PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 86, No. 5
May 2016
pp. 1055–1066

Bohr Hamiltonian with different mass parameters applied to band structures of Eu isotopes built on Nilsson orbitals

M J ERMAMATOV[1,*], H YÉPEZ-MARTÍNEZ[2] and P C SRIVASTAVA[3]
[1]Institute of Nuclear Physics, Ulughbek, Tashkent 100214, Uzbekistan
[2]Universidad Autónoma de la Ciudad de México, Prolongación San Isidro Col. San Lorenzo Tezonco, Del. Iztapalapa, 09790 México D.F., Mexico
[3]Department of Physics, Indian Institute of Technology, Roorkee 247 667, India
*Corresponding author. E-mail: ermamat@inp.uz

MS received 7 October 2014; revised 3 March 2015; accepted 19 June 2015
DOI: 10.1007/s12043-015-1133-2; ***e*Publication:** 25 January 2016

Abstract. The band structure of the proton-odd nuclei ^{153,155}Eu, built on Nilsson orbitals, is investigated within the framework of a recently developed extended Bohr Hamiltonian model. The relative distance between spherical orbitals is taken into account by considering single-particle energies as a parameter which changes with increasing neutron number. Energy levels of each band and $B(E2)$ values inside the ground-state band are calculated and compared with the available experimental data. Thus, more comprehensive information on the structure of deformed nuclei can be obtained by studying the rotation–vibration spectra of odd nuclei built on Nilsson single-particle orbitals.

Keywords. Mass parameter; deformed nuclei; spectra; electromagnetic properties of nuclei.

PACS Nos 21.10.Tg; 23.20.Lv; 27.70.+q

1. Introduction

The band structures of several europium isotopes, built from Nilsson orbitals, have been well studied experimentally, and a large amount of information is available on them [1]. ^{153}Eu in particular has been experimentally investigated extensively. In ref. [2], two opposite-parity bands were observed for this nucleus, and the bandheads are determined by the Nilsson model. In refs [3–5], knowledge of both positive- and negative-parity states was extended, and intraband electromagnetic transition probabilities were measured. Subsequently, the experimental bands built from Nilsson orbitals were discussed [6].

The Bohr Hamiltonian [7] has long been used in the study of nuclear structure, some important examples are refs [8–13], and this field has been developed by the recent works of refs [14–28]. The collective single-particle structure of the deformed odd nuclei has been studied in our earlier works [29–31] using the same mass parameter for all the vibration and rotation modes. The idea of different mass parameters for different modes of motion in a nucleus, originating from ref. [25], has been used in a simple model where quantum numbers of the projection of angular momentum of a nucleus, K, and that of an external nucleon, Ω, are good quantum numbers as shown in refs [32,33]. In ref. [34], the Coriolis interaction, to which a number of earlier interesting works are devoted [35,36], and its effects on spectra and reduced $E2$ transition probabilities, have been studied in the case that the projection of the angular momentum to the third axis connected with a nucleus and that of the external nucleon are not conserved.

It is well known that in the Nilsson model, single-particle energies are calculated by solving the Schrödinger equation for a particle moving in a deformed potential. The model and its features are discussed in detail, for example, in ref. [37]. The corresponding interacting boson fermion model is discussed in ref. [38]. In particular it has been shown that the Nilsson model corresponds to the classical limit of the interacting boson–fermion model with a pure quadrupole boson–fermion interaction. The relationship between the Nilsson model and the interacting boson model is discussed in detail in refs [36,38,39].

Bands built from the Nilsson model when it considers only one spherical orbital have been considered in our previous works [29–34]. The Nilsson model can be applied to determine the ordering of levels in odd nuclei, placing particles in each K-level [38]. In this work, we utilize the model which we developed in ref. [34] to describe the band structure of the ^{153,155}Eu isotopes. Owing to the restriction to just one orbital in the Nilsson model in ref. [34], single-particle energies did not play a role in the results, though interaction with the core and Coriolis interaction contributions were essential.

In refs [23–28,32,33], importance of using different mass parameters has been shown. They have stronger influence, especially on the interband $E2$ transition probabilities. At higher spins also spectra and intraband $E2$ transition probabilities are affected by these differences. In this work, we stress on the bands originated by not only considering one j which gave possibility to describe bands with ground-state parity, but several j which give the possibility of including other opposite-parity bands observed in the experiment by including different single-particle energies in the Hamiltonian, which takes into account the relative distance of spherical orbits.

2. Model

We write the Schrödinger equation in the following form:

$$(H_{\mathrm{v}} + H_{\mathrm{rot}} + H_{\mathrm{p}} + H_{\mathrm{int}})\Psi = E\Psi. \tag{1}$$

The explicit forms of H_{v}, H_{rot}, $H_{\mathrm{p}} + H_{\mathrm{int}}$ are given in [34]. H_{p} takes into account the central-symmetrical part and the corresponding energy ε_{p} in units of $\hbar^2/B_\beta\beta_0^2$ is used as a parameter which determines the distance between the single-particle spherical orbits.

The same potential of eq. (6) in refs [32–34] is considered. The eigenvalues of the Hamiltonian in eq. (1) are determined by the following expression:

$$E_{n_\beta n_\gamma L|m|j\tau} = [2n_\beta + q_{n_\gamma}^{j\tau}(L,|m|) + 3/2]\sqrt{2g_\beta} + \varepsilon_{\mathrm{p}}, \tag{2}$$

where

$$q_{n_\gamma}^{j\tau}(L,|m|) = \sqrt{\Lambda - \Lambda_0 + 2g_\beta + 1/4} - 1/2 \tag{3}$$

and

$$\Lambda - \Lambda_0 = \frac{2}{g}\frac{B_\beta}{B_\gamma}\left(2n_\gamma + |m| + \frac{m^2}{3}\right) + \varepsilon_{|m|Lj\tau} - \varepsilon_{0L_0j1}, \tag{4}$$

where Λ is the eigenvalue of the γ-vibrational part of the Hamiltonian plus the third term of the rotational section of the Hamiltonian, Λ_0 is that of the ground state, L_0 is the lowest state for each Nilsson band, τ distinguishes between different states of the same L and n_β and n_γ are the quantum numbers of β and γ rotations, respectively. Values of m are connected with K and Ω through the condition $K - \Omega = 2m$, where it should take integer values ± 1, ± 2, ..., [12], $g_\beta = B_\beta \beta_0^2 V_0/\hbar^2$ and $g = (1/\beta_0^2)(\hbar^2/\sqrt{B_\gamma C_\gamma})$. Reduced $E2$ transition probabilities also depend on the same parameters as in the case of spectra [34].

3. Shell-model predictions for ground-state properties of ^{153,155}Eu

To know about the structure of the ground state, we have performed shell-model calculation by taking an inert ^{132}Sn core with valence protons in 50-82 space and valence neutrons in 82-126 space. We performed calculation with CWG Hamiltonian, which is based on the CD-Bonn force [40] using shell-model code ANTOINE [41]. The single-particle neutron energies of the $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$ and $1i_{13/2}$ orbitals are -0.894, -2.455, -0.450, -1.601, -0.799 and 0.250 MeV, and the single-particle proton energies of the $1g_{7/2}$, $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$ and $1h_{11/2}$ orbitals are -9.663, -9.000, -7.323, -7.223 and -6.870 MeV, respectively.

In ^{153,155}Eu, we allowed protons to fill the $g_{7/2}$ and $d_{5/2}$ orbitals and neutrons in the $f_{7/2}$, $p_{3/2}$, $h_{9/2}$, $p_{1/2}$ and $f_{5/2}$ orbitals. The dimensions of matrices with this truncation involved in m-scheme are ~5 millions (^{153}Eu) and ~26 millions (^{155}Eu). Under these conditions, for ^{153}Eu, the ground state $7/2^+$ (~7%) has a dominant configuration of $\pi(g_{7/2}^7 d_{5/2}^6) \otimes \nu(f_{7/2}^2 p_{3/2}^4 h_{9/2}^2)$. The average occupancy of the $g_{7/2}$ orbital is 7.00 and that of $d_{5/2}$ orbital is 5.99. The occupancies of neutron orbitals are 1.67($f_{7/2}$), 2.59($p_{3/2}$), 1.95($h_{9/2}$), 0.75($p_{1/2}$) and 1.04($f_{5/2}$). Thus, the contribution of the proton $g_{7/2}$ orbitals is dominant in comparison to other orbitals. For ^{155}Eu, the ground state $7/2^+$ (~8%) has a dominant configuration of $\pi(g_{7/2}^7 d_{5/2}^6) \otimes \nu(f_{7/2}^2 p_{3/2}^4 h_{9/2}^2 f_{5/2}^2)$. The occupancies of $g_{7/2}$ orbital is 7.01 and that of $d_{5/2}$ orbital is 5.98. The occupancies of neutron orbitals are 2.5($f_{7/2}$), 2.86($p_{3/2}$), 2.35($h_{9/2}$), 0.92($p_{1/2}$) and 1.37($f_{5/2}$). The contribution of the proton $g_{7/2}$ orbitals is dominant in comparison to other orbitals.

4. Results and discussions

In the region of europium, experimental bands are built mainly on the Nilsson single-particle levels originating from the $1g_{7/2}$, $1h_{11/2}$ and $2d_{5/2}$ orbitals. Hence, we consider here the bands which originate from these spherical orbitals. Experimentally, many bands of ^{153}Eu built on the Nilsson orbitals are studied using single-proton transfer reactions in ref. [6].

When $m = 0$, the angular momentum vector of the prolate core is perpendicular to the axis of symmetry, and thus it cannot contribute to the value of K. Then, the value of K is determined by the projection of the angular momentum of the last proton (see, for example, refs [12,13,34]. We have determined the $5/2^+[413]$ ground state of the Eu isotopes from the Nilsson model. As this state corresponds to $j = 7/2$ spherical orbital, $K = 1/2, 3/2, 5/2$ and $7/2$ values are possible.

The bands of the Eu isotopes are calculated by diagonalizing rotational part with all possible K taking into account the $K \pm 1$ mixture of non-diagonal elements. According to both experiment and the Nilsson model, the $7/2^+[404]$ band should be much higher than the $5/2^+[413]$ band. This experimental band is close to the $7/2^+[404]$ band which is obtained by building a rotational band on a single-particle β phonon state (for brevity, high-energy β bands are not shown in figures, though they have been calculated together with the bands shown, and thus their mixtures have been taken into account). Therefore, the calculated results lead to the conclusion that the appearance of the $7/2^+[404]$ band is connected with building a rotational band on the $7/2^+[404]$ single-particle β phonon state. The relative positions of the K bands belonging to particular values of j are fixed by the parameter ε_{p}. The value of ε_{p} does not change for each K band, but it changes for each $g_{7/2}$, $d_{5/2}$ and $h_{11/2}$ spherical orbitals. Therefore, it has the same value, for example, for the bands $5/2^+[413]$, $3/2^+[422]$, $1/2^+[431]$ and the $7/2^+[404]$ β bands, which originate from $g_{7/2}$ spherical orbital.

The values of parameters used in the calculation are given in table 1. Parameters are fixed for each isotope and each spherical orbital assuming that single-particle energies and the interaction of the last valence nucleon with the core will change by changing the neutron number.

4.1 *The bands originating from* $1g_{7/2}$

The $5/2^+[413]$ band exists for all four isotopes of europium considered. According to experiment [1], as well as the $5/2^+[413]$ rotational band, there is a 55–91% chance that there exists another higher energy $5/2^+[413]$ band for all the considered nuclei, and furthermore there is a 5–40% probability that there is a contribution from the $5/2^+[402]$ orbital in the case of ^{153}Eu. For ^{153}Eu, this band is a β vibration band based on the $5/2^+[413]$ ground state [42]. This level is not observed in the single-proton transfer experiments because such a complex configuration should not be populated by a single-nucleon transfer reaction on an even–even target. This band also is a β band for ^{155}Eu [1].

Burke [6] confirmed the results of several experiments that suggested that for the level at ∼569 keV in this mass region, the only Nilsson orbitals expected to have large $l = 4$

Table 1. The values of the parameters used in calculations.

Nuclei	Orbitals	ξ	g_β	B_β/B_{rot}	$\Delta\varepsilon_{\mathrm{p}}$
^{153}Eu	$g_{7/2}$	0.100	160	3.2	0.00
	$d_{5/2}$	0.053	160	3.2	4.74
	$h_{11/2}$	0.077	160	3.2	4.45
^{155}Eu	$g_{7/2}$	0.070	160	3.2	0.00
	$d_{5/2}$	0.035	160	3.2	13.15
	$h_{11/2}$	0.044	160	3.2	5.60

strengths originate from the $1g_{7/2}$ shell. Therefore, the band built on this level is adopted as the $7/2^+[404]$ band. The type of this band is not yet determined [1] in experiment, but this model suggests that it is a β vibrational band.

In figures 1 and 2, we show all the calculated rotational single-particle bands ($n_\beta = n_\gamma = 0 = |m| = 0$) as well as rotational single-particle β bands ($n_\beta = 1 n_\gamma = 0 = |m| = 0$), originating from the $1g_{7/2}$ orbital, for ^{153}Eu and ^{155}Eu.

For the relevant results, figures 1 and 2, it is meaningful to discuss the quality of calculation almost exclusively in the case of the $5/2^+[413]$ ground state bands, as these are the only ones for which a significant number of states have been measured. In the case of ^{153}Eu (figure 1), in which every state in this band is measured from the $5/2^+$ ground state up to $45/2^+$, an interesting pattern emerges for every alternate state beginning with the $5/2^+$ state, that is $((5+4n)/2)^+$, where n is an integer greater than or equal to 0 and agreement with the experiment is exceptionally good, with the greatest deviation being 0.04 in $E(7/2^+_{\text{g.s.}})$ units. The other set of alternates, starting with $7/2^+$, or $((7+4n)/2)^+$, begin in close agreement with experiment, but increasingly underestimate as angular momentum increases. This breaks the harmonic oscillator spacing pattern seen in the experiment, creating an artificial pairing of states. A similar pattern is seen in the case of ^{155}Eu (figure 2), where all states up to $29/2^+$ have been measured. The same pairing of states, not seen in experiment, is observed in the calculated spectrum, and the alternating set $(7+4n)/2)^+$ still increasingly underestimates energy as angular momentum increases, but the set $((5+4n)/2)^+$ now also underestimates energy increasingly, though not as much as the other set.

Indeed, all calculated bands in figures 1 and 2, with the exception of the $7/2^+[404]$ bands, show this pattern. This effect is greater in the bands of smaller j and for the bands for which the value K is less than j, because parameters were fixed to create the best match with lower spin states. In fact, both the strength of the interaction of the external nucleon with the core and the deformation of the core change as spin increases. For example, for ^{153}Eu we have fixed parameters in such a way that the $9/2^+$ level in the $5/2^+[413]$ ground-state band has an energy of 2.32. If parameters are fixed in such a way that it is not 2.32, as in the experiment, but 2.28, better spacing between higher spins can be obtained at the expense of the low-energy fit.

4.2 *The bands originating from* $2d_{5/2}$

In figures 3 and 4, all the calculated rotational bands originating from $2d_{5/2}$ and $1h_{11/2}$ are compared with the available experimental data. As we have discussed in the previous section, we have compared the $3/2^+[422]$ band of ^{155}Eu in the experiment with the $3/2^+[411]$ β band, as in the calculated $3/2^+[411]$ β band is higher than the $3/2^+[422]$ band. Experiments in ref. [6] show an admixture of the $1/2^+[420]$ and $5/2^+[402]$ orbitals, as well as other types of components. In the calculation, the 1/2, 3/2 and 5/2 components mix within the $2d_{5/2}$ spherical orbital.

As seen from figures 3 and 4, the first three levels for the $3/2^+[411]$ band are in good agreement with the experiment. Then, the $11/2^+$ state is underestimated by the calculation, while the $13/2^+$ state is close to the experiment. This alternation, with every second state being underestimated, continues up to $25/2^+$ for ^{153}Eu and up to $23/2^+$ for ^{155}Eu. The $11/2^+$, $15/2^+$, $19/2^+$ and $23/2^+$ levels are close to the experiment in ^{155}Eu. For

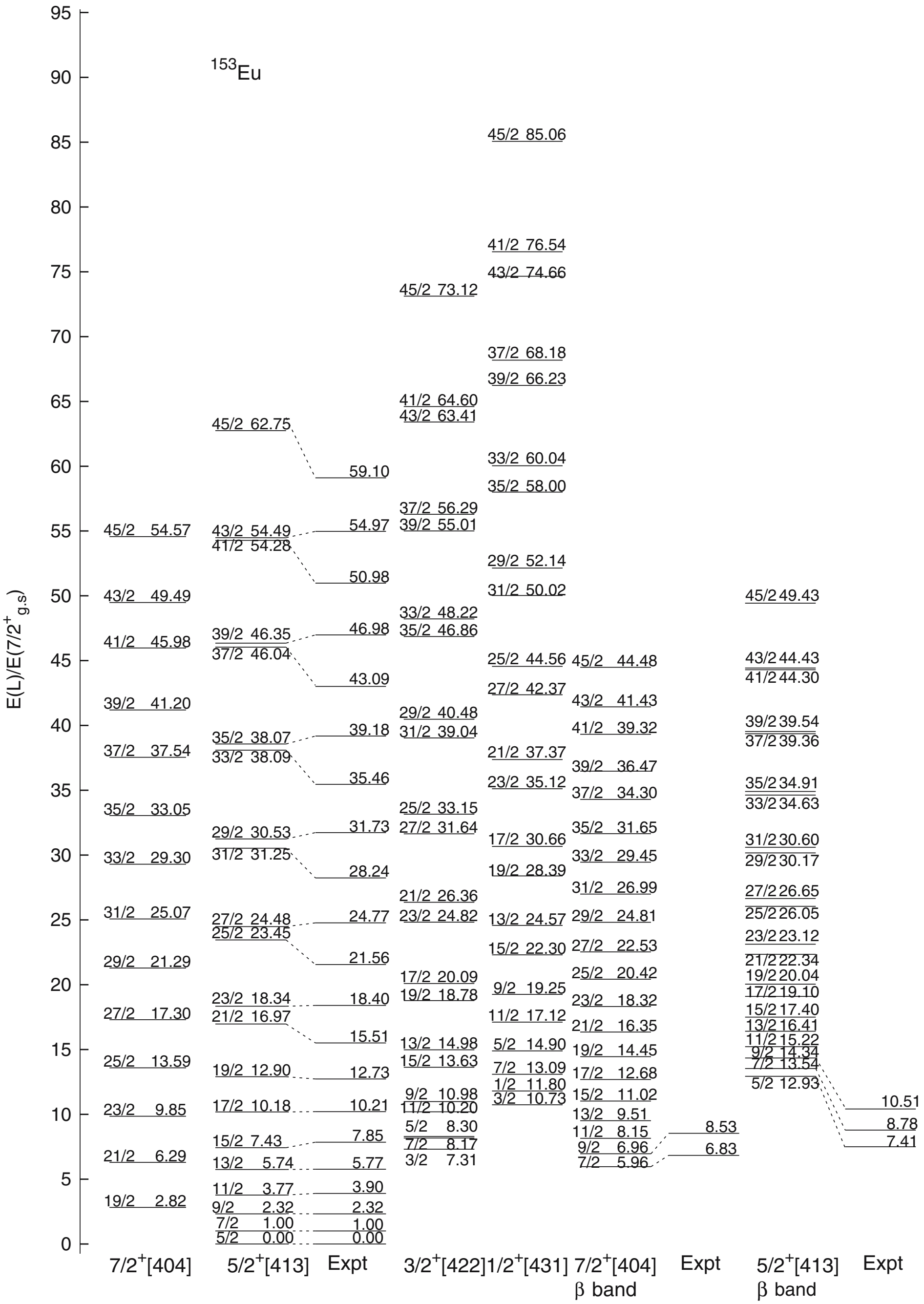

Figure 1. Comparison of the calculated values of energy levels originating from $1g_{7/2}$ Nilsson orbital in units of $E(7/2^+_{\rm g.s.})$ for ^{153}Eu with the experimental data from ref. [1].

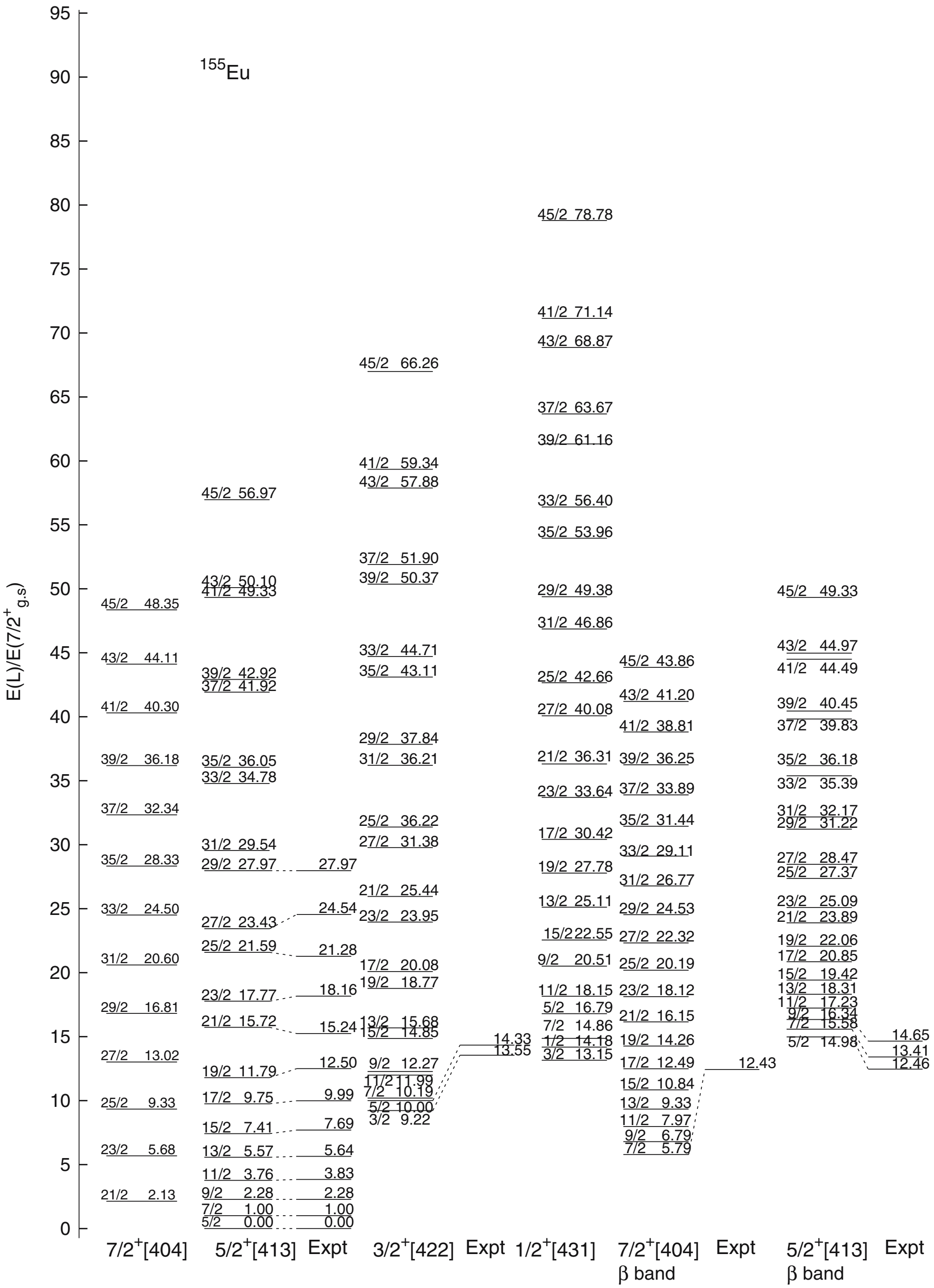

Figure 2. Comparison of the calculated values of energy levels originating from $1g_{7/2}$ Nilsson orbital in units of $E(7/2^+_{\text{g.s.}})$ for ^{155}Eu with the experimental data from ref. [1].

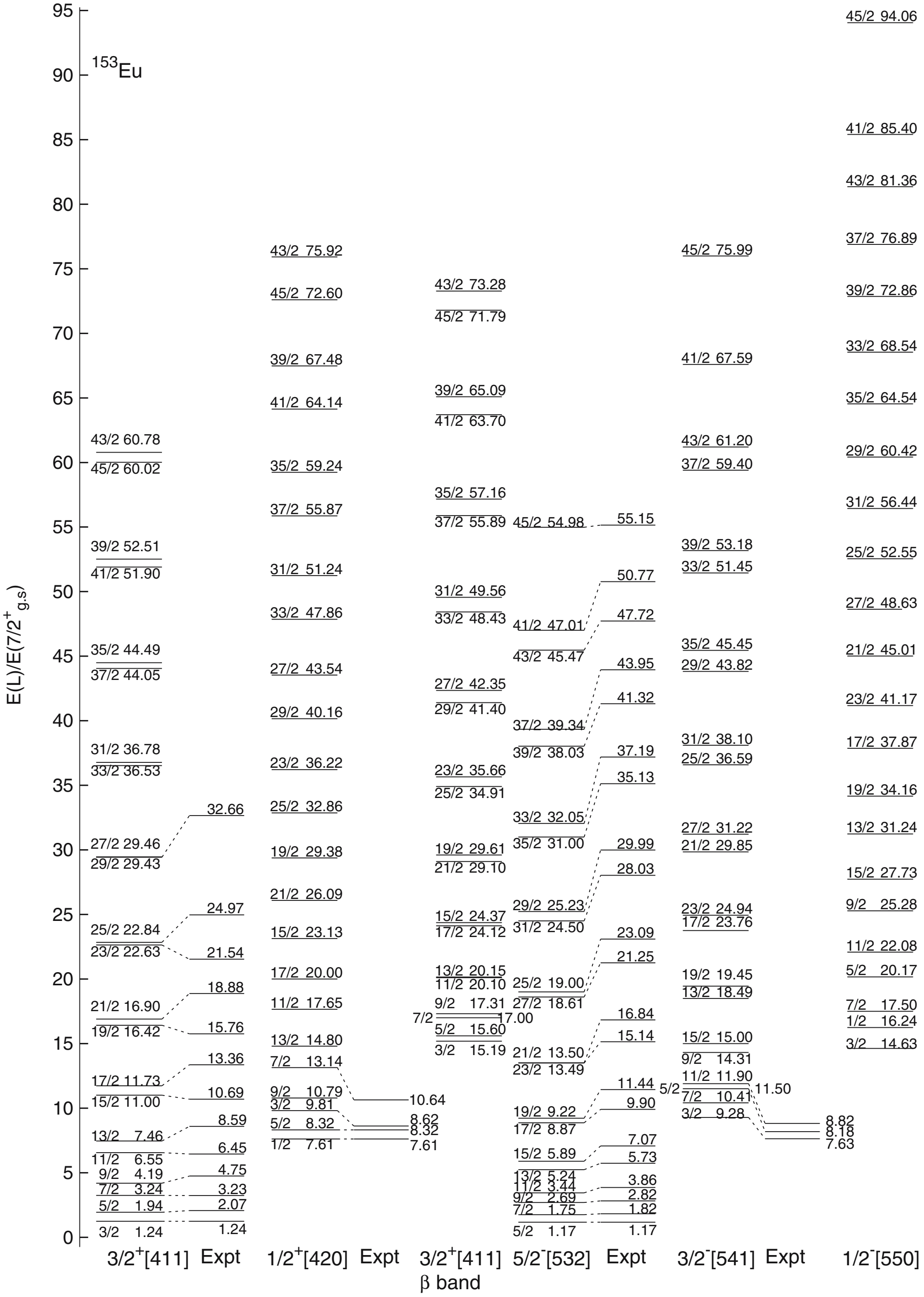

Figure 3. Comparison of the calculated values of energy levels originating from $2d_{5/2}$ and $1h_{11/2}$ Nilsson orbitals in units of $E(7/2^+_{\mathrm{g.s.}})$ for ^{153}Eu with the experimental data from ref. [1].

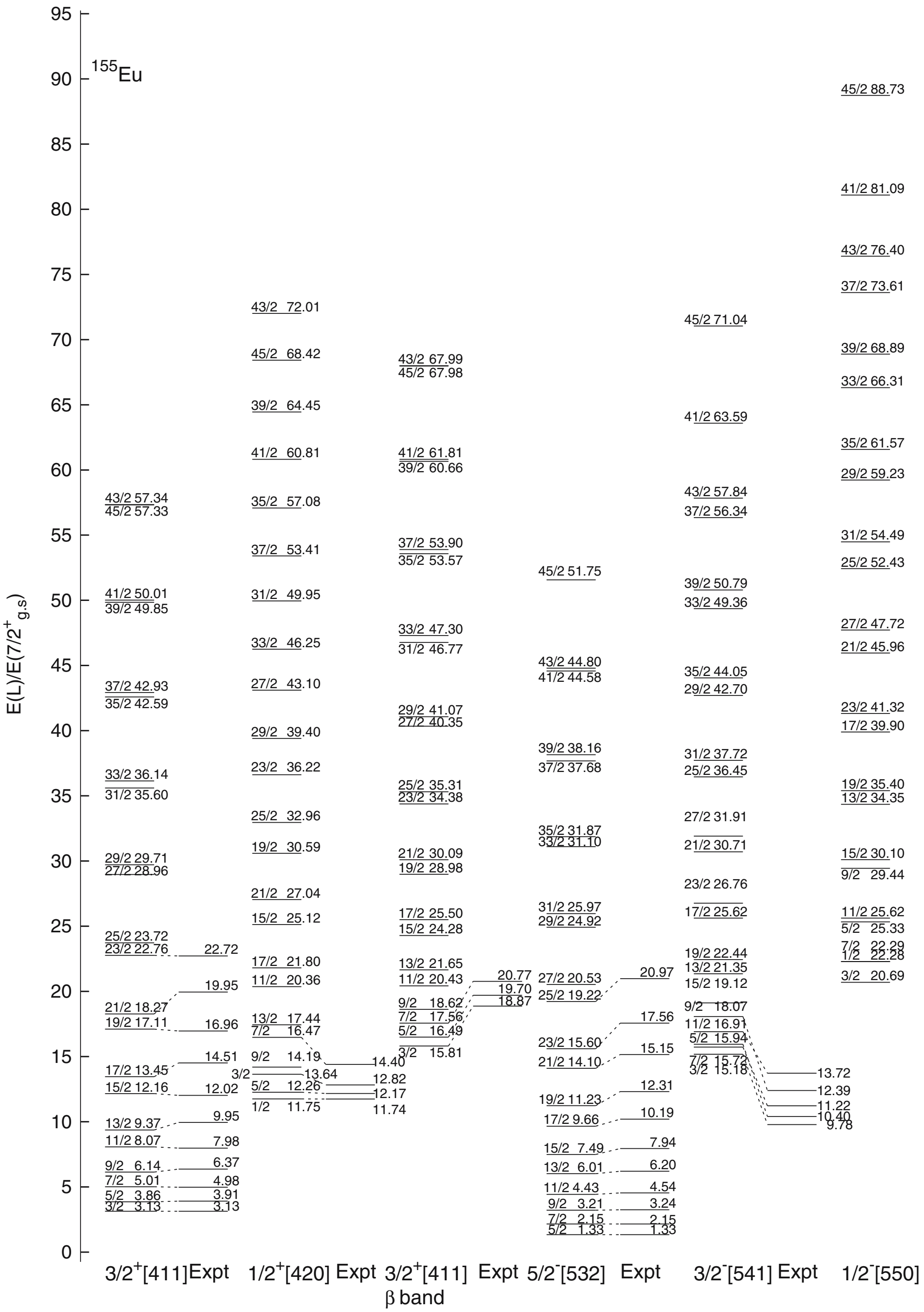

Figure 4. Comparison of the calculated values of energy levels originating from $2d_{5/2}$ and $1h_{11/2}$ Nilsson orbitals in units of $E(7/2^+_{\mathrm{g.s.}})$ for ^{155}Eu with the experimental data from ref. [1].

Table 2. The calculated and experimental values of the $B(E2; L_{\text{g.s.}}+2 \rightarrow L_{\text{g.s.}})$ in units of $B(E2; 9/2^+_{\text{g.s.}} \rightarrow 5/2^+_{\text{g.s.}})$ for ^{153}Eu. The experimental values are taken from ref. [4].

$L_{\text{g.s.}}$	$7/2^+$	$9/2^+$	$11/2^+$	$13/2^+$	$15/2^+$	$17/2^+$	$19/2^+$	$21/2^+$
Calc.	1.55	1.86	2.07	2.22	1.88	2.05	2.07	2.19
Expt.	1.50(7)	1.36(4)	1.92(4)	1.82(6)	1.98(9)	2.02(6)	1.97(8)	2.03(10)

^{153}Eu, they start to arrange lower than in the experiment. The Coriolis effect of the diagonal elements arranges the levels of $1/2^+[420]$ exactly as per the experiment for the first three levels of the isotopes. The calculated $9/2^-$ level does not exist in the experiment. The $3/2^+[413]$ β band, which has been measured only for ^{155}Eu, is higher than in the experiment.

4.3 *The bands originating from* $1h_{11/2}$

Two of the three bands, $5/2^-[532]$, $3/2^-[541]$ and $1/2^-[550]$ that originate from $1h_{11/2}$, have been measured for ^{153}Eu and ^{155}Eu. There are experimental levels up to $45/2^-$ for ^{153}Eu and up to $25/2^-$ for ^{155}Eu. It is seen from figures 3 and 4 that the interchange of the numerical sequence for pairs of levels of the $5/2^-[532]$ band at higher spins is also observed in the experimental levels for ^{153}Eu after $19/2^-$, and this behaviour does not exist for ^{155}Eu.

The three experimentally measured levels of the $1/2^-[550]$ band of ^{153}Eu are lower than the ones mentioned in the theory. The $7/2^-$ level, which is not observed in the experiment, is located between the $3/2^-$ and $5/2^-$ levels in the calculation. For ^{155}Eu, the pairs of the calculated levels $5/2^-$ and $7/2^-$, and $11/2^-$ and $9/2^-$, are interchanged with respect to experiment. The agreement between the calculated and the experimental values of the excited state energies in some $K = 1/2$ bands is not very good. As is known, $K = 1/2$ bands are strongly affected by Coriolis term. Various arguments of the possible reasons are discussed in connection with the systematics of Coriolis decoupling parameter in ref. [43]. Very large values of decoupling parameter in ^{155}Eu, for example, is explained by octupole deformation which is not included in this work. Moreover, it has been mentioned in ref. [43] that there can be an admixture of γ bands in A=153–163 region.

Comparison of the calculated $B(E2)$ values of ^{153}Eu within the ground-state band with those of the available experimental data is shown in table 2. As we concentrate on the changes of the spectra with changes to the interaction of the last nucleon with the core, we have changed only the parameters corresponding to this case, and did not change the parameters which belong to the core part. Therefore, $B(E2)$ values within the ground-state band will not change drastically for the other isotopes.

5. Conclusions

Band structures built on Nilsson orbitals of four europium isotopes have been studied using different mass parameters for each allowed collective mode. Calculated energy levels and reduced $B(E2)$ transition probabilities are compared with the existing experimental data. It is shown that both deformation of the core and the interaction of the last odd proton with the core have a significant influence on the spectra of these nuclei. Also these results suggest the necessity to complicate the model with the inclusion of triaxiality.

Acknowledgements

The authors are grateful to Dr P R Fraser for fruitful discussions. MJE acknowledges support through grant F2-FA-F177 of Uzbekistan Academy of Sciences.

References

[1] http://www.nndc.bnl.gov/ensdf
[2] C J Pearson *et al*, *Phys. Rev. C* **49**, R1239 (1994)
[3] S Basu *et al*, *Phys. Rev. C* **56**, 1756 (1997)
[4] J F Smith *et al*, *Phys. Rev. C* **58**, 3171 (1998)
[5] J F Smith *et al*, *Phys. Rev. C* **62**, 034312 (2000)
[6] D Burke, *Nucl. Phys. A* **747**, 131 (2005)
[7] A Bohr, *Mat. Fys. Medd. K. Dan. Vidensk. Selsk.* **26**, 1 (1952)
[8] J M Eisenberg and W Greiner, *Nuclear models: Collective and single-particle phenomena* (North Holland, Amsterdam, 1970)
[9] A S Davydov and A F Filipov, *Nucl. Phys.* **8**, 237 (1958)
[10] A S Davydov and A A Chaban, *Nucl. Phys.* **20**, 499 (1958)
[11] A Faessler and W Greiner, *Z. Phys.* **168**, 425 (1962)
[12] A S Davydov and R A Sardaryan, *Nucl. Phys.* **37**, 106 (1962)
[13] V V Pashkevich and R A Sardaryan, *Nucl. Phys.* **65**, 401 (1965)
[14] F Iachello, *Phys. Rev. Lett.* **87**, 052502 (2001)
[15] R F Casten and N V Zamfir, *Phys. Rev. Lett.* **87**, 052503 (2001)
[16] D Bonatsos *et al*, *Phys. Rev. C* **69**, 014302 (2004)
[17] D Bonatsos, P E Georgoudis, D Lenis, N Minkov and C Quesne, *Phys. Rev. C* **83**, 044321 (2011)
[18] I Yigitoglu and D Bonatsos, *Phys. Rev. C* **83**, 014303 (2011)
[19] D Bonatsos, E A McCutchan, N Minkov, R F Casten, P Yotov, D Lenis, D Petrellis and I Yigitoglu, *Phys. Rev. C* **76**, 064312 (2007)
[20] L Fortunato, *Phys. Rev. C* **70**, 011302 (2004)
[21] D J Rowe, *Nucl. Phys. A* **735**, 372 (2004)
[22] M A Caprio, *Phys. Rev. C* **83**, 064309 (2011)
[23] R V Jolos and P von Brentano, *Phys. Rev. C* **74**, 064307 (2006)
[24] R V Jolos and P von Brentano, *Phys. Rev. C* **76**, 024309 (2007)
[25] R V Jolos and P von Brentano, *Phys. Rev. C* **78**, 064309 (2008)
[26] R V Jolos and P von Brentano, *Phys. Rev. C* **79**, 044310 (2009)
[27] D Bonatsos, P E Georgoudis, N Minkov, D Petrellis and C Quesne, *Phys. Rev. C* **88**, 034316 (2013)
[28] R Jolos, N Shirikova and A Sushkov, *Phys. At. Nucl.* **76**, 557 (2013)
[29] S Sharipov and M J Ermamatov, *Int. J. Mod. Phys. E* **12**, 41 (2003)
[30] S Sharipov, M J Ermamatov and J K Bayimbetova, *Int. J. Mod. Phys. E* **15**, 951 (2006)
[31] S Sharipov, M J Ermamatov and J K Bayimbetova, *Phys. At. Nucl.* **71**, 215 (2008)
[32] M J Ermamatov and P R Fraser, *Phys. Rev. C* **84**, 044321 (2011)
[33] M J Ermamatov, P C Srivastava, P R Fraser and P Stránský, *Euro. Phys. J. A* **48**, 1 (2012)

[34] M J Ermamatov, P C Srivastava, P R Fraser, P Stránský and I O Morales, *Phys. Rev. C* **85**, 034307 (2012)

[35] A Frank, S Pittel, D Warner and J Engel, *Phys. Lett. B* **182**, 233 (1986)

[36] A Arima, A Gelberg and O Scholten, *Phys. Lett. B* **185**, 259 (1987)

[37] R F Casten, *Nuclear structure from a simple perspective* (Oxford University Press, 1990)

[38] F Iachello and P V Isacker, *The interacting boson–fermion model* (Cambridge University Press, Cambridge, 1991)

[39] A Frank, P V Isacker and D Warner, *Phys. Lett. B* **197**, 474 (1987)

[40] B A Brown, N J Stone, J R Stone, I S Towner and M Hjorth-Jensen, *Phys. Rev. C* **71**, 044317 (2005)

[41] E Caurier, G Martinez-Pinedo, F Nowacki, A Poves and A P Zucker, *Rev. Mod. Phys.* **77**, 427 (2005)

[42] R Helmer, *Nuclear Data Sheets* **83**, 285 (1998)

[43] A K Jain, R K Sheline, P C Sood and Kiran Jain, *Rev. Mod. Phys.* **62**, 393 (1990)