



Inelastic surface vibrations versus energy-dependent nucleus–nucleus potential in sub-barrier fusion dynamics of ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system

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Abstract. Limitations of the static Woods–Saxon potential and the applicability of the energy-dependent Woods–Saxon potential (EDWSP) model within the framework of one-dimensional Wong formula to explore the sub-barrier fusion data are highlighted. The inelastic surface excitations of the fusing nuclei are found to be dominating in the enhancement of sub-barrier fusion excitation function data and the effects of such dominant vibrational states are exploited through the coupled channel calculations obtained by using the code CCFULL. It is worth mentioning here that the influence of multiphonon vibrational states of the reactants can be simulated by introducing the energy dependence in the nucleus–nucleus potential.

Keywords. Heavy-ion near-barrier fusion reactions; depth and diffuseness of Woods–Saxon potential; coupled channel equations; diffuseness anomaly.

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In recent years, the dynamics of sub-barrier fusion reactions are extensively studied because various dynamical aspects of the fusion process are still unexplored. Heavy-ion fusion reactions can be used to probe the interplay of nuclear structure and nuclear interactions between colliding nuclei and also seem to be an intermediate step for the production of nuclei away from the valley of stability. The substantially large fusion enhancement at sub-barrier energies over the calculations of one-dimensional barrier penetration model attracts theorists and experimentalists to explore different features of reaction mechanisms between the reactants. The coupling between the relative motion and internal structure degrees of freedom influences the quantum mechanical tunnelling in such a way that it brings an anomalously large sub-barrier fusion enhancement. The major factors that can be attributed to this fusion enhancement are inelastic surface excitations of the projectile (target) or permanent deformation of the projectile (target) or nucleon (multinucleon) transfer channels [1–7]. The role of static deformation and the inelastic surface vibrations of colliding pairs on fusion dynamics are well addressed by the various coupled

channel models [1–7]. However, the impacts of neutron transfer channels have not been fully explored because transferring of neutrons is insensitive to fusion barrier. The neutrons can flow between collision partners at large internuclear separation which makes fusion process a complex rearrangement of nucleons [8–10].

The complex interactions between fusing nuclei are often described by the effective nucleus–nucleus potential, which is the primary ingredient to examine the various aspects of reaction dynamics. The correct knowledge of nucleus–nucleus potential is extremely desirable for the good understanding of fusion dynamics. Generally, the influences of internal structure degrees of freedom are addressed through this potential. The Coulomb and centrifugal terms are well understood but the large ambiguities in the nuclear part of nucleus–nucleus potential restrict basic understanding of different properties of nuclear interactions. The elastic scattering and inelastic scattering are mainly sensitive to surface region of the nuclear potential, while the fusion reactions are quite sensitive to the inner part of the nucleus–nucleus potential and thus these nuclear processes provide complementary information with regard to the optimum form of nucleus–nucleus potential [11–13]. In this regard, various parametrizations of the nuclear potential were used to explore a variety of nuclear phenomena in connection with heavy-ion reactions wherein many attempts have been made to extract intimate link with the accurate picture of nuclear potential by analysing large set of experimental data. The different coupled channel formulations make use of the standard energy-independent Woods–Saxon potential for explaining the dynamics of fusion process [1–7]. Among three parameters of static Woods–Saxon potential, the diffuseness parameter is related to the slope of nuclear potential in the tail region of the Coulomb barrier wherein fusion starts to take place [12]. In the past few years, large values of diffuseness parameters ranging from $a = 0.75$ fm to $a = 1.5$ fm were used to describe the behaviour of sub-barrier fusion data. Surprisingly, such values are much larger than $a = 0.65$ fm deduced from the elastic scattering data. This diffuseness anomaly, which might be an artifact of various kinds of static and dynamical physical effects, reflects the systematic failure of the static Woods–Saxon potential for simultaneous description of elastic scattering process and the fusion process [1–13]. Different kinds of static and dynamical channel coupling effects, which generally occur in the surface region of the nuclear potential or in the tail region of the Coulomb barrier, are responsible for the modification of potential parameters but the clarifications of such facts require more intensive theoretical and experimental studies.

In order to resolve such issues of heavy-ion fusion dynamics, several efforts were undertaken in previous works [14–26] to describe the experimental data of various heavy-ion fusion reactions by using the energy-dependent nucleus–nucleus potential. Recently, the energy-dependent Woods–Saxon potential (EDWSP) model [14–26] was successfully used to explore the fusion dynamics of stable nuclei such as ${}^{40,48}_{20}\text{Ca} + {}^{90,96}_{40}\text{Zr}$, ${}^{32,36}_{16}\text{S} + {}^{90,96}_{40}\text{Zr}$, ${}^{40,48}_{20}\text{Ca} + {}^{48}_{20}\text{Ca}$, ${}^{32}_{16}\text{S} + {}^{112,116,120}_{50}\text{Sn}$, ${}^{32,36}_{16}\text{S} + {}^{110}_{46}\text{Pd}$ and ${}^{28}_{14}\text{Si} + {}^{90,94}_{40}\text{Zr}$ reactions, wherein the effects of either the vibrational character of the colliding nuclei or/and the nucleon (multinucleon) transfer channels were found to be dominating. The influences of such dominant channels and other static and dynamical physical effects which arise due to nuclear structure degrees of freedom of colliding pairs were adequately analysed within the context of the EDWSP model. From the previous work, one can realize that the closely similar features of heavy-ion fusion dynamics, which arise due to internal structure of colliding pairs, can be reproduced by entertaining energy dependence

in the real part of the nucleus–nucleus potential in such a way that it becomes more attractive at sub-barrier energies. The energy dependence in nucleus–nucleus potential effectively decreases the height of fusion barrier between colliding nuclei and hence predicts larger sub-barrier fusion cross-section with respect to the energy-independent one-dimensional barrier penetration model. Similar conclusions are also evident from the coupled channel calculations of a wide range of projectile–target combinations [1–7]. The necessity of introducing energy dependence in real part of the nucleus–nucleus potential is evident from the nucleon–nucleon interactions and also from the non-local quantum effects [27–30]. To address the issues related to the optimum form of nucleus–nucleus potential, the present work is mainly focussed on the limitations of static Woods–Saxon potential and the applicability of the energy-dependent Woods–Saxon potential for exploration of sub-barrier fusion dynamics. For this, the fusion dynamics of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system has been considered in the present analysis [31]. Theoretical calculations have been performed by using the static Woods–Saxon potential and the EDWSP model in conjunction with one-dimensional Wong formula [32]. The underlying reason to choose the fusion dynamics of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system is the rich interplay of multiphonon vibrational states of the colliding pairs [31]. For coupled channel calculations, the static Woods–Saxon potential is used in the coupled channel code CCFULL [33] wherein the effects of multiphonon vibrational states of the colliding nuclei can be accurately included. In the coupled channel calculations, the projectile is considered as inert while the role of inelastic surface vibrations of the target nucleus has been properly entertained. The static Woods–Saxon potential along with Wong formula fails to accurately describe the fusion enhancement of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system and thus reflects the inconsistency of static Woods–Saxon potential for exploring heavy-ion fusion dynamics. However, the scenario of fusion enhancement of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system has been successfully explored by the EDWSP model [14–26], wherein the energy dependence in Woods–Saxon potential simulates the coupling effects of the dominant channels.

The partial wave fusion cross-section is given by the following expression:

$$\sigma_F = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_{\ell}^F. \quad (1)$$

Based on the parabolic approximation of effective interaction potential between the colliding nuclei, Hill and Wheeler proposed the following expression for tunnelling probability (T_{ℓ}^F) [34]:

$$T_{\ell}^{\text{HW}} = \frac{1}{1 + \exp[(2\pi/\hbar\omega_{\ell})(V_{\ell} - E)]}. \quad (2)$$

This expression was further simplified by Wong using the following assumptions for the barrier position, barrier curvature and barrier height [32]:

$$\begin{aligned} R_{\ell} &= R_{\ell=0} = R_B, \\ \omega_{\ell} &= \omega_{\ell=0} = \omega, \\ V_{\ell} &= V_B + \frac{\hbar^2}{2\mu R_B^2} \left[\ell + \frac{1}{2} \right]^2. \end{aligned}$$

Using these assumptions and substituting eq. (2) into eq. (1), the fusion cross-section can be written as

$$\sigma_F = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} \frac{(2\ell + 1)}{[1 + \exp(2\pi/\hbar\omega)(V_\ell - E)]}. \quad (3)$$

According to Wong, infinite numbers of partial waves are participating in the fusion process. So changing the summation over ℓ into integral with respect to ℓ in eq. (3) and by solving the integral, one can obtain the following final expression of the Wong formula [32]:

$$\sigma_F = \frac{\hbar\omega R_B^2}{2E} \ell n \left[1 + \exp\left(\frac{2\pi}{\hbar\omega}(E - V_B)\right) \right]. \quad (4)$$

Theoretically, the standard way to address the effects of coupling between relative motion and the intrinsic degrees of freedom of colliding nuclei is to solve the coupled channel equations by including all the relevant channels [33,35,36]. Thus, the set of coupled channel equation can be written as

$$\left[\frac{-\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N(r) + \frac{Z_P Z_T e^2}{r} + \varepsilon_n - E_{c.m.} \right] \psi_n(r) + \sum_m V_{nm}(r) \psi_m(r) = 0. \quad (5)$$

Here, \vec{r} represents the relative motion of the collision partners and μ is the reduced mass of the colliding nuclei. The quantities E_{cm} and ε_n denote the bombarding energy in the centre of mass frame and the excitation energy of the n th channel respectively. V_{nm} is the matrix element of the coupling Hamiltonian, which in the collective model consists of Coulomb and nuclear components. The coupled channel code CCFULL [33] uses the following basic approximations to solve the coupled channel equations. The no-Coriolis or rotating frame approximation is used to reduce the number of coupled channel equations [33,35,36]. In this approximation, the z -axis is chosen in the direction of the relative separation \vec{r} of the colliding nuclei so that $\theta = 0$ and the spherical harmonics become $Y_{\lambda\mu}^*(\hat{r}) = \sqrt{(2\lambda+1)/4\pi} \delta_{\mu,0}$. This implies that the excited states of each nucleus will have the same spin projection on the rotating z -axis as in the respective ground states. Under this approximation, the numbers of the coupled equations to be solved are significantly reduced. For instance, the multipole excitation $0^+ \rightarrow \lambda$ is represented by only one channel in this approximation, whereas $(\lambda + 1)$ channels are required in the complete problem. Specifically, if one wants to include 0^+ , 2^+ , 4^+ and 6^+ rotational states of the deformed nucleus, a set of 16 coupled channel equations are to be solved but in no-Coriolis approximation this reduces to only four coupled channel equations [35,36]. Another approximation is the ingoing wave boundary conditions (IWBC) which are applicable for heavy-ion fusion reactions. According to IWBC, there are only incoming waves at the minimum position of potential pocket inside the Coulomb barrier, while there are only outgoing waves at infinity for all channels except for the entrance channel ($n = 0$). The code CCFULL [33] makes use of energy-independent Woods–Saxon potential which is defined as

$$V_N(r) = \frac{-V_0}{[1 + \exp(\frac{r-R_0}{a})]} \quad (6)$$

with $R_0 = r_0(A_P^{1/3} + A_T^{1/3})$. The quantity V_0 is the depth and a is the diffuseness parameter of nuclear potential. By including all the relevant channels, the fusion cross-section can be written as

$$\sigma_F(E) = \sum_J \sigma_J(E) = \frac{\pi}{k_0^2} \sum_J (2J + 1) P_J(E), \quad (7)$$

where $P_J(E)$ is the total transmission coefficient corresponding to the angular momentum J .

The nucleus–nucleus potential, which highlights the different aspects of nuclear interactions and reaction dynamics between the colliding nuclei, is the fundamental characteristic of heavy-ion fusion reactions. The present article makes use of both the static Woods–Saxon potential and the energy-dependent Woods–Saxon potential in conjunction with one-dimensional Wong formula to describe the fusion dynamics of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system. In energy-dependent Woods–Saxon potential (EDWSP) model, the depth of the real part of the Woods–Saxon potential is defined as [14–26]

$$V_0 = \left[A_P^{2/3} + A_T^{2/3} - (A_P + A_T)^{2/3} \right] \times \left[2.38 + 6.8(1 + I_P + I_T) \frac{A_P^{1/3} A_T^{1/3}}{(A_P^{1/3} + A_T^{1/3})} \right] \text{MeV}, \quad (8)$$

where

$$I_P = \left(\frac{N_P - Z_P}{A_P} \right) \quad \text{and} \quad I_T = \left(\frac{N_T - Z_T}{A_T} \right)$$

are the isospin asymmetry of collision partners. The EDWSP model includes the effects of surface energy as well as the isospin asymmetry of the colliding nuclei [14–26]. The first term in the square bracket of eq. (8) is directly proportional to the surface energy of the nucleus and hence strongly depends on the collective motion of all the nucleons inside the nucleus. Various channel coupling effects, which are responsible for the fusion enhancement at sub-barrier energies, are the surfacial effects and these surfacial effects modify the surface diffuseness as well as the surface energy of collision partners. For instance, the colliding nuclei overlap in the neck region wherein the densities of collision partners get fluctuated and such dynamical physical effects are responsible for the modification of diffuseness parameter. This brings the necessity for a larger value of the diffuseness parameter for reproducing fusion excitation function data [1–7]. The second term inside the square bracket of eq. (8) is directly related to the isospin asymmetry effects of the colliding nuclei. The isospin asymmetry is different for different isotopes of a particular element and hence isotopic effects are also included in the nucleus–nucleus potential through this term. In literature, abnormally large values of diffuseness parameters ranging from $a = 0.75$ fm to $a = 1.5$ fm have been used to account for the fusion dynamics of a wide range of projectile–target combinations [1–7]. This abnormally large diffuseness might be an artifact of various kinds of static and dynamical physical effects such as fluctuation of densities and surface energy of the colliding pairs. In the essence of requirement for a larger value of diffuseness parameter and the energy dependence in nucleus–nucleus potential [27–30], the energy dependence in the

Woods–Saxon potential is introduced via its diffuseness parameter and hence given by the following expression:

$$a(E) = 0.85 \left[1 + \frac{r_0}{13.75(A_p^{-1/3} + A_T^{-1/3})(1 + \exp(\frac{E/V_{B0}-0.96}{0.03}))} \right] \text{ fm.} \quad (9)$$

In this expression, the range parameter (r_0) is treated as a free parameter and varied to adjust the values of diffuseness parameter required for addressal of fusion data. It is very important to note that the range parameter strongly depends upon the nature of fusing systems under consideration. In EDWSP model calculations, eq. (9) provides a wide range of diffuseness depending upon the value of r_0 and the bombarding energy of collision partners. The coupled channel calculations based on static Woods–Saxon potential with large diffuseness parameters ranging from $a = 0.75$ fm to $a = 1.5$ fm have an effect that is closely similar to that of the shallow M3Y + repulsion potential in low-energy region [37–39]. Ghodsi *et al* [40] have showed that the M3Y + repulsion and static Woods–Saxon potential with large diffuseness parameter accurately reproduce the fusion dynamics of various heavy-ion fusion reactions and M3Y+repulsion can be accurately reproduced in sub-barrier energy region by static Woods–Saxon potential with large diffuseness parameter. In the present work, it was found that the predictions of the energy-dependent Woods–Saxon potential have close resemblance with that of the static Woods–Saxon potential with large diffuseness. This indicates that the EDWSP model also has close resemblance with that of the shallow M3Y+repulsion potential in their predictive power. It will be shown later that the theoretical calculations based on static Woods–Saxon potential (CCFULL calculations) must include couplings to inelastic surface excitations of the colliding nuclei and multinucleon transfer channels or other static and dynamical physical effects for complete description of the fusion data. However, the energy dependence in the nucleus–nucleus potential induces similar kind of channel coupling effects that arise due to intrinsic degrees of freedom of the colliding nuclei and hence accurately reproduces the sub-barrier fusion excitation function data of various projectile–target combinations as evident from the earlier works [14–26].

Before going into details of coupled channel calculations, the fusion dynamics of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system has been discussed within the framework of the energy-independent and the energy-dependent Woods–Saxon potential model in conjunction with the one-dimensional Wong formula. For this system, the experimental data are substantially larger than the calculations of the one-dimensional Wong formula obtained by using static Woods–Saxon potential as shown in figure 1. The failure of static Woods–Saxon potential to provide accurate description of fusion data mirrors the significance of introduction of energy dependence in the real part of the nucleus–nucleus potential. In EDWSP model, the energy-dependent diffuseness parameter produces a distribution of barrier of varying heights. The barriers whose heights are smaller than that of an uncoupled barrier are responsible for the maximum flux lost from elastic channel to fusion channel and hence ultimately predict the larger sub-barrier fusion cross-section over the expectations of energy-independent one-dimensional Wong formula as evident from figure 1. Furthermore, in the EDWSP model, the variation of diffuseness parameter is effectively equivalent to the increase of capture radii of the colliding nuclei and this increase of capture radii of fusing nuclei indicates that the fusion process starts at

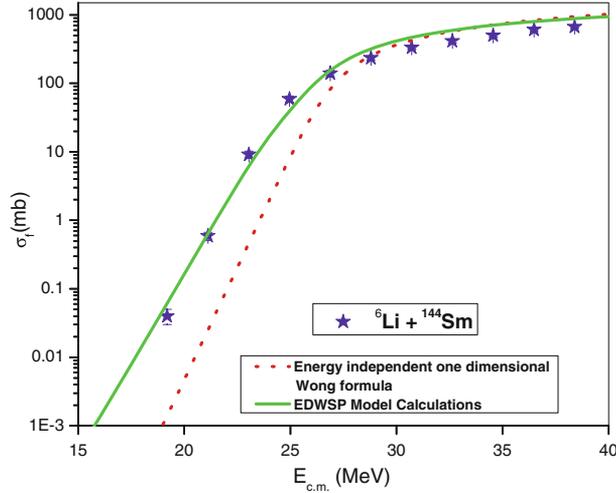


Figure 1. The fusion excitation function of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system obtained by using static Woods–Saxon potential and the energy-dependent Woods–Saxon potential (EDWSP) model along with Wong formula. The theoretical results are compared with the available experimental data [31].

much larger internuclear separation between the collision partners [16]. Similar physical effects are also evident from the coupled channel analysis of this system which will be discussed in figure 2. Similarly, the coupling between the elastic channel and internal degrees of freedom of fusing nuclei effectively increases the capture radii of the reactants and significantly enhances the fusion cross-section in near-barrier and sub-barrier energy regions. In EDWSP model calculations, at below-barrier energies, the value of the diffuseness parameter is the largest ($a = 0.93$ fm) resulting in the lowest fusion barrier. This lowest fusion barrier is responsible for the maximum flux lost from the elastic channel to the fusion channel. As the incident energy increases, the diffuseness parameter decreases resulting in an increase in height of the corresponding fusion barrier. In the above barrier energy regions, wherein the fusion cross-section is almost insensitive to various channel coupling effects (internal structure of the colliding nuclei), the value of diffuseness parameter gets saturated to its minimum value ($a = 0.85$ fm) and hence produces the largest fusion barrier. In EDWSP model calculations, for this system the diffuseness parameter a varies from $a = 0.93$ fm to 0.85 fm in the energy range from $E_{c.m.} = 15$ MeV to 40 MeV. The value of depth parameter (V_0) comes out to be 32.63 MeV, while the range parameter r_0 is kept fixed at 1.040 fm. Barrier characteristics such as barrier height, barrier position and barrier curvature are 25.51 MeV, 9.80 fm and 5.04 MeV respectively.

The coupled channel analysis of fusion of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system, wherein inelastic surface excitations are dominating, is discussed in figure 2. In the coupled channel approach, the depth of 48 MeV is used instead of 47 MeV as taken by Rath *et al* [31], so that coupled channel calculations do not overpredict the experimental data in sub-barrier energy regions. The low-lying surface vibrations in the target nucleus, which are included in the coupled channel calculations, are $\beta_2 = 0.110$, $E_2 = 1.660$ MeV and $\beta_3 = 0.210$,

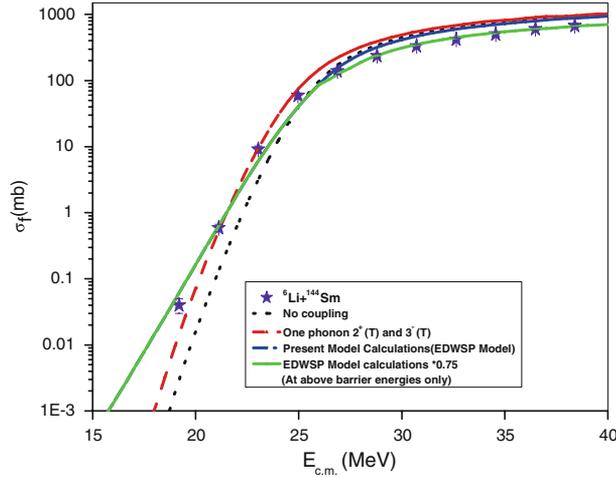


Figure 2. The fusion excitation function of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system obtained by using coupled channel model and the EDWSP model. The theoretical results are compared with the available experimental data [31].

$E_3 = 1.810$ MeV. Keeping the projectile as inert, the coupling to the inelastic surface excitations such as one-phonon couplings 2^+ and 3^- vibrational states along with their mutual coupling of the target nuclei enhances the fusion cross-section to that of one-dimensional barrier penetration model. Such couplings reasonably reproduce the fusion excitation functions data of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system in the whole range of energy. Further addition of two or more phonons of the octupole vibrations in the target seems to be undesirable and the sub-barrier fusion enhancement with respect to one-dimensional barrier penetration model can be ascribed to single phonon vibrational states of the target nucleus. It is worth mentioning here that there are large deviations between coupled channel calculations and the experimental data in the above-barrier energies, while such discrepancies are minimized in the EDWSP model calculations. The suppression of fusion data at the above-barrier energies can be correlated with the break-up effects of the projectile due to its low binding energy ($BE = 1.475$ MeV). The projectile is the loosely bound nucleus and because of the break-up of the projectile in the entrance channel, there is reduction of incoming flux going into fusion channel. Therefore, the above barrier fusion data are suppressed with respect to the coupled channel calculations by a factor of 32%, while this suppression factor is minimized in the EDWSP model calculations. The above barrier fusion data are suppressed with respect to EDWSP model calculations by a factor of 25% which is smaller than that of 32% reported in ref. [31].

In literature, various coupled channel models predicted that the couplings of the relative motion with the internal nuclear degrees of freedom of the reactants produce a distribution of barriers of varying heights and the passage through the barriers whose heights are smaller than that of uncoupled Coulomb barrier is more probable. This ultimately enhances sub-barrier fusion cross-section by several orders of magnitude over the expectations of the one-dimensional barrier penetration model. Similarly, the EDWSP model produces a set of barriers of varying heights and reasonably addresses the fusion

enhancement at sub-barrier energies. Furthermore, the energy dependence in Woods–Saxon potential simulates various kinds of static and dynamical physical effects that arise due to the internal structure of the colliding nuclei. The failure of the standard Woods–Saxon potential model in conjunction with the one-dimensional Wong formula to provide accurate description of sub-barrier fusion dynamics looks after the significance of introduction of energy dependence in nucleus–nucleus potential. Therefore, different kinds of channel coupling effects with regard to the sub-barrier fusion enhancement, whether mirrors an accurate picture of fusion process or simply mocks at the inconsistency of static Woods–Saxon potential, is still not clear and requires more intensive theoretical and experimental investigations.

This paper summarizes the limitations of static Woods–Saxon potential model and the applicability of the EDWSP model to describe fusion dynamics of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system. The systematic failure of the static Woods–Saxon potential to provide an accurate explanation of fusion enhancement of this system clearly indicates that the energy-dependent nucleus–nucleus potential (EDWSP model) seems to be an alternative approach to address fusion enhancement in below-barrier energy regions. Furthermore, the above barrier fusion data are suppressed with respect to the EDWSP model calculations by a factor of 25% which is smaller than that of 32% with respect to the coupled channel calculations as reported in ref. [31]. This suppression of fusion data at above-barrier energies can be correlated with the break-up effect of the projectile due to its loosely bound nature. In EDWSP model calculations, larger values of diffuseness parameter ranging from $a = 0.85$ fm to 0.93 fm are required to account for the observed fusion enhancement of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ system. In literature, it has been well established that M3Y + repulsion and static Woods–Saxon potential with large diffuseness parameter induce similar physical effects in sub-barrier energy regions and thus the effects of M3Y + repulsion potential can be accurately entertained by the static Woods–Saxon potential with an abnormally large diffuseness parameters ranging from $a = 0.75$ fm to 1.5 fm. The successful explanation of fusion dynamics of the ${}^6_3\text{Li} + {}^{144}_{62}\text{Sm}$ reaction within the framework of the EDWSP model calculations indicate that the energy dependence in the Woods–Saxon potential reflects similar characteristics of heavy-ion fusion reactions as deduced from the M3Y+repulsion potential and the static Woods–Saxon potential with large diffuseness parameters ranging from $a = 0.75$ fm to 1.5 fm.

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References

- [1] M Beckerman, *Rep. Prog. Phys.* **51**, 1047 (1988)
- [2] W Reisdorf, *J. Phys. G* **20**, 1297 (1994)
- [3] M Dasgupta *et al*, *Annu. Rev. Nucl. Part. Sci.* **48**, 401 (1998)
- [4] A B Balantekin and N Takigawa, *Rev. Mod. Phys.* **70**, 77 (1998)
- [5] L F Canto *et al*, *Phys. Rep.* **424**, 1 (2006)
- [6] K Hagino and N Takigawa, *Prog. Theor. Phys.* **128**, 1061 (2012)

- [7] B B Back *et al*, *Rev. Mod. Phys.* **86**, 317 (2014)
- [8] V I Zagrebaev, *Phys. Rev. C* **67**, 061601 (2003)
- [9] G Montagnoli *et al*, *Eur. Phys. J. A* **15**, 351 (2005)
- [10] N Rowley, *Phys. Lett. B* **282**, 276 (1992)
- [11] C H Dasso and G Pollarolo, *Phys. Rev. C* **68**, 054604 (2003)
- [12] K Hagino *et al*, *Phys. Rev. C* **67**, 054603 (2003)
- [13] G Pollarolo and A Winther, *Phys. Rev. C* **62**, 054611 (2000)
- [14] M Singh, Sukhvinder and R Kharab, *Mod. Phys. Lett. A* **26**, 2129 (2011)
- [15] M Singh, Sukhvinder and R Kharab. Atti Della “Fondazione Giorgio Ronchi” Anno LXV **6**, 751 (2010)
- [16] M Singh, Sukhvinder and R Kharab, *Nucl. Phys. A* **897**, 179 (2013)
- [17] M Singh, Sukhvinder and R Kharab, *Nucl. Phys. A* **897**, 198 (2013)
- [18] M Singh, *Analysis of high energy heavy ion collisions (Fusion reactions)*, M.Phil. Dissertation (unpublished) (Kurukshetra University, Kurukshetra, Haryana, India, 2009)
- [19] M Singh, *Analysis of heavy ion fusion excitation function data through energy-dependent Woods–Saxon potential in near barrier energy region*, Ph.D. Thesis (unpublished) (Kurukshetra University, Kurukshetra, Haryana, India, 2013)
- [20] M Singh, Sukhvinder and R Kharab, *AIP Conf. Proc.* **1524**, 163 (2013)
- [21] M Singh and R Kharab, *EPJ Web of Conferences* **66**, 03043 (2014)
- [22] M S Gautam, *Phys. Rev. C* **90**, 024620 (2014)
M S Gautam *et al*, *Phys. Rev. C* **93**, 054605 (2015); *AIP* **1675**, 020052 (2015)
- [23] M S Gautam, *Nucl. Phys. A* **933**, 272 (2015); *Can. J. Phys.* **93**, 1343 (2015); *Chin. Phys. C* **39**, 114102 (2015)
- [24] M S Gautam, *Mod. Phys. Lett. A* **30**, 1550013 (2015); *Acta Phys. Pol. B* **46**, 1055 (2015)
- [25] M S Gautam, *Phys. Scr.* **90**, 025301 (2015); *Phys. Scr.* **90**, 125301 (2015)
- [26] M S Gautam, *Phys. Scr.* **90**, 055301 (2015)
- [27] L C Chamon *et al*, *Phys. Rev. C* **66**, 014610 (2002)
- [28] K Washiyama and D Lacroix, *Phys. Rev. C* **74**, 024610 (2008)
- [29] C Simenel *et al*, *Phys. Rev. C* **88**, 064604 (2013)
- [30] A S Umar, C Simenel and V E Oberacker, *Phys. Rev. C* **89**, 034611 (2014)
- [31] P K Rath *et al*, *Phys. Rev. C* **79**, 051601 (2009)
- [32] C Y Wong, *Phys. Rev. Lett.* **31**, 766 (1973)
- [33] K Hagino, N Rowley and A T Kruppa, *Comput. Phys. Commun.* **123**, 143 (1999)
- [34] D L Hill and J A Wheeler, *Phys. Rev.* **89**, 1102 (1953)
- [35] T Rumin, K Hagino and N Takigawa, *Phys. Rev. C* **61**, 014605 (1999)
- [36] H Esbensen *et al*, *Phys. Rev. C* **36**, 1216 (1987)
- [37] G Montagnoli *et al*, *Phys. Rev. C* **85**, 024607 (2012)
- [38] H Esbensen, C L Jiang and A M Stefanini, *Phys. Rev. C* **82**, 054621 (2010)
- [39] A M Stefanini *et al*, *Phys. Lett. B* **679**, 95 (2009)
- [40] O N Ghodsi and V Zanganeh, *Nucl. Phys. A* **846**, 40 (2010)