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Signature splitting in two quasiparticle rotational bands of ^{180,182}Ta

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Abstract. The signature splittings in $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$, $K^\pi = 0^-$: $9/2[514]_\pi \otimes 9/2[624]_\nu$ bands of ^{180}Ta and $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$, $K^\pi = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$, $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ bands of ^{182}Ta are analysed within the framework of two-quasiparticle rotor model. The phase as well as magnitude of the experimentally observed signature splitting in $K^\pi = 1^+$ band of ^{180}Ta, which could not be explained in earlier calculations, is successfully reproduced. The conflict regarding placement of a 12^+ level in $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ ground-state rotational band of ^{180}Ta is resolved and tentative nature of $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$, $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ bands observed in ^{182}Ta is confirmed. As a future prediction for experimentalists, these two-quasiparticle structures observed in ^{180}Ta and ^{182}Ta are extended to higher spins.

Keywords. Signature splitting; odd–odd nuclei; rotational bands; particle rotor model.

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1. Introduction

The study of nuclear structure of the odd–odd ^{180}Ta isotope and understanding the nucleosynthesis process are of contemporary importance [1]. The most prominent aspect of this nuclide is the concern with $K^\pi = 9^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ isomeric state lying at 78.0 keV energy [2,3]. This isomeric state lives more than 1.2×10^{15} yr [4,5] which is much large as compared to the $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ ground state whose half-life is only 8.18 h [6,7]. During the last few decades the structures of tantalum nuclides, namely ^{180}Ta and ^{182}Ta, have been explored by various experimental studies [8–15].

Among all the $2qp$ rotational bands observed in both the ^{180}Ta and ^{182}Ta isotopes, there are only two rotational bands, namely $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ and $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ in ^{180}Ta and two bands namely $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$, $K^\pi = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$ in ^{182}Ta, which show pronounced signature splitting and hence these four $2qp$ bands could be effectively examined using the two-quasiparticle rotor model (TQPRM) approach. The main motivations behind the present study are:

(i) The resolution of ambiguity regarding the placement of 12^+ level observed by Saitoh *et al* [8] and Dracoulis *et al* [12,13], in the case of $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ band observed in ^{180}Ta.
(ii) Theoretical explanation of signature splitting observed in $2qp$ bands namely, $K^\pi = 0^-$, $K^\pi = 1^+$ of ^{180}Ta and $K^\pi = 0^-$, $K^\pi = 1^-$ of ^{182}Ta.
(iii) Estimation of Newby shift energies for the bands $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$, $K^\pi = 0^+$: $7/2^+[404]_\pi \otimes 7/2^+[633]_\nu$ of ^{180}Ta and $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$, $K^\pi = 0^-$: $3/2[411]_\pi \otimes 3/2[512]_\nu$, $K^\pi = 0^+$: $7/2[404]_\pi \otimes 7/2[633]_\nu$ of ^{182}Ta, which actively take part in Coriolis mixing.
(iv) To confirm spin, parity and configuration assignment of $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$ and $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ bands observed in ^{182}Ta.

In §2, we elaborate the basic formulation of the theoretical model used in the present calculations.

In §3 and 4, results and discussions along with inferences drawn from the present study are presented.

2. The model and methodology

In order to reproduce the experimentally observed signature splitting in $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$, $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ rotational bands of ^{180}Ta and $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$ and $K^\pi = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$ bands of ^{182}Ta, we used the TQPRM [16,17] approach. As formulation of the TQPRM is well known, we present here only its important formulas. The total Hamiltonian in the framework of the TQPRM can be written as

$$H_{\text{tot}} = H_{\text{in}} + H_{\text{rot}}, \tag{1}$$

where

$$H_{\text{in}} = H_{\text{av}} + H_{\text{pair}} + H_{\text{vib}} + V_{\text{np}}, \tag{2}$$

$$H_{\text{rot}} = H^0_{\text{rot}} + H_{\text{cor}} + H_{\text{ppc}} + V_{\text{irrot}}. \tag{3}$$

The intrinsic part (H_{in}) of the total Hamiltonian (H_{tot}) is constructed by the axially symmetric average field (H_{av}) plus various Hamiltonian operators corresponding to pairing (H_{pair}), vibrational (H_{vib}) and neutron–proton (H_{np}) interactions. Similarly, the rotational part (H_{rot}) of the total Hamiltonian is composed of various terms such as pure rotation (H^0_{rot}), Coriolis coupling (H_{cor}), particle–particle coupling (H_{ppc}) and irrotational component (H_{irrot}). The various components appearing in the total Hamiltonian are given as

$$H^0_{\text{rot}} = \frac{\hbar^2}{2\Im}\left(I^2 - I_z^2\right), \tag{4}$$

$$H_{\text{cor}} = -\frac{\hbar^2}{2\Im}\left(I_+ j_- + I_- j_+\right), \tag{5}$$

$$H_{\text{ppc}} = \frac{\hbar^2}{2\Im}\left(j_p^+ j_n^- + j_p^- j_n^+\right), \tag{6}$$

$$H_{\text{irrot}} = \frac{\hbar^2}{2\Im}\left[\left(j_p^2 - j_{pz}^2\right) + \left(j_n^2 - j_{nz}^2\right)\right]. \tag{7}$$

In this formulation, the vibrational component (H_{vib}) of the total Hamiltonian (H_{tot}) is neglected. The basis states used to solve the total Hamiltonian (H_{tot}) can be written in terms of the symmetrized Wigner function and intrinsic wave function as [16,17]

$$|IMK\alpha\rangle = \sqrt{\left[\frac{2I+1}{16\pi^2\,(1+\delta_{K0})}\right]} \times \left[D^I_{MK}\,|K\alpha\rangle + (-1)^{I+K} D^I_{M-K} R_i\,|K\alpha\rangle\right]. \tag{8}$$

The index $|K\alpha\rangle$ characterizes the configuration of odd neutron and odd proton. The symbols D^I_{MK} and R_i are Wigner functions and rotation operator respectively. In odd–odd deformed nuclei, projection of odd proton (Ω_{p}) and projection of odd neutron (Ω_{n}) can couple either in parallel (triplet) or in antiparallel (singlet) fashion. This neutron–proton coupling in odd–odd nuclei gives rise to the Gallagher–Moszkowski doublet $K^\pm = \left|\Omega_{\text{p}} \pm \Omega_{\text{n}}\right|$, with triplet state lying lower in energy as compared to singlet state [18] and reverse is the situation in even–even nuclei [19]. This energy splitting between triplet and singlet states may be attributed to the neutron–proton residual interaction (V_{np}) in $K \neq 0$ rotational bands. In case of $K = 0$ rotational bands, the residual interaction results in additional relative displacement of odd spin members of a given band with respect to even spin members of the same band. This shifting in $K = 0$ bands caused by residual interaction among odd neutron and odd proton is known as Newby shift [20].

In the present study, H_{av} is calculated using Nilsson model [21]. As the Newby shift of $K = 0$ band can be transferred to $K \neq 0$ bands through $\Delta K = 1$ Coriolis coupling, in order to perform complete Coriolis mixing calculations one has to consider mixing of various interacting $2qp$ rotational bands. Each $2qp$ band taking part in these calculations is characterized by three parameters: band-head energy ($E_{\alpha\pm}$), inertia parameter ($\hbar^2/2\Im$) and Newby shift energy (E_{N}). However, we did not allow all these parameters to vary quite freely because these variations are bound by certain physical limits. $2qp$ Coriolis mixing calculations require many $2qp$ interacting bands, which constitute a basis set for the given TQPRM calculations. In order to estimate the band-head energies of these interacting bands taking part in the present Coriolis mixing calculations, we used formulation by Hoff [22]. The values of inertia parameter ($\hbar^2/2\Im$) have been approximated from the experimental data of neighbouring nuclei [23] and for the bands where experimental data are not available, we have taken these values as 10.5 keV and 10.0 keV for K^+ and K^- bands, respectively. The values of the Newby shifts for the $K = 0$ bands have been adopted from experimental data compiled by Jain *et al* [23] and for the bands where Newby shifts are not available, values are adjusted within defined limits. The matrix elements ($j_p^\pm$, $j_n^\pm$) are calculated using Nilsson model [21] and attenuated within the defined limits [24]. In order to obtain the optimized set of parameters, we used the functional minimization subroutine (MINUIT) [25].

3. Results and discussion

3.1 *The $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ ground state band of ^{180}Ta*

Dracoulis *et al* [12,13] observed $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ rotational band up to spin 12^+ with tentative placement of this 12^+ level at $E = 1774.0$ keV and Saitoh *et al* [8] extended this band up to $I = 16^+$ with 12^+ level at 1629.6 keV, which leads to an ambiguity in the above-mentioned level placement. In order to resolve this ambiguity regarding 12^+ level placement by Saitoh *et al* [8] and Dracoulis *et al* [12,13], we performed the TQPRM calculations for $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]$ rotational band. The basis set involved in the present TQPRM calculations consists of all rotational bands appearing from $g_{7/2} \otimes i_{13/2}$ orbital couplings. The single proton and neutron wavefunctions of ^{180}Ta are calculated using Nilsson model with modified harmonic oscillator potential [21]. The optimized potential parameters, $k = 0.0620$ and $\mu = 0.614$ for protons and $k = 0.0636$ and $\mu = 0.393$ for neutrons, used in the present calculations are taken from Jain *et al* [26]. The equilibrium deformation parameters $\varepsilon_2 = 0.242$ and $\varepsilon_4 = 0.087$ are adopted from Moller *et al* [27]. In figure 1, we present a comparison among the experimental excitation energies reported by Saitoh *et al* [8], Dracoulis *et al* [12,13] and the present TQPRM calculations.

From this figure, it is clear that there is an excellent agreement between the present TQPRM calculated

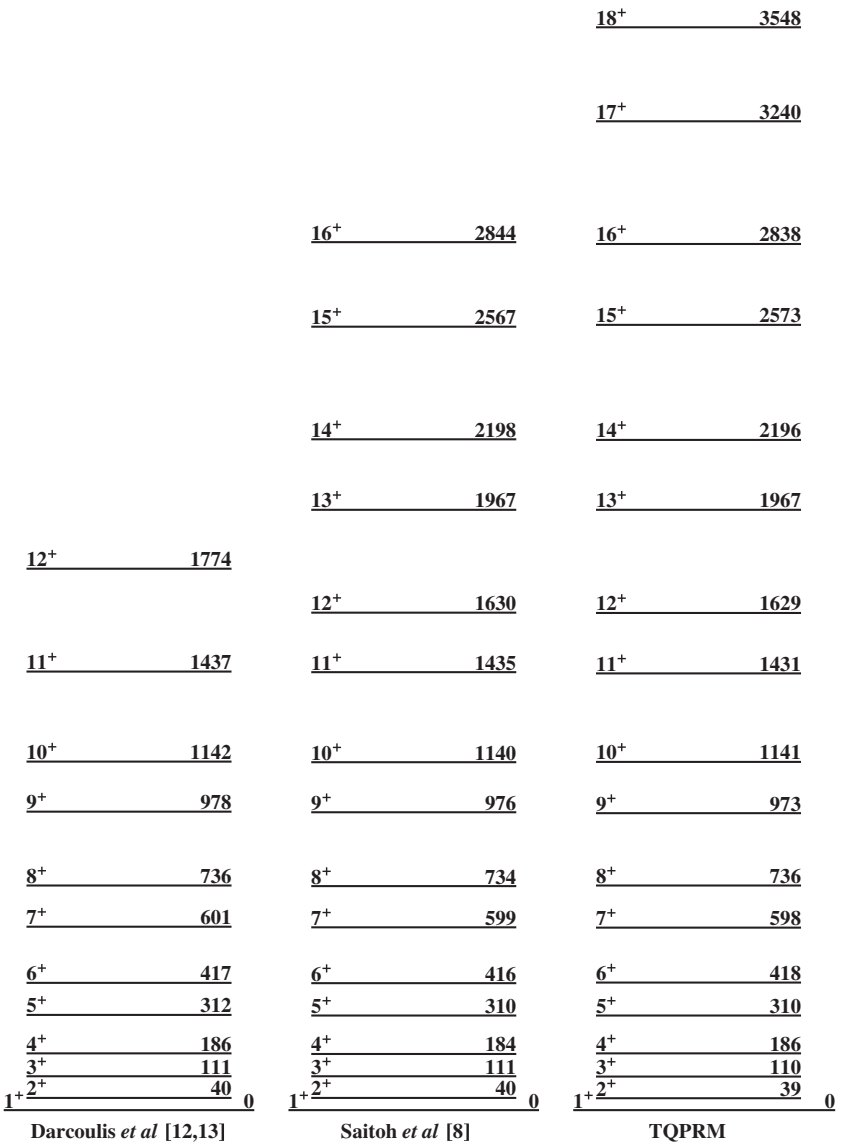

Figure 1. The comparison of the calculated excitation energies and experimental data for $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]$ rotational band.

excitation energies and the corresponding experimental values reported by Saitoh *et al* [8] and Dracoulis *et al* [12,13] except for 12^+ level which is placed at 1774 keV by Dracoulis *et al* and at 1630 keV by Saitoh *et al*. The 12^+ level placement at 1630 keV by Saitoh *et al* [8] is consistent with our TQPRM calculations. Thus, our TQPRM calculations further strengthen the interpretation of Saitoh *et al* [8] according to which, the energy levels corresponding to the spins 13^+ and 11^+ are connected by a cascade of 338 keV, 195 keV and a cross-over 533 keV $E2$ transitions. Therefore, the placement of 12^+ level at 1774 keV by Dracoulis *et al* [12,13] is not correct.

In addition to this ambiguity of 12^+ level placement, the $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ band under discussion exhibits pronounced signature splitting, the magnitude of which increases with spin and then starts decreasing beyond spin 12^+ as shown in figure 2a. Because of the unknown nature of Newby shift for $K^\pi = 0^+$: $7/2^+[404]_\pi \otimes 7/2^+[633]_\nu$ band, which plays a significant role in explaining the observed signature splitting of $K^\pi = 1^+$ band under discussion, Saitoh *et al* [8] could not explain the observed signature effects in this band. Although Goel *et al* [28] tried to explain this signature splitting, but they could not reproduce the initial phase as well as magnitude of this splitting at higher spins.

Moreover, these calculations by Goel *et al* [28] are completely silent about the above-mentioned ambiguity of 12^+ level placement, which we successfully resolved in the present calculations. As the magnitude of signature splitting depends on various factors such as Nilsson orbitals occupied by the unpaired nucleons, matrix elements connecting $K = 0$ and $K = 1$ bands, magnitude of the Newby shift and energy difference among interacting bands, we exercised a special care in constructing the basis set and also in the variation of matrix elements ($j_{\rm p}^{\pm}$, $j_{\rm n}^{\pm}$) during functional minimization [25]. In figure 3, we present the results of the TQPRM calculations along with experimental data of Dracoulis *et al* [12,13] and Saitoh *et al* [8].

From this figure, it is clear that

(i) the present TQPRM calculations successfully reproduced the experimentally observed signature splitting and these results are also consistent with experimental observations by Saitoh *et al* [8],
(ii) the phase as well as magnitude of the observed signature splitting at lower and higher spins are successfully reproduced in the present study, which could not be explained in the earlier calculations by Goel *et al* [28].

The present Coriolis mixing calculations reveal that, out of all the bands defined in the given basis set ($g_{7/2} \otimes i_{13/2}$), there are mainly two bands, namely, $K^\pi = 0^+$: $7/2[404]_\pi \otimes 7/2[633]_\nu$ and $K^\pi = 2^+$: $7/2[404]_\pi \otimes 11/2[615]_\nu$, which have substantial contribution in explaining experimentally observed signature splitting in the ground-state $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$, rotational band. This further suggests that the first-order Coriolis mixing plays a major role in explaining the observed signature splitting in $K^\pi = 1^+$ band under discussion. In table 1, we present the admixture of wave functions for $K^\pi = 1^+$ rotational band and from this table it is clear that, at higher spins ($I > 13\hbar$), there is appreciable mixing among $K^\pi = 0^+$: $A \otimes B$, $K^\pi = 1^+$: $A \otimes C$, $K^\pi = 2^+$: $A \otimes D$ bands, which might be the possible reason for decrease in signature splitting at higher spins. The optimized set of parameters, bandhead energies, inertia parameters and Newby shift energy obtained in the present calculations are listed in table 2.

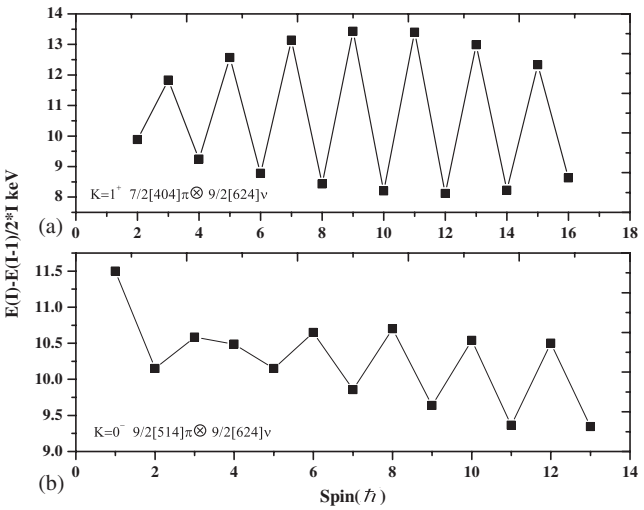

Figure 2. The experimental signature splitting observed in $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ (**a**) and $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ (**b**) rotational bands.

3.2 *The* $K^\pi = 0^-$*:* $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ *band of* ^{180}*Ta*

There are two motivations behind the present TQPRM calculations for $K^\pi = 0^-$: $9/2[514]_\pi \otimes 9/2[624]_\nu$ band. Firstly, this band exhibits pronounced signature splitting at higher spins ($I \geq 5\hbar$) as depicted in figure 2b. Hence, it could be studied through the TQPRM approach used in this paper. Secondly, the Newby shift energy for this band has not been calculated so far, as a result of which the observed signature splitting could not be studied.

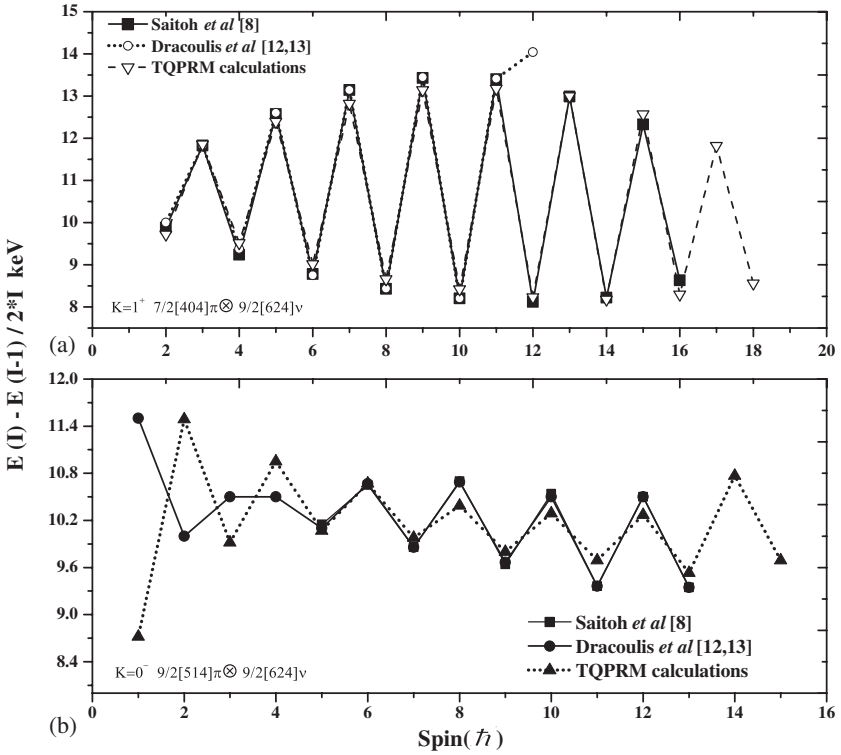

Figure 3. The comparison of the present TQPRM calculations and the experimentally observed signature splitting observed in (**a**) $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ and (**b**) $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ rotational bands.

Table 1. The admixture of wave functions taking part in $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ Coriolis mixing.

	Admixed configurations[a]		
Spin (I) ($\hbar$)	$K^\pi = 0^+$ $A \otimes B$	$K^\pi = 1^+$ $A \otimes C$	$K^\pi = 2^+$ $A \otimes D$
1	0.077	0.997	
2	−0.161	0.985	0.061
3	0.186	0.977	0.102
4	−0.283	0.949	0.135
5	0.287	0.941	0.175
6	−0.389	0.900	0.196
7	0.376	0.893	0.242
8	−0.477	0.842	0.249
9	0.453	0.836	0.306
10	−0.548	0.782	0.295
11	0.512	0.770	0.379
12	−0.600	0.718	0.351
13	0.544	0.692	0.473
14	0.627	−0.651	−0.426
15	0.534	0.597	0.597
16	0.620	−0.575	−0.532
17	0.472	0.482	0.061
18	0.562	−0.482	0.102

[a]Configurations: A: $7/2^+[404]_\pi$, B: $7/2^+[633]_\nu$, C: $9/2^+[624]_\nu$, D: $11/2^+[615]_\nu$.

Table 2. The optimized set of free parameters pertaining to the $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+$ $[624]_\nu$, rotational bands obtained in the present calculations.

Band 1: $K = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$				
Configuration	K^π	E_α (keV)	$\hbar^2/2\Im$ (keV)	$E_{\rm N}$ (keV)
$7/2^+[404]_\pi \otimes 7/2^+[633]_\nu$	0^+	943.78	10.60	94.9
$7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$	1^+	0.0	15.07	
$7/2^+[404]_\pi \otimes 11/2^+[615]_\nu$	2^+	649.87	9.12	

In order to explain the experimentally observed signature splitting in $K^\pi = 0^-$: $9/2[514]_\pi \otimes 9/2[624]_\nu$ band, we constructed a basis comprising $2qp$ bands appearing from $h_{11/2} \otimes i_{13/2}$ orbital couplings. The single proton and neutron wave functions of ^{180}Ta are calculated using the Nilsson model with modified harmonic oscillator potential [21] and with the same potential and equilibrium deformations parameter as used in the case of $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2$ $[624]_\nu$, rotational band discussed in §3.1. In figure 3b, we present comparison of the present TQPRM calculations along with experimental data [8,12,13] for the $K^\pi = 0^-$ band under discussion. From this figure, it is clear that our calculations successfully reproduced the experimentally observed signature splitting at higher spins ($I \geq 5\hbar$). The lower lying levels of this band (i.e., $I \leq 4\hbar$) are populated only through $E1$ transitions [8,12,13], a well known feature ascribed to particle vibrational octupole coupling [29–31]. As the present version of TQPRM does not incorporate vibrational octupole terms, a few initial spins (i.e., $I \leq 4\hbar$) are not considered during the optimization of physical parameters and hence there is a little disagreement between the TQPRM calculations and the experimental data.

In table 3, we present the admixture of wave functions of the bands contributing substantially to $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ band. From this table, it is clear that the mixing strength among $K^\pi = 1^-$: $A \otimes C$, $K^\pi = 2^-$: $A \otimes D$, $K^\pi = 0^-$: $B \otimes C$ and $K^\pi = 1^-$: $B \otimes D$ rotational bands increases as the spin increases with a major contribution from $K^\pi = 1^-$: $B \otimes D$ band at higher spin values. The optimized set of parameters for this band is listed in table 4.

In figure 4, we present a comparison between the calculated and experimental level energies. From this figure, it is clear that there is an excellent agreement

Table 3. The admixture of wave functions taking part in $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ Coriolis mixing.

	Admixed configurations[a]			
Spin (I) ($\hbar$)	$K^\pi = 1^-$ $A \otimes C$	$K^\pi = 2^-$ $A \otimes D$	$K^\pi = 0^-$ $B \otimes C$	$K^\pi = 1^-$ $B \otimes D$
0			0.997	
1	0.007		0.994	0.072
2	−0.013	−0.002	0.989	−0.126
3	0.022	0.006	0.981	0.177
4	−0.032	−0.013	0.970	−0.229
5	0.052	0.025	0.955	0.277
6	−0.074	−0.042	0.938	−0.326
7	0.127	0.079	0.912	0.367
8	−0.185	−0.123	0.879	−0.406
9	0.265	0.181	0.830	0.430
10	−0.288	−0.202	0.797	−0.462
11	0.353	0.248	0.737	0.469
12	−0.322	−0.228	0.714	−0.501
13	0.351	0.247	0.601	0.454
14	−0.214	−0.150	0.675	−0.566
15	0.248	0.172	0.647	0.580

[a]Configurations: A: $7/2^-[523]_\pi$, B: $9/2^-[514]_\pi$, C: $9/2^+[624]_\nu$, D: $11/2^+[615]_\nu$.

Table 4. The optimized set of free parameters pertaining to the $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ rotational band.

Band 2: $K^\pi = 0^-$: $9/2[514]_\pi \otimes 9/2[624]_\nu$				
Configuration	K^π	E_α (keV)	$\hbar^2/2\Im$ (keV)	E_{N} (keV)
$7/2^-[523]_\pi \otimes 9/2^+[624]_\nu$	1^-	934.40	15.03	
$7/2^-[523]_\pi \otimes 11/2^+[615]_\nu$	2^-	1793.17	14.04	
$9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$	0^-	120.16	13.47	−1.45
$9/2^-[514]_\pi \otimes 11/2^+[615]_\nu$	1^-	1173.62	9.66	

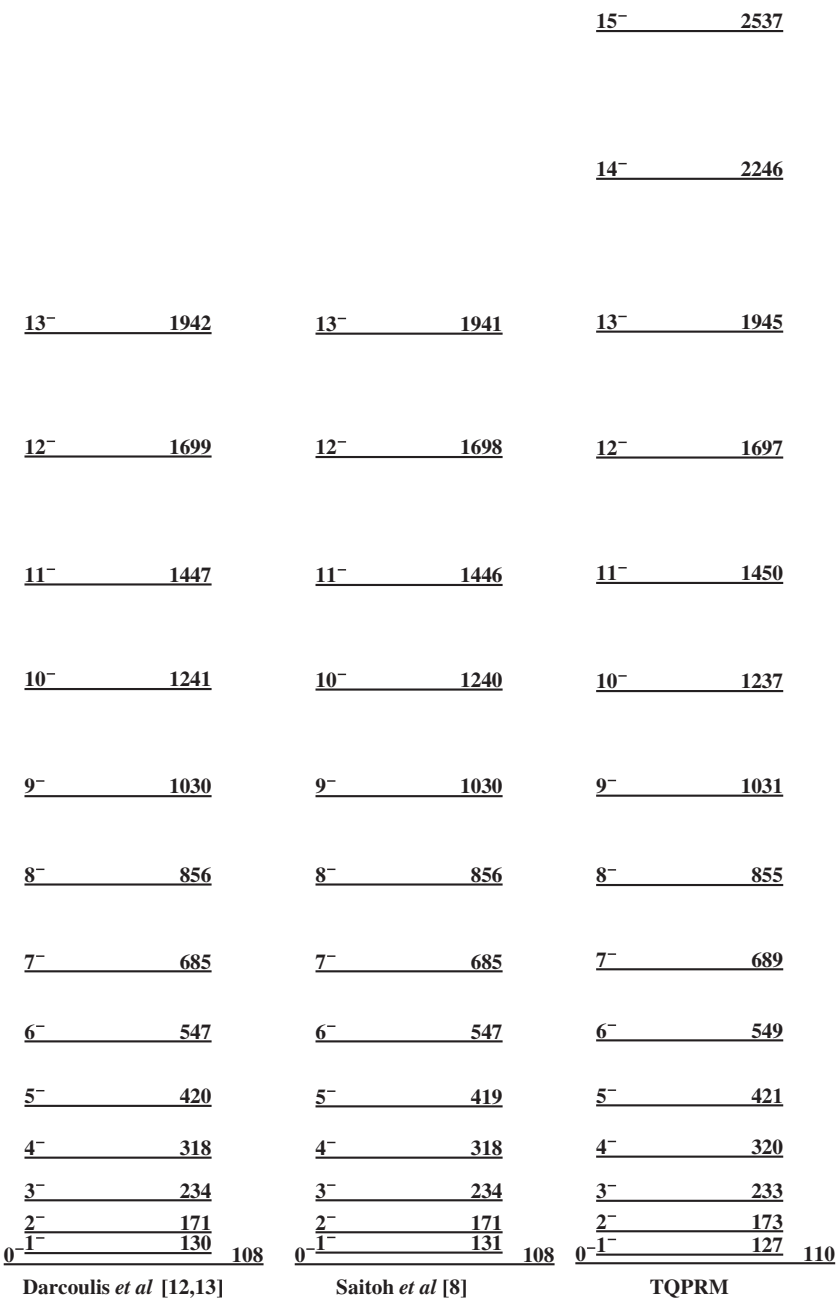

Figure 4. The comparison of the calculated excitation energies and the experimental data for $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ rotational band.

between the theoretical and experimental results. Although this band has been observed up to $I = 13\hbar$ spin [8,12,13], in our calculations we extended this band up to $I = 15\hbar$ as a future prediction for experimentalists.

3.3 *The* $K^\pi = 0^-$*:* $7/2[404]_\pi \otimes 7/2[503]_\nu$ *band of* ^{182}Ta

The band $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$, based on parallel coupling of $7/2^+[404]_\pi$ with $7/2^-[503]_\nu$ Nilsson configurations, is observed up to spin $5\hbar$ [14]. In figure 5, we present experimental level energies along with TQPRM results. From this figure, it is clear that 1^- level of this band is pushed down to the 0^- level and Newby shift is assumed to be a possible cause for this type of shifting [20]. The motivations behind the present TQPRM calculations for this band are:

(1) The Coriolis mixing explanation of the observed signature splitting.
(2) To test the role of Newby shift energy for the above-mentioned shifting of odd spin members with respect to even spin members.
(3) To confirm the tentative nature of this band which could not be firmly established in the earlier studies [14,15].
(4) To extend this band up to higher spin levels so that future prediction for the experimentalists could be made.

In the present calculations for this band, the basis set consists of 25 bands appearing from the coupling of $\pi(g_{7/2}, d_{5/2})$ and $\nu(h_{9/2}, p_{3/2})$ Nilsson orbitals. The equilibrium deformation parameters $\varepsilon_2 = 0.233$ and $\varepsilon_4 = 0.100$ used in the present calculations are adopted from Moller *et al* [27]. The single proton and neutron wave functions are calculated using Nilsson model with modified harmonic oscillator potential [21] and appropriate potential strength parameters are taken as $k = 0.0620$ and $\mu = 0.614$ for protons and $k = 0.0636$ and $\mu = 0.393$ for neutrons [26]. In figure 6a, we present the comparison of theoretical and experimental results. From this figure, it is clear that the present TQPRM calculations excellently reproduce the phase as well as magnitude of the experimentally observed signature splitting. The admixture of wave functions having significant contribution in the wave functions of

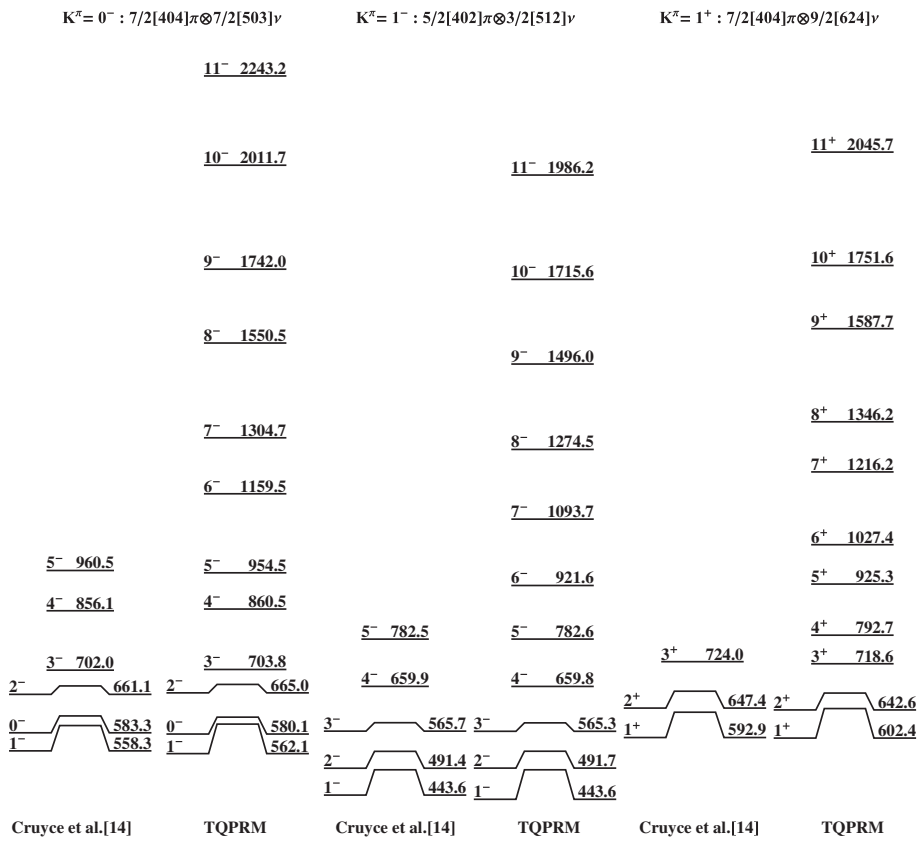

Figure 5. The comparison of the experimental and theoretical excitation energies obtained in the present TQPRM calculations for $K = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$, $K = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$ and $K = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ bands observed in ^{182}Ta.

the band under discussion is presented in table 5. From this table, it is clear that particle–particle and rotor–particle couplings play major roles in explaining the observed signature splitting and also the mixing among $K^\pi = 1^-$: $7/2[404]_\pi \otimes 9/2[505]_\nu$ and $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$ bands goes on increasing at higher spins. The optimized set of parameters such as band-head energies, inertia parameters and Newby shift energies of the strongly interacting bands are presented in table 6. From the present calculations, we also suggest that the Newby shift energy for the $K^\pi = 0^-$ band under study is -23.6 keV, which is in reasonable agreement with the previously reported value (-26.5 keV) by Jain *et al* [17] and this Newby shift energy plays a vital role for the explanation of observed signature splitting. From figure 6a, it is clear that there is an excellent agreement between the present TQPRM calculations and the experimental results, which further strengthen the spin, parity and configuration assignment to this band by Cruyce *et al* [14]. We also extend this band to higher spins (up to $11\hbar$) as a future prediction for experimentalists and this extension could be useful to the experimentalists for fixing the level energies at higher spins.

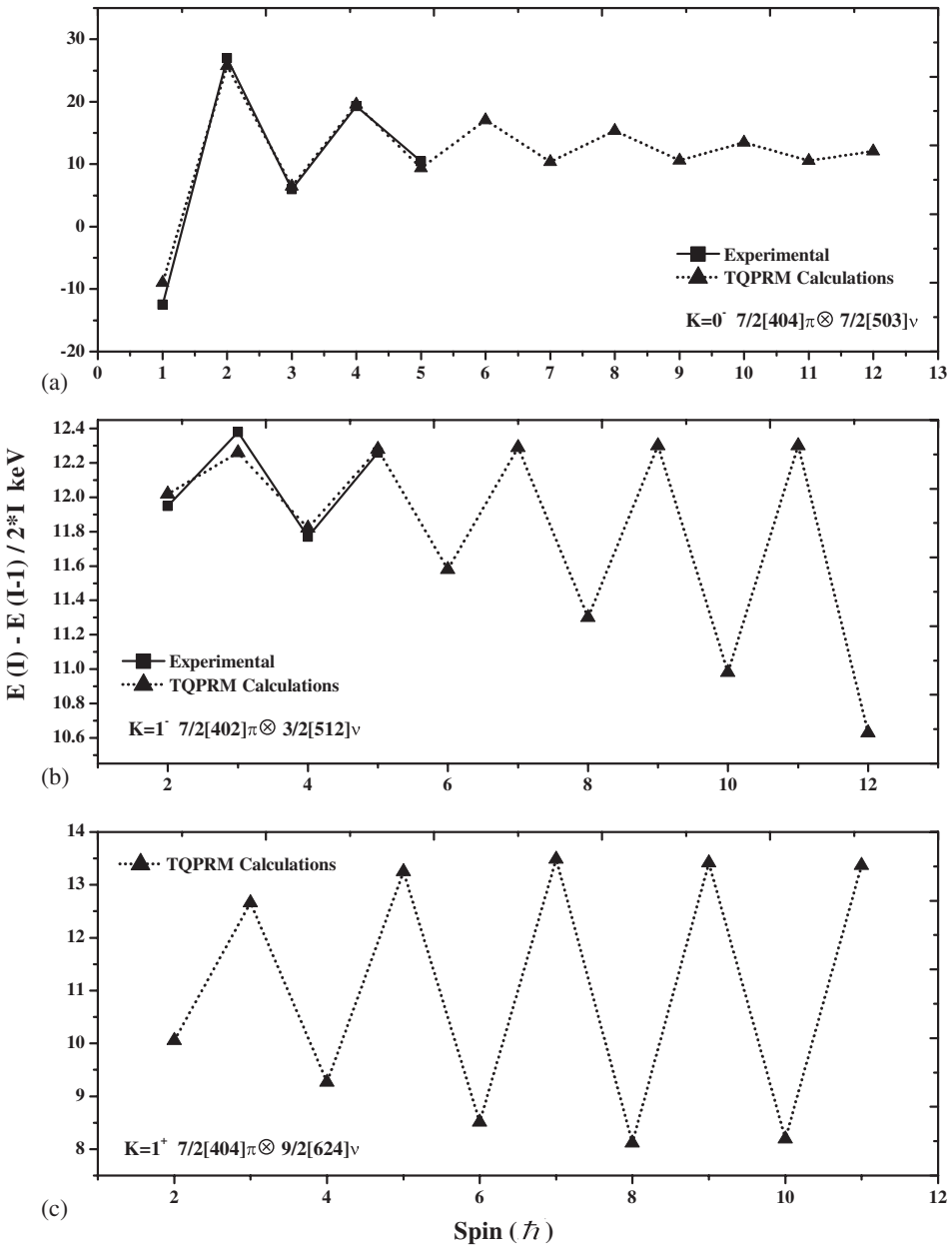

Figure 6. The comparison of experimental (solid line) and theoretical (dotted line) signature splitting observed in (**a**) $K = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$, (**b**) $K = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$ and (**c**) TQPRM-predicted signature splitting in $K = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ bands.

Table 5. Admixture of wave functions of $K = 0^-$: 7/2[404]$_\pi \otimes$7/2[503]$_\nu$, $K = 1^-$: 5/2[402]$_\pi \otimes$3/2[512]$_\nu$ and $K = 1^+$: 7/2[404]$_\pi \otimes$9/2[624]$_\nu$ bands obtained in the present calculations.

$K = 0^-$: 7/2[404]$\pi \otimes$7/2[503]ν				$K = 1^-$: 5/2[402]$\pi \otimes$3/2[512]ν			$K = 1^+$: 7/2[404]$\pi \otimes$9/2[624]ν			
Spin ($\hbar$)	Configurations[a]			Spin ($\hbar$)	Configurations[a]		Spin ($\hbar$)	Configurations[a]		
	$K^\pi = 0^-$ $A \otimes D$	$K^\pi = 1^-$ $A \otimes E$	$K^\pi = 1^-$ $A \otimes F$		$K^\pi = 1^-$ $B \otimes G$	$K^\pi = 0^-$ $C \otimes G$		$K^\pi = 1^+$ $A \otimes H$	$K^\pi = 0^+$ $A \otimes I$	$K^\pi = 2^+$ $A \otimes J$
0	0.997			1	0.999	0.047	1	0.995	0.105	
1	0.996	0.055	0.011	2	0.994	0.091	2	0.969	−0.222	−0.111
2	0.991	−0.101	0.027	3	0.990	0.116	3	0.952	0.250	−0.177
3	0.987	0.139	0.031	4	0.981	0.167	4	0.894	−0.379	−0.241
4	0.975	−0.195	0.058	5	0.977	0.186	5	0.880	0.379	−0.287
5	0.966	0.236	0.061	6	0.962	0.248	6	0.793	−0.498	−0.350
6	0.937	−0.304	0.116	7	0.958	0.262	7	0.790	0.484	−0.378
7	0.912	0.347	0.118	8	0.935	0.333	8	0.680	−0.575	−0.456
8	0.866	−0.428	0.210	9	0.932	0.343	9	0.688	0.558	−0.464
9	0.845	0.482	0.190	10	0.898	0.422	10	0.554	−0.603	−0.574
10	0.739	−0.534	0.369	11	0.895	0.428	11	0.572	0.592	−0.568
11	0.731	0.604	0.281	12	0.839	0.505				
12	−0.622	0.634	−0.414							

[a]Configurations: A: 7/2$^+$ [404]$_\pi$, B: 5/2$^+$ [402]$_\pi$, C: 3/2$^+$ [411]$_\nu$, D: 7/2$^-$ [503]$_\nu$, E: 9/2$^-$ [505]$_\pi$, F: 5/2$^-$ [512]$_\pi$, G: 3/2$^+$ [512]$_\nu$, H: 9/2$^+$ [624]$_\nu$, I: 7/2$^+$ [633]$_\nu$, J: 11/2$^+$ [615]$_\nu$.

Table 6. The optimized values of band-head energies, inertia parameters and Newby shift energies of strongly interacting bands indicated in table 5.

Configuration	K^π	E_α (keV)	$\hbar^2/2\Im$ (keV)	$E_{\rm N}$ (keV)
Band 1: $K = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$				
$7/2^+[404]_\pi \otimes 7/2^-[503]_\nu$	0^-	566.70	15.75	−23.6
$7/2^+[404]_\pi \otimes 9/2^-[505]_\nu$	1^-	1520.00	9.50	
$7/2^+[404]_\pi \otimes 5/2^+[512]_\nu$	1^-	846.90	12.0	
Band 2: $K = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$				
$5/2^+[402]_\pi \otimes 3/2^-[512]_\nu$	1^-	432.53	13.02	
$3/2^+[411]_\pi \otimes 3/2^-[512]_\nu$	0^-	1252.79	10.50	45.1
Band 3: $K = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$				
$7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$	1^+	596.11	14.50	
$7/2^+[404]_\pi \otimes 7/2^+[633]_\nu$	0^+	1250.00	10.10	76.2
$7/2^+[404]_\pi \otimes 11/2^+[615]_\nu$	2^+	235.00	12.50	

3.4 *The $K^\pi = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$ band of ^{182}Ta*

The $K^\pi = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$ band appears from parallel coupling of $5/2^+[402]_\pi$ with $3/2^-[512]_\nu$ neutron Nilsson configurations and observed up to spin $5\hbar$ [14]. The objective behind the present TQPRM calculations for this band is to explain the experimentally observed signature splitting in the spin range $1\hbar \leq I \leq 5\hbar$ and estimation of level energies at higher spin values as a future prediction for experimentalists. In the present calculations, we used the same basis set as used for the above-mentioned $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$ band. From figure 6b it is clear that the present TQPRM calculations successfully explain the experimentally observed signature splitting in this band under discussion (i.e., $K^\pi = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$). We also extend this band up to higher spin values (figure 5) as a future prediction for experimentalists. On the basis of the present calculations, we also suggest that, unlike the $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$ band, the band under discussion exhibits pronounced signature splitting at higher spins and this splitting goes on increasing with increase in spin values. This predicted staggering at higher spins ($I \geq 5\hbar$) could be useful for experimentalists for fixing higher spin level energies of this band. The admixture of wave functions having significant contribution in the wave function of this band is presented in table 5 and from this table it is clear that the rotor particle coupling plays a major role in explaining the observed splitting. The optimized set of parameters for this band is listed in table 6.

3.5 *The $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ band of ^{182}Ta*

The $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ band is also observed in ^{180}Ta and exhibits the pronounced signature splitting throughout the whole spin range (figure 2a). But in ^{182}Ta, only band head is confirmed for this band and two higher levels namely 2^- at 647.4 keV and 3^- at 724.0 keV are completely tentative in nature [14]. Hence, it is a good example for the present calculations for prediction of band structure at higher spins. The comparison among calculated and experimental level energies for this band is presented in figure 5. We extended the theoretical level energies up to spin $11\hbar$ as a future prediction for experimentalists and the TQPRM staggering pattern obtained in the present calculations is presented in figure 6c. From this figure it is clear that, the $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ band exhibits pronounced signature splitting at higher spins. This TQPRM prediction regarding pronounced signature splitting at higher spins in this band was further strengthened by experimentally observed signature splitting in the similar configuration (i.e., $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$) observed in ^{180}Ta (figure 3a). The admixture of wave functions of the bands having substantial contribution in the band under discussion and optimized set of parameters for these bands are presented in tables 5 and 6, respectively.

4. Conclusions

In this paper, we present TQPRM results for two quasiparticle bands of ^{180}Ta and ^{182}Ta which exhibit pronounced signature effects. We successfully reproduced

the magnitude as well as phase of experimentally observed signature splitting in $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$, $K^\pi = 0^-$: $9/2^-[514]_\pi \otimes 9/2^+[624]_\nu$ bands of ^{180}Ta and $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$, $K^\pi = 1^-$: $5/2[402]_\pi \otimes 3/2[512]_\nu$ bands of ^{182}Ta. On the basis of present Coriolis mixing calculations, we also confirm the tentative nature of the $K^\pi = 0^-$: $7/2[404]_\pi \otimes 7/2[503]_\nu$ and $K^\pi = 1^+$: $7/2[404]_\pi \otimes 9/2[624]_\nu$ bands observed in ^{182}Ta which could not be firmly established in the earlier studies. In $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ band of ^{182}Ta, the pronounced signature splitting at higher spins is predicted, which remains an open challenge for experimentalists to confirm the predicted splitting in this band. The ambiguity regarding placement of the 12^+ level in $K^\pi = 1^+$: $7/2^+[404]_\pi \otimes 9/2^+[624]_\nu$ ground state rotational band of ^{180}Ta is successfully resolved.

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