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Hybrid synchronization of two independent chaotic systems on complex network

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Abstract. The real network nodes are always interfered by other messages. So, how to realize the hybrid synchronization of two independent chaotic systems based on the complex network is very important. To solve this problem, two other problems should be considered. One is how the same network node of the complex network was affected by different information sources. Another is how to achieve hybrid synchronization on the network. In this paper, the theoretical analysis and numerical simulation on various complex networks are implemented. The results indicate that the hybrid synchronization of two independent chaotic systems is feasible.

Keywords. Hybrid synchronization; complex network; information source; chaotic system.

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1. Introduction

In the past several decades, synchronization has attracted increasing attention in the field of complex network. The chaotic synchronization on a complex network has been investigated extensively [1–11] and it has been applied in various fields such as life sciences [12,13], mechanical engineering, secure communications [14–16], etc. The concept of chaotic synchronization was proposed by Pecora and Carroll from the navy lab of the United States [17] in 1990. And then, many effective methods of synchronization were proposed in succession, such as pinning synchronization method [18–20], adaptive synchronization method [21,22], local synchronization method [23–26] and pulse synchronization method [27,28]. However, almost all the methods mentioned above, have a presupposition that the signal received by all nodes on the networks come from a single information source. But, in practice, the nodes of network are always interfered by various signals, and it is difficult to ensure a pure and single information source. In other words, the same node is usually affected by more than two information sources at the same time. Thus, it is very important to know how to realize hybrid synchronization on complex networks. To solve this problem, the synchronization of two independent chaotic systems was investigated on different networks, and an adaptive controller was designed. Theoretical analysis and numerical simulations indicated that the proposed method was reasonable and feasible.

In secure communication, generally, two same chaotic systems were adopted to realize encryption and decryption through synchronization. However, the system is not secure enough because of the short secret key which relates to the parameters of the chaotic system. Certainly, if the two systems are different, the security would be improved. How to realize synchronization of two different chaotic systems is the focus of this paper.

The rest of the paper is organized as follows: in the next section, some basic concepts and problems are described, In §3, the local synchronization of hybrid state is investigated. In §4, the global synchronization of hybrid state is also investigated. In §5, numerical simulations on different networks are implemented. Finally, conclusions are given in §6.

2. Problem description

Suppose, two nonlinear dynamic systems are described as follows:

$$\dot{X} = f(X), \tag{1}$$

$$\dot{Y} = g(Y) + \Gamma. \tag{2}$$

Here, $X \in \Re^n$, $Y \in \Re^n$ are state vectors. $f\colon B \subseteq \Re \to \Re$, $g\colon B \subseteq \Re \to \Re$ are nonlinear smooth vector fields. $\Gamma \in [-1, 1]$ is a noise system. Therefore, the dynamics network system can be described by

$$\dot{X}_i = f(X_i) + \sum_{j=1}^{N} C_{ij}\phi(t, X_j), \quad i = 1, 2, \ldots, N, \tag{3}$$

$$\dot{Y}_i = g(Y_i) + \sum_{j=1}^{N} C_{ij}\phi(t, Y_j) + \Gamma(t), \quad i = 1, 2, \ldots, N. \tag{4}$$

Here, $X_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in B$ and $Y_i = (y_{i1}, y_{i2}, \ldots, y_{in})^T \in B$ represent two different signals of the ith vertex. $\phi\colon B \in \Re$ is an unknown nonlinear smooth diffusion coupling function. $\sum_{j=1}^{N} C_{ij}\phi(t, X_j)$ and $\sum_{j=1}^{N} C_{ij}\phi(t, Y_j)$ characterize the couple-term of the ith vertex which has been affected by other vertices through the network connections. The network is composed of N vertices, and every vertex is identical just like the description of eqs (3) and (4). $C_{ij} \in \Re$ is the coupling configuration matrix which represents coupling strength and topological structure on the network. It can also be called an outer-coupling matrix. Here, C_{ij} is defined as: if the ith vertex and the jth vertex have a connection, $C_{ij} \neq 0 (i \neq j)$; otherwise, $C_{ij} = 0 (i \neq j)$. The element of opposite angles C_{ii} is defined in the following. Here, $|\Upsilon|$ stands for the degree of the ith vertex.

$$C_{ii} = -\sum_{\substack{j=1 \\ i \neq j}}^{N} C_{ij} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} C_{ji} = \Upsilon, \quad i = 1, 2, \ldots, N. \tag{5}$$

Suppose that the network is connected in such a way that there are no isolated clusters, that is, C_{ij} is an irreducible matrix. If all vertices have equal coupling strength, C_{ij} would only describe the topological structure of the network. For simplicity, we suppose, in this paper, that all the coupling strengths are identical and that coupling strength of any two vertices is 1, i.e., $c = 1$.

Considering the coupling strengths, the systems eqs (3) and (4) also can be rewritten as follows:

$$\dot{X}_i = f(X_i) + c\sum_{j=1}^{N} C_{ij}\phi(t, X_j), \quad i = 1, 2, \ldots, N \tag{6}$$

$$\dot{Y}_i = g(Y_i) + c\sum_{j=1}^{N} C_{ij}\phi(t, Y_j) + \Gamma(t), \quad i = 1, 2, \ldots, N. \tag{7}$$

In reality, the nodes of the network are not independent, and they are interfered by all sorts of information. Therefore, the hybrid synchronization of two independent chaotic systems on complex network should be studied. We have to consider the interaction between two chaotic systems, and at the same time, observe and measure the hybrid state of the chaotic systems. In some aspects, these are worth to research and discuss.

Suppose the hybrid state can be expressed as $Z = M(Z)$. In this paper, we only consider the linear superposition of two-systems state, thus Z can be described as

$$Z = M(Z) = X + Y. \tag{8}$$

Here, $Z = (z_1, z_2, \ldots z_m)^T \in \Re$ is the hybrid state vector. Because both X and Y are nonlinear dynamic systems, Z is a new nonlinear dynamic system.

For studying synchronization, this paper denotes time as a variable, so X and Y can be denoted by $X_i(t, X_0)$ and $Y_i(t, Y_0)$ $(i = 1, 2, \ldots N)$. $M(Z_1(t)), M(Z_2(t)), \ldots, M(Z_N(t))$ and $M(S(t))$ represent hybrid status, and the nonautonomous complex dynamic network is described by eqs (3) and (4). If eq. (9) is workable, $M(Z_1(t)), M(Z_2(t)), \ldots, M(Z_N(t))$ and $M(S(t))$ can achieve hybrid state synchronization.

$$M(Z_1(t)) \to M(Z_2(t)) \to \cdots \to M(Z_N(t)) \to M(S(t)). \tag{9}$$

In other words:

$$\lim_{t\to\infty} \|M(Z_i(t)) - M(S(t))\|_2 = 0, \quad i = 1, 2, \ldots, N. \tag{10}$$

Here, $S = (s_1, s_2, \ldots, s_n)^T \in W$ is the solution of an isolated node and satisfies $\dot{S} = k(S)$. Besides, $S(t)$ can be represented as an equilibrium point, a periodic orbit or an arbitrary chaotic attractor.

3. The local synchronization of hybrid states of two independent chaotic systems

In order to achieve the synchronization of a hybrid state, a controller u_i is designed, which can control two systems at the same time. Here, we describe the control power system as follows:

$$\dot{X}_i = f(X_i) + \sum_{j=1}^{N} C_{ij}\phi(t, X_j) + u_i, \quad i = 1, 2, \ldots, N, \tag{11}$$

$$\dot{Y}_i = g(Y_i) + \sum_{j=1}^{N} C_{ij}\phi(t, Y_j) + \Gamma(t) + u_i, \quad i = 1, 2, \ldots, N. \tag{12}$$

System error vector is:

$$e_i(t) = Z_i(t) - S(t) = (e_{i,1}, e_{i,2}, \ldots, e_{i,n})^T, \quad i = 1, 2, \ldots, N. \tag{13}$$

Then error dynamic system is:

$$\begin{aligned} \dot{e}_i &= \dot{Z}_i(t) - \dot{S}(t) \\ &= f(X_i) - k(S) + \sum_{j=1}^{N} C_{ij}[\phi(X_i) - \phi(S)] + u_i. \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{e}_i' &= \dot{Z}_i(t) - \dot{S}(t) \\ &= g(Y_i) - k(S) + \sum_{j=1}^{N} C_{ij}[\phi(X_i) - \phi(S)] + \Gamma(t) + u_i. \end{aligned} \tag{15}$$

The deviation vector is defined as

$$\begin{aligned} \Delta_i &= M(Z_i(t)) - M(S(t)) = (\delta_{i,1}, \delta_{i,2}, \delta_{i,3}, \ldots, \delta_{\mathrm{i},m})^{\mathrm{T}} \\ &= \{[M(X_i(t)) - M(S(t))] + [M(Y_i(t)) - M(S(t))]\} \\ &= e_i(t) + e_i(t)'. \end{aligned} \tag{16}$$

If the system achieves synchronization, then

$$\lim_{i\to\infty} \|\Delta_i\|_2 = \lim_{i\to\infty} \|e_i(t) + e_i(t)'\|_2 = 0. \tag{17}$$

Linearizing the error dynamical systems formula (14) and (15) around zero, which can be described by

$$\dot{e}_i = Ae_i(t) + \sum_{j=1}^{N} C_{ij}(\phi(X_j) - \phi(S)) + u_i, \quad i = 1, 2, \ldots, N, \tag{18}$$

$$\dot{e}_i' = De_i(t) + \sum_{j=1}^{N} C_{ij}(\phi(Y_j) - \phi(S)) + \Gamma(t) + u_i, \quad i = 1, 2, \ldots, N. \tag{19}$$

Here, $Df(S) = A \in R^{N\times N}$ is the Jacobian of f evaluated at $X_i = S(t)$ and $Dg(S) = D \in R^{N\times N}$ is the Jacobian of g evaluated at $Y_i = S(t)$.

From the above formulas, finding that $\|A\|_2$ and $\|D\|_2$ have a limit, we can suppose

$$\|A\|_2 \le \delta \tag{20}$$

$$\|D\|_2 \le \delta. \tag{21}$$

Here, δ is a non-negative constant. Because $\|\phi(X_j) - \phi(S)\|_2$, $\|\phi(Y_j) - \phi(S)\|_2$ and $\|\Delta_i\|_2$ are bounded, there must exist a non-negative constant η satisfying

$$\|\phi(X_j) - \phi(S)\|_2 \le \eta \|\Delta_i\|_2, \quad i = 1, 2, \ldots, N, \tag{22}$$

$$\|\phi(Y_j) - \phi(S)\|_2 \le \eta \|\Delta_i'\|_2, \quad i = 1, 2, \ldots, N. \tag{23}$$

From $Z = M(Z) = X + Y$, an adaptive controller can be designed:

$$u_i = -\sigma_i \Delta_i = -\sigma_i(\dot{e}_i + \dot{e}_i'), \quad i = 1, 2, \ldots, N, \tag{24}$$

and

$$\dot{\sigma}_j = 2\|\Delta_i\|_2^2 = 2\|(\dot{e}_i + \dot{e}_i')\|_2^2, \quad i = 1, 2, \ldots, N. \tag{25}$$

From eqs (13) and (16), let $\gamma = \min\left\{e_{i,1}, e_{i,2}, \ldots, e_{i,n}\right\}$ denotes the minimum nontrivial eigenvalue and γ is a constant.

$$\|\Delta_i\|_2 = \|(\dot{e}_i + \dot{e}_i')\|_2 \ge \gamma \|e_i\|_2 \quad \text{and} \quad \|e_i\|_2 \le \frac{\|\Delta_i\|_2}{\gamma}, \tag{26}$$

$$\|\Delta_i\|_2 = \|(\dot{e}_i + \dot{e}_i')\|_2 \ge \gamma \|\dot{e}_i'\|_2 \quad \text{and} \quad \|\dot{e}_i'\|_2 \le \frac{\|\Delta_i\|_2}{\gamma}. \tag{27}$$

Define a Lyapunov candidate function as follows:

$$V = \sum_{i=1}^{N} \left[\Delta_i^T \Delta_i + \left(\sigma_i - 2\eta|C_{ii}| - \frac{\delta}{\gamma} - \theta - \frac{\Gamma(t)}{2\|\Delta_i\|_2}\right)^2\right], \quad i = 1, 2, \ldots, N. \tag{28}$$

Here, $\theta > 0$ is a constant. From eqs (11)–(28):

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{N} (\dot{\Delta}_i^T \Delta_i + \Delta_i^T \dot{\Delta}_i) + 2\sum_{i=1}^{N} \left(\sigma_i - 2\eta|C_{ii}| - \frac{\delta}{\gamma} - \theta - \frac{\Gamma(t)}{2\|\Delta_i\|_2}\right) \dot{\sigma}_i \\
&= \sum_{i=1}^{N} [(\dot{e}_i + \dot{e}_i')^T \Delta_i + \Delta_i^T (\dot{e}_i + \dot{e}_i')] \\
&\quad + 2\sum_{i=1}^{N} \left(\sigma_i - 2\eta|C_{ii}| - \frac{\delta}{\gamma} - \theta - \frac{\Gamma(t)}{2\|\Delta_i\|_2}\right) \dot{\sigma}_i, \\
&= \sum_{i=1}^{N} \Bigg[(A+D)e_i(t) + \sum_{j=1}^{N} C_{ij}(\phi(X_j) + \phi(Y_j) - 2\phi(S)) + \Gamma(t) + 2u_i)^T \Delta_i \\
&\quad + \Delta_i^T ((A+D)e_i(t) + \sum_{j=1}^{N} C_{ij}(\phi(X_j) + \phi(Y_j) - 2\phi(S)) + \Gamma(t) + 2u_i)\Bigg] \\
&\quad + 2\sum_{i=1}^{N} \left(\sigma_i - 2\eta|C_{ii}| - \frac{\delta}{\gamma} - \theta - \frac{\Gamma(t)}{2\|\Delta_i\|_2}\right) \dot{\sigma}_i
\end{aligned}$$

$$= \sum_{i=1}^{N}\Bigg[(A+D)e_i(t)+\sum_{j=1}^{N}C_{ij}(\phi(X_j)+\phi(Y_j)-2\phi(S))+\Gamma(t)-2\sigma_i\Delta_i)^T\Delta_i$$

$$+\Delta_i^T((A+D)e_i(t)+\sum_{j=1}^{N}C_{ij}(\phi(X_j)+\phi(Y_j)-2\phi(S))+\Gamma(t)-2\sigma_i\Delta_i)\Bigg]$$

$$+2\sum_{i=1}^{N}\left(\sigma_i-2\eta|C_{ii}|-\frac{\delta}{\gamma}-\theta-\frac{\Gamma(t)}{2\|\Delta_i\|_2}\right)\dot{\sigma}_i$$

$$= \sum_{i=1}^{N}\Bigg[-4\sigma_i\Delta_i^T\Delta_i+2\Delta_i^T(A+D)e_i(t)$$

$$+2\sum_{j=1}^{N}C_{ij}(\phi(X_j)+\phi(Y_j)-2\phi(S))\Delta_i^T+2\Gamma(t)\Delta_i\Bigg]$$

$$+2\sum_{i=1}^{N}\left(\sigma_i-2\eta|C_{ii}|-\frac{\delta}{\gamma}-\theta-\frac{\Gamma(t)}{2\|\Delta_i\|_2}\right)\dot{\sigma}_i$$

$$\leq \sum_{i=1}^{N}\left[-4\sigma_i\|\Delta_i\|_2^2+4\frac{\delta}{\gamma}\|\Delta_i\|_2^2+2\Gamma(t)\|\Delta_i\|_2+4\eta\|\Delta_i\|_2^2\sum_{j=1}^{N}C_{ij}\right]$$

$$+2\sum_{i=1}^{N}\left(\sigma_i-2\eta|C_{ii}|-\frac{\delta}{\gamma}-\theta-\frac{\Gamma(t)}{2\|\Delta_i\|_2}\right)\dot{\sigma}_i$$

$$= \sum_{i=1}^{N}\left[-4\sigma_i\|\Delta_i\|_2^2+4\frac{\delta}{\gamma}\|\Delta_i\|_2^2+2\Gamma(t)\|\Delta_i\|_2+4\eta\|\Delta_i\|_2^2\sum_{j=1}^{N}C_{ij}\right]$$

$$+4\sum_{i=1}^{N}\left(\sigma_i-2\eta|C_{ii}|-\frac{\delta}{\gamma}-\theta-\frac{\Gamma(t)}{2\|\Delta_i\|_2}\right)\|\Delta_i\|_2^2$$

$$= -4\sum_{i=1}^{N}\theta\|\Delta_i\|_2^2 \leq 0.$$

When $t \to \infty$, deviation vector tends to be zero, i.e., $\Delta_i \to 0$, and $\Delta_i \to 0$ means the local hybrid synchronization of complex dynamic network system was realized.

4. The global synchronization of hybrid states of two independent chaotic systems

Suppose that dynamical system consists of a linear part and a nonlinear part. Then, the system can be described as follows:

$$f(X,t) = BX(t) + h(X,t), \tag{29}$$

$$g(Y,t) = EY(t) + h(Y,t) + \Gamma(t), \tag{30}$$

where both $B \in \Re^{n\times n}$ and $E \in \Re^{n\times n}$ are constant matrixes. $h\colon R^n \times R^+ \to R^n$ is a nonlinear function and R^+ denotes time as a variable. $\Gamma(t) \in [-1,1]$ is the noise system. According to eqs (11) and (12), the vertex dynamic system can be rewritten as follows:

$$\dot{X}_i = BX_i(t) + h(X_i, t) + \sum_{j=1}^{N} C_{ij}\phi(X_j, t) + u_i, \quad i = 1, 2, \ldots, N, \tag{31}$$

$$\dot{Y}_i = EY_i(t) + h(Y_i, t) + \sum_{j=1}^{N} C_{ij}\phi(Y_j, t) + \Gamma(t) + u_i, \quad i = 1, 2, \ldots, N. \tag{32}$$

The dynamic system of isolated vertexes is:

$$\dot{S} = BS_i(t) + h(S, t), \quad i = 1, 2, \ldots, N. \tag{33}$$

When $\tilde{h}(X_i, S, t) = h(X_i, t) - h(S, t), \tilde{\phi}(X_j, S) = \phi(X_j) - \phi(S), \tilde{h}(Y_i, S, t) = h(Y_i, t) - h(S, t)$ and $\tilde{\phi}(Y_j, S) = \phi(Y_j) - \phi(S)$, the error dynamic systems are described by

$$\dot{e}_i = Be_i(t) + \tilde{h}(X_i, S, t) + \delta\sum_{j=1}^{N} C_{ji}\tilde{\phi}(X_j, S) + u_i, \quad i = 1, 2, \ldots, N, \tag{34}$$

$$\dot{e}'_i = Ee_i(t) + \tilde{h}(Y_i, S, t) + \delta\sum_{j=1}^{N} C_{ji}\tilde{\phi}(Y_j, S) + \Gamma(t) + u_i, \quad i = 1, 2, \ldots, N. \tag{35}$$

Here, the coupling strength $\delta = 1$. Because $B \in \Re^{n\times n}$ and $E \in \Re^{n\times n}$ are constant matrixes, there must exist a non-negative constant β satisfying

$$\|B\|_2 \le \beta, \tag{36}$$

$$\|E\|_2 \le \beta. \tag{37}$$

Besides, $\|\tilde{h}(X_i, S, t)\|_2$, $\|\tilde{h}(Y_i, S, t)\|_2$ and $\|\Delta_i\|_2$ are bounded, and so there must exist a non-negative constant ψ satisfying

$$\|\tilde{h}(X_i, S, t)\|_2 \le \psi\|\Delta_i\|_2, \quad i = 1, 2, \ldots, N, \tag{38}$$

$$\|\tilde{h}(Y_i, S, t)\|_2 \le \psi\|\Delta_i\|_2, \quad i = 1, 2, \ldots, N. \tag{39}$$

Define a Lyapunov candidate function as

$$V = \sum_{i=1}^{N}\left[\Delta_i^T\Delta_i + \left(\sigma_i - 2\eta|C_{ii}| - \frac{\delta}{\gamma} - \frac{\Gamma(t)}{2\|\Delta_i\|_2} - \psi - \theta\right)^2\right], \quad i = 1, 2, \ldots, N. \tag{40}$$

Here, $\theta > 0$ is a constant. From eqs (16), (18)–(40):

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{N}(\dot{\Delta}_i^T\Delta_i+\Delta_i^T\dot{\Delta}_i)+2\sum_{i=1}^{N}\left(\sigma_i-2\eta|C_{ii}|-\frac{\delta}{\gamma}-\frac{\Gamma(t)}{2\|\Delta_i\|_2}-\psi-\theta\right)\dot{\sigma}_i\\
&= \sum_{i=1}^{N}[(\dot{e}_{i\partial}+\dot{e}_i')^T\Delta_i+\Delta_i^T(\dot{e}_{i\partial}+\dot{e}_i')]\\
&\quad +2\sum_{i=1}^{N}\left(\sigma_i-2\eta|C_{ii}|-\frac{\delta}{\gamma}-\frac{\Gamma(t)}{2\|\Delta_i\|_2}-\psi-\theta\right)\dot{\sigma}_i\\
&= \sum_{i=1}^{N}\Bigg\{\Bigg[\Bigg(Be_i(t)+\tilde{h}(X_i,S,t)+\delta\sum_{j=1}^{N}C_{ji}\tilde{\phi}(X_j,S)+u_i+Ee_i(t)\\
&\qquad +\tilde{h}(Y_i,S,t)+\sum_{j=1}^{N}C_{ji}\tilde{\phi}(Y_j,S)+\Gamma(t)+u_i\Bigg)^T\Delta_i\Bigg]\\
&\quad +\Bigg[\Delta_i^T\Bigg(Be_i(t)+\tilde{h}(X_i,S,t)+\delta\sum_{j=1}^{N}C_{ji}\tilde{\phi}(X_j,S)+u_i+Ee_i(t)\\
&\qquad +\tilde{h}(Y_i,S,t)+\sum_{j=1}^{N}C_{ji}\tilde{\phi}(Y_j,S)+\Gamma(t)+u_i\Bigg)\Bigg]\Bigg\}\\
&\quad +2\sum_{i=1}^{N}\left(\sigma_i-2\eta|C_{ii}|-\frac{\delta}{\gamma}-\frac{\Gamma(t)}{2\|\Delta_i\|_2}-\psi-\theta\right)\dot{\sigma}_i.\\
&= \sum_{i=1}^{N}\Bigg\{\Bigg[\Bigg((B+E)e_i(t)+\tilde{h}(X_i,S,t)+\tilde{h}(Y_i,S,t)\\
&\quad +\sum_{j=1}^{N}C_{ji}[\tilde{\phi}(X_j,S)+\tilde{\phi}(Y_j,S)]+\Gamma(t)+2u_i\Bigg)^T\Delta_i\Bigg]\\
&\quad +\Bigg[\Delta_i^T\Bigg((B+E)e_i(t)+\tilde{h}(X_i,S,t)+\tilde{h}(Y_i,S,t)\\
&\quad +\sum_{j=1}^{N}C_{ji}[\tilde{\phi}(X_j,S)+\tilde{\phi}(Y_j,S)]+\Gamma(t)+2u_i\Bigg)\Bigg]\Bigg\}\\
&\quad +2\sum_{i=1}^{N}\left(\sigma_i-2\eta|C_{ii}|-\frac{\delta}{\gamma}-\frac{\Gamma(t)}{2\|\Delta_i\|_2}-\psi-\theta\right)\dot{\sigma}_i.\\
&= \sum_{i=1}^{N}\Bigg[(B+E)e_i(t)(\Delta_i+\Delta_i^T)+\tilde{h}(X_i,S,t)(\Delta_i+\Delta_i^T)\\
&\quad +\tilde{h}(Y_i,S,t)(\Delta_i+\Delta_i^T)
\end{aligned}$$

$$+2\Gamma(t)\Delta_i + (\Delta_i + \Delta_i^T)\sum_{j=1}^{N} C_{ji}[\tilde{\phi}(X_j, S) + \tilde{\phi}(Y_j, S)] + 2u_i(\Delta_i + \Delta_i^T)\Bigg]$$

$$+2\sum_{i=1}^{N}\left(\sigma_i - 2\eta|C_{ii}| - \frac{\delta}{\gamma} - \frac{\Gamma(t)}{2\|\Delta_i\|_2} - \psi - \theta\right)\dot{\sigma}_i$$

$$\leq \sum_{i=1}^{N}\Bigg[4\frac{\beta}{\gamma}\|\Delta_i\|_2^2 + 2\psi\|\Delta_i\|_2^2 + 2\psi\|\Delta_i\|_2^2 + 2\Gamma(t)\|\Delta_i\|_2$$

$$+4\eta\sum_{j=1}^{N} C_{ij}\|\Delta_i\|_2^2 - 4\sigma_i\|\Delta_i\|_2^2\Bigg]$$

$$+2\sum_{i=1}^{N}\left(\sigma_i - 2\eta|C_{ii}| - \frac{\delta}{\gamma} - \frac{\Gamma(t)}{2\|\Delta_i\|_2} - \psi - \theta\right)\dot{\sigma}_i$$

$$= \sum_{i=1}^{N}\left[4\frac{\beta}{\gamma}\|\Delta_i\|_2^2 + 4\psi\|\Delta_i\|_2^2 + 2\Gamma(t) + 4\eta\sum_{j=1}^{N} C_{ij}\|\Delta_i\|_2^2 - 4\sigma_i\|\Delta_i\|_2^2\right]$$

$$+2\sum_{i=1}^{N}\left(\sigma_i - 2\eta|C_{ii}| - \frac{\delta}{\gamma} - \frac{\Gamma(t)}{2\|\Delta_i\|_2} - \psi - \theta\right)\dot{\sigma}_i$$

$$= \sum_{i=1}^{N}\left[4\frac{\beta}{\gamma} + 4\psi + 8\eta|C_{ii}| + \frac{2\Gamma(t)}{\|\Delta_i\|_2} - 4\sigma_i\right]\|\Delta_i\|_2^2$$

$$+4\sum_{i=1}^{N}\left(\sigma_i - 2\eta|C_{ii}| - \frac{\beta}{\gamma} - \frac{\Gamma(t)}{2\|\Delta_i\|_2} - \psi - \theta\right)\|\Delta_i\|_2^2$$

$$= -4\theta\sum_{i=1}^{N}\|\Delta_i\|_2^2 \leq 0.$$

When $t \to \infty$, deviation vector tends to be zero, i.e., $\Delta_i \to 0$. $\Delta_i \to 0$ means that the global hybrid synchronization of complex dynamic network system was realized.

5. The numerical simulation

Let Lorenz system ($a = 0$) and Lü system ($b = 0.8$) be two different and unified chaotic systems, and the network be composed of 100 identical nodes. The node dynamics are respectively described as follows:

$$\begin{cases} \dot{x}_{i1} = (25a + 10)(x_{i2} - x_{i1}) \\ \dot{x}_{i2} = (28 - 35a)x_{i1} - x_{i1}x_{i3} + (29a - 1)x_{i2} \\ \dot{x}_{i3} = x_{i1}x_{i2} - \dfrac{a+8}{3}x_{i3} \end{cases} \tag{41}$$

$$\begin{cases} \dot{y}_{i1} = (25b + 10)(y_{i2} - y_{i1}) \\ \dot{y}_{i2} = (28 - 35b)y_{i1} - y_{i1}y_{i3} + (29b - 1)y_{i2} \\ \dot{y}_{i3} = y_{i1}y_{i2} - \dfrac{b+8}{3}y_{i3} \end{cases}. \tag{42}$$

Here $X_i = (x_{i1}, x_{i2}, x_{i3})^T$, $Y_i = (y_{i1}, y_{i2}, y_{i3})^T$. When $a \in [0, 1]$ and $b \in [0, 1]$, the dynamical system of node i would achieve chaos.

Due to $Z = M(Z) = X + Y$, $\phi(X_i) = X_i$ and $\phi(Y_i) = Y_i$, the network system can be defined as

$$\begin{cases} \dot{X}_i = f(X_i) + \sum_{j=1}^{N} c_{ij}X_j + u_i \\ \dot{Y}_i = g(Y_i) + \sum_{j=1}^{N} c_{ij}Y_j + \Gamma(t) + u_i \\ u_i = -\sigma_i \Delta_i = e_i + e_i' \\ \dot{\sigma}_i = 2\|\Delta_i\|_2^2 \end{cases}. \tag{43}$$

The initialization value is $\sigma_i = 5$; $s(1) = 4$; $s(2) = 6$; $s(3) = 8$; $x(i, 1) = 20$*Rand; $x(i, 2) = 20$*Rand; $x(i, 3) = -20$*Rand; $y(i, 1) = 10$*Rand; $y(i, 2) = 10$*Rand; $y(i, 3) = -10$*Rand; and $\Gamma(t) = \sin(t)$. Here, Rand $\in (0, 1)$ is a random number.

Case 1. Globally coupled network

Select the outer coupling matrix as follows:

$$c = \begin{pmatrix} -99 & 1 & \cdots & 1 \\ 1 & -99 & \cdots & 1 \\ \vdots & \vdots & \ddots & 1 \\ 1 & 1 & \cdots & -99 \end{pmatrix}. \tag{44}$$

Case 2. Ring network

Select the outer coupling matrix as follows:

$$c = \begin{pmatrix} -2 & 1 & \cdots & 1 \\ 1 & -2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & -2 \end{pmatrix}. \tag{45}$$

Case 3. Nearest-neighbour network

Select the outer coupling matrix as follows:

$$c = \begin{pmatrix} -10 & 1 & \cdots & 1 \\ 1 & -10 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & -10 \end{pmatrix}. \tag{46}$$

Case 4. Scale-free network

The algorithm of BA scale-free network is designed as follows: first, when m_0, the number of vertex a network, is very small, then one new vertex is added to the network with every step, and the new one is connected to other m-nodes. Thus, the probability of one border connected to the old vertex (also called parent vertex) is $p_i = k_i/\sum_j k_j$, k_i is the level of the vertex i. We select $\langle k \rangle = 2m = 10$, and the total number of the vertex $N = 100$.

Case 5. Small-world network

The algorithm of network is designed as follows: Consider that there is a ring network that has N-nodes. Each node connects to its neighbouring and nearest m-nodes by undirected edge, the border of each node and other nodes of network are reconnected randomly, and the probability of reconnection is p. They also need to avoid repetitive borders. When $p = 1$, the network can be completely random. Here, we select $k = m = 10$, probability $p = 0.5$ and the total number of the vertex $N = 100$.

In the simulations, the abscissa represents time, the ordinate represents the hybrid state error of chaotic systems, and each line represents the status of each node. Here, x_1, x_2 and x_3 respectively are the linear accumulation of x_i and y_i on chaotic system; here $i = 1, 2, 3$. When the hybrid state error of chaotic systems tends to be zero, it means that the two independent chaotic systems achieve synchronization. Figure 1 shows the hybrid state error evolution on global network, figure 2 shows the hybrid state error evolution on ring network, figure 3 shows the hybrid state error evolution on nearest-neighbour network, figure 4 shows the hybrid state error evolution on scale-free network and figure 5

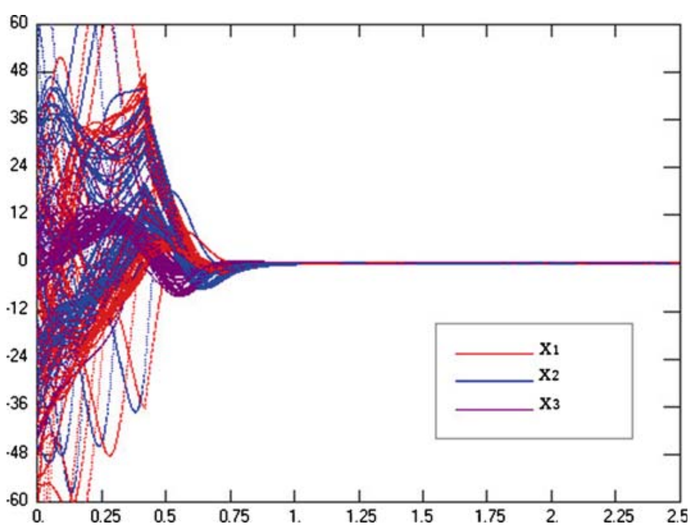

Figure 1. Hybrid state error evolution of two independent chaotic systems on global network.

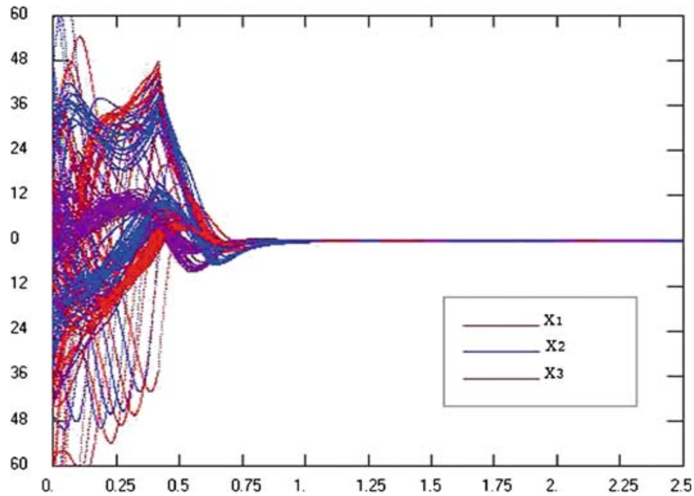

Figure 2. Hybrid state error evolution of two independent chaotic systems on ring network.

shows the hybrid state error evolution on small-world network. From figure 1 to figure 5, we can see that the hybrid synchronization of two independent chaotic systems were realized on global network, ring network, nearest-neighbour network, scale-free network and small-world network respectively. At the same time, we find that the noise system has an effect on the chaotic system, but does not affect the hybrid synchronization of the two independent chaotic systems.

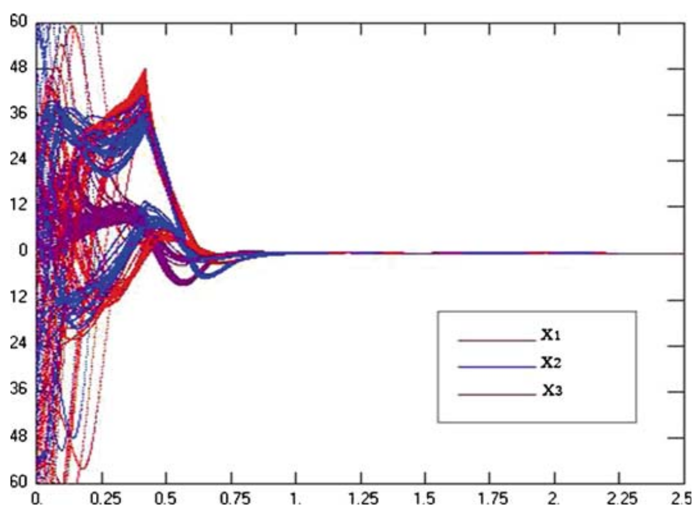

Figure 3. Hybrid state error evolution of two independent chaotic systems on nearest-neighbour network.

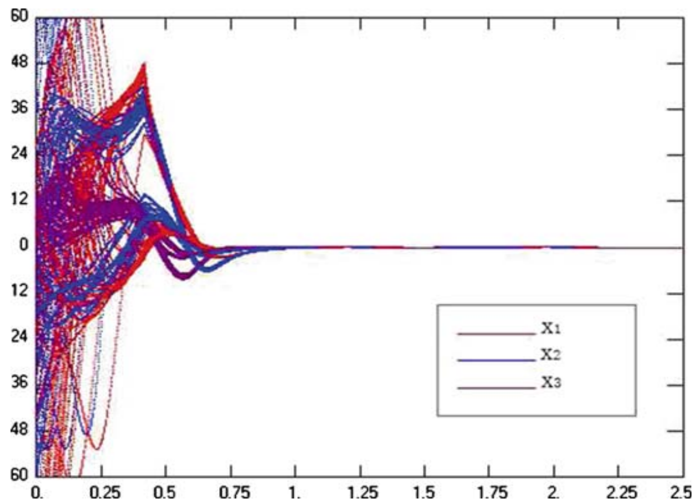

Figure 4. Hybrid state error evolution of two independent chaotic systems on scale-free network.

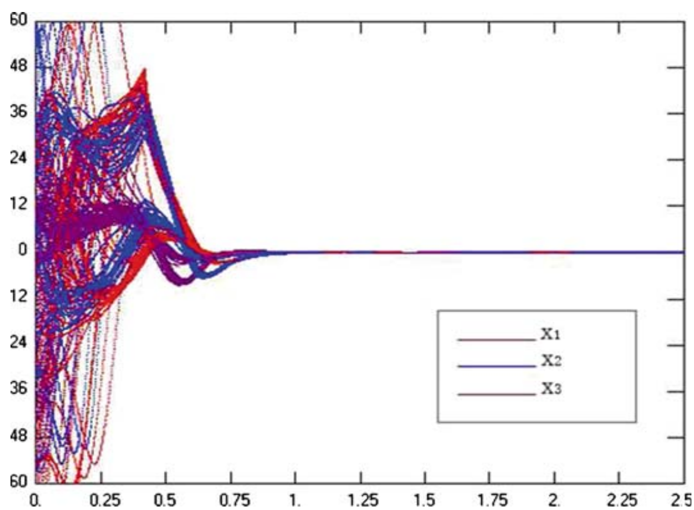

Figure 5. Hybrid state error evolution of two independent chaotic systems on small-world network.

6. Conclusion

In the past, the synchronization problem of single linear system or nonlinear system was investigated extensively. However, as a network node is often interfered by all kinds of signals, it is difficult to maintain single information source. There exist two or more than two signal sources affecting one node in the network. Thus, two problems were considered in this paper. One is how the same network node of the complex network was affected by different information sources. Another is how to achieve hybrid synchronization on the network. The simulations were implemented on the globally coupled network, ring network, nearest-neighbour network, scale-free network and small-world network respectively. Theoretical analysis and numerical simulations are implemented. The results show that the noise system has an effect on the chaotic system, but does not affect the hybrid synchronization of the two independent chaotic systems, and the hybrid synchronization of two independent chaotic systems is feasible.

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