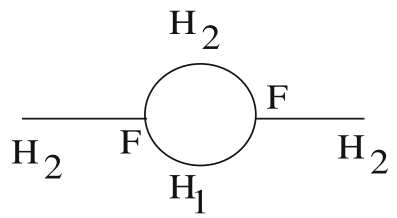

Figure 2. Large log divergent graph.

Gildener and Weinberg [1] who realized that the relative stability of the smaller scale f as against the larger scale F cannot be maintained under radiative quantum effects and this was given the name: *Gauge Hierarchy Problem.*

Quadratically divergent graphs for the two-point correlations of light scalar fields give large corrections ($\sim F^2$) to its M_2^2 ($\sim f^2$). Besides these, there are also large logarithmically divergent contributions from the graphs involving large ($\sim F$) three-point coupling [3]. These come from the mixed light–heavy field interaction terms of the type:

$$\begin{aligned}\mathcal{L}_{\text{int}} \sim \kappa\Phi^2\phi^2 &= \kappa(H_1+F)^2\,(H_2+f)^2\\ &\sim \cdots + cH_1^2H_2 + dH_1H_2^2 + \cdots;\\ &c \sim f,\ \ d\sim F. \end{aligned} \tag{8}$$

The relevant three-point vertex is from the interaction term $dH_1H_2^2$ with coupling strength proportional to the heavy mass scale, $d \sim F$. As shown in figure 2, this leads to a logarithmically divergent two-point graph with light fields H_2 on the external lines and one heavy (H_1) and one light (H_2) fields propagating on the internal lines in the loop. This diagram contributes a correction to the light mass square, M_2^2, given by

$$\Delta M_2^2 \sim F^2\ln(F^2/\mu^2) \tag{9}$$

which is proportional to the square of the larger mass scale due the presence of factor F^2 coming from the two interaction vertices. This results in a destabilization of the light–heavy mass hierarchy.

4. A window to physics beyond SM: Supersymmetry

In a Grand Unified Theory, perturbative quantum corrections tend to draw the smaller electroweak scale ($M_{\text{EW}} \sim 100$ GeV) towards the GUT scale ($M_{\text{GUT}} \sim 10^{16}$ GeV). Even without grand unification, now with the established non-zero masses for neutrinos, though very small, as indicated by the neutrino oscillations, the see-saw mechanism for these masses also suggests a new physical high scale of the order of 10^{11} GeV or so linked to the mass of the right-handed neutrino. This would imply that the radiative corrections would drag the Higgs mass to such high values. Even if we ignore both these sources of possible high mass scales, there is yet another high physical mass scale, $M_{\text{Pl}} = 10^{19}$ GeV in Nature, associated with quantum gravity. Radiative corrections would draw the masses of electroweak theory to this high scale and hence their natural values would be $\sim 10^{19}$ GeV and not the physical values characterized by the low SM scale! All these suggest that there has to be some new physics beyond 1 TeV such that the SM with its characteristic scale of 100 GeV stays natural beyond this scale.

There are several proposed solutions to the Naturalness Problem (for a review, see [4]). Of these, with Higgs mass at 126 GeV, supersymmetry is the most promising solution.

An elementary property of quantum field theory which gives an extra minus sign for the radiative diagram with a fermion as against a boson field going around in the loop allows for the possibility that naturalness-violating effects due to bosonic and fermionic quantum fluctuations can be arranged to cancel against each other. For this to happen, the various couplings and masses of bosons and fermions have to be related to each other in a highly restrictive manner. Further, for such a cancellation to hold at every order of perturbation, a symmetry between bosons and fermions would be imperative. This is what supersymmetry does indeed provide.

A historical note: Supersymmetric solution of the Naturalness Problem (or Non-decoupling Problem or Gauge Hierarchy Problem) was discovered in 1981 in Bangalore, requiring supersymmetry to become operative at about 1 TeV for the masses of Standard Model to be stable against radiative corrections:

(i) In ref. [5], the quadratic divergences were shown to be absent in a supersymmetric theory with spontaneously broken anomaly-free $U(1)$ gauge symmetry.

(ii) In [3], absence of the naturalness-spoiling quadratic divergences as well as the large logarithmic divergences was shown in a supersymmetric theory with two distinctly different scales, heavy F and light f, associated with sequential gauge symmetry breaking through vacuum expectation values of two sets of scalar fields. This demonstrated that the hierarchy $f^2/F^2 \ll 1$ is radiatively maintained even when quantum corrections are included in a supersymmetric framework.

(iii) In ref. [6], it was demonstrated that (a) in a supersymmetric theory with anomalous $U(1)$ gauge invariance (where the $U(1)$ charges do not add up to zero, $\sum Q_{U(1)} \neq 0$), quadratic divergences are not absent; but in a theory which is anomaly-free ($\sum Q_{U(1)} = 0$), these are absent and (b) in a supersymmetric theory with anomaly-free $SU(2) \times U(1)$ gauge invariance, quadratic divergences due to the boson and fermion fields in the loops cancel out completely with no net quadratic divergences and hence such a theory is perfectly natural.

(iv) In ref. [7], technicolour and supersymmetric solutions of the Naturalness Problem of SM are reviewed.

(v) It was argued in [8] that, in a general GUT with two distinct mass scales, the decoupling of the high mass scale from the low mass scale is spoiled by the same features of elementary scalar fields as are responsible for the Coleman–Weinberg radiative symmetry breaking. In a supersymmetric theory, Coleman–Weinberg mechanism is not operative and hence, the low–high mass scale distinction holds even when quantum corrections are incorporated.

(vi) In supersymmetric theories with spontaneously broken $U(1)$ gauge symmetry even when trace of $U(1)$ charges is zero, the D term can get one-loop corrections, but that these are only logarithmically divergent was proved in ref. [9].

5. Supersymmetric extension of the Standard Model

Supersymmetric theories with non-Abelian gauge invariances are always free of quadratic divergences. On the other hand, those with $U(1)$ gauge invariance have quadratically divergent radiative corrections proportional to the sum of $U(1)$ charges of all the fields. If the $U(1)$ charges sum to zero, quadratic divergences are absent even in these theories. Supersymmetrized Standard Model is one such theory.

Also for theories with two widely separated scales such as a supersymmetric GUT, the large logarithmic divergences are also separately absent.

In supersymmetric theories with spontaneously broken gauge symmetries through non-zero vacuum expectation value (VEV) of elementary scalar fields, the limit VEV $\rightarrow 0$ does lead to an enhanced symmetry even at the quantum level (provided, in the presence of a $U(1)$ gauge symmetry, all the $U(1)$ charges add up to zero). Coleman–Weinberg mechanism does not produce radiative breaking of the gauge symmetry in such theories.

Supersymmetry requires that bosons and fermions come in families. For supersymmetric model building, see ref. [10]. Simplest supersymmetric model is the minimal supersymmetric Standard Model (MSSM) where every SM particle has a superpartner with opposite statistics: for the photon we have a Majorana fermion, the photino, as its supersymmetric partner; for the leptons we have scalar sleptons; for the quarks scalar squarks; etc.

Exact supersymmetry requires that all properties, except the spin, of particles in a supermultiplet are the same: masses are equal and so are the couplings; electroweak and colour quantum numbers are identical. But, supersymmetry cannot be an exact symmetry of Nature: otherwise we should already have seen a scalar superpartner of an electron with the same mass and charge. So supersymmetry has to be broken in a way that the superpartners are much heavier than the SM particles. Yet naturalness-violating effects should not appear: in particular the quadratic divergences should not reappear in the radiative corrections. This indeed happens if supersymmetry is spontaneously broken or explicitly broken by the so-called soft terms (i.e., broken by masses only and not by dimensionless couplings) in the action at a scale $M_{\mathrm{SUSY}} \sim 1$ TeV.

MSSM has a whole variety of possible new interactions: a large number of new free parameters ($\sim$100) with all possible soft supersymmetry breaking terms. This makes it difficult to make any robust and easily verifiable predictions. An important requirement in supersymmetric theories is the suppression of unwanted flavour changing neutral current (FCNC) which are otherwise generically present in large sizes in such theories. Sometimes, people make certain assumptions about the nature of the new interactions which reduces the number of the extra parameters. Different possible choices of these parameters lead to predictions with different possible masses and also different decay patterns. One such model, the constrained minimal supersymmetric model (cMSSM), has only a few extra free parameters, five in all.

So far, no evidence for supersymmetry has emerged from the 8 TeV data collected at LHC. This may change over time when more data become available. But, it is perfectly possible that the simplest form of supersymmetric model, i.e., cMSSM, is not the right picture. More involved supersymmetric models may have to be explored. A MSSM with more parameters or a next-to-minimal supersymmetric Standard Model (NMSSM) [6,11] or even a non-minimal model with more structure may be required. A recent example of a more involved model is the gauge-mediated supersymmetry breaking (GMSB) with an unconventional messenger content [12] as against the 5 and $\bar{5}$ multiplets of the grand unification gauge group $SU(5)$ in the minimal conventional GMSB model.

It is important to realize that, except for the compelling naturalness argument which predicts supersymmetry as operative at about 1000 GeV, so far there are really not enough strong theoretical or experimental constraints available to guide us to a reliable supersymmetric model. Besides, the properties of the Standard Model including the fact that Higgs mass is now known to be around 126 GeV, other important and stringent restrictions for supersymmetry model building come from the requirement of sufficient suppression of flavour changing neutral currents (FCNCs) which otherwise can tend to be generically large in the supersymmetric theories. Hopefully, more experimental results from the LHC will provide enough discriminating guidance that will finally result in the correct supersymmetric model.

6. Other non-decoupling problems: Lorentz non-invariance

There can be many other physical situations where presence of elementary scalar fields can cause the same non-decoupling problems. These again would be cured by supersymmetry. We shall now discuss such an example which concerns possible Lorentz non-invariance generated by quantum gravity effects at the Planck scale.

Large quantum fluctuations in the gravitational field would introduce granularity of space at extremely short distances ($\sim \ell_{\text{Planck}} = 10^{-33}$ cm). This would imply a minimum spatial length beyond which no physical process can penetrate. This is in conflict with Lorentz invariance (LI) because LI implies that we can make an arbitrarily large boost transformation which would result in Lorentz contraction of lengths to arbitrarily small values. This violation of LI would reflect itself through a change of the dispersion relation for a particle. These features are known to emerge in theories of quantum gravity such as the loop quantum gravity (LQG) as well as the string theory [13].

Do these Planck scale effects decouple from low-energy physics? In quantum field theory of fermions and gauge fields where only low–high mass scales decoupling holds, this would indeed be realized. But, as discussed by Collins *et al* [14], theories containing elementary scalar fields would not exhibit such a property. We now present their argument for this behaviour in the following.

Lorentz invariance implies a unique form of dispersion relation for a particle: $E^2 - \vec{p}^2 - m^2 = 0$, in units where velocity of light $c = 1$. A Lorentz non-invariance effect would change the dispersion relation to: $E^2 - \vec{p}^2 - m^2 - \Pi(E, \vec{p}) = 0$, where Π represents the result of all the self-energy graphs with a small Lorentz-violating contribution from the quantum gravity effects.

We may parametrize the Lorentz violation through a dimensionless parameter [14]:

$$\xi = \lim_{p\to 0}\left[\left(\frac{\partial}{\partial p_0}\frac{\partial}{\partial p_0} + \frac{\partial}{\partial|\vec{p}|}\frac{\partial}{\partial|\vec{p}|}\right)\Pi(p)\right]. \quad (10)$$

For exact Lorentz invariance $\Pi(p)$ would be a function of the Lorentz-invariant combination $p_0^2 - \vec{p}^2$ implying that ξ is zero. Thus, $\xi \neq 0$ would provide a measure of violation of Lorentz invariance.

Now, as an example, consider a Yukawa theory of a fermion and a scalar field with their small masses given by the low scale m_{low}. Let us study the contribution to ξ for the scalar field from the correction to the scalar two-point function $\Pi(p_0, \vec{p})$ due to a fermion loop in this theory:

$$\begin{aligned}\Pi(p) &= -4iy^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}[k\cdot(k+p) + m_{\text{low}}^2] \\ &\qquad \times[k^2 - m_{\text{low}}^2 + i\epsilon]^{-1} \\ &\qquad \times[(p+k)^2 - m_{\text{low}}^2 + i\epsilon]^{-1} \\ &= -2iy^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m_{\text{low}}^2 + i\epsilon}\right. \\ &\qquad \left. + \frac{1}{(k+p)^2 + m_{\text{low}}^2 + i\epsilon}\left(1 + \frac{4m_{\text{low}}^2 - p^2}{k^2 - m_{\text{low}}^2 + i\epsilon}\right)\right],\end{aligned}$$

where y is the Yukawa coupling. This integral has a quadratic divergence which gets converted to a logarithmic divergence in ξ due to the momentum derivatives in its definition above:

$$\begin{aligned}\xi &= -16iy^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\frac{k_0^2 + \frac{1}{3}\vec{k}^2}{(k^2 - m_{\text{low}}^2 + i\epsilon)^3} \\ &\qquad \times\left[1 + \frac{4m_{\text{low}}^2}{k^2 - m_{\text{low}}^2 + i\epsilon}\right].\end{aligned}$$

This integral can be evaluated by Euclidean continuation of k_0 to imaginary values ik_4:

$$\xi = 16y^2\int\frac{(\mathrm{d}^4k)_E}{(2\pi)^4}\ \frac{k_4^2-\frac{1}{3}\vec{k}^2}{(k^2+m_{\text{low}}^2)^3}\left[1-\frac{4m_{\text{low}}^2}{k^2+m_{\text{low}}^2}\right]. \quad (11)$$

We may use an ultraviolet cut-off Λ for the Euclidean internal loop momentum $k = \sqrt{k^\mu k_\mu} = \sqrt{k_4^2+\vec{k}^2}$ which is invariant under four-dimensional Euclidean rotations. It is straightforward to check that such a calculation yields the Lorentz symmetric answer $\xi = 0$.

However, due to the possible Lorentz violations from the Planck scale physics, the free fermion propagator used in this calculation would get significantly modified at high scales. The momentum cut-off would have to be Lorentz violating, introducing different cut-offs for the k_0 and $|\vec{k}|$ integrations. One way to introduce this order-one non-invariance is by introducing a Lorentz non-invariant cut-off by multiplying the free fermion propagator by a smooth function $f(|\vec{k}|/\Lambda)$ which for the momenta much below the Planck-scale cut-off Λ ($\sim M_{\text{Pl}}$) goes to 1, $f(0) = 1$, so that the low-energy propagator stays largely unaffected, and for high momenta this function goes to zero, $f(\infty) = 0$, to tame the ultraviolet behaviour in a Lorentz non-invariant manner. This would lead to a one-loop contribution to the two-point scalar function as

$$\Pi(p) = -2iy^2\int\frac{\mathrm{d}^4k}{(2\pi)^4}\left[\frac{f\left(\frac{|\vec{k}|}{\Lambda}\right)f\left(\frac{|\vec{k}+\vec{p}|}{\Lambda}\right)}{k^2-m_{\text{low}}^2+i\epsilon} + \frac{f\left(\frac{|\vec{k}|}{\Lambda}\right)f\left(\frac{|\vec{k}+\vec{p}|}{\Lambda}\right)}{(k+p)^2-m_{\text{low}}^2+i\epsilon} \times\left(1+\frac{4m_{\text{low}}^2-p^2}{k^2-m_{\text{low}}^2+i\epsilon}\right)\right].$$

A simple example of such a cut-off function is $f(|\vec{k}|/\Lambda) = (1+(\vec{k}^2/\Lambda^2))^{-1}$. Such a regulator yields a large Lorentz violation at low energies as can be seen by evaluating ξ from this $\Pi(p)$ after Euclidean continuation of k_0 to ik_4:

$$\begin{aligned}\xi = {} & 16y^2\int\frac{(\mathrm{d}^4k)_E}{(2\pi)^4}\left[\frac{f\left(\frac{|\vec{k}|}{\Lambda}\right)f\left(\frac{|\vec{k}|}{\Lambda}\right)}{(k^2+m_{\text{low}}^2)^3}\right]\left[1-\frac{4m_{\text{low}}^2}{k^2+m_{\text{low}}^2}\right] \times\left(k_4^2-\frac{1}{3}\vec{k}^2\right)\\
&-\frac{4y^2}{3}\int\frac{(\mathrm{d}^4k)_E}{(2\pi)^4} \times\left[\frac{f\left(\frac{|\vec{k}|}{\Lambda}\right)f'\left(\frac{|\vec{k}|}{\Lambda}\right)+\frac{|\vec{k}|}{2\Lambda}f\left(\frac{|\vec{k}|}{\Lambda}\right)f''\left(\frac{|\vec{k}|}{\Lambda}\right)}{|\vec{k}|\Lambda(k^2+m_{\text{low}}^2)}\right]\\
&-\frac{4y^2}{3}\int\frac{(\mathrm{d}^4k)_E}{(2\pi)^4} \times\left[\frac{f\left(\frac{|\vec{k}|}{\Lambda}\right)f'\left(\frac{|\vec{k}|}{\Lambda}\right)+\frac{|\vec{k}|}{2\Lambda}f\left(\frac{|\vec{k}|}{\Lambda}\right)f''\left(\frac{|\vec{k}|}{\Lambda}\right)}{|\vec{k}|\Lambda(k^2+m_{\text{low}}^2)} - \frac{2|\vec{k}|f\left(\frac{|\vec{k}|}{\Lambda}\right)f'\left(\frac{|\vec{k}|}{\Lambda}\right)}{\Lambda(k^2+m_{\text{low}}^2)^2}\right] \times\left(1-\frac{4m_{\text{low}}^2}{k^2+m_{\text{low}}^2}\right)\\
= {} & -\frac{y^2}{3\pi^2}\int_0^\infty \mathrm{d}x\,\left(f(x)f'(x)+xf(x)f''(x)\right) + O\left(\frac{m_{\text{low}}^2}{\Lambda^2}\right)\\
= {} & \frac{y^2}{3\pi^2}\int_0^\infty \mathrm{d}x\ x\left(f'(x)\right)^2 + O\left(\frac{m_{\text{low}}^2}{\Lambda^2}\right),\end{aligned} \quad (12)$$

where prime denotes derivative with respect to the argument. Note that the leading effect is independent of the Planck scale cut-off Λ ($\sim M_{\text{Pl}}$). That is, there is no $m_{\text{low}}^2/M_{\text{Pl}}^2$ factor in the leading term and we have only a coupling constant suppression. Hence, there is no decoupling of the Planck scale effects from light mass scale! This result implies a low-energy violation of Lorentz invariance of a size which is very large compared to the measured limits on such non-invariance.

It is important to emphasize that this non-decoupling behaviour again emerges from those parts of the radiative corrections in the two-point scalar correlation which, without the cut-off, are quadratically divergent.

The amount of non-invariance of Lorentz symmetry at low energies depends on the exact cut-off function $f(x)$. Clearly, if we replace $f(x) = 1$ in the above expression for ξ, we obtain, as expected, the Lorentz symmetric answer $\xi = 0$.

We emphasize again that in theories with only gauge fields and fermions and no elementary scalar fields, where there are no quadratic divergences, this order, one quantum gravity-induced Lorentz violation at the Planck scale would radiatively percolate down to low energies in a highly suppressed form, with only order $\left(m_{\text{low}}^2/M_{\text{Pl}}^2\right)$ effects.

Obviously, in theories with elementary scalar fields, supersymmetry again provides a protection mechanism against the above discussed large radiative transmission of Lorentz violation from Planck scale to low scales: there are no quadratic divergences in the scalar self-energy graphs in supersymmetric theories. In the supersymmetrized version of field theory example discussed above, exact supersymmetry will completely cancel out the quadratically divergent contributions in the two-point scalar correlator from graphs with boson fields going around the loops with those with fermion fields going around the loops. Consequently, Planck scale violations of Lorentz invariance will leave behind in ξ only a highly suppressed low-energy effect [15] of a size $O(m_{\text{low}}^2/M_{\text{Pl}}^2)$. But as supersymmetry is softly broken at low energies below a scale M_{SUSY} in Nature, the Bose–Fermi cancellation will not be exact, but will happen up to logarithmic effects: $\xi \sim y^2(M_{\text{SUSY}}^2/M_{\text{Pl}}^2)\ln(M_{\text{Pl}}^2/M_{\text{SUSY}}^2)$. In the Standard Model, radiative stability of the Higgs mass requires $M_{\text{SUSY}} \sim 10^3$ GeV. Though approximate supersymmetry provides a suppression, yet this discussion implies a profound result that quantum gravity effects predict a tiny violation of Lorentz invariance at low-energy scales given by: $\xi \sim y^2(M_{\text{SUSY}}^2/M_{\text{Pl}}^2) \sim (100)^{-1}\ (10^3/10^{19})^2 \sim 10^{-34}$. Non-zero value of ξ modifies the Lorentz symmetry respecting dispersion relation $\Pi_0(p) \equiv -p^2 + m^2c^2 = -(E^2/c^2) + \vec{p}^2 + m^2c^2 = 0$ by a change of velocity of light c by an amount given by $\Delta c/c = \xi/4 + O(\xi^2)$. The estimate of violation of Lorentz invariance here may be contrasted with the present day observational/experimental limits on this violation as represented by the varying velocity of light as: $\Delta c/c < 10^{-22}$. The violation of Lorentz invariance suggested above is significantly smaller, by some 12 orders magnitude, than this limit.

7. Conclusion

Quantum field theories with elementary scalar fields do not exhibit low–high energy decoupling behaviour: such theories are not natural. The various low mass parameters are not stable under quantum radiative corrections which tend to drag them to the highest mass scale.

The Naturalness Problem of the SM has proved to be an ideational fountain-head for a whole variety of new beyond Standard Model (BSM) ideas over last several decades. Supersymmetry is the most promising of these. Now is the time to confront these with experiments at LHC. Surely, experimental search for supersymmetry and related phenomenological developments are the present day frontier of high-energy physics.

Hopefully, experimental discovery of supersymmetry, though very likely not in the simplest version as represented by the cMSSM, but as in a more general MSSM framework, or even perhaps in a non-minimal form, may happen in the near future.

Besides the naturalness issues related to the masses in the Standard Model, there are other places where similar problems arise. For example, generic non-decoupling of the (possible) Planck scale violation of Lorentz invariance due to quantum gravity effects in theories with elementary scalar fields has the same origin. Supersymmetry again can ensure decoupling of this Planck scale violation from the low-energy physics. This implies a suppressed low-energy violation of Lorentz invariance as reflected by a variable velocity of light of a size $\xi \sim 4(\Delta c/c) \sim y^2(M_{\text{SUSY}}^2/M_{\text{Pl}}^2) \sim 10^{-34}$ for a supersymmetry breaking scale of $M_{\text{SUSY}} \sim 10^3$ GeV (which is required by the radiative stability of the 100 GeV scale of electroweak theory).

Acknowledgements

Comments by Gautam Bhattacharyya are gratefully acknowledged. This write-up is based on a talk at the conference on *Supersymmetry and Dark Matter*, Centre for High Energy Physics, Indian Institute of Science, Bangalore, Indian, October 3–5, 2013. The author would like to thank the organizers of this meeting for excellent hospitality at Bangalore. The author also acknowledges the support of Department of Science and Technology, Government of India, through a J C Bose National Fellowship.

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