



## Analysis of the normal optical, Michel and molecular potentials on the $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$ reaction

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**Abstract.** Full finite-range (FFR) distorted-wave Born approximation (DWBA) method has been applied to analyse the angular distributions of cross-sections of the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  reaction at 28 MeV incident energy for the 22 transitions involving both the bound and unbound states of  $^{44}\text{Ti}$  by using the normal optical, Michel and molecular potentials. The extracted spectroscopic factors for the three optical potentials are compared with those of some previous studies of zero-range (ZR) calculations of the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  reaction using the normal optical potential. The  $\chi^2$  values of all the levels are obtained for the three optical potentials to estimate the quality of the fits. Molecular and Michel potentials have been used for the first time to analyse the four-nucleon transfer reaction and it seems that the molecular potential fits the experimental data more satisfactorily for some of the states than the normal optical and Michel potentials.

**Keywords.** Full finite-range; distorted-wave Born approximation; optical potential; spectroscopic factors.

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### 1. Introduction

The characteristic features of a large number of states for most of the nuclei lighter than  $A = 40$  can be investigated by  $\alpha$ -clustering. For nuclei heavier than  $A = 40$ , especially for the fp shell nuclei, many experiments of  $\alpha$  transfer have also been performed [1–3]. The theory of  $\alpha$ -cluster [4–7] structure is studied using an effective interaction which cannot bind the ground-state band. The  $\alpha$ -cluster transfer of the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  reaction at  $E_{\text{Li}} = 28$  MeV and 50 MeV was analysed by Yeal Kim *et al* [8] using only the normal optical potential. But it is well known that there is a limitation of the normal optical potential to describe the anomalous large angle scattering (ALAS) for elastic and inelastic processes including transfer reactions induced by  $\alpha$ -particles [9–13]. Two other types

of potentials have been proposed to explain ALAS. The first one, proposed by Michel *et al* [4,14,15], is a special type of optical potential with a squared Woods–Saxon (Michel) geometry. The second one is a molecular type of complex potential suggested by Block and Malik [16]. By using the Michel and molecular potentials, this problem can be solved successfully in the elastic scattering of  $\alpha$ -particle [14,17–23] and of  ${}^6\text{Li}$  [24]. The success of Michel and molecular potentials on various reactions starting from single-nucleon transfer to three-nucleon transfers [25–31] has encouraged the researchers to undertake the present study. Moreover, to the best of our knowledge, this reaction data are being analysed using Michel and molecular potentials for the first time.

The aim of this study is to investigate the  ${}^{40}\text{Ca}({}^6\text{Li}, d){}^{44}\text{Ti}$  reaction at  $E_{\text{Li}} = 28$  MeV by using the normal optical, Michel and molecular potentials on the basis of full finite-range (FFR) distorted-wave Born approximation (DWBA) method. All forms of potentials must be set by fitting the elastic data and its acid test will be the reaction analysis. It is well-known that in reaction analysis, there is no opportunity to play with the parameters for better fit. It is a one-shot process. Moreover, if it is found that the fits of the reaction data are of the same quality for all types of potential, it will be interesting to find any equivalence or supersymmetric relation amongst the potentials as mentioned by Baye [32]. The experimental angular distribution data for the  ${}^{40}\text{Ca}({}^6\text{Li}, d){}^{44}\text{Ti}$  reaction are taken from the  $E_{\text{Li}} = 28$  MeV incident energy, involving 22 transitions for both the bound and unbound states covering the total band i.e., positive- and negative-parity bands [2]. For unbound states, the resonance form factor formulated by Vincent and Fortune [33,34] is used. The form of three types of  ${}^6\text{Li}$ -nucleus potentials are presented in §2. The DWBA formalism and analyses are discussed in §3 and §4, respectively. Section 5 deals with the discussion of the present work. Finally, the conclusion is given in §6.

## 2. The form of ${}^6\text{Li}$ -nucleus potentials

The standard form of the normal optical potential for the  ${}^6\text{Li}$ -nucleus system including the Coulomb term is given by

$$V(r) = V_c - Vf(x_R) - i \left[ Wf(x_W) - 4W_D \frac{d}{dx} f(x_D) \right]. \quad (1)$$

Here,  $f(x_i) = (1 + e^{x_i})^{-1}$ , where  $x_i = (r - R_i)/a_i$  with  $R_i = r_i A_T^{1/3}$  ( $i = 0, W, D$ ).

The complex squared Woods–Saxon (Michel) potential [4,14] including the Coulomb term  $V_c(r)$  comprises the following forms [14] of the real  $V_M(r)$  and imaginary  $W_M(r)$  parts:

$$V_M(r) = -V_0 \left[ 1 + \alpha \exp \left\{ - \left( \frac{r}{\rho} \right)^2 \right\} \right] \left[ 1 + \exp \left( \frac{r - R_0}{2a_0} \right) \right]^{-2} + V_c(r), \quad (2)$$

$$W_M(r) = -W_0 \left[ 1 + \exp \left( \frac{r - R_W}{2a_W} \right) \right]^{-2}, \quad (3)$$

with

$$V_c(r) = \left[ \frac{Z_1 Z_2 e^2}{2R_c} \right] \left[ 3 - \frac{r^2}{R_c^2} \right] \quad \text{for } r \leq R_c, \quad (4)$$

$$= \frac{Z_1 Z_2 e^2}{r} \quad \text{for } r > R_c. \quad (5)$$

In eqs (2)–(5),  $R_i = r_i A_T^{1/3}$  ( $A_T$  is the mass number of the target and  $i = 0, W, C$ ) and  $a_i$  ( $i = 0, W$ ).  $V_0$  is the real well depth,  $W_0$  is the imaginary well depth,  $\rho$  is the geometry parameter and  $\alpha$  represents the strength of the potential.  $Z_1$  and  $Z_2$  are the charges of the projectile and the target, respectively.

The molecular potential is a well-known complex potential [20], which has its root in the energy density functional (EDF) formalism [16,18,35]. The potential comprises the following forms for the real  $V_m(r)$  and imaginary  $V_m(r)$  parts:

$$V_m(r) = -V_0 \left[ 1 + \exp\left(\frac{r - R_0}{a_0}\right) \right]^{-1} + V_1 \exp\left[-\left(\frac{r}{R_1}\right)^2\right] + V_c(r) \quad (6)$$

and

$$W_m(r) = -W_0 \exp\left[-\left(\frac{r}{R_W}\right)^2\right]. \quad (7)$$

The real part is non-monotonic due to the second term in eq. (6), which has a short-range repulsion. Here  $V$ 's and  $W_0$  are the depth,  $V_c(r)$  is the Coulomb potential,  $R$ 's is the half-way radius and  $a_0$  is the surface diffuseness.

### 3. Formalism for the DWBA method

The differential cross-section (eq. (8)) for a (<sup>6</sup>Li, d) reaction in the DWBA theory [36,37] with a full finite-range (FFR) interaction is given by

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(2\pi \hbar^2)^2} \frac{k_f}{k_i} \frac{1}{(2J_i + 1)(2s_a + 1)} \sum |T_{fi}|^2. \quad (8)$$

In eq. (8),  $\mu$ 's and  $k$ 's are the reduced masses and wave numbers, respectively.  $J_i$  is the spin of the target and  $s_a$  is the spin of the projectile. The subscripts  $i$  and  $f$  indicate the incident and outgoing channels, respectively.  $\sum$  and  $T_{fi}$  denote, respectively, the sum over all magnetic substates and the transition amplitude.

In the isospin representation, the calculation of the cross-section for the stripping reaction in FFR can be represented by the expression [38]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \frac{2J_f + 1}{2J_i + 1} C^2 S_s \left(\frac{d\sigma}{d\Omega}\right)_{\text{DWUCK5}}. \quad (9)$$

Here  $(d\sigma/d\Omega)_{\text{exp}}$  and  $(d\sigma/d\Omega)_{\text{DWUCK5}}$  are respectively, the experimental cross-section and the cross-section calculated by the computer code DWUCK5 [38].  $J_i$  and  $J_f$  are the total spins of the initial and final nuclei, respectively.  $C^2$  is the isospin Clebsch–Gordon coefficient and  $S$  is the heavy-particle and  $s = 2.0$  is the light-particle spectroscopic factors, respectively.

The differential cross-section for the unbound states of the final nucleus is given [34] by

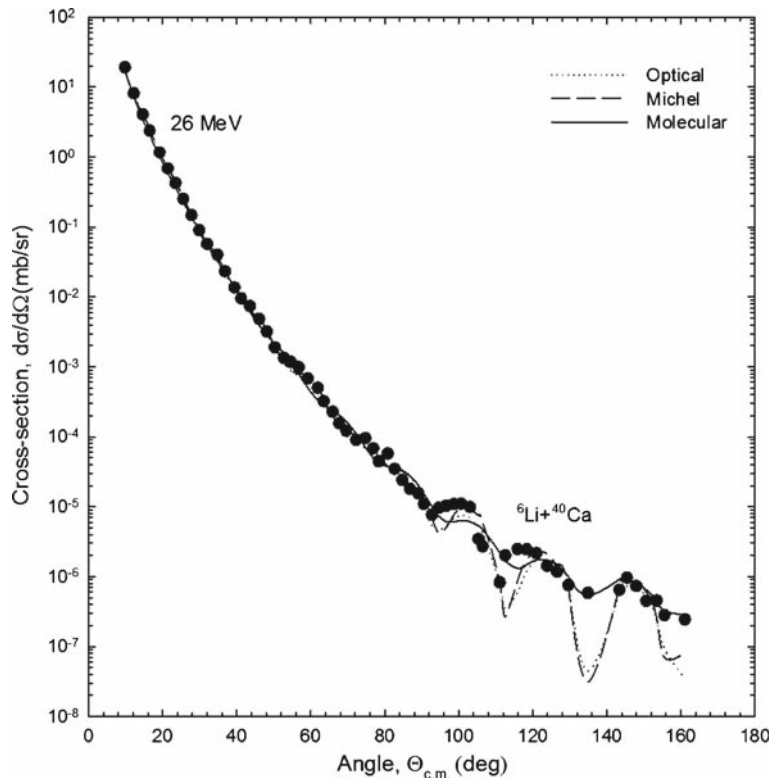
$$\frac{d\sigma}{d\Omega} = \frac{\Gamma \mu k}{\hbar} \left(\frac{d\sigma^F}{d\Omega}\right). \quad (10)$$

Here  $d\sigma^F/d\Omega$  is the cross-section predicted at the resonance energy,  $\Gamma$  is the resonance width,  $k$  is the wave number of the transferred particle at the resonance energy and  $\mu$  is the reduced mass of the transferred particle and the target nucleus.

#### 4. DWBA analysis

The computer code DWUCK5 [38] is used for describing angular distribution of the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  reaction at  $E_{\text{Li}} = 28$  MeV incident energy using full finite-range DWBA method for the bound and unbound state transitions. The code is modified for the Michel and molecular potentials. Here, non-locality [7,38–40] of the potentials in the conventional form has been applied to include non-locality parameters  $\beta(\text{Li}) = 0.1$ ,  $\beta(d) = 0.54$  and  $\beta(\alpha) = 0.20$ . The FFR analyses have been performed for both the bound and unbound regions using the normal optical, Michel and molecular forms of the potential. For the inclusion of parameters of the Michel and molecular potentials in the entrance channel, the optical model code SCAT2 [41] has also been modified.

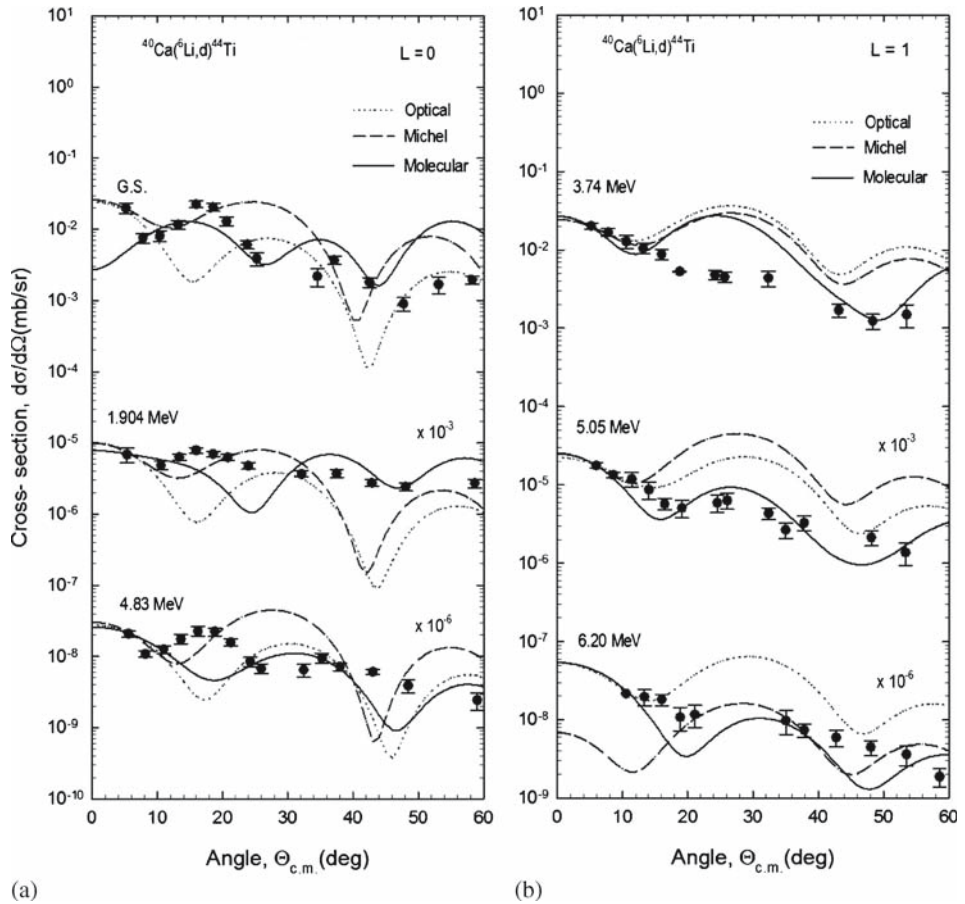
At the very beginning of the analyses, the accuracy parameters are fixed first and then the non-locality parameters are chosen. The parameters of the three optical potentials are



**Figure 1.** Fits to the  $^6\text{Li}+^{40}\text{Ca}$  elastic scattering data of [42] at 26 MeV with the normal optical, Michel and molecular potentials.

taken from Hossain [42] in the entrance channel. Its one-shot fit for the elastic scattering is displayed in figure 1. In the exit channel, a single set of parameters are taken from Daehnick *et al* [43] for the normal optical, Michel and molecular potentials. The spin-orbit parameters are obtained from Yamaya *et al* [3], which is a modification of Umeda *et al* [44]. The first and the second bound state parameters are taken from Delbar *et al* [45] and Kubo *et al* [46], respectively. It is to be noted that for the bound state of  $^{44}\text{Ti}$  for the FFR calculations,  $\alpha$ -transfer wave function has been computed by adjusting the normal optical potential well depth so that its eigenvalue equals the separation energy [47].

As the target has zero spin, the orbital angular momentum of the bound  $\alpha$  is  $L = J$  and the spin of the residual nucleus and parity,  $\pi = (-1)^L$  is normal. The number of nodes  $n_r$  in the bound-state radial wave function are found from the Talmi–Moshinsky relationship,  $N = 2n_r + L$ , where  $N$  is the primary quantum number of harmonic oscillator states. In  $\alpha + ^{40}\text{Ca}$  system,  $N = 12$  and  $13$  [2,3] correspond to the positive-parity and negative-parity

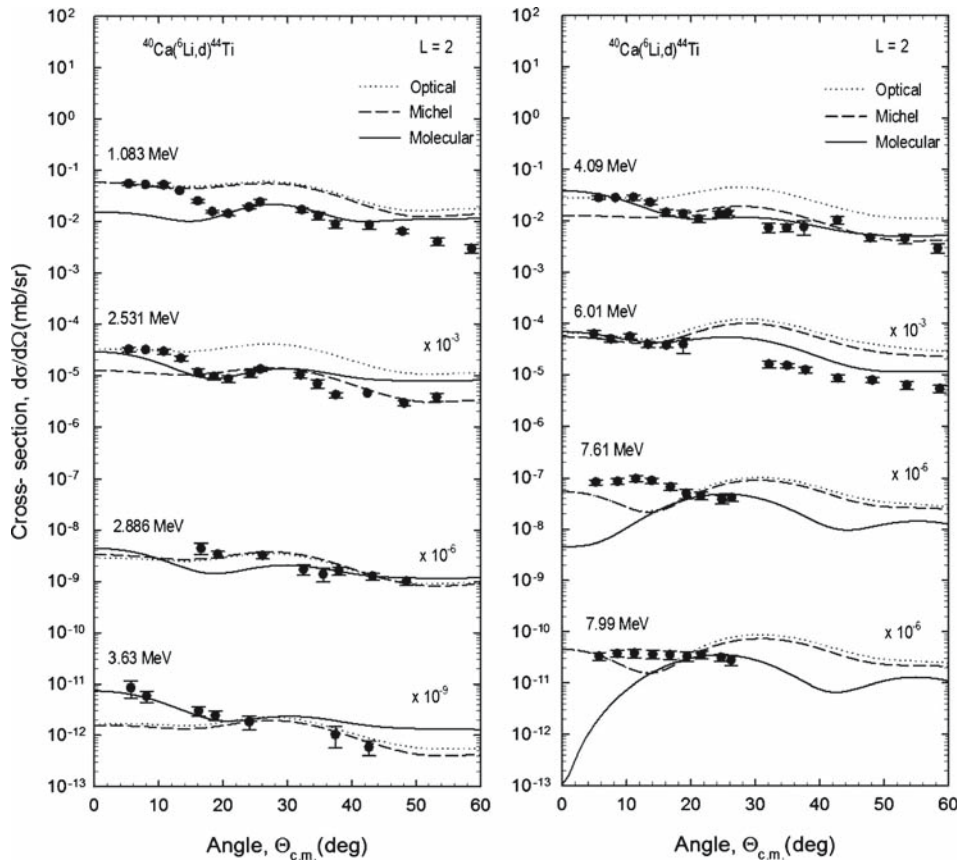


**Figure 2.** Full finite-range DWBA fits to the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  angular distribution using normal optical (dotted curves), Michel (dashed curves) and molecular (solid curves) potentials associated with (a)  $L = 0$  and (b)  $L = 1$  transfers.

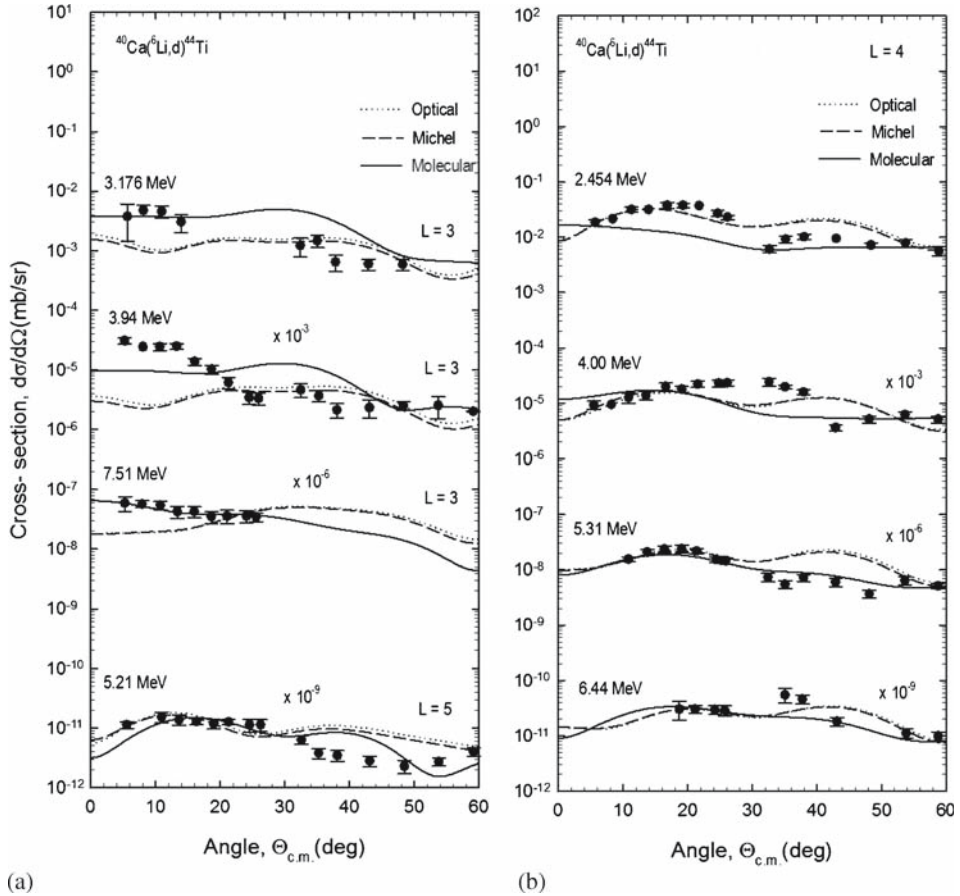
band states, respectively. Starting from  $L = 0$  transfer for  $n_r = 6$  in the ground-state transition, we have proceeded systematically towards  $L = 5$  transfer for higher excited states to reproduce the experimental data after choosing suitable  $n_r$ ,  $Q$  and binding energy values. The angular distributions for the data using the three optical potentials such as, the normal optical, Michel and molecular potentials for different  $L$  transfers are given in figures 2–4 without using lower cut-off radius in the  $\alpha$ -channel.

### 5. Results and discussion

In the present study, 22 transitions for both the bound and unbound states have been analysed using the normal optical, Michel and molecular potentials within the framework of the full finite-range DWBA method for the  $^{40}\text{Ca}(^6\text{Li},d)^{44}\text{Ti}$  reaction at  $E_{\text{Li}} = 28$  MeV incident energy. The analyses involve (i) three transitions with  $L = 0$  transfer leading



**Figure 3.** Full finite-range DWBA fits to the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  angular distribution using normal optical (dotted curves), Michel (dashed curves) and molecular (solid curves) potentials associated with  $L = 2$  transfers.



**Figure 4.** Full finite-range DWBA fits to the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  angular distribution using normal optical (dotted curves), Michel (dashed curves) and molecular (solid curves) potentials associated with (a)  $L = 3, 5$  and (b)  $L = 4$  transfers.

to the ground state (GS), 1.904 and 4.83 MeV states, (ii) three transitions with  $L = 1$  transfer leading to the 3.74, 5.05 and 6.20 MeV states, (iii) eight transitions with  $L = 2$  transfer leading to the 1.083, 2.531, 2.886, 3.63, 4.09, 6.01, 7.61 and 7.99 MeV states, (iv) three transitions with  $L = 3$  transfer for the 3.176, 3.94 and 7.51 MeV states, (v) four transitions with  $L = 4$  transfer for the 2.454, 4.00, 5.31 and 6.44 MeV states and (vi) one transition with  $L = 5$  transfer leading to the 5.21 MeV state. The parameters of different optical potentials are given in table 1. The extracted spectroscopic factors and  $\chi^2$  values of different levels for the normal optical, Michel and molecular potentials are given in tables 2 and 3, respectively.

In this study, a single set of the normal optical, Michel and molecular potentials has been used in the entrance channel to analyse the data. The fits to the elastic scattering data [42] for three optical potentials are shown in figure 1. It has been observed that the molecular potential fits the elastic scattering data at almost all the angles. The other two forms of optical potentials such as, the normal optical and Michel potentials, fit the elastic

**Table 1.** Parameters of the normal optical, Michel and molecular potentials.  $V$  is adjusted to give the separation energy.

| Channel<br>Potential type | ${}^6\text{Li} + {}^{40}\text{Ca}$ |                     |                        | $d + {}^{44}\text{Ti}$ | $\alpha + {}^{40}\text{Ca}$ | $\alpha + d$             |
|---------------------------|------------------------------------|---------------------|------------------------|------------------------|-----------------------------|--------------------------|
|                           | Optical <sup>a</sup>               | Michel <sup>a</sup> | Molecular <sup>a</sup> | Optical <sup>b</sup>   | Bound state <sup>c</sup>    | Bound state <sup>d</sup> |
| $V_0$ (MeV)               | 214.8                              | 241                 | 54.2                   | 86.72                  | $V$                         | $V$                      |
| $R_0$ (fm)                | 3.31                               | 4.31                | 5.46                   |                        |                             |                          |
| $r_0$ (fm)                |                                    |                     |                        | 1.17                   | 1.37                        | 1.20                     |
| $a_0$ (fm)                | 0.968                              | 0.78                | 0.46                   | 0.757                  | 1.20                        | 0.65                     |
| $V_1$ (MeV)               |                                    |                     | 53.5                   |                        |                             |                          |
| $R_1$ (fm)                |                                    |                     | 3.70                   |                        |                             |                          |
| $\alpha$                  |                                    | 1.00                |                        |                        |                             |                          |
| $\rho$ (fm)               |                                    | 1.00                |                        |                        |                             |                          |
| $W_0$ (MeV)               | 9.40                               | 11                  | 7.20                   | 0.975                  | 0.50                        |                          |
| $R_W$ (fm)                | 6.771                              | 7.18                | 6.16                   |                        |                             |                          |
| $r_1$ (fm)                |                                    |                     |                        | 1.325                  | 1.75                        |                          |
| $a_1$ (fm)                | 0.66                               | 0.44                |                        | 0.762                  | 1.00                        |                          |
| $W_D$ (MeV)               |                                    |                     |                        |                        |                             |                          |
| $r_D$ (fm)                |                                    |                     |                        |                        |                             |                          |
| $a_D$ (fm)                |                                    |                     |                        |                        |                             |                          |
| $V_{SO}$ (MeV)            |                                    |                     |                        | 3.50                   |                             |                          |
| $r_{SO}$ (fm)             |                                    |                     |                        | 1.05                   |                             |                          |
| $a_{SO}$ (fm)             |                                    |                     |                        | 0.85                   |                             |                          |
| $r_c$ (fm)                | 1.30                               | 1.30                |                        | 1.30                   | 1.30                        | 1.00                     |
| $R_c$ (fm)                |                                    |                     | 9.00                   |                        |                             |                          |

<sup>a</sup>ref. [42].

<sup>b</sup>ref. [43].

<sup>c</sup>ref. [45].

<sup>d</sup>ref. [46].

data as the same quality of the molecular potential at forward angles but at large scattering angles above  $80^\circ$ , the fits are not satisfactory. This is because, the optical potentials that have been used in the analysis are no longer global potentials. The fits to the experimental data for various  $L$  transfers using all three optical potentials are compared in figures 2–4 for 22 levels, which are grouped according to the associated  $L$  transfers. In the case of normal optical potential, eight transitions out of 22 fit the data quite satisfactorily for all the angles. For most of the other transitions, the normal optical potential fits the data satisfactorily at forward angles  $<15^\circ$ , but at large scattering angles this potential fits the experimental data with less accuracy. The Michel and normal optical potentials fit the data with the same accuracy in all the transitions except the ground state (GS), 1.904 and 4.83 MeV states for  $L = 0$  transition (figure 2a). The full-finite range DWBA calculation, using the molecular potential, satisfies the experimental data of 14 transitions out of 22 with eight other states fitting moderately (figures 2–4). The analyses show that no potential completely fits all the states. It is a four-nucleon transfer reaction analysis. The mismatch of different states for the fits to the transfer data may be originated by the compound nucleus contributions as mentioned by Jankowski *et al* [13] in the two-nucleon transfer reaction. But considering the quality of fits to the elastic and



**Table 2.** States of  $^{44}\text{Ti}$  observed in the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  reaction at  $E_{\text{Li}} = 28$  MeV.

| $E_x$<br>(MeV) | $L$ | $J^\pi$        | $(2J_f + 1)C^2S_s$ |        |           |          |          |          |
|----------------|-----|----------------|--------------------|--------|-----------|----------|----------|----------|
|                |     |                | Optical            | Michel | Molecular | Ref. [8] | Ref. [2] | Ref. [5] |
| GS             | 0   | 0 <sup>+</sup> | 0.03               | 0.06   | 0.08      | 0.94     | 1.00     | 0.20     |
| 1.083          | 2   | 2 <sup>+</sup> | 0.28               | 0.25   | 0.14      | 0.48     | 0.46     | 0.20     |
| 1.904          | 0   | 0 <sup>+</sup> | 0.02               | 0.02   | 0.04      | 0.32     | 0.27     |          |
| 2.454          | 4   | 4 <sup>+</sup> | 0.15               | 0.13   | 0.07      | 0.50     | 0.29     | 0.18     |
| 2.531          | 2   | 2 <sup>+</sup> | 0.19               | 0.06   | 0.08      | 0.32     | 0.26     |          |
| 2.886          | 2   | 2 <sup>+</sup> | 0.02               | 0.02   | 0.01      |          | 0.046    |          |
| 3.176          | 3   | 3 <sup>-</sup> | 0.01               | 0.01   | 0.01      | 0.01     | 0.032    |          |
| 3.63           | 2   | 2 <sup>+</sup> | 0.01               | 0.01   | 0.01      |          | 0.024    |          |
| 3.74           | 1   | 1 <sup>-</sup> | 0.09               | 0.07   | 0.06      | 0.14     | 0.17     |          |
| 3.94           | 3   | 3 <sup>-</sup> | 0.03               | 0.03   | 0.04      | 0.01     | 0.052    |          |
| 4.00           | 4   | 4 <sup>+</sup> | 0.08               | 0.08   | 0.06      | 0.56     | 0.34     |          |
| 4.09           | 2   | 2 <sup>+</sup> | 0.21               | 0.08   | 0.06      | 0.21     | 0.15     |          |
| 4.83           | 0   | 0 <sup>+</sup> | 0.07               | 0.11   | 0.03      | 1.00     | 0.67     |          |
| 5.05           | 1   | 1 <sup>-</sup> | 0.10               | 0.12   | 0.07      |          | 0.072    |          |
| 5.21           | 5   | 5 <sup>-</sup> | 0.08               | 0.06   | 0.06      | 0.01     | 0.046    |          |
| 5.31           | 4   | 4 <sup>+</sup> | 0.17               | 0.14   | 0.07      | 0.30     | 0.083    |          |
| 6.01           | 2   | 2 <sup>+</sup> | 0.84               | 0.56   | 0.25      | 0.80     |          |          |
| 6.20           | 1   | 1 <sup>-</sup> | 0.38               | 0.05   | 0.10      | 0.25     |          |          |
| 6.44           | 4   | 4 <sup>+</sup> | 0.28               | 0.25   | 0.16      | 0.56     | 0.72     |          |
| 7.51           | 3   | 3 <sup>-</sup> | 0.48               | 0.40   | 0.23      |          |          |          |
| 7.61           | 2   | 2 <sup>+</sup> | 0.88               | 0.66   | 0.44      |          |          |          |
| 7.99           | 2   | 2 <sup>+</sup> | 0.82               | 0.60   | 0.45      |          |          |          |

reaction data it has been found that the molecular potential provides a better description to the experimental data for some states as compared to the other two forms of optical potentials.

The spectroscopic factors (table 2) extracted using the normal optical, Michel and molecular potentials are compared with those obtained from Fullbright *et al* [2], Yeal Kim *et al* [8] and Michel *et al* [5], using the normal optical potential for the zero-range (ZR) approximation. The spectroscopic factors of the normal optical potential are almost higher than the molecular potential in all the levels except three transitions including the ground state (GS). The molecular potential spectroscopic factor is comparable to Michel potential but an inconsistency is found in the 4.83 MeV state for the  $L = 0$  transfer. The spectroscopic factor deduced from the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  reaction, using the zero-range calculation by Yeal Kim *et al* [8], has a large value compared to the three optical potentials except the 6.01 and 6.20 MeV states for the normal optical potential and the 3.94 and 5.21 MeV states for all the potentials. In the case of Fullbright *et al* [2], the spectroscopic factor extracted for some levels for this reaction are higher again than the normal optical, Michel and molecular potential in all the levels, without the 4.09 MeV state for the normal optical potential, the 5.05 and 5.31 MeV states for both the normal optical and Michel potentials and the 5.21 MeV state for all the optical potentials. For Michel *et al* [5], the comparison of spectroscopic factors for some levels are good for the three optical potentials except the ground state (GS). Generally there may be difference in magnitudes

**Table 3.**  $\chi^2$  values of the fits for different levels using the normal optical, Michel and molecular potentials.

| $E_x$<br>(MeV) | $L$ | $\chi^2$ |        |           |
|----------------|-----|----------|--------|-----------|
|                |     | Optical  | Michel | Molecular |
| GS             | 0   | 15.78    | 292.46 | 240.43    |
| 1.083          | 2   | 188      | 143    | 40.2      |
| 1.904          | 0   | 18.3     | 10.9   | 13.9      |
| 2.454          | 4   | 24.9     | 19.5   | 10.1      |
| 2.531          | 2   | 228      | 8.68   | 24.4      |
| 2.886          | 2   | 13.7     | 51.1   | 6.86      |
| 3.176          | 3   | 32.2     | 21.2   | 126       |
| 3.63           | 2   | 11.1     | 11.0   | 21.4      |
| 3.74           | 1   | 658      | 375    | 215       |
| 3.94           | 3   | 31.9     | 19.4   | 92.1      |
| 4.00           | 4   | 21.9     | 20.5   | 8.46      |
| 4.09           | 2   | 451      | 33.3   | 11.4      |
| 4.83           | 0   | 31.9     | 194    | 11.7      |
| 5.05           | 1   | 235      | 1160   | 7.48      |
| 5.21           | 5   | 23.6     | 20.4   | 11.9      |
| 5.31           | 4   | 46.9     | 46.6   | 3.55      |
| 6.01           | 2   | 447      | 381    | 27.1      |
| 6.20           | 1   | 447      | 17.5   | 13.0      |
| 6.44           | 4   | 8.62     | 5.19   | 3.35      |
| 7.51           | 3   | 9.71     | 10.7   | 0.19      |
| 7.61           | 2   | 20.9     | 44.2   | 17.8      |
| 7.99           | 2   | 13.5     | 11.9   | 13.8      |

for all levels. It is now accepted that the spectroscopic factors extracted by the method of full finite-range calculations are generally more reliable than zero-range calculations.

In order to estimate the quality of fits more quantitatively, the  $\chi^2$  values for the fits to the experimental data are calculated by the standard  $\chi^2$  formula. The calculations are performed for the normal optical, Michel and molecular potentials in all the 22 transitions for both the bound and unbound states of the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  reaction. In about 68% of the levels, the  $\chi^2$  values are lower for the molecular potential than for the normal optical and Michel potentials.

## 6. Conclusion

The present work of the  $^{40}\text{Ca}(^6\text{Li}, d)^{44}\text{Ti}$  reaction at  $E_{\text{Li}} = 28$  MeV has been analysed using the computer code DWUCK5 within the framework of the full finite-range DWBA method of direct reaction to analyse  $\alpha$  transfer data using three types of optical potentials such as, normal optical, Michel and molecular potentials for both the bound and unbound states of  $^{44}\text{Ti}$ . Here, the concentration on the comparison of the outputs by three different potentials are all done using the same method. So, the trends of the magnitude of the spectroscopic factors extracted by three different potentials have been mentioned in the

analysis. In the entrance channel, the three optical potential parameters are obtained from the  $^6\text{Li}+^{40}\text{Ca}$  elastic scattering at 26 MeV incident energy. These parameters are best for the  $^6\text{Li}+^{40}\text{Ca}$  elastic scattering data in this energy and obtained by minimizing the  $\chi^2$  values [42]. In this study, it has been observed that the normal optical, Michel and molecular potentials can moderately describe the angular distribution of the transitions involving both the bound and unbound states of  $^{44}\text{Ti}$ , but the molecular potential shows a better description of angular distributions for the reproduction of the experimental data for some of the states.

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