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Synchronization analysis of coloured delayed networks under decentralized pinning intermittent control

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Abstract. This paper investigates synchronization of coloured delayed networks under decentralized pinning intermittent control. To begin with, the time delays are taken into account in the coloured networks. In addition, we propose a decentralized pinning intermittent control for coloured delayed networks, which is different from that most of pinning intermittent controls are only applied to the nodes from 1 to l or centralized nodes. Moreover, sufficient conditions are derived to guarantee the synchronization of coloured delayed networks based on Lyapunov stability theorem. Finally, numerical simulations are provided to verify the validity of the obtained results.

Keywords. Coloured networks; decentralized pinning control; intermittent control; time delay.

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1. Introduction

Recently, synchronization, as a collective behaviour of complex networks, has been a hot research area in various fields including physics [1,2], engineering science [3] and biology [4]. A lot of research is being conducted in this direction [2–7], due to two main reasons – truly reflect the phenomena in real world and the potential application in various areas.

Up to now, many network models on synchronization have been put forward, such as, the small-world network, directed network, neural network etc. Previous efforts were mainly to study the outer relationship between the nodes. But, the inner interaction is always overlooked. Afterwards, the coloured network model has been brought in this scope by Wu *et al* [8]. A brief introduction of coloured network is reviewed as follows: In social networks, there are many relationships between individuals, e.g., between schoolmates, relatives and collaborators. Individuals i and j may be either schoolmates or collaborators but have no relative relationship, while individuals i and k $(k \neq j)$, may only have collaborative relationship. Similarly, the interaction between nodes in real complex systems also shows diverse properties.

By virtue of the new properties, it is an interesting scheme to achieve the synchronization of coloured networks. In ref. [8], pinning control and adaptive coupling strength methods were first proposed to make the presented network achieve synchronization with an arbitrarily given orbit. Afterwards, Su *et al* [9] employed two discrete controls such as intermittent control and an impulsive controller to synchronize two coloured networks. That paves the way for understanding the synchronization behaviours of coloured networks via different control schemes. However, some significant factors are not taken into account, such as the time delays and the more economical control, which motivates the current study.

In the real world, the time delay cannot be avoided for modelling real complex systems because of the finite speed of switching or information transfer [10–12]. Hence, this should be considered as an important index to analyse synchronization. On the other hand, complex networks are composed of a large number of nodes. That means, it is difficult to apply control techniques to every node. According to the facts, pinning control, which is applied to a small fraction of network nodes, is an effective control pattern. Many literatures have even reported the synchronization of complex networks via pinning control [13–16]. In addition, pinning intermittent control was also proposed to achieve synchronization of complex networks [15,16]. However, a common feature of the works presented in earlier literature is that there are fixed pinned nodes from 1 to l or centralized nodes. Although some research was done about this problem in ref. [17], few works concern the decentralized pinning intermittent control, and especially the application to coloured networks. Therefore, a novel technique is presented to explore the synchronization of coloured networks.

Inspired by the above discussions, this paper aims at the synchronization of coloured delayed network via decentralized pinning intermittent control. The time delay in the coloured network is first taken into account. In addition, a new control scheme for reducing control energy is presented. Moreover, sufficient conditions for ensuring synchronization of coloured delayed networks are derived by virtue of Lyapunov stability theory. Finally, two examples are shown to mimic the proposed methods, and validate the correctness of the results obtained.

The rest of this paper is organized as follows. In §2, a general coloured delayed network is presented. In §3, some sufficient conditions for general synchronization are derived by decentralized pinning intermittent control. Numerical examples are given in §4. Finally, the conclusions are drawn in §5.

2. Modelling description

In this section, the dynamic model of coloured delayed network, simple introduction to coloured network and useful conditions are presented.

Consider a general coloured delayed network consisting of N linearly and diffusively coupled nodes:

$$\dot{x}_i(t) = F_i(t, x_i(t), x_i(t-\tau(t))) + \varepsilon \sum_{j=1, j\neq i}^{N} a_{ij} H_{ij}(x_j(t) - x_i(t)), \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), ..., x_{in}(t))^{\mathrm{T}} \in R^n$ $(i = 1, 2, ..., N)$ captures the state vector of the ith node at a certain time t; $F_i(t, x_i(t), x_i(t-\tau(t))) : R \times R^n \times R^n$ denotes the local dynamic or individual behaviour, which is continuous differentiable. The conditions of

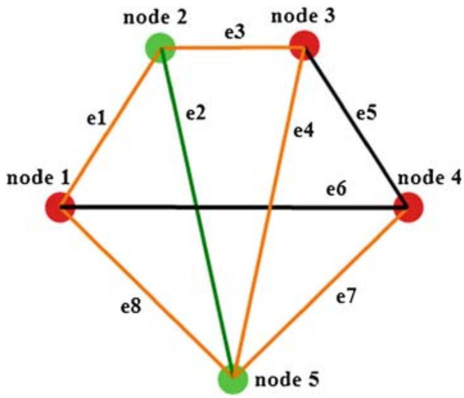

Figure 1. A coloured network consisting of five coloured nodes and eight coloured edges.

time delay is only bounded but not limited to be differentiable. The matrix $A = (a_{ij})_{N\times N}$ indicates the network topology. If there is an interaction or information transfer between nodes i and j, then $a_{ij} > 0$, otherwise, $a_{ij} = 0$; $H_{ij} = \mathrm{diag}(h^1_{ij}, h^2_{ij}, ..., h^n_{ij})$ is the inner coupling matrix defined as follows: if the ζth component of node i is affected by that of node j, then $h^{\zeta}_{\mathrm{in}} \neq 0$, otherwise, $h^{\zeta}_{\mathrm{in}} = 0$.

Here a succinct description of coloured network is given in order to understand the inner structure of the coloured network. Figure 1 describes a coloured network consisting of five coloured nodes and eight coloured edges. It also indicates $F_1 = F_3 = F_4$, $F_2 = F_5$, $H_{12} = H_{32} = H_{35} = H_{45} = H_{15}$ and $H_{14} = H_{34}$. The inner connection of three nodes is shown in figure 2. Then we can obtain $H_{13} = \mathrm{diag}(1, 0, 1)$ and $H_{34} = \mathrm{diag}(0, 1, 0)$. That is, the first and third components of node 1 are affected by those of node 3; similarly, the second component of node 3 is affected by that of node 4.

Now let $c_{ij} = \mathrm{diag}(c^1_{ij}, c^2_{ij}, ..., c^n_{ij})$, where $c^k_{ij} = a_{ij}h^k_{ij}$ for $i \neq j$, and $c^k_{ii} = -\sum_{j=1, j\neq i}^{N} c^k_{ij}$. Then the coloured delayed network (1) can be rewritten as follows.

$$\dot{x}_i(t) = F_i(t, x_i(t), x_i(t-\tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} x_j(t). \tag{2}$$

Let $C_{\xi} = (c^{\xi}_{ij}) \in R^{N\times N}$, $\xi = 1, 2, ..., n$, the coloured delayed network (2) can be regarded as a combination of n-component subnetworks with a topology determined by C_{ξ}. The main objective of this paper is to apply decentralized pinning intermittent control to make the coloured delayed networks (2) synchronize with $s(t)$, i.e., $\lim_{t\to\infty} \|x_i(t) - s(t)\| = 0, i = 1, 2, \ldots, N$, where $s(t) = s(t, \tau(t); t_0, x_0) \in R^n$ be

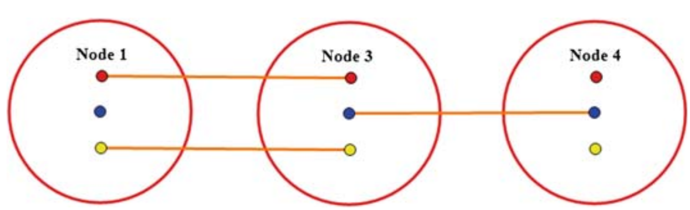

Figure 2. The inner connection of nodes 1, 3 and 4.

a solution of dynamics of the isolated node $\dot{s}(t) = F(t, s(t), s(t-\tau(t)))$. $s(t)$ may be an equilibrium point, a limit cycle, or even a chaotic attractor. To begin with, define the error system $e_i(t) = x_i(t) - s(t)$, $i = 1, 2, ..., N$. On account of the proposed model, then the networks (2) under pinning intermittent controls are described:

$$\dot{x}_i(t) = F_i(t, x_i(t), x_i(t-\tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} x_j(t) + d_i(t) h_i(x_i(t) - s(t)), \quad (3)$$

where $h_i = 1$, if node i is selected to be pinned, otherwise, $h_i = 0$; $d_i(t)$ is the intermittent feedback control gain, which is defined as

$$d_i(t) = \begin{cases} -d_i, & t \in [mT, mT+\delta); \\ 0, & t \in [mT+\delta, (m+1)T), \qquad m = 0, 1, 2, \ldots, \end{cases} \quad (4)$$

where $d_i > 0$ is a positive control gain, $T > 0$ is the control period, $\delta > 0$ is the control width. Then the error dynamical system is derived as

$$\begin{cases} \dot{e}_i(t) = \hat{F}_i(t, x_i(t), x_i(t-\tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} x_j(t) - h_i d_i(t) e_i(t), \\ mT \leq t < (m+\theta)T; \\ \dot{e}_i(t) = \hat{F}_i(t, x_i(t), x_i(t-\tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} x_j(t), \\ (m+\theta)T \leq t < (m+1)T, \end{cases} \quad (5)$$

where

$$\hat{F}_i(t, x_i(t), x_i(t-\tau(t))) = F_i(t, x_i(t), x_i(t-\tau(t))) - F_i(t, s(t), s(t-\tau(t))).$$

Before presenting the main results, the following assumption and lemma are required.

ASSUMPTION 1

For any $x, y \in R^n$, the vector-valued function $F(t, x(t), x(t-\tau(t)))$ is uniformly continuous and there exist constants L_1 and $L_2 > 0$ satisfying

$$\begin{aligned} &[x(t) - y(t)]^{\mathrm{T}}[F(t, x(t), x(t-\tau(t))) - F(t, y(t), y(t-\tau(t)))] \\ &\leq L_1[x(t) - y(t)]^{\mathrm{T}}[x(t) - y(t)] \\ &+ L_2[x(t-\tau(t)) - y(t-\tau(t))]^{\mathrm{T}}[x(t-\tau(t)) - y(t-\tau(t))]. \end{aligned} \quad (6)$$

Lemma 1 [16]. *Let* $0 \leq \tau(t) \leq \tau$. *If* $y(t)$ *is a continuous and non-negative function and satisfies the following conditions:*

$$\begin{cases} \dot{y}(t) \leq -\gamma_1 y(t) + \gamma_2 y(t-\tau(t)), & nT \leq t < (n+\theta)T, \\ x(t) = \varphi(t), \quad -\tau \leq t \leq 0; & \\ \dot{y}(t) \leq \gamma_3 y(t) + \gamma_2 y(t-\tau(t)), & (n+\theta)T \leq t < (n+1)T, \\ y_i(t) = \Phi(t), \quad -\tau \leq t \leq 0; & n \in \{0, 1, 2, \ldots\}, \end{cases}$$

where $\gamma_1 > \gamma_2 > 0$, $\delta = \gamma_1 + \gamma_3 > 0$ *and* $\eta = \lambda - \delta(1-\theta) > 0$, *in which* $\lambda > 0$ *is the only positive solution of function* $\lambda - \gamma_1 + \gamma_2 \exp(\lambda\tau) = 0$, *we can obtain* $y(t) \leq \sup_{-\tau \leq s \leq 0} y(s) \exp(-\eta t)$, $t \geq 0$.

3. Main results

In this section, the synchronization criteria of error system (5) are investigated by employing the Lyapunov stability theory. Before presenting the results, we state that the Assumption 1 is satisfied.

Theorem 1. *The coloured delayed network can gradually achieve synchronization with s(t) if the following conditions hold*:

(1) $a_1 = -(L_1 + \lambda_{\max}(\varepsilon C - M \otimes I_{N\times N})) > 0$;
(2) $a_3 - a_1 < 0$;
(3) $\bar{w} = \varphi - 2a_2(1-\theta) > 0$,

where $M = (h_i d_i)_{n\times n}$, $a_2 = \lambda_{\max}(M)$, $a_3 = L_2$, $\varphi > 0$ is the unique positive solution of equation $\varphi - 2a_1 + 2a_3 \exp\{\varphi\tau\} = 0$ and $\lambda_{\max}(\bullet)$ denotes the maximum eigenvalue of the matrix.

Proof. Construct a non-negative Lyapunov function with respect to time t:

$$V(t) = \frac{1}{2} e^{\mathrm{T}}(t) e(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t).$$

As the definition of $s(t)$, it is not hard to obtain $\sum_{j=1}^{N} c_{ij} s(t) = 0$ for $i = 1, 2, \ldots, N$. Therefore,

$$\sum_{j=1}^{N} c_{ij} x_j(t) = \sum_{j=1}^{N} c_{ij}(x_j(t) - s(t)) = \sum_{j=1}^{N} c_{ij} e_j(t).$$

When $mT \le t < (m+\theta)T$, $m = 1,2,\ldots$, take the derivative of $V(t)$ along the error systems with respect to t. Combined with Assumption 1, we can obtain

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) \\
&= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\left[\hat{F}_i(t, x_i(t), x_i(t-\tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} x_j(t) - h_i d_i(t) e_i(t)\right] \\
&\le (L_1 - h_i d_i)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) + L_2 \sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau(t)) e_i(t-\tau(t)) \\
&\quad + \varepsilon \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij} e_i^{\mathrm{T}}(t) e_j(t).
\end{aligned}$$

$$\begin{aligned}
\dot{V}(t) &\le (L_1 + \lambda_{\max}(\varepsilon C - M \otimes I_{N\times N}))\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \\
&\quad + L_2 \sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau(t)) e_i(t-\tau(t)) \\
&\le -2a_1 V(t) + 2a_3 V(t-\tau(t)),
\end{aligned}$$

where $M = (h_i d_i)_{n\times n}$, $a_1 = -(L_1 + \lambda_{\max}(\varepsilon C - M \otimes I_{N\times N})$ and $a_3 = L_2$.

When $(m+\theta)T \leq t < (m+1)T, m = 1, 2, \ldots,$

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) \\
&= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)[\hat{F}_i(t, x_i(t), x_i(t-\tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} e_j(t) \\
&\leq L_1 \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) + L_2 \sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau(t)) e_i(t-\tau(t)) + \varepsilon \sum_{i=1}^{N} c_{ij} e_i^{\mathrm{T}}(t) e_j(t) \\
&\leq 2(a_2 - a_1)V(t) + 2a_3 V(t-\tau(t)),
\end{aligned}$$

where $a_2 = \lambda_{\max}(M)$.

According to Lemma 2 and the conditions in Theorem 1, naturally we can obtain $V(t) \leq \sup_{-\tau \leq s \leq 0} V(s)\exp(-\bar{w}t), t \geq 0$. As $\bar{w} > 0$, the Lyapunov function converges to zero when time tends to infinity. That is, the coloured delayed network can achieve synchronization with $s(t)$ under the proposed control. That completes the proof. □

Remark 1. Although the pinning control and intermittent control are mature to guarantee synchronization of complex networks, the decentralized pinning intermittent control techniques are firstly considered on the convergence behaviours of coloured networks. To our best knowledge, very few papers have been written in this area of research. This work especially can be extended to general networks such as BA networks, small-world models and so forth.

4. Numerical simulation

In this section, two quintessential examples are adopted to demonstrate the validity and to reduce conservatism of the obtained results.

Example 1. Consider an edge-coloured network with ten coupled time-delayed Lorenz systems:

$$F(t, x(t), t(t-\tau(t))) = Ax(t) + g_1(x(t)) + g_2(x(t-\tau(t))),$$

where

$$x(t) = (x_1, x_2, x_3)^{\mathrm{T}}, \quad g_1(x(t)) = (0, -x_1x_3, x_1x_2)^{\mathrm{T}},$$

$$g_2(x(t)) = (0, \sigma_0 x_2(t), 0)^{\mathrm{T}}$$

$$A = \begin{pmatrix} -a_0 & a_0 & 0 \\ r_0 & \sigma_0 - 1 & 0 \\ 0 & 0 & -b_0 \end{pmatrix},$$

and

$$a_0 = 10, \quad b_0 = 8/3, \quad r_0 = 28, \quad \sigma_0 = 5, \quad \tau = 0.1.$$

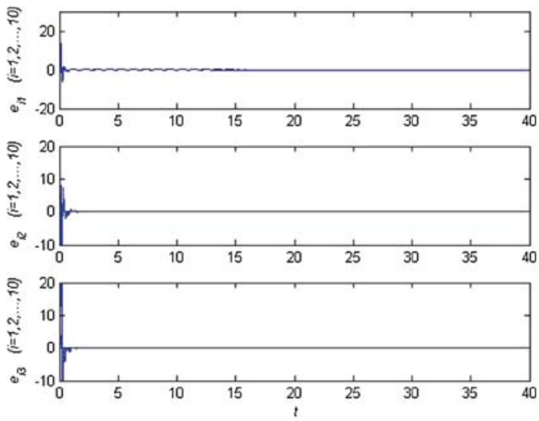

Figure 3. Synchronization errors of the edge-coloured network.

In numerical simulation, the initial values of drive–response system are chosen as $x_i(0) = (0.3 + 0.1i, 0.3 + 0.1i, 0.3 + 0.1i)^{\mathrm{T}}$. The control period, control width and control strength are set as $T = 0.7$, $\delta = 0.97$, $d_i = 8$. Figure 3 depicts the synchronization errors of the edge-coloured delayed network. It indicates that the error system addressed via the proposed controller can converge to zero. In other words, the coloured network under decentralized pinning intermittent control can globally synchronize with the isolated node $s(t)$.

Example 2. Consider a general coloured network, whose topology is coupled with four time-delayed Lorenz systems and four time-delayed Chua systems. The Chua system is described as follows:

$$\dot{x}(t) = F(t, x(t), t - \tau(t)) = Ax(t) + h_1(x(t)) + h_2(x(t - \tau(t))),$$

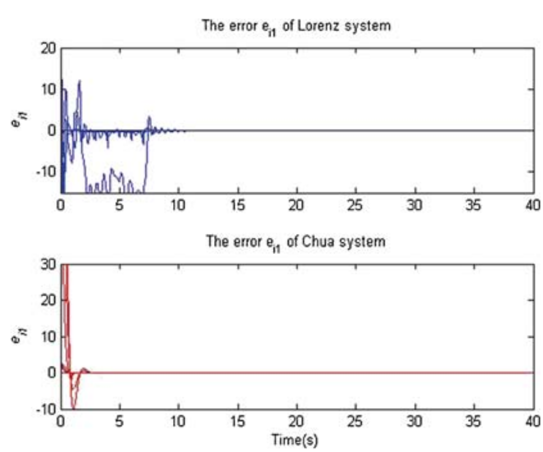

Figure 4. Synchronization error e_1 of a general coloured network.

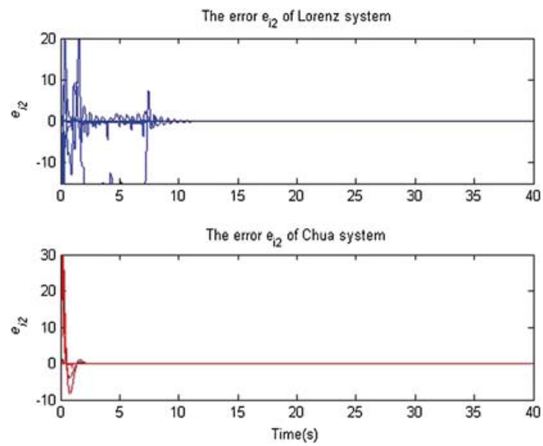

Figure 5. Synchronization error e_2 of a general coloured network.

where

$$x(t) = (x_1, x_2, x_3)^{\mathrm{T}},$$

$$h_1(x(t)) = (-1/2\alpha(m_1 - m_2)(|x_1(t) + 1| - |x_1(t) - 1|), 0, 0)^{\mathrm{T}},$$

$$h_2(x(t)) = (0, 0, -\beta\rho_0 \sin(\nu x_1(t - \tau(t))))^{\mathrm{T}},$$

$$A = \begin{pmatrix} -\alpha(1 + m_2) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & -\omega \end{pmatrix}.$$

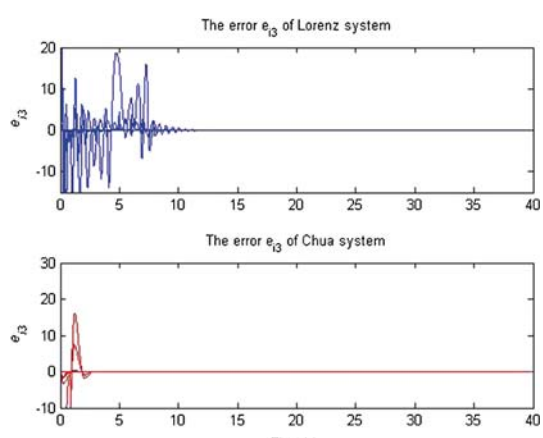

Figure 6. Synchronization error e_3 of a general coloured network.

In addition, $\alpha = 10, \beta = 19.53, \omega = 0.1636, m_1 = -14.325, m_2 = -0.7831, \nu = 0.5, \rho_0 = 0.2$ and $\tau(t) = 0.02$. The control period, control width and control strength are set as $T = 1.0$, $\delta = 0.90$, $d_i = 10$. The initial values of the drive–response system are chosen the same as in Example 1. The synchronization errors of a general coloured network are plotted in figures 4–6, particularly, the errors of the Lorenz system and Chua system. From the figures we can see that the given controller can enforce the general coloured delayed network gradually to the desired objective. Therefore, we can draw the conclusions that the general coloured delayed network under the given controller can achieve synchronization with the isolated node.

5. Conclusion

In this paper, decentralized pinning intermittent control is employed to investigate synchronization of coloured delayed networks. This application weakens the restrictions of pinned nodes only from 1 to l or central nodes via intermittent control. Specially, we consider a coloured delayed network model. There are only a few papers which consider the time delay in coloured network. Furthermore, simple and useful criteria for the synchronization of coloured delayed networks have been established. Two typical examples are provided to support the effectiveness of the theoretical results.

Acknowledgements

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