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Effect of dust size distribution and dust charge fluctuation on dust ion-acoustic shock waves in a multi-ion dusty plasma

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Abstract. The effects of dust size distribution and dust charge fluctuation of dust grains on the small but finite amplitude nonlinear dust ion-acoustic shock waves, in an unmagnetized multi-ion dusty plasma which contains negative ions, positive ions and electrons, are studied in this paper. A Burgers equation and its stationary solutions are obtained by using the reductive perturbation method. The analytical and numerical results show that the height with polynomial dust size distribution is larger than that of the monosized dusty plasmas with the same dust grains, but the thickness in the case of different dust grains is smaller than that of the monosized dusty plasmas. Furthermore, the moving speed of the shock waves also depend on different dust size distributions.

Keywords. Dusty plasma; dust-acoustic shock wave; dust size distribution; adiabatic dust charge variation; negative ions.

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1. Introduction

The low-frequency dust ion-acoustic waves are typical acoustic modes in unmagnetized and collisionless dusty plasma with a weak Coulomb coupling between the charged dust grains. The existence of dust ion-acoustic waves was first reported by Shukla and Silin theoretically [1]. Then, laboratory experiments confirmed its existence [2,3]. In a collisionless dusty plasma, dissipation which originates from the dust charging processes has relation to some phenomena including Landau damping, collisions between charged particles and neutrals and kinematic viscosity, among the plasma constituents and results in a new kind of dust ion-acoustic shock wave [4,5]. The study of dust ion-acoustic shock waves in collisionless plasmas by taking into account the dust charge fluctuation received wide attention in the past few years [6–15]. Recently, Mamun *et al* investigated the dust negative ion-acoustic shock waves in a multi-ion dusty plasma containing extremely massive negatively charged fluctuation dust [16]. Kim and Merlino observed that negative ions can cause the dust charge to have a transition from negative to positive in laboratory experiments [17,18]. So, Duha studied the positive dust charging fluctuation in the multi-ion dusty plasma based on the work of Mamun *et al* [19]. Alinejad *et al* investigated the nonlinear propagation of dust ion-acoustic shock waves in a charge-varying dusty plasma with electrons having kappa velocity distribution [20]. However, most of these investigations are restricted to monosized dusty plasmas where charges and sizes of dust grains are the same. In fact, in realistic situations, the dust grains in space dusty plasma and laboratory situations have different size and mass [21–26]. It is well known that in space plasma, the dust size follows a power law distribution [21–23,26,27], while in laboratory plasma, the dust size distribution satisfies the Gaussian distribution [28]. The dust size distribution depends on the situation of the environment and the experimental condition. Hence, the aim of the present work is to study the effect of dust size distribution on the propagation of small but finite-amplitude dust ion-acoustic shock waves in a multi-ion dust plasma with dust charge variation. Inspired by the work of Duan *et al* [24,29], we consider a more general situation in which the dust size follows a polynomial distribution function: $n(r)\mathrm{d}r = (a_0 + a_1 r + a_2 r^2 + a_3 r^3 + \cdots)\mathrm{d}r$ ($r_{\min} < r < r_{\max}$).

The paper is organized as follows. The set of basic equations for the propagation of shock waves are given in §2, and in §3, the adiabatic dust charge variation is studied. A Burgers equation and its shock solution are obtained by the reductive perturbation method in §4. The analytical and numerical results are presented in §5.

2. Governing equations

Here, we consider a four-component unmagnetized dusty multi-ion plasma consisting of stationary negative charge dust grains, Boltzmann-distributed electrons, as well as single-charge positive ions and single-charge negative ions. We also assume that there are N number of different dust grain sizes ($j = 1, 2, \ldots, N$). The quasineutrality condition in equilibrium state can be written as $n_{e0}-n_{p0}+n_{n0}+\sum_{j=1}^{N} n_{d0j} Q_{d0j}/e = 0$, where n_{e0}, n_{p0}, n_{n0} and n_{d0j} refer to the unperturbed number densities of electrons, positive ions, negative ions and jth dust grains, respectively. Q_{d0j} is the charge of the jth dust grain. The normalized governing equations for the negative and positive ion fluids are given by [30]

$$\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x}(n_p u_p) = 0, \tag{1}$$

$$\frac{\partial n_n}{\partial t} + \frac{\partial}{\partial x}(n_n u_n) = 0, \tag{2}$$

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + \frac{\partial \phi}{\partial x} + 3\sigma n_p \frac{\partial n_p}{\partial x} - \eta \frac{\partial^2 u_p}{\partial x^2} = 0, \tag{3}$$

$$\frac{\partial u_n}{\partial t} + u_n \frac{\partial u_n}{\partial x} - \frac{\partial \phi}{\partial x} + 3\sigma n_n \frac{\partial n_n}{\partial x} - \eta \frac{\partial^2 u_n}{\partial x^2} = 0, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \exp(\phi) + \alpha n_n - (1+\alpha+\beta) n_p + \frac{\beta}{N_{\text{tot}} \bar{Q}_d} \sum_{j=1}^{N} n_{dj} q_{dj}, \tag{5}$$

where n_s, u_s and m_s denote the number density, velocity and mass of s, $s = e, n, p, dj$ stands for electrons, negative ions, positive ions and the jth dust grains, respectively. ϕ is the electrostatic potential, $\alpha = n_{n0}/n_{e0}$, $\beta = \sum_{j=1}^{N} n_{d0j} Q_{d0j}/(e n_{e0})$, the total number density of dust grains $N_{\text{tot}} = \sum_{j=1}^{N} n_{d0j}$ and $\bar{Q}_d$ is the average charge of the dust grains. $\sigma = T_i/T_e$, where T_e and T_i are the electron and ion temperatures, $\eta = \mu_{id}/\lambda_{\text{D}}^2 \omega_{pi}$, where μ_{id} is the kinematic viscosity of the dusty plasma, λ_{D}, the effective Debye length $= (k_{\text{B}} T_e / 4\pi n_{n0} e^2)^{1/2}$ and the effective dusty plasma frequency $\omega_{pi} = (4\pi n_{n0} e^2/m_n)^{1/2}$, $q_{dj} = Q_{dj}/n_{d0j} Q_{d0j}$. The different physical quantities are normalized as follows: n_p and n_n are normalized by n_{p0} and n_{n0}, time t, space coordinate x, velocity $u_{p,n}$ and electrostatic potential ϕ are normalized by ω_{pi}^{-1}, λ_{D}, ion sound speed $C_i = (k_{\text{B}} T_e/m_{n0})^{1/2}$ and $k_{\text{B}} T_e/e$, respectively.

3. The adiabatic dust charge variation

In the theory of adiabatic charge variations, it is assumed that the ratio of the dust plasma frequency and the charging frequency is very small. Then the charging equation for the jth spherical dust grain is given by [26]

$$\frac{\partial Q_{dj}}{\partial t} = I_{ej} + I_{pj} + I_{nj}. \tag{6}$$

According to the orbit motion limited theory, the charge of the dust grains $Q_{dj} = C_j \phi_d$, where the capacitance C_j of the spherical dust grain is $C_j = r_{dj} \exp(-r_{dj}/\lambda_d) \approx r_{dj}$ for $\lambda_d \geq r_{dj}$, r_{dj} is the radius of the jth dust grain, ϕ_d is the potential difference between the grain potential and the plasma potential. The current carried by the Boltzmann-distributed electrons, positive ions and negative ions, I_{ej}, I_{pj} and I_{nj}, for the negative charge dust grains are given by

$$I_{ej} = -4\pi r_{dj}^2 n_e e \left(\frac{T_e}{2\pi m_e}\right)^{1/2} \exp\left(\frac{e Q_{dj}}{r_{dj} T_e}\right), \tag{7}$$

$$I_{pj} = 4\pi r_{dj}^2 n_p e \left(\frac{T_p}{2\pi m_p}\right)^{1/2} \left(1 - \frac{e Q_{dj}}{r_{dj} T_p}\right), \tag{8}$$

$$I_{nj} = -4\pi r_{dj}^2 n_n e \left(\frac{T_n}{2\pi m_n}\right)^{1/2} \exp\left(\frac{e Q_{dj}}{r_{dj} T_n}\right). \tag{9}$$

It is obvious that $|I_{ej}| \gg I_{pj}$ and I_{nj} (because $m_i \gg m_e$) and the jth dust grain surface becomes negatively charged. This increases the ion current and decreases the electron and negative ion current until $I_{ej} + I_{pj} + I_{nj} = 0$.

The current at equilibrium condition is defined as

$$I_{e0j} + I_{p0j} + I_{n0j} = 0, \tag{10}$$

where $I_{e0j} = I_{ej}$ ($\phi_j = 0, Q_{dj} = Q_{d0j}$), $I_{pj0} = I_{pj}$ ($\phi_j = 0, Q_{dj} = Q_{d0j}$) and $I_{n0j} = I_{nj}$ ($\phi_j = 0, Q_{dj} = Q_{d0j}$). Let $Q_{dj} = Q_{d0j} + q_{dj}$, $I_{ej} = I_{e0j} + I'_{ej}$, $I_{pj} = I_{p0j} + I'_{pj}$, $I_{nj} = I_{n0j} + I'_{nj}$. We obtain

$$\frac{\partial q_{dj}}{\partial t} = I'_{ej} + I'_{pj} + I'_{nj}, \tag{11}$$

where

$$I'_{ej} = I_{e0j}\left[\phi + \frac{eq_{dj}}{r_{dj}T_e} + \frac{1}{2}\left(\phi + \frac{eq_{dj}}{r_{dj}T_e}\right)^2 + \cdots\right] \tag{12}$$

$$I'_{pj} = I_{p0j}\left[n_p\left(1 - \frac{eq_{dj}}{r_{dj}T_p f_e}\right) - 1\right] \tag{13}$$

$$I'_{nj} = I_{n0j}\left[n_n \exp\left(\frac{eq_{dj}}{r_{dj}T_n}\right) - 1\right] \tag{14}$$

for $Q_{dj} < 0$, where $f_e = 1 - (eQ_{d0j})/(r_{dj}T_i)$.

4. Derivation of the Burgers equation

In order to study the dust ion-acoustic shock wave, reductive perturbation method is used and the stretched space-time coordinates are taken as $\xi = \epsilon(x - v_0 t)$ and $\tau = \epsilon^2 t$, where v_0 is the phase velocity of the wave and ϵ is a small parameter measuring the weakness of the nonlinearity. The dependent variables n_p, n_n, u_p, u_n, q_{dj} and ϕ are expanded as power series

$$n_p = 1 + \epsilon n_{p1} + \epsilon^2 n_{p2} + \cdots, \tag{15}$$

$$n_n = 1 + \epsilon n_{n1} + \epsilon^2 n_{n2} + \cdots, \tag{16}$$

$$u_p = \epsilon u_{p1} + \epsilon^2 u_{p2} + \cdots, \tag{17}$$

$$u_n = \epsilon u_{n1} + \epsilon^2 u_{n2} + \cdots, \tag{18}$$

$$q_{dj} = \epsilon q_{d1j} + \epsilon^2 q_{d2j} + \cdots, \tag{19}$$

$$\phi = \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots. \tag{20}$$

Substituting eqs (15)–(20) to eqs (1)–(5) and (11)–(14), for the lowest order in ϵ, we have

$$n_{p1} = \frac{1}{v_0^2 - 3\sigma}\phi_1, \quad u_{p1} = \frac{v_0}{v_0^2 - 3\sigma}\phi_1, \tag{21}$$

$$n_{n1} = \frac{1}{3\sigma - v_0^2}\phi_1, \quad u_{n1} = \frac{v_0}{3\sigma - v_0^2}\phi_1, \tag{22}$$

$$q_{d1j} = \frac{B_j}{A_j}\frac{r_{dj}}{e}\phi_1, \tag{23}$$

$$v_0^2 = 3\sigma + \frac{1 + \beta - \frac{\beta}{N_{\text{tot}}\bar{Q}_d}\sum_{j=1}^{N}\frac{n_{d0j}(I_{n0j} - I_{p0j})}{A_j}\frac{r_{dj}}{e}}{1 - \frac{\beta}{N_{\text{tot}}\bar{Q}_d}\sum_{j=1}^{N}\frac{n_{d0j}I_{e0j}}{A_j}\frac{r_{dj}}{e}}, \tag{24}$$

where

$$A_j = \frac{I_{n0j}}{T_n} + \frac{I_{e0j}}{T_e} - \frac{I_{p0j}}{f_e T_i}$$

and

$$B_j = \frac{1}{(v_0^2 - 3\sigma)}(I_{n0j} - I_{p0j}) - I_{e0j}.$$

To the next higher order in ϵ, we derive the Burgers equation for dust ion-acoustic waves

$$\frac{\partial\phi_1}{\partial\tau} + M^{-1}N\phi_1\frac{\partial\phi_1}{\partial\xi} + M^{-1}P\frac{\partial^2\phi_1}{\partial\xi^2} = 0, \tag{25}$$

where

$$M = \frac{2\beta}{N_{\text{tot}}\bar{Q}_d}\sum_{j=1}^{N} n_{d0j}(I_{p0j} - I_{n0j})v_0\gamma_1^2 + \frac{2v_0\gamma_1^2 A_j e}{r_{dj}}(1 + 2\alpha + \beta), \tag{26}$$

$$N = \frac{\beta}{N_{\text{tot}}\bar{Q}_d}\sum_{j=1}^{N} n_{d0j}\left[I_{n0j}\left(\frac{B_j^2}{A_j^2 T_N^2} + \frac{2\gamma_1 B_j}{A_j T_n}\right) + I_{e0j}\left(1 + \frac{B_j^2}{A_j^2 T_e^2}\right) - I_{p0j}\frac{2\gamma_1 B_j}{f_e T_i A_j} + \left(I_{n0j} + I_{p0j}\right)C\right] + \frac{A_j e}{r_{dj}}[(1+\beta)C - 1], \tag{27}$$

$$P = -\frac{\beta\eta}{N_{\text{tot}}\bar{Q}_d}\sum_{j=1}^{N} n_{d0j}(I_{p0j} - I_{n0j})v_0\gamma_1^2 - \frac{\eta v_0\gamma_1^2 A_j e}{r_{dj}}(1+2\alpha+\beta) + v_0[\gamma_1(1+\beta) - 1], \tag{28}$$

where $\gamma_1 = 1/(v_0^2 - 3\sigma)$, $C = 3\gamma_1^3(v_0^2 + \sigma)$.

The stationary solution of the Burgers equation (25) is of the form

$$\phi_1 = \phi_m\left[1 - \tanh\left(\frac{\xi - U_0\tau}{\omega}\right)\right]. \tag{29}$$

Here, the height of the shock waves is $\phi_m = U_0/M^{-1}N$ and its thickness is $\omega = -2M^{-1}P/U_0$.

5. Numerical results and discussion

Here, assume that radius r of the dust grains is in a given range $[r_1, r_2]$ and dust size distribution expressed as a polynomial has the form $n(r)\mathrm{d}r = (a_0 + a_1 r + a_2 r^2 + a_3 r^3 + \cdots)\mathrm{d}r$, $r_1 < r < r_2$. The total number density of the dust grains is given by $N_{\text{tot}} = \int_{r_1}^{r_2} n(r)\mathrm{d}r$. Outside the limit of $r > r_2$ and $r < r_1$, $n(r) = 0$. The average radius of the dust grains is $\bar{r}_d = \int_{r_1}^{r_2} n(r) r\mathrm{d}r / \int_{r_1}^{r_2} n(r)\mathrm{d}r$.

In the following, we can calculate the dispersion relation v_0^2, the height ϕ_m and thickness ω of the shock waves. To compare the results with that of the monosized dusty plasma with the same dust size, we introduce $\bar{v}_0^2$, $\bar{\phi}_m$ and $\bar{\omega}$ to represent the dispersion relation, height and thickness of the shock waves in the monosized dusty plasmas, respectively, and use the following relations:

$$\bar{I}_{e0} = \frac{\bar{r}_d^2}{r_{dj}^2} I_{e0j}, \quad \bar{I}_{n0} = \frac{\bar{r}_d^2}{r_{dj}^2} I_{n0j}, \quad \bar{I}_{p0} = \frac{\bar{r}_d^2}{r_{dj}^2} I_{p0j}, \tag{30}$$

$$\bar{A} = \frac{\bar{r}_d^2}{r_{dj}^2} A_j = \left(\frac{\bar{I}_{n0}}{T_n} + \frac{\bar{I}_{e0}}{T_e} - \frac{\bar{I}_{p0}}{T_i - e\phi_0} \right), \tag{31}$$

$$\bar{B} = \frac{\bar{r}_d^2}{r_{dj}^2} B_j = \gamma_1 (\bar{I}_{n0} - \bar{I}_{p0}) - \bar{I}_{e0}, \tag{32}$$

where $\bar{I}_{e0}$, $\bar{I}_{n0}$ and $\bar{I}_{p0}$ respectively denote the electron, negative ion and positive ion currents to the dust grain with average size at equilibrium state. In the numerical process, the same values of dusty plasma parameters are selected [30]: $n_{n0} = 5 \times 10^{13}$ m^{-3}, $n_{p0} = 2.5 \times 10^{14}$ m^{-3}, $n_{d0} = 1.2 \times 10^{10}$ m^{-3}, $Q_{d0} = 1.5 \times 10^4\, e$, $T_e = 1.5$ eV, $T_n = T_p = 0.1$ eV, $m_n = m_p$ and $r \sim 0.01$–1 um. Remember that all the physical quantities are normalized. Now, we shall discuss two special cases.

Case 1. $n(r)\mathrm{d}r = a_0 \mathrm{d}r, r_1 < r < r_2$

This is the simplest case. In this case, the total number dust density $N_{\text{tot}} = a_0(r_2 - r_1)$ and the linear dispersion relation can be rewritten as

$$v_0^2 = 3\sigma + \frac{1 + \beta - \frac{\beta}{N_{\text{tot}} \bar{Q}_d} \frac{\bar{I}_{n0} - \bar{I}_{p0}}{\bar{A}e} \bar{r}_d N_{\text{tot}}}{1 - \frac{\beta}{N_{\text{tot}} \bar{Q}_d} \frac{\bar{I}_{e0}}{\bar{A}e} \bar{r}_d N_{\text{tot}}}. \tag{33}$$

Obviously, the speed is the same as that for the monosized dusty plasmas.

The dependence of the height ϕ_m and the ratio $\phi_m/\bar{\phi}_m$ on the parameter a_0 are depicted in figure 1. It can be seen that ϕ_m increases as a_0 increases. Figure 1 also shows that the height in Case 1 of the polynomial distribution is larger than that of the monosized dusty plasmas. Furthermore, the ratio $\phi_m/\bar{\phi}_m$ decreases with an increase in a_0 (similar conclusions can be found in ref. [29]).

Figure 2 displays the variation of ω and the ratio $\omega/\bar{\omega}$ with respect to a_0. it is clear from figure 2 that the thickness of the shock waves decreases as a_0 increases. Figure 2 also shows that increasing a_0 leads to increase in the ratio of $\omega/\bar{\omega}$, which indicates that the thickness

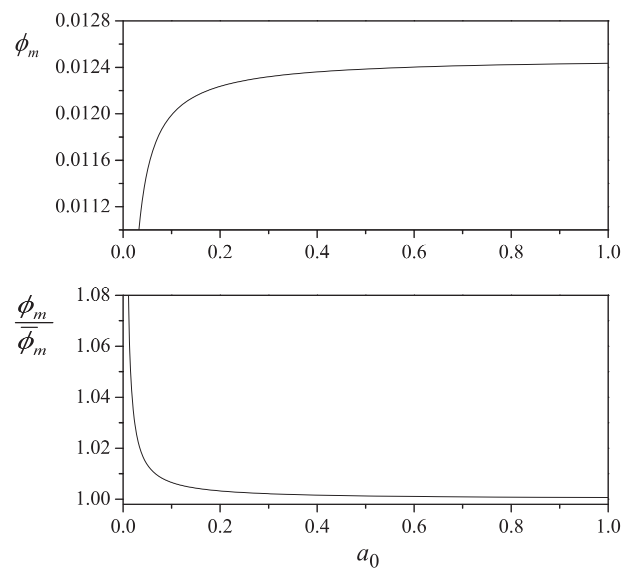

Figure 1. ϕ_m and $\phi_m/\bar{\phi}_m$ vs. a_0 in Case 1. Here, $r_2 = 1$, $r_1 = 0.01$.

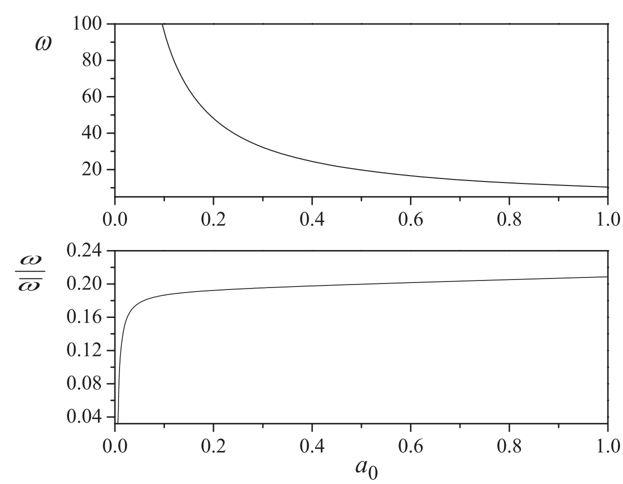

Figure 2. ω and $\omega/\bar{\omega}_m$ vs. a_0 in Case 1. Here, $\eta = 1$ and the other parameters are the same as in figure 1.

in the case of dust size distribution is smaller than that of the monosized dusty plasmas.

Case 2. $n(r)\mathrm{d}r = (a_0 + a_1 r)\mathrm{d}r, r_1 < r < r_2$

In this case, $N_{\rm tot} = [a_0 + \frac{1}{2}a_1(r_2 - r_1)](r_2 - r_1)$. Using eqs (26)–(28), it can be seen that v_0^2, ϕ_m and ω are functions of parameters a_0 and a_1. We will numerically analyse the variation of the quantities on a_0 and a_1 in the following discussion.

Figure 3 shows the variation of v_0^2 and the ratio $v_0^2/\bar{v}_0^2$ with a_1 for three different values of a_0. It is seen that v_0^2 increases as a_0 increases, and for a given a_0, v_0^2 increases as a_1 increases. It should be mentioned that $a_1 < 0$ means that the part of the number density of smaller dust grains is greater than that of the larger dust grains, which is the usual case of dusty plasma, such as the famous power law dust size distribution in space plasma [23,26,27]. Otherwise, $a_1 > 0$ means that the part of the number density of the smaller dust grains is smaller than that of larger dust grains, which is unusual in the case of dusty plasmas. Therefore, we can conclude that v_0^2 for the usual case $a_1 < 0$ is smaller than that of the unusual case $a_1 > 0$. Figure 3 also shows that the ratio $v_0^2/\bar{v}_0^2$ decreases as a_0 increases, and for a given a_0, the ratio will increase first, and then decrease as a_1 increases. Moreover, the maximum value of the ratio is not at the point $a_1 = 0$. For example, when $a_0 = 0.7$ and $a_1 = -0.463$, the ratio gets its the maximum value 1.08793, and when $a_0 = 0.9$, the maximum value of the ratio is 1.0, which can be found at the point $a_1 = -0.012$. This indicates that enhancing the parameter a_0, makes the maximum value of the ratio $v_0^2/\bar{v}_0^2$ shift to larger a_1.

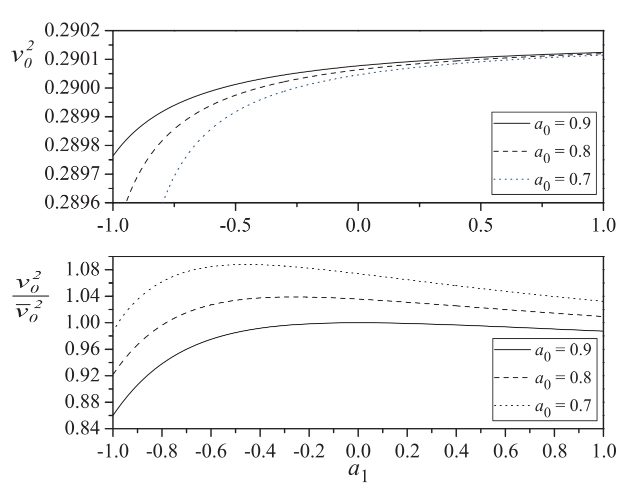

Figure 3. v_0^2 and $v_0^2/\bar{v}_0^2$ vs. a_1 in Case 2. Here, $r_2 = 1$, $r_1 = 0.01$.

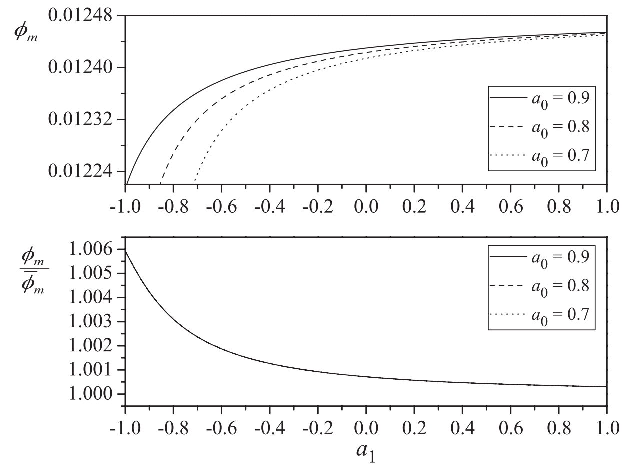

Figure 4. ϕ_m and $\phi_m/\bar{\phi}_m$ vs. a_1 in Case 2. Here, the other parameters are the same as in figure 3.

Figure 4 displays the variation of the height ϕ_m and the ratio $\phi_m/\bar{\phi}_m$, with respect to a_1 for three different values of a_0. It is observed that ϕ_m increases with the increase of a_0 and a_1. It is also observed that the ratio $\phi_m/\bar{\phi}_m$ decreases as a_1 increases, but it is not influenced by a_0. Figure 4 also indicates that the height is larger than that of the monosized dusty plasmas by considering the dust size distribution, as $\phi_m/\bar{\phi}_m > 1$.

The effects of a_1 on the thickness ω and the ratio $\omega/\bar{\omega}$ for three different values of a_0 are shown in figure 5. It seems that ω decreases as a_0 and a_1 increase. Furthermore, as the ratio $\omega/\bar{\omega} < 1$, we find that the thickness of dusty plasmas with polynomial dust size distribution is smaller than that of the monosized dusty plasmas with the same dust size.

In summary, it can be seen from the present study that the dust size distribution and adiabatic dust charge

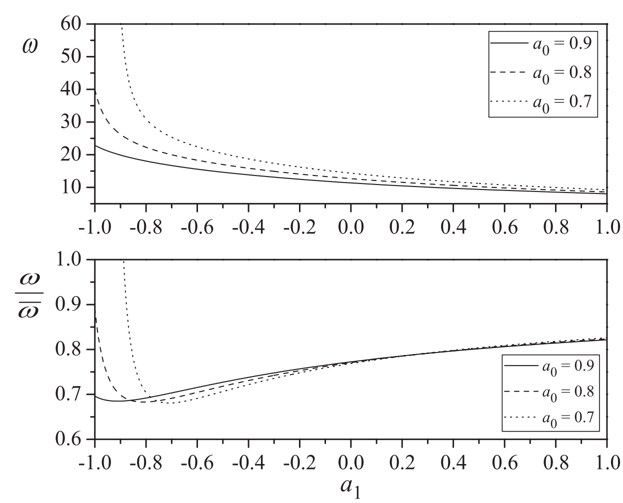

Figure 5. ω and $\omega/\bar{\omega}$ vs. a_1 in Case 2. Here, $\eta = 1$, and other parameters are the same as in figure 3.

variation in a multi-ion dusty plasma plays important roles on the nature of dust ion-acoustic shock wave motions, especially the thickness. As the dust size distribution is considered, which in turn enhances the dust plasma frequency [31], compared to a monosized dusty plasma, the plasma with a polynomial distribution with different sizes has a larger height, but a smaller thickness. The adiabatic dust charge fluctuation leads to further strong dissipation in the plasma system [4], both of which can significantly influence the magnitude of the dissipative coefficient. Therefore, the thickness is more highly sensitive to the size and charge fluctuation of the dust grains. Our results also indicate that different dust grains with different sizes have different speeds in such dusty plasma.

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