Pramana – J. Phys. (2016) 87: 19
DOI 10.1007/s12043-016-1219-5

Angular momentum and the electromagnetic top

GIANFRANCO SPAVIERI[1,*] and GEORGE T GILLIES[2]

[1]Centro de Física Fundamental, Facultad de Ciencias, Universidad de Los Andes, Mérida, 5101-Venezuela
[2]Department of Physics, University of Virginia, Charlottesville, VA 22901-4714, USA
*Corresponding author. E-mail: spavieri@ula.ve

MS received 15 November 2014; revised 30 August 2015; accepted 1 October 2015; published online 6 July 2016

Abstract. The electric charge–magnetic dipole interaction is considered. If $\Gamma_{\rm em}$ is the electromagnetic and $\Gamma_{\rm mech}$ the mechanical angular momentum, the conservation law for the total angular momentum $\Gamma_{\rm tot}$ holds: $\Gamma_{\rm tot} = \Gamma_{\rm em} + \Gamma_{\rm mech} = {\rm const.}$, but when the dipole moment varies with time, $\Gamma_{\rm mech}$ is not conserved. We show that the non-conserved $\Gamma_{\rm mech}$ of such a macroscopic isolated system might be experimentally observable. With advanced technology, the strength of the interaction hints to the possibility of novel applications for gyroscopes, such as the electromagnetic top.

Keywords. Electromagnetic interaction; magnetic dipole; nonconservation of mechanical angular momentum; Faraday's law.

PACS Nos 03.30.+p; 03.50.De; 07.55.Db; 45.20.da; 45.20.df

1. Introduction

The violation of the action and reaction principle in classical electrodynamics and special relativity (SR) has been discussed in several contexts in the literature [1–3]. However, there is a widespread belief that it is confined to microscopic systems and has little practical consequence. Among many others in the literature, Page and Adams [2] have pointed out the existence of the violation of mechanical linear and angular momentum of interacting point-charged particles in motion, while Feynman [4] has provided an example of the nonconservation of mechanical angular momentum of an isolated system. Of course, as also pointed out by these authors, classical electrodynamics foresees that the total (mechanical+electromagnetic) linear and angular momentum of the isolated system is conserved.

We discuss here the nonconservation of mechanical angular momentum $\Gamma_{\rm mech}$ in the context of the electromagnetic (em) interaction between an electric charge q and a magnetic dipole $\mathbf{m}$. With regard to the behaviour of a magnetic dipole $\mathbf{m}$ that varies with time in the presence of an electric field, one must consider the 'paradox' pointed out by Shockley and James [5]. Its resolution requires an extra term to be added to the standard expression of the force on $\mathbf{m}$, in order to conserve the linear mechanical momentum of an isolated system. This problem of classical electrodynamics has particular relevance in quantum mechanics in the context of the nonlocality of Aharonov–Bohm [6] effects. Specifically, key to understanding the nonlocality of the Aharonov–Casher effect [6] is the modification of the standard force on $\mathbf{m}$, as shown by Aharonov *et al* [7] for a nonconducting magnetic dipole, and also by Spavieri [8] for both conducting and nonconducting magnetic dipoles.

Discussion of the angular momentum (torque) exchange between a time-dependent magnetic moment and a ring of charges appears in the literature and provides a comprehensive theoretical explanation of this issue. Nevertheless, no special attention has been paid to the related nonconservation of the mechanical angular momentum, $\Gamma_{\rm mech}$, of this isolated system. Hence, the purpose of our paper is to elaborate on the following two aspects. One is related to the possibility of actually testing the nonconservation of the mechanical angular momentum and the other hints to possible technological applications based on the conservation of $\Gamma_{\rm tot}$ when the mechanical angular momentum $\Gamma_{\rm mech} = 0$, but $\Gamma_{\rm em} \neq 0$. After a brief review of the related theory, we consider here the feasibility of testing the em forces and torques involved in the interaction between q and $\mathbf{m}$. We show that these tests are viable and, in particular, the acquired angular momentum is observable even for a macroscopic system. An experiment proposed for this purpose indicates that the strength of this self-driven rotation is such that it may lead to novel and interesting applications for gyroscopes.

2. The charge–magnetic dipole interaction

2.1 *Linear momentum*

Consider the system shown figure 1, where a ring of charges encircles a solenoid, consisting ideally of a linear array of magnetic dipoles. Let us consider the charge q of one element of the ring and its interaction with the highlighted magnetic dipole of moment $\mathbf{m}$. The time derivative of the em momentum is usually related to em force density as

$$-\frac{1}{4\pi c}\partial_t(\mathbf{E}\times\mathbf{B})^i + \partial_k\Theta^{ik} = \left(\rho\mathbf{E} + \frac{1}{c}\mathbf{j}\times\mathbf{B}\right)^i. \quad (1)$$

However, when the magnetic moment of the dipole is switched off, there is a force $\mathbf{f}_q = q\mathbf{E} = -c^{-1}q\partial_\mathbf{t}\mathbf{A}$ acting on the charge q, while an equal and opposite force $\mathbf{f}_m = c^{-1}\dot{\mathbf{m}}\times\mathbf{E}_q = -\mathbf{f}_q$ acts on the dipole [5,7,8]. By taking into account the term $\mathbf{f}_m$ in the expression of the force on $\mathbf{m}$, there is no net force on the charge–magnetic dipole system, assuring that its mechanical linear momentum is conserved. In the literature, the extra force $\mathbf{f}_m = c^{-1}\dot{\mathbf{m}}\times\mathbf{E}_q$, to be added to the standard expression of the force on the magnetic dipole, has been shown to be due to the time variation of the so-called hidden momentum $\mathbf{P}_h = c^{-1}\mathbf{m}\times\mathbf{E}_q$ [5,7–9]. The hidden momentum arises when the em tensor $\Theta^{\mu\upsilon}$ in (1) is complemented with the stress tensor $S^{\mu\upsilon}$ to provide the total tensor $T^{\mu\upsilon} = \Theta^{\mu\upsilon} + S^{\mu\upsilon}$, which, together with the continuity equation $\partial_\upsilon T^{\mu\upsilon} = 0$, is suitable for describing systems with internal structure such as this [5,7,8]. $\mathbf{f}_m$ appears only when fields are time-varying, e.g., when the time-varying $\mathbf{m}$ is in the presence of the external electric field $\mathbf{E}_q$. Here, we have adopted Maxwell–Lorentz's theory, according to which the hidden momentum has been transferred to the magnet. We mention, as pointed out by Mansuripur [9], that supporters of the Einstein–Laub viewpoint do not need to invoke the hidden momentum and may explain the experimental results on the basis of the force/torque exerted on the magnet while it is being switched off. The end results of the two interpretations are the same.

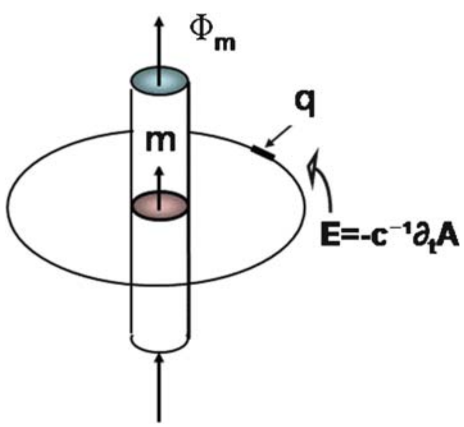

Figure 1. In the interaction between the charge element q of the ring of radius r and a time-varying magnetic dipole $\mathbf{m}$ of the solenoid, a force $\mathbf{f}_q = q\mathbf{E} = -c^{-1}q\partial_\mathbf{t}\mathbf{A}$ acts on q and an equal and opposite force $\mathbf{f}_m = c^{-1}\dot{\mathbf{m}}\times\mathbf{E}_q = -\mathbf{f}_q$ acts on $\mathbf{m}$. Due to the action of the local field $\mathbf{E}$, a torque $\mathbf{r}\times q\mathbf{E} = c^{-1}Q\,\mathbf{r}\times(-\partial_\mathbf{t}\mathbf{A})$ acts on the charged ring but no countertorque acts on $\mathbf{m}$.

The two forces $\mathbf{f}_m$ and $\mathbf{f}_q$ involved in the charge–magnetic dipole system, are equal and opposite but not collinear. Therefore, there is a net couple (or torque) acting on the system as $\mathbf{m}$ varies with time. A test of $\mathbf{f}_m$ is possible with the present technology but its description is left to a future contribution. The equation of motion of the magnetic dipole in the presence of the external fields $\mathbf{E}$ and $\mathbf{B}$ has been derived in refs [7,8] assuming the existence of $\mathbf{P}_h$ and considering that the moving $\mathbf{m}$ possesses an electric dipole moment $\mathbf{d} = c^{-1}\mathbf{v}\times\mathbf{m}$. A simpler approach consists of introducing the Lagrangian

$$L = T - U = \frac{1}{2}mv^2 + \mathbf{v}\cdot\mathbf{P}_h + \mathbf{m}\cdot\mathbf{B}, \quad (2)$$

where in (2), the potential energy $\mathbf{v}\cdot\mathbf{P}_h = c^{-1}\mathbf{v}\cdot\mathbf{m}\times\mathbf{E}$. From this Lagrangian it is straightforward to obtain the equation of motion

$$-\frac{1}{c}\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{m}\times\mathbf{E}) + \frac{1}{c}\nabla(\mathbf{v}\cdot(\mathbf{m}\times\mathbf{E})) + \nabla(\mathbf{m}\cdot\mathbf{B}) = \frac{\mathrm{d}}{\mathrm{d}t}(m\mathbf{v}) \quad (3)$$

which, by means of vector identities and the use of Maxwell's equations, yields

$$-\frac{1}{c}\dot{\mathbf{m}}\times\mathbf{E} + \frac{1}{c}\mathbf{v}\times(\mathbf{m}\cdot\nabla)\mathbf{E}] + (\mathbf{m}\cdot\nabla)\mathbf{B} = \frac{\mathrm{d}}{\mathrm{d}t}(m\mathbf{v}). \quad (4)$$

Expression (4) coincides with the force expression derived [7] in the special case of $\mathbf{m}$ moving with velocity $\mathbf{v}$ in the presence of the electric field $\mathbf{E}$.

2.2 *Angular momentum*

The time derivative of the em angular momentum density is obtained by multiplying vectorially by $\mathbf{x}$ expression (1). If the fields vanish for $r \to \infty$ as $1/r^2$ or faster, as in our case, the terms related to the divergence $\partial_k \Theta^{ik}$ do not contribute to the em angular momentum Γ_{em}. Thus, apart from a constant term, integrating over volume and time, eq. (1) yields the conservation law

$$\Gamma_{\text{em}} = \frac{1}{4\pi c}\int \mathbf{r}\times(\mathbf{E}\times\mathbf{B})\mathrm{d}V = -\frac{1}{c}\int \rho\, \mathbf{r}\times(-\partial_{\mathbf{t}}\mathbf{A})\mathrm{d}V\mathrm{d}t = -\Gamma_{\text{mech}}, \quad (5)$$

i.e.,

$$\Gamma_{\text{em}} + \Gamma_{\text{mech}} = \text{const.} \quad (6)$$

Expression (5) is in agreement with the result [10] that, for a point charge q, the em momentum is $c^{-1}q\mathbf{A} = c^{-1}q\mathbf{m}\times\mathbf{r}/r^3 = -c^{-1}\mathbf{m}\times\mathbf{E}_q$, where $\mathbf{A}$ is the vector potential and $c^{-1}\mathbf{m}\times\mathbf{E}_q$ is the momentum due to internal stresses [5,7,8].

Let us consider Faraday's law for the solenoid of figure 1. When $\mathbf{m}$ varies with time, the induction field $\mathbf{E} = -c^{-1}\partial_{\mathbf{t}}\mathbf{A}$ yields an observable em$f = -c^{-1}\oint_C \partial_t\mathbf{A}\cdot\mathrm{d}\mathbf{l} = -(\mathrm{d}/\mathrm{d}t)c^{-1}\int \mathbf{B}\cdot\mathrm{d}\mathbf{a}$, where $\int \mathbf{B}\cdot\mathrm{d}\mathbf{a} = \Phi_m$ is the linked magnetic flux of the solenoid. Due to the action of the local field $\mathbf{E}$, a torque $\mathbf{r}\times q\mathbf{E} = c^{-1}Q\,\mathbf{r}\times(-\partial_{\mathbf{t}}\mathbf{A})$ acts on the charged ring but, as considered below, no counter torque acts on the dipoles $\mathbf{m}$ forming the solenoid. As an integral law, Faraday's has been tested in the common case where Γ_{mech} is conserved and what is being measured is the current induced in an external neutral coil, on which there is no net torque. However, if the external coil is replaced by the charged ring of figure 1, there is a net torque $\tau = \mathbf{r}\times\mathbf{f} = c^{-1}Q\,\mathbf{r}\times(-\partial_{\mathbf{t}}\mathbf{A}) = -(\mathrm{d}/\mathrm{d}t)\mathbf{\Gamma}_{\text{em}}$ that provides the angular impulse $\Gamma_{\text{mech}} = \int |\mathbf{r}\times\mathbf{f}|\,\mathrm{d}t = c^{-1}QrA = c^{-1}Qm/r$, where Q is the total charge of the ring. As the electrostatic field $\mathbf{E}_q$ of this ring of charges is zero at the location of the dipole on the plane of the ring, there are no internal stresses or induced charges on the dipole, i.e., no hidden angular momentum. Moreover, the torque on a magnetic dipole is $\mathbf{m}\times\mathbf{B}$ and, as the magnetic field of the ring of charges is $\mathbf{B} = 0$, the resulting torque on the whole solenoid is zero by even considering all the other dipoles that compose it. Thus, the conservation law (6) implies that the isolated system achieves a net mechanical angular momentum at the expense of the variation of the em angular momentum! Of course, when the dipole current is reversed, an opposite torque is acting on it and no net angular impulse is acquired over a full cycle. In the next section, we describe an experiment that can yield evidence of the nonconservation of mechanical angular momentum as a direct consequence of the field $\mathbf{F}/q = c^{-1}(-\partial_{\mathbf{t}}\mathbf{A})$, thus testing in this special condition the locality of Faraday's law.

3. Nonconservation of mechanical angular momentum

3.1 *An experiment for testing the nonconservation of* Γ_{mech}

For a realistic test of Γ_{mech}, an arrangement similar to that of figure 1 appears convenient. Consider then a charged cylindrical capacitor, divided into vertical sectors, of inner radius a, outer radius b, length L, linear charge density λ and with an applied potential difference V. The capacitor is suspended vertically by a torsion

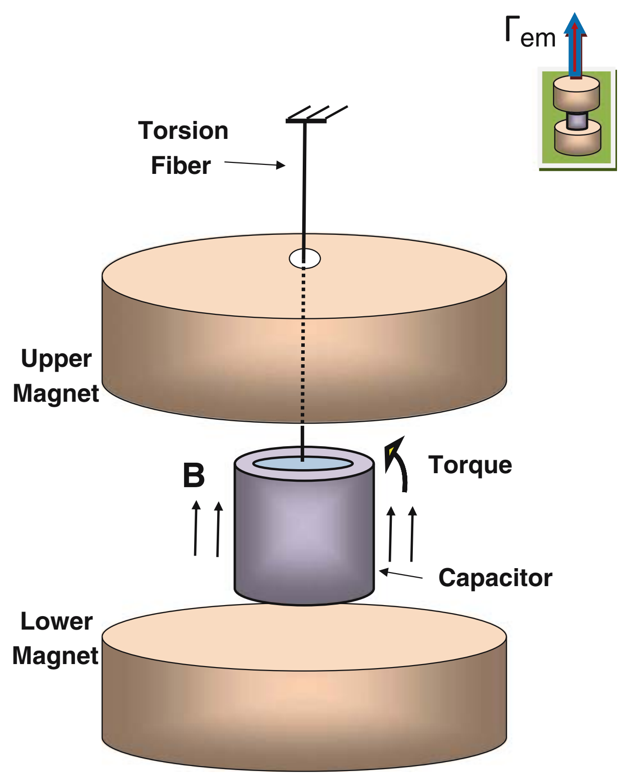

Figure 2. A charged capacitor is suspended on a torsion fibre and is placed within the uniform magnetic field $\mathbf{B}$ produced by two magnets. An electromagnetic angular momentum Γ_{em} is stored in the system. When the field $\mathbf{B}$ is switched off, the induction field $\mathbf{E} = -c^{-1}\partial_{\mathbf{t}}\mathbf{A}$ acts on the capacitor charges producing an observable rotation of the capacitor. No countertorque acts on the magnets. On the top of the figure, to the right, the em top is represented, storing a constant em angular momentum Γ_{em}.

fibre along its symmetry axis and placed, ideally, in a uniform magnetic field **B**, which is also directed vertically. The field **B** may be produced by a solenoid or electromagnets placed directly above and below the capacitor, as suggested schematically in figure 2. There are fringe effects at the ends of the cylindrical capacitor where the electric field due to the positive and negative charges leaks outside and induces polarization charges on the magnet. However, the field intensity decreases rapidly and its overall effect on the magnet is negligible (if we wish to eliminate fringe effects completely, we may use instead a spherical capacitor where the electric field is completely confined). Thus, to a first approximation, we may disregard fringe effects and assume that the electric field $E_q = \lambda/2r$ is radial and confined inside the capacitor.

In the expression $(1/4\pi c)(\mathbf{E}_q \times \mathbf{B})$ of the interaction em momentum density, $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic induction field that, when varying in time, yields the radiation field $\mathbf{E}_m = -c^{-1}\partial_t \mathbf{A}$ acting on the capacitor charges. $\mathbf{E}_q$ is the constant electrostatic field inside the charged capacitor. Therefore, the term $(1/4\pi c)(\mathbf{E}_q \times \mathbf{B})$ is not associated with a pure radiation momentum density like, for example, that of the usual em waves (light or photons). Conversion of em energy into mechanical energy occurs in ordinary em devices and electric motors. However, in these devices and motors there is no static electric field ($\mathbf{E}_q = 0$ and $\Gamma_{\mathrm{em}} = 0$) and, for the magnetic interaction forces between current elements, action and reaction holds between the stator and the rotor: mechanical angular momentum is conserved. In the present experiment, $\mathbf{E}_q$ and $(1/4\pi c)(\mathbf{E}_q \times \mathbf{B})$ do not vanish. As $\mathbf{E}_q$ is an electrostatic field, the strengths of $\mathbf{E}_q$ and **B** are much greater that that of the radiation fields. For this reason, the em interaction leading to nonconservation of Γ_{mech} can be observed experimentally in principle, as shown below.

Using the expression for the torque $\tau = c^{-1} q r \partial_t A_\varphi$ that acts on the charges of one of the capacitor surfaces when **B** varies with time, application of Stokes' theorem leads to the following value of the angular impulse per unit of length:

$$\int \tau \, \mathrm{d}t = \frac{\lambda}{2\pi c} \oint \mathbf{A} \cdot \mathrm{d}l = \frac{r\lambda B S}{c} = \frac{\lambda B r^2}{2c}, \tag{7}$$

where BS is the linked magnetic flux. As the charge on the inner and outer surfaces is the same, the total angular impulse on the capacitor is

$$\Gamma_{\mathrm{mech}} = \int \tau_b \mathrm{d}t - \int \tau_a \mathrm{d}t = QB(b^2 - a^2)/2c. \tag{8}$$

The total charge is given by $Q = VC \simeq 2\pi^{-1} LV / \ln(b/a)$, where C is the capacitance. When the current of the magnet is slowly switched off, the impulse received by the capacitor of mass M generates an angular velocity ω such that $\Gamma_{\mathrm{mech}} = I\omega$, where $I \simeq Mb^2$ is the moment of inertia. If k is the torsion constant of the suspension fibre and θ the maximum angular displacement, then from conservation of energy, $(1/2)I\omega^2 = (1/2)k\theta^2$, we obtain

$$\theta = \frac{LVB}{\pi c \sqrt{kM}} \frac{b}{\ln(b/a)} \left(1 - \frac{a^2}{b^2}\right). \tag{9}$$

For illustrative purposes, consider a capacitor with $L = 0.5$ m, $a = 0.25$ m, $b = 1.05a$, a suspension fibre with torsion constant $k = 10^{-7}$ Nm/rad and, for the other experimental parameters, let $M = 0.1$ kg and $B = 1$ T (without the need for superconducting technology). For a detection system with an optical lever that can resolve $\theta \gtrsim 10^{-4}$ rad, the corresponding threshold voltage would be $V \gtrsim 1 \times 10^2$ V. The value of θ may be enhanced by a factor of 10 or more by using resonance techniques, where **B** is made to vary periodically at the resonant frequency of oscillation. In this case, the effects of damping must be included in the model (especially near the resonance frequency) in order to achieve more realistic results. Given that V can be increased (perhaps by up to a factor of 6, without electrical breakdown concerns), an experimental test appears viable. Open questions regarding the design of such an experiment would include, among others, the nature and size of competing effects (e.g., rotations induced via vibrational couplings), interference from transient and static background fields, possible need of an evacuated housing, and associated grounding and shielding requirements.

3.2 *Possible applications of the nonconservation of* Γ_{mech} *and conservation of* Γ_{tot}

If the experiment proves the nonconservation of Γ_{mech}, it also proves the validity of the theory, the reality of Γ_{em} and the conservation law for the total angular momentum $\Gamma_{\mathrm{tot}} = \Gamma_{\mathrm{em}} + \Gamma_{\mathrm{mech}} = \mathrm{const}$. In figure 2, $\Gamma_{\mathrm{mech}} = 0$ and $\Gamma_{\mathrm{tot}} = \Gamma_{\mathrm{em}}$. Thus, it is interesting to consider what value of Γ_{em} might be achievable by means of special techniques capable of increasing the charge stored in the capacitor. For example, if one inserts a material with a very high relative dielectric constant ($\epsilon \simeq 10^3$–10^4) between the cylindrical plates of the capacitor, with the same applied voltage V, the primary charge Q would be increased by the factor ϵ, but a secondary charge would be stored in the dielectric. However, if the source of V is disconnected while keeping the capacitor charged and the dielectric with its secondary charge is removed, only the increased primary charge Q is kept in the capacitor. Moreover, with appropriate materials and experimental conditions, in principle the potential could be raised to $V = 10^7$ V and, in a very high B-field environment, e.g., $B \sim 6$ T, the resulting em angular momentum might be as high as $\Gamma_{\mathrm{em}} \simeq 1$ N m s.

If it were to turn out that a system storing such a Γ_{em} is technologically feasible, it would likely display uncommon features. A detailed experimental plan, possibilities and consequences of applications for such a system will be discussed in a future contribution. Speculatively, we only mention here an interesting feature of such a system possessing a stored Γ_{em}: if tilted, a reaction torque appears in order to conserve the total angular momentum $\Gamma_{\mathrm{em}} + \Gamma_{\mathrm{mech}}$. Thus, the capacitor–magnet system would behave like a common mechanical spinning top. Hence, instead of a mechanical spinning top, in principle one would realize an electromagnetic top, as sketched in figure 2. Obviously, the basic physical principle of the electromagnetic top is the same as that of the mechanical one, and of course this principle forms the basis of several rotational instrument applications related to gyroscopes, such as pointing devices, gyrocompasses, antiroll devices/stabilizers, artificial horizons and autopilots for aircraft, robotics and other technologies that employ precision angle and rate sensors. An advantage over such usual devices is that an electromagnetic top is a purely electromagnetic system and would be far less mechanically complex.

4. Conclusions

We have considered the charge–magnetic dipole system and described the interaction force $\mathbf{f}_m$ acting on the dipole $\mathbf{m}$ when the fields vary with time and the force $\mathbf{f}_q$ acting on the charge q. A test of $\mathbf{f}_m$ and $\mathbf{f}_q$ is possible in principle and in such a test, the action and reaction principle and conservation of the linear em and mechanical momentum hold. The situation regarding the em angular momentum Γ_{em} is more complex. We indicate here the possible logical outcomes for an experimental arrangement of the type described in §3, and note therein the predictions of classical electrodynamics. The relevant cases are:

(i) No torque on the capacitor and on the magnets: In this case, Γ_{mech} of the isolated system is conserved. However, this is contrary to the prediction of the theory.
(ii) Opposite torques on the capacitor and on the magnets: The torque on the capacitor is predicted by the theory, but that on the magnets is not. Possibly, this unlikely result could imply that the standard expression $\mathbf{m} \times \mathbf{B}$ of the torque on $\mathbf{m}$ has to be modified. Therefore, an experiment of this type is potentially an indirect test of the standard torque expression $\mathbf{m} \times \mathbf{B}$.
(iii) Torque on the capacitor but not on the magnets: As foreseen by classical electrodynamics, the experiment would confirm the locality of the force $\mathbf{f}_q = -c^{-1} q\, \partial_t \mathbf{A}$, implying that the mechanical angular momentum, Γ_{mech}, of this isolated system is NOT conserved, while the sum $\mathbf{\Gamma}_{\mathrm{tot}} = \mathbf{\Gamma}_{\mathrm{em}} + \mathbf{\Gamma}_{\mathrm{mech}}$ is conserved. Thus, an application such that of the electromagnetic top would also be viable.

In conclusion, the interaction forces of our system should be experimentally observable. In particular, the arrangement of figure 2 can test the nonconservation of Γ_{mech} for this isolated system. The conservation law $\mathbf{\Gamma}_{\mathrm{tot}} = \mathbf{\Gamma}_{\mathrm{em}} + \mathbf{\Gamma}_{\mathrm{mech}}$ may lead to novel em devices and applications, such as the electromagnetic top.

Although foreseen by classical electrodynamics, the notion that a macroscopic isolated system could acquire a self-driven angular impulse certainly represents a remarkable dynamic feature that has never been tested and may have appealing uses. We believe that fundamental features of electrodynamics with such potential should be subject to dedicated experimental research and testing.

Acknowledgements

This work was supported in part by the CDCHT, ULA, Mérida, Venezuela.

References

[1] L Zangwill, *Modern electrodynamics*, Section 15.5 (Cambridge University Press, 2013)
[2] L Page and N I Adams, *Am. J. Phys.* **13**, 141 (1945)
[3] G Cavalleri, G Spavieri and G Spinelli, *Nuovo Cimento B* **102**, 495 (1988)

[4] R P Feynman, R B Leighton and M Sands, *The Feynman lectures on physics* (Addison-Wesley, New York, 1964) Vol. 2
[5] W Shockley and R R James, *Phys. Rev. Lett.* **18**, 876 (1967)
[6] Y Aharonov and D Bohm, *Phys. Rev.* **115**, 485 (1959)
Y Aharonov and A Casher, *Phys. Rev. Lett.* **53**, 319 (1984)
[7] Y Aharonov, P Pearle and L Vaidman, *Phys. Rev. A* **37**, 4052 (1988)
[8] G Spavieri, *Nuovo Cimento B* **109**, 45 (1994)
G Spavieri, *Found. Phys. Lett.* **3**, 291 (1990)
[9] M Mansuripur, *Optical trapping and optical micromanipulation X* edited by K Dholakia and G C Spalding, *Proc. SPIE* **8810**, 88100K-1:18 (2013)
[10] T H Boyer, *Phys. Rev. D* **8**, 1667 (1973)
X Zhu and W C Henneberger, *J. Phys. A* **23**, 3983 (1990)
G Spavieri, *Phys. Rev. Lett.* **82**, 3932 (1999)
G Spavieri, *Phys. Rev. A* **59**, 3194 (1999)
G Spavieri, *Eur. Phys. J. D* **37**, 327 (2006)