



Ion waves driven by shear flow in a relativistic degenerate astrophysical plasma

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Abstract. We investigate the existence and propagation of low-frequency (in comparison to ion cyclotron frequency) electrostatic ion waves in highly dense inhomogeneous astrophysical magnetoplasma comprising relativistic degenerate electrons and non-degenerate ions. The dispersion equation is obtained by Fourier analysis under mean-field quantum hydrodynamics approximation for various limits of the ratio of rest mass energy to Fermi energy of electrons, relevant to ultra-relativistic, weakly-relativistic and non-relativistic regimes. It is found that the system admits an oscillatory instability under certain condition in the presence of velocity shear parallel to ambient magnetic field. The dispersive role of plasma density and magnetic field is also discussed parametrically in the scenario of dense and degenerate astrophysical plasmas.

Keywords. Relativistic degenerate plasma; dispersion; velocity shear.

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1. Introduction

The recent surge of interest in Fermi degenerate (quantum) plasmas has different backgrounds but mainly driven by prospective applications of such plasmas in contemporary science and technology. Such plasmas are exotic because interelectron distance is of the order of electron de Broglie wavelength owing to the extreme number density and temperature. In this state, the electron Fermi energy surpasses the thermal energy due to restrictions of Pauli exclusion principle and quantum mechanics comes into play. Such plasmas are naturally found in compressed astrophysical objects, for instance, the interiors of white dwarfs and neutron stars, magnetars, core of Jovian planets (Jupiter, Saturn), etc. [1–3]. The examples of quantum plasmas in laboratory are the quantum electron

gas in metals and semiconductor nanostructures [4,5], electron–hole plasmas in semiconductors [4,6] and dense plasmas suited for inertial fusion experiments [1,7]. The latter can also be seen in the laboratory where the astrophysical conditions are produced as a result of impinging ultraintense (petawatt) laser beams on highly compressed targets [8,9], commonly known as warm dense matter (WDM). The density, magnetic field and temperature of these degenerate plasmas vary over a wide range of values. For instance, the bulk electron number density in white dwarfs is $\sim 10^{30} \text{ cm}^{-3}$ and the magnetic field varies from a few kilogauss to gigagauss range. Plasma under such condition becomes quantum degenerate. However, the relativity effects depend upon the ratio of electron Fermi energy and rest mass energy. In non-relativistic degenerate limit, $E_{\text{Fe}} < m_e c^2$. However, for $E_{\text{Fe}} > m_e c^2$, the plasma becomes relativistic and degenerate [1,2].

Collective oscillations can provide effective probe for various dynamical properties of matter inside dense stellar media [10,11]. Various studies on low-frequency dynamics of dense and relativistic degenerate plasmas have been performed in recent years. These include the investigation of electrostatic and electromagnetic modes in dense and relativistic degenerate plasmas and applications to white dwarfs by Shukla *et al* [12,13], modulational instability and non-linear evolution of electrostatic wave packets in a degenerate ultrarelativistic plasma using Schrödinger-like model by Misra and Shukla [14], existence of ion solitary waves in Fermi–Dirac electron–positron–ion plasmas using Poincaré–Lighthill–Kuo formalism by Moghanjoughi [15], low-frequency coupled modes in relativistic degenerate astrophysical plasmas by Khan [16,17], and so on. Mamun and Shukla [18] have investigated the non-planar shock wave dynamics on ion time-scale in a strongly-coupled degenerate plasma and described the process of shock formation for correlated ions. Formation of ion solitary waves in ultrarelativistic and degenerate plasma have been studied by Mamun and Shukla [19]. Similarly, ion-acoustic Korteweg–de Vries solitons in a relativistic degenerate plasma have been studied by Nahar *et al* [20] and Kadomtsev–Petviashvili-type solitary structures in relativistic degenerate astrophysical plasmas by Rahman *et al* [21]. The equation of state considered in all the foregoing studies of relativistic degenerate plasmas is based on the concept of electron degeneracy pressure given by Chandrasekhar in his classic work [22,23]. Chandrasekhar’s momentous discovery that white dwarfs had a maximum mass of $1.4M_{\odot}$ (also known as Chandrasekhar limit or Chandrasekhar mass where M_{\odot} is the solar mass) was based on the idea of degeneracy pressure of the relativistic electron Fermi gas which actually balances the gravitational pull of the star.

The non-uniform magnetized degenerate plasma in the presence of density inhomogeneity gives rise to drift mode [24,25]. In drift approximation, Shukla and Stenflo [26] have studied the propagation of low-frequency electromagnetic ion modes in a degenerate quantum plasma. In equilibrium, if velocity shear and density inhomogeneity are present in a magnetized plasma, the drift mode gets modified, which become unstable under certain conditions, as mentioned by D’Angelo long ago [27]. For highly dense plasmas, average interelectron distance (or Wigner–Seitz radius), $\bar{r} \sim n_e^{-1/3}$ is comparable to the thermal de Broglie wavelength, $\lambda_{\text{Be}} = \hbar/m_e v_{\text{te}}$ and the degeneracy parameter $\chi (= n_e \lambda_{\text{Be}}^3) \geq 1$. The plasma exhibits ideal behaviour when the Brueckner parameter is much smaller than unity [28,29]. For relativistic and degenerate plasmas, the ratio $p_{\text{Fe}}/m_e c \cong (n_e/5.9 \times 10^{29} \text{ cm}^{-3})^{1/3}$, where $p_{\text{Fe}} = (3\hbar^3 n_e/8\pi)^{1/3}$ is the Fermi relativistic momentum [15]. It can be seen that when $n_e \geq 10^{29} \text{ cm}^{-3}$, the Brueckner

parameter becomes ~ 0.01 which decreases with increase in electron density. The electron degeneracy pressure is so large that it dominates over thermal effects and the regime $E_{Fe} > m_e c^2$ can be described by quantum hydrodynamics.

In the present paper, our purpose is to investigate the electrostatic waves driven by parallel velocity shear in a non-uniform relativistic degenerate plasma. In the presence of free energy in inhomogeneous plasma, the parallel velocity gradient drives an oscillatory instability. We employ mean-field linearized quantum hydrodynamics and discuss the results for typical plasma parameters. The paper is organized as follows; In §2, the equations of the quantum hydrodynamics for electron–ion plasma are given. In §3, dispersion relation is obtained for relativistic as well as non-relativistic degenerate plasma cases. In §4, the results are discussed numerically, while in §5, the conclusion is given.

2. Mathematical formulation

Let us consider a highly dense electron–ion magnetoplasma which consists of fully degenerate and relativistic electrons and non-degenerate ions with ambient magnetic field $B_0(\mathbf{z})$ parallel to the z -axis, where \mathbf{z} is the unit vector. At equilibrium, we assume that the plasma contains gradients of ion velocity ∇v_0 and number density, ∇n_0 , where v_0 and n_0 are unperturbed quantities, i.e., streaming speed and number density, respectively. For simplicity, we assume singly-charged ions, unidirectional parallel velocity gradient $\partial v_0/\partial x$ and density gradient $\partial n_0/\partial x$. The low-frequency (in comparison with the ion cyclotron frequency $\Omega_{ci} = (e\mathbf{B}_0/m_i c)$, where m_i is the ion mass and c is the speed of light in vacuum) electrostatic electric field perturbations is given by $\mathbf{E} = -\nabla\phi$ with ϕ being the electrostatic wave potential. In such low-frequency limit ($\partial_t \ll \Omega_{ci}$), the quantum hydrodynamics equations, the momentum balance and continuity equations for the j th species can be written as

$$m_j n_j (\partial_t + \mathbf{v}_j \cdot \nabla) \mathbf{v}_j = n_j \mathbf{F}_j - \nabla P_j, \quad (1)$$

$$\partial_t n_j + \nabla \cdot (n_j \mathbf{v}_j) = 0, \quad (2)$$

where $j = e, i$ denotes the plasma species i.e., electrons and ions, respectively. The first term on the right-hand side of eq. (1) gives the Lorentz force, while the second term results from the Fermi degeneracy pressure. It is to be noted that the diffraction term in the hydrodynamic equation is neglected due to its smallness in very high-density plasmas [6]. The ions are assumed to follow classical dynamics, hence the pressure of ions as compared to the pressure of relativistic degenerate electrons is negligible.

The mean rate of electron momentum transport per unit area gives rise to degeneracy pressure. In fully degenerate and relativistic limit, this pressure can be given by [22,23]

$$P_j = A f(r), \quad (3)$$

where

$$A = \frac{\pi}{3} \left(\frac{mc}{h} \right)^3 mc^2 (= 6 \times 10^{22} \text{ (cgs)}),$$

$$f(r) = r\sqrt{1+r^2}(2r^2-3) + 3 \ln \left(r + \sqrt{1+r^2} \right),$$

$$r = p_{Fj}/mc \quad \text{and} \quad p_{Fj} = (3h^3 n_j / 8\pi)^{1/3}.$$

Equation (3) for electrons can be written as

$$P_e = K^\alpha n_e^\gamma, \quad (4)$$

where $\hbar = h/2\pi$ and $T_e \ll T_{Fe} (= E_{Fe}/k_B)$ with T_{Fe} being the Fermi temperature. The parameters K^α and γ vary for ultrarelativistic, weakly relativistic and non-relativistic degenerate electrons. In ultrarelativistic limit denoting $\alpha = ur$, eq. (4) leads to

$$P_e = K^{ur} n_e^{4/3} \quad (5)$$

with $\gamma = 4/3$ and $K^{ur} = (3\hbar c/4)(\pi^2/9)^{1/3}$. Similarly, for weakly relativistic case ($\alpha = wr$), $\gamma = 5/3$, $K^{wr} = (3\pi\hbar^2/5m_e)(\pi/3)^{1/3}$ and we get

$$P_e = K^{wr} n_e^{5/3}. \quad (6)$$

For non-relativistic case ($\alpha = nr$), $\gamma = 5/3$, $K^{nr} = (\hbar^2/5m_e)(3\pi^2)^{2/3}$ and eq. (4) reduces to

$$P_e = K^{nr} n_e^{5/3}. \quad (7)$$

3. Linearized quantum hydrodynamic relations

The momentum eq. (1) and an appropriate equation of state, i.e., eq. (5) for ultrarelativistic pressure, eq. (6) for weakly relativistic pressure or eq. (7) for non-relativistic pressure of electrons give rise to relations for linear components of the velocity of electrons and ions in drift approximation $|\partial_t| \ll \Omega_{cj}$, ck . In the direction perpendicular to z -axis, we have

$$\mathbf{v}_{e\perp 1} \cong \frac{c}{\mathbf{B}_0} \left(\mathbf{z} \times \nabla_\perp \phi_1 + \frac{\gamma K^\alpha n_{e0}^{\gamma-2}}{e} (\nabla n_{e1} \times \mathbf{z}) \right), \quad (8)$$

$$\mathbf{v}_{i\perp 1} \cong \frac{c}{\mathbf{B}_0} \left(\mathbf{z} \times \nabla_\perp \phi_1 - \frac{1}{\Omega_{ci}} (\partial_t + v_0 \partial_z) \nabla_\perp \phi_1 \right). \quad (9)$$

The value of γ is either 4/3 for ultrarelativistic electrons or 5/3 for weakly or non-relativistic electrons. The drift velocity for electrons due to polarization has been neglected in low-frequency regime of our interest, while it is included for non-degenerate ions. The components of velocity for electrons and ions in a direction parallel to $\hat{\mathbf{z}}$ are respectively given by

$$(\partial_t + v_0 \partial_z) \mathbf{v}_{ez1} \cong \frac{e}{m_e} \partial_z \left(\phi_1 - \frac{\gamma K^\alpha n_{e0}^{\gamma-2}}{m_e} n_{e1} \right) + \frac{c}{\mathbf{B}_0} v'_0 \partial_y \phi_1, \quad (10)$$

$$(\partial_t + v_0 \partial_z) \mathbf{v}_{iz1} \cong -\frac{e}{m_i} \partial_z \phi_1 + \frac{c}{\mathbf{B}_0} v'_0 \partial_y \phi_1, \quad (11)$$

where $v'_0 = \partial_x v_0$. By considering electron continuity equation in linearized form, eqs (8) and (10) under approximations $(\partial_t + v_0 \partial_z)^2 n_{e1} \ll (\gamma K^\alpha n_{e0}^{\gamma-1}/m_e) \partial^2 n_{e1}/\partial z^2$ and $v'_0 \partial^2 \phi_1/\partial z \partial y$, $(\partial_t + v_0 \partial_z)(\partial \ln n_0/\partial x) \partial \phi_1/\partial y \ll \Omega_{ce} \partial^2 \phi_1/\partial z^2$, gives

$$\nabla^2 n_{e1} = \frac{e}{\gamma K^\alpha n_{e0}^{\gamma-2}} \nabla^2 \phi_1. \quad (12)$$

In the same manner, the ion equations lead to

$$(\partial_t + v_0 \partial_z) \left(1 - \frac{C_s^2}{\Omega_{ci}^2} \nabla_{\perp}^2 \right) \nabla^2 \phi_1 + \frac{C_s^2}{\Omega_{ci}} \kappa_n \partial_y \nabla^2 \phi_1 + \frac{m_i C_s^2}{e} \partial_z \nabla^2 v_{iz1} = 0, \quad (13)$$

where $C_s = (\gamma K^\alpha n_{e0}^{\gamma-1} / m_i)^{1/2}$ is the electrostatic ion mode speed and $\kappa_n = \partial \ln n_0 / \partial x$ is the inverse of density inhomogeneity scale length. Equations (12) and (13) can be solved to give

$$(\partial_t + v_0 \partial_z) v_{iz1} = -\frac{e}{m_i} (\partial_z - S \partial_y) \phi_1, \quad (14)$$

where $S = v_0' / \Omega_{ci}$ is the shear flow parameter. Let the fluctuations be proportional to $e^{i(k_{\perp} y + k_{\parallel} z - \omega t)}$ with k_{\perp} and k_{\parallel} being the perpendicular and parallel components of the wave vector \mathbf{k} and ω the plasma wave frequency. We assume $\kappa_n \ll k_{\perp}$ so that local approximation remains valid. On Fourier transformation with the notations $\Omega_k = \omega - v_0 k$, $\rho_L = C_s / \Omega_{ci}$ and $b_s = \rho_L^2 k_{\perp}^2$, eqs (13) and (14) lead to the dispersion equation of the form

$$\Omega_k^2 - \omega_* \Omega_k - \frac{C_s^2 \Omega_{ci}^2 k_{\parallel}^2}{(1 + b_s)} \left(1 - S \frac{k_{\perp}}{k_{\parallel}} \right) = 0, \quad (15)$$

where ρ_L is the ion Larmor radius and $\omega_* = \rho_L^2 \Omega_{ci} \kappa_n k_{\perp} / (1 + b_s)$ is the drift wave frequency. The above equation satisfies the solution

$$\omega = v_0 k_{\parallel} + \frac{1}{2} \omega_* \pm \frac{1}{2} \left(\omega_*^2 + \frac{4 C_s^2 \Omega_{ci}^2 k_{\parallel}^2}{(1 + b_s)} \left(1 - S \frac{k_{\perp}}{k_{\parallel}} \right) \right)^{1/2}. \quad (16)$$

It is worth mentioning that the solution (16) in general is valid for all three cases of interest, i.e., for the relevant equation of state described by eqs (5)–(7). The parameters α and γ for appropriate physical condition is given in eq. (6) for ultrarelativistic, weakly relativistic and non-relativistic degenerate electrons. One can see that if

$$v_0' > \frac{\Omega_{ci} k_{\parallel}}{k_{\perp}} \left(1 + \frac{\rho_L^4 \kappa_n^2 k_{\perp}^2}{4 C_s^2 k_{\parallel}^2 (1 + b_s)} \right), \quad (17)$$

eq. (16) leads to an unstable mode. Thus, the system admits an oscillatory instability under such condition. It is worth mentioning that the shear flows are also responsible for drawing kinetic energy from mean flow in plasmas through more general forms of small-scale perturbations. Then the plasma dynamics is driven by the injected energy. This is essentially an irrotational phenomenon which can give rise to other shear flow modes like interchange or Rayleigh–Taylor or Kelvin–Helmholtz-type instabilities. Such interfacial instabilities are significant in non-linear evolution and transport of plasmas. However, our analysis is limited to the simple case of unidirectional density and velocity gradients. In what follows, we perform a parametric analysis to elaborate our results under various conditions.

4. Results and discussion

In the impressive progress of next generation superintense lasers, it is revealed that overdense plasmas with conditions consistent with astrophysical regimes can be created in

a laboratory [7,9]. The counterpropagating superstrong lasers under specific conditions have the ability to generate very high transient magnetic fields ($\sim 10^9$ G), overlapping with the field of dense astrophysical objects [30]. Although it is not simple to replicate the physical conditions of end stage stars, the progress made in this direction is remarkable.

In order to examine the results obtained in §3 numerically, we consider parameters relevant to the dense plasmas found in compact astrophysical objects e.g., neutron stars [12,18]. These plasmas are exotic in nature due to extreme values of density and magnetic field. The coupling strength between electrons is very weak due to large number density thus well suited for quantum hydrodynamics. We use the physical constants in cgs units, i.e., $e = 4.80 \times 10^{-10}$ esu, $\hbar = 1.05 \times 10^{-27}$ erg-s, $c = 3.00 \times 10^{10}$ cm s $^{-1}$, $m_e = 9.11 \times 10^{-28}$ g, $m_i = 1.67 \times 10^{-24}$ g and $k_B = 1.38 \times 10^{-16}$ erg K $^{-1}$. For relativistic and degenerate electrons, assuming streaming speed $v_0 = 10^4$ cm s $^{-1}$, we consider $\kappa_n = 0.02k_\perp$, $k_\perp = 5 \times 10^5$ cm $^{-1}$ and the shear flow parameter $S = 0.0001$, so that the local approximation remains valid. Then, we obtain $C_s = 7.33 \times 10^8$ cm s $^{-1}$ for ultrarelativistic

Table 1. The ion mode speed ($\times 10^8$ cm s $^{-1}$) and drift wave frequency ($\times 10^{-11}$ s $^{-1}$) for $B_0 \cong 10^{12}$ G and different number densities n_{e0} ($\times 10^{30}$ cm $^{-3}$) and corresponding Fermi temperatures T_{Fe} ($\times 10^{10}$ K). Relevant α values are denoted by superscripts on C_s and ω_* .

| n_{e0} | T_{Fe} | C_s^{ur} | C_s^{wr} | C_s^{nr} | ω_*^{ur} | ω_*^{wr} | ω_*^{nr} |
|----------|----------|------------|------------|------------|-----------------|-----------------|-----------------|
| 5.0 | 1.58 | 7.33 | 9.29 | 9.36 | 2.80 | 4.50 | 4.56 |
| 6.0 | 1.79 | 7.56 | 9.88 | 9.95 | 2.98 | 5.08 | 5.17 |
| 7.0 | 1.98 | 7.76 | 10.40 | 10.48 | 3.14 | 5.63 | 5.75 |
| 8.0 | 2.17 | 7.94 | 10.87 | 10.98 | 3.29 | 6.16 | 6.28 |

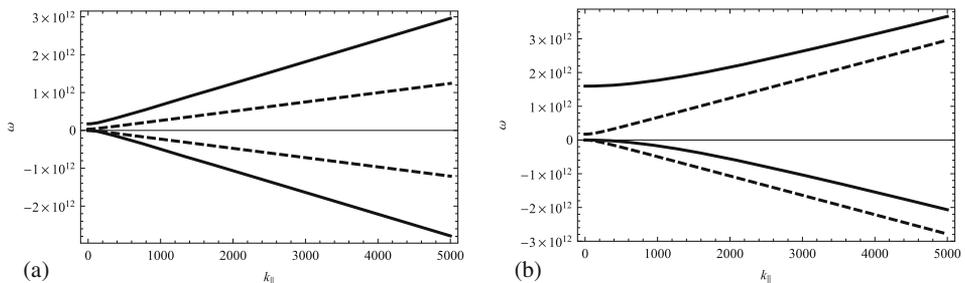


Figure 1. The ion wave dispersion relation ω vs. k_z is plotted in an electron–ion dense plasma for the ultrarelativistic case. (a) Corresponds to the positive and negative roots of eq. (16). In (a), the variation of electron number density from 3×10^{28} cm $^{-3}$ (dashed curve) to 5×10^{30} cm $^{-3}$ (solid curve) is shown. In (b), the strength of the ambient magnetic field is shown to vary from $B_0 \cong 10^{11}$ G (solid curve) to $B_0 \cong 10^{12}$ G (dashed curve) for $n_{e0} \cong 5 \times 10^{30}$ cm $^{-3}$.

and $9.29 \times 10^8 \text{ cm s}^{-1}$ for weakly relativistic degenerate plasma. The ion mode speed for a range of plasma density is given in table 1. The variation of density and magnetic field modifies the wave speed as expected. In case of non-relativistic and degenerate electron plasmas, the ion mode speed becomes $9.36 \times 10^8 \text{ cm s}^{-1}$.

To demonstrate the results graphically, we have performed the parametric analysis of the dispersion relation (16). In figure 1, the dispersion equation is shown for ultra-relativistic degenerate plasma for certain plasma density and magnetic field which can vary over a wide range in dense astrophysical objects. It is noticed that the response of linearized mode is more pronounced with magnetic field as evident from figure 1b. This effect is dominant in all the cases. So, the influence of magnetic field is much stronger and change is more rapid. In the case of weakly relativistic and degenerate plasma as seen in figure 2, an increase in the wave speed is noted. This means that the wave slows down with increase in relativistic effects. For non-relativistic plasma under the same conditions of density and ambient magnetic field, the speed of ion mode increases further as shown in figure 3. This shows that as the relativistic effects come into play, the wave speed starts decreasing.

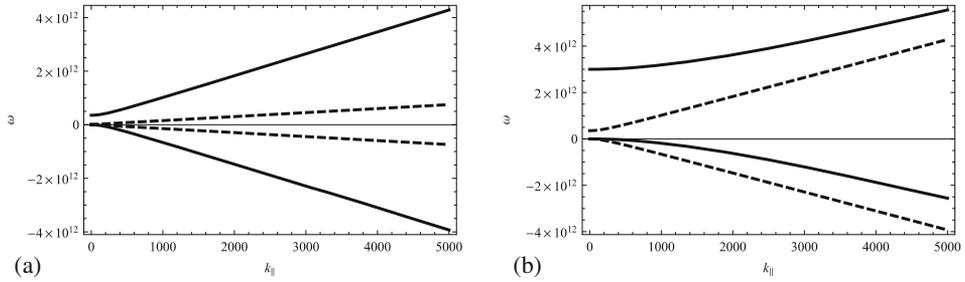


Figure 2. The electrostatic ion mode frequency ω (rad s^{-1}) is plotted vs. the parallel wave number k_z (cm^{-1}) for weakly relativistic plasma. The variation of number density of electrons and magnetic field is the same as in figure 1 with (a) (b) showing the number density (magnetic field) variations.

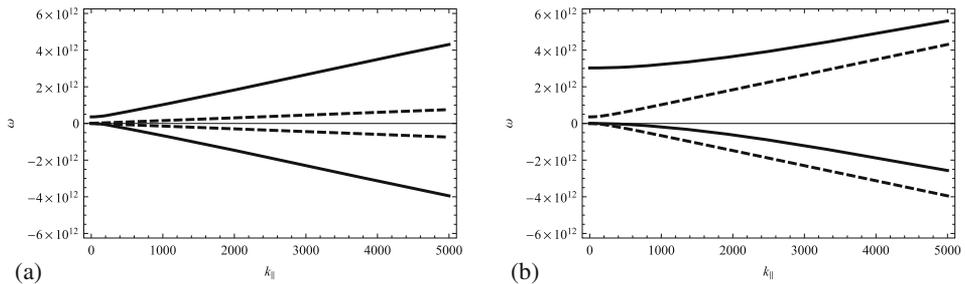


Figure 3. In non-relativistic and degenerate electron cases, the frequency is plotted vs. the wave number for the same parameters as in figure 2. The increase in wave frequency is more rapid than the other two cases. Density (magnetic field) variations are shown in (a) and (b).

In all the cases, the dispersive effects are very weak. This is a general trend on the low-frequency waves in a relativistic and degenerate plasma as the variation is over very short length and time-scales. The drift wave frequency varies over the range of 10^{-11}s^{-1} as seen in table 1.

5. Conclusion

To conclude, we have investigated the features of low-frequency electrostatic waves in the presence of equilibrium parallel velocity shear and a unidirectional density gradient in dense non-uniform plasma containing relativistic degenerate electrons and non-degenerate ions. The mean-field quantum hydrodynamics model has been employed with the equation of state consistent for relativistic and degenerate plasmas predominantly found in dense astrophysical regimes. In the drift approximation, dispersion relations are obtained in linearized case for three different cases i.e., for ultrarelativistic and degenerate, weakly relativistic degenerate as well as non-relativistic degenerate electrons. The wave mode shows weak dispersive effects and strong influence of density and ambient magnetic field. It is found that the influence of magnetic field on wave dispersion is stronger than the density fluctuations. This work aims to understand some features of low-frequency electrostatic oscillations driven by shear flows and density gradients in very high-density plasmas which exist naturally in dense astrophysical objects under extreme conditions.

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