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Exact solutions of certain nonlinear chemotaxis diffusion reaction equations

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Abstract. Using the auxiliary equation method, we obtain exact solutions of certain nonlinear chemotaxis diffusion reaction equations in the presence of a stimulant. In particular, we account for the nonlinearities arising not only from the density-dependent source terms contributed by the particles and the stimulant but also from the coupling term of the stimulant. In addition to this, the diffusion of the stimulant and the effect of long-range interactions are also accounted for in the constructed coupled differential equations. The results obtained here could be useful in the studies of several biological systems and processes, e.g., in bacterial infection, chemotherapy, etc.

Keywords. Nonlinear diffusion reaction equation; chemotaxis; auxiliary equation method; solitary wave solutions.

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1. Introduction

Chemotaxis – a chemically directed movement – is a nonlocal mechanism in which, unlike diffusion, the motion is directed towards the chemical due to its concentration gradient. In other words, chemotaxis is defined as the movement of an organism in response to the gradient created by a chemical substrate or stimulant (which, in fact, can be an attractant or a repellant). Further, in this mechanism there is a random movement of particles or cells depending upon the nature of the phenomenon under study.

Besides that, the importance of this mechanism is highlighted in the context of animal and insect ecology [1], its account, at times, becomes crucial in the study of some biophysical systems [2–4] as well. For example, when a bacterial infection invades the body, the former at the same time is also attacked by the cells of the body as a result of chemotaxis mechanism. Another example is that of chemotherapy [1,5] in which this mechanism works to eliminate the cancerous cells from the body. In fact, the study of chemotaxis has become very important in a variety of situations. The mathematical equations constructed here are rich enough to accomodate some of these situations.

Note that the directed movement is the characteristic feature of this chemotaxis mechanism and there are various ways and means of creating this directed movement which in turn allow the introduction of different models in different contexts. For example, Dallon and Othmer [6,7] developed a model in which the cells are considered as discrete entities and the chemoattractant concentrations are continuous. The results are found to agree well with many of the extant experimental results. Similarly, when pheromone is released by a particular insect in a given population, then the directed movement of the remaining members takes place in the population itself due to the release of the chemical [1]. It may be mentioned that theoretical studies have been carried out for several species and several stimulants [8–13]. In this paper, however, we shall limit our studies to one species and one stimulant with some plausible generalizations.

Here, we attempt to solve rather exactly the coupled pair of nonlinear differential equations which account for several new features pertaining to this mechanism of chemotaxis. In particular, the following features of the chemotaxis mechanism are accounted for in the present work: (i) The source terms for both particles and stimulants are considered as density-dependent in a nonlinear fashion. (ii) When considering the source term for the stimulant, both linear and nonlinear decays in it are found to conform the exact nature of the solutions of the derived equations. Similarly, for the particle source term, a different type of nonlinearity is found to be consistent with the derived solutions. In general, these choices of the source terms are model-dependent. In fact, some features in the source terms used here, are already speculated in the literature (see, e.g., [1,2]). (iii) Certain types of nonlocal effects arising from next-to-nearest neighbour interactions are also accounted for in the constructed nonlinear equations.

In the next section, we construct the chemotaxis diffusion reaction equation in a general manner accounting for all the above-mentioned effects. However, we proceed only for the exact solutions of equations corresponding to the case of one species and one stimulant. Exact solutions of these equations for the two choices of the source function for the stimulant are obtained in §3 and §4 using the so-called 'auxiliary equation method' [14–20]. Finally, the results are discussed in §5.

2. Construction of chemotaxis-diffusion-reaction equations

Horstmann [8], when studying the case of N species and M stimulants, however could not account for certain effects in his scheme of study. For example, next-to- nearest neighbour or for that matter long-range interaction [21] could not be accomodated in his work. In what follows, we account for such details and recast his equations for the ith species and jth stimulant in the following form:

$$\frac{\partial n_i}{\partial t} = F_i(n,s) - \nabla \cdot \left[\sum_{j=1}^{M} C_{ij}(n,s)\nabla s_j\right] + \nabla \cdot \left[\left(\sum_{k=1}^{N} D_{1_{ik}}(n,s)\nabla n_j\right)\right]$$
$$-\nabla \cdot \left[\nabla \cdot \sum_{l=1}^{N} D_{2_{il}}(n,s)\nabla^2 n_j\right], \qquad (1)$$

$$\frac{\partial s_j}{\partial t} = \nabla \cdot \left[\left(\sum_{u=1}^{M} D_{0_{uj}}(n, s) \nabla s_i \right) \right] + G_j(n, s), \tag{2}$$

where $n_1, \ldots, n_N$ and $s_1, \ldots, s_M$ are the number densities of N species and M stimulants, respectively; n_j with $j \neq i = 1, \ldots, N$ represents the remaining $N - 1$ interacting sites. Note that the coefficient matrices D_1, D_2, C and D_0 are the functions of number densities of both species and stimulant in general. Here $F_i(n, s)$ and $S_j(n, s)$, respectively are the source terms defining a highly particular situation in which both species as well as stimulants in general, contribute to each other's population. But in reality it is not so; normally species do contribute but not the stimulants.

In the present case of one species and one stimulant, we consider D_1, D_2 and D_0 as constants and take $C(n, s) = n\chi_0$ (here onwards the subscripts to n and s are dropped). In general, χ is a function of s, viz., $\chi \equiv \chi(s)$. It is the measure of chemotactic strength but we resort to choose it as a constant. In eq. (1), while the second term on the right-hand side accounts for the chemotactic flux, the third and fourth terms respectively account for the diffusion and for the long-range interactions [1,20–22] as an attribute of next-to-nearest neighbour averages. In fact, a variety of nonlinear diffusion reaction equations has been studied earlier with such long-range interactions [20]. With these choices, eqs (1) and (2) can be written as

$$\frac{\partial n}{\partial t} = F(n, s) - \nabla \cdot \left(n\chi(s) \frac{\partial s}{\partial x} \right) + \nabla \cdot \left(D_1 \nabla n - D_2 \frac{\partial^3 n}{\partial x^3} \right), \tag{3}$$

$$\frac{\partial s}{\partial t} = G(n, s) + \frac{\partial}{\partial x} \left(D_0 \frac{\partial s}{\partial x} \right). \tag{4}$$

Next, we obtain exact solution of these coulped equations for the case when

$$F(n, s) \equiv f(n) = \alpha n - \beta n (n_x)^2 \tag{5}$$

and for the two choices of $G(n, s)$, namely

$$\text{(a)} \quad G(n, s) = hn^3 - ks \quad \text{and} \quad \text{(b)} \quad G(n, s) = hn^2 - ks^2, \tag{6}$$

where h, k are positive constants. The first terms in these choices represent the spontaneous production of the stimulant whereas terms like $-ks$ and $-ks^2$ in them account for the decay of the stimulant density. With regard to the choice of $f(n)$, note that the second term in its expression accounts for the nonlocal effects.

For the choice (5), eq. (3) in one dimension takes the form

$$n_t - D_1 n_{xx} + D_2 n_{xxxx} + \chi_0 n_x s_x + \chi_0 n s_{xx} = \alpha n - \beta n (n_x)^2, \tag{7}$$

whereas for the forms (6a) and (6b), eq. (4) gives

$$s_t - D_0 s_{xx} - hn^3 + ks = 0 \tag{8}$$

and

$$s_t - D_0 s_{xx} - hn^2 + ks^2 = 0, \tag{9}$$

respectively. For travelling wave solutions, we use the wave variable $\xi = x - wt$ and write $n(x,t) = u(\xi)$ and $s(x,t) = v(\xi)$, which, in turn, lead eqs (7)–(9) to the form

$$-wu' - D_1u'' + D_2u'''' + \chi_0 u'v' + \chi_0 uv'' - \alpha u + \beta u(u')^2 = 0, \tag{10}$$

$$-wv' - D_0v'' - hu^3 + kv = 0, \tag{11}$$

$$-wv' - D_0v'' - hu^2 + kv^2 = 0. \tag{12}$$

It may be mentioned that the stability of the solutions of equations like eqs (3) and (4) have also been studied in the literature by way of carrying out a kind of stability analysis [8–11]. In this method, one normally linearizes the system often by neglecting the higher-order nonlinear terms; or at the most by accounting for some simplified versions of nonlinearity. On the other hand, the exact solutions of the underlying equations, if obtained, will provide a deeper insight into the nature of solutions as far as the parametric dependence of the latter is concerned. With this spirit in mind, we proceed for exact solutions of the above equations, of course with certain restrictions on the underlying parameters. In the next two sections we investigate such solutions of the pair-wise coupled differential equations, namely the pairs of eqs (10) and (11), and (10) and (12).

3. Exact solutions of eqs (10) and (11)

For the pair of eqs (10) and (11), we make an ansatz [19]

$$u(\xi) = \sum_{i=0}^{l} a_i z^i(\xi), \quad v(\xi) = \sum_{i=0}^{m} b_i z^i(\xi), \tag{13}$$

where a_i and b_i are real constants to be determined later. Here, l and m are positive integers which can be obtained by balancing [19] the highest-order derivative terms with the highest-order nonlinear terms in eqs (3) and (4). The function $z(\xi)$ is chosen in accordance with the solutions of the differential equation [19] of the form

$$\frac{\mathrm{d}z}{\mathrm{d}\xi} = b + z^2(\xi), \tag{14}$$

where b is a constant to be determined later. Equation (14) admits the solutions,

(i) $z(\xi) = -\sqrt{-b}\tanh\left(\sqrt{-b}\xi\right)$ or $z(\xi) = -\sqrt{-b}\coth\left(\sqrt{-b}\xi\right)$, for $b < 0$,

(ii) $z(\xi) = \sqrt{b}\tan(\sqrt{b}\xi)$ or $z(\xi) = -\sqrt{b}\cot(\sqrt{b}\xi)$, for $b > 0$,

(iii) $z(\xi) = \frac{-1}{\xi}$, for $b = 0$.

Note that the case $b < 0$ provides the kink–antikink or solitary wave class of solutions and the case corresponding to $b > 0$ provides periodic solutions which in the present context are of least physical interest. The balancing procedure, on the other hand, immediately yields $l = m = 1$ in eq. (13). This leads to the forms of $u(\xi)$ and $v(\xi)$ as

$$u(\xi) = a_0 + a_1 z(\xi), \tag{15}$$

$$v(\xi) = b_0 + b_1 z(\xi). \tag{16}$$

Using these results along with (14) in eqs (10) and (11) and then setting the coefficients of $z^j(\xi)$ for $j = 0, 1, \ldots, 5$ separately to zero in the resultant expression, one obtains the following set of algebraic equations:

$$\beta a_1^3 + 24D_2 a_1 = 0, \tag{17}$$

$$\beta a_0 a_1^2 + 3\chi_0 a_1 b_1 = 0, \tag{18}$$

$$-2D_1 a_1 + 40ba_1 D_2 + 2\chi_0 b_1 a_0 + 2\beta b a_1^3 = 0, \tag{19}$$

$$-wa_1 + 4\chi_0 a_1 b_1 b + 2\beta a_0 a_1^2 b = 0, \tag{20}$$

$$-2D_1 a_1 b + 16b^2 a_1 D_2 + 2\chi_0 b_1 a_0 b - \alpha a_1 + \beta a_1^3 b^2 = 0, \tag{21}$$

$$-wa_1 b + \chi_0 a_1 b_1 b^2 - \alpha a_0 + \beta b^2 a_0 a_1^2 = 0, \tag{22}$$

corresponding to eq. (10), and

$$-ha_1^3 - 2D_0 b_1 = 0, \tag{23}$$

$$-wb_1 - 3ha_0 a_1^2 = 0, \tag{24}$$

$$-3ha_0^2 a_1 + kb_1 - 2D_0 b_1 b = 0, \tag{25}$$

$$-wa_1 b - ha_0^3 + kb_0 = 0, \tag{26}$$

corresponding to eq. (11). Next, we solve these equations for the arbitrary constants appearing in eqs (14)–(16). Note that eqs (17), (18), (23) and (24) immediately yield the values of a_0, a_1, b_1, w as

$$a_0 = -\frac{36\chi_0 h D_2}{D_0 \beta^2}; \quad a_1 = \sqrt{\frac{-24D_2}{\beta}};$$

$$b_1 = -\frac{h}{2D_0}\left[\left(\frac{-24D_2}{\beta}\right)^{3/2}\right]; \quad w = -\frac{108\chi_0 h}{\beta}\sqrt{\frac{-D_2}{6\beta}}. \tag{27}$$

Further use of these values in eqs (20) and (26) yields the values of b and b_0 as

$$b = \frac{3D_0}{8D_2}; \quad b_0 = -\frac{972\chi_0 h^2 D_2}{k\beta^3}\left[\frac{48\chi_0^2 h^2 D_2}{D_0^3 \beta^3} + 1\right]. \tag{28}$$

Now using these values in eq. (22) yields $\alpha = 0$, the remaining eqs (19), (21) and (25) uniquely give the following constraining relations among various parameters, namely $D_1, D_2, D_0, \beta, h, k$ and χ_0:

$$3D_0^2\beta^3 - 4D_2 D_0 k\beta^3 + 1296\chi_0^2 h^2 D_2^2 = 0; \tag{29}$$

$$3\beta^3 D_0^3 + 2\beta^3 D_0^2 D_1 + 864\chi_0^2 h^2 D_2^2 = 0. \tag{30}$$

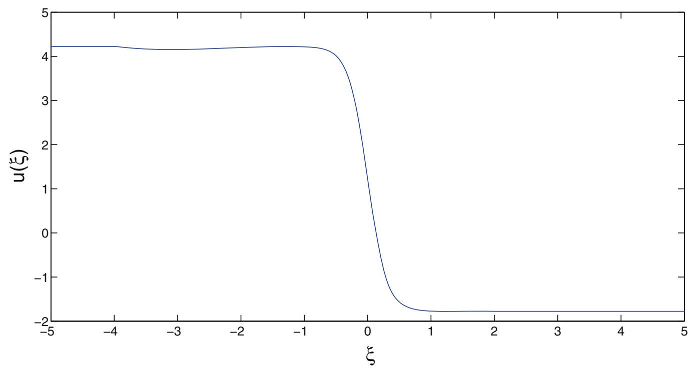

Figure 1. Solution $u(\xi)$ of coupled eqs (10) and (11) (cf. eq. (31)) corresponding to $\chi_0 = h = 1$, $D_2 = 0.034$, $k = -11.0924$ and $D_0 = \beta = -1$.

Finally, the solutions $u(\xi)$ and $v(\xi)$ of eqs (10) and (11) corresponding to $b < 0$ turn out to be

$$u(\xi) = \left[-\frac{36\chi_0 h D_2}{D_0 \beta^2} - 3\sqrt{\frac{D_0}{\beta}} \tanh\left(\sqrt{\frac{-3D_0}{8D_2}} \xi \right) \right], \tag{31}$$

$$v(\xi) = \left[-\frac{972\chi_0 h^2 D_2}{k\beta^3} \left(\frac{48\chi_0^2 h^2 D_2}{D_0^3 \beta^3} + 1 \right) - \frac{36hD_2}{\beta} \sqrt{\frac{D_0}{\beta}} \tanh\left(\sqrt{\frac{-3D_0}{8D_2}} \xi \right) \right], \tag{32}$$

for $D_0 < 0$, $D_2 > 0$, which clearly represent a kink or antikink class of solitary wave solutions. In this case the populations of both particles and stimulant will either increase or decrease in the beginning and attain constant values later. The choice of the model in terms of the signs of β, D_0 and D_2, however, becomes crucial for this purpose as both

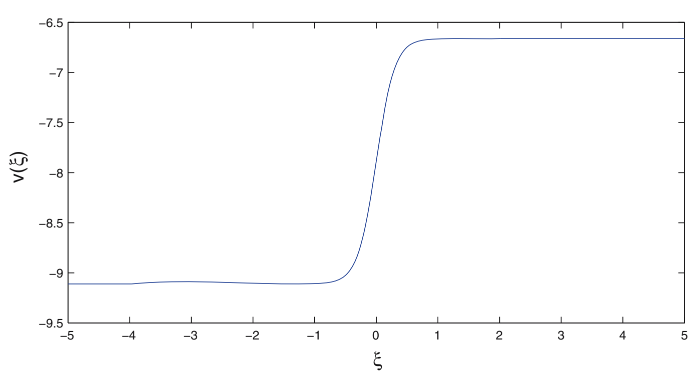

Figure 2. Solution $v(\xi)$ of coupled eqs (10) and (11) (cf. eq. (32)) corresponding to $\chi_0 = h = 1$, $D_2 = 0.034$, $k = -11.0924$ and $D_0 = \beta = -1$.

$u(\xi)$ and $v(\xi)$ depend on the tanh function. As a matter of fact, these claims are supported by the representative plots shown in figures 1 and 2 corresponding to the solutions (31) and (32), respectively.

4. Exact solutions of eqs (10) and (12)

We use the balancing procedure again with the ansatz (13) and obtain $l = 1$ and $m = 2$ in the present case. This yields the choice for $u(\xi)$ and $v(\xi)$ as

$$u(\xi) = a_0 + a_1 z(\xi), \tag{33}$$

$$v(\xi) = b_0 + b_1 z(\xi) + b_2 (z(\xi))^2. \tag{34}$$

As before, substituting (33) and (34) along with (14) in eqs (10) and (12) and then equating the coefficients of powers of $z(\xi)$ to zero in the resultant expression, one obtains another set of algebraic equations

$$\beta a_1^3 + 24 D_2 a_1 + 8\chi_0 b_2 = 0, \tag{35}$$

$$\beta a_0 a_1^2 + 3\chi_0 a_1 b_1 + 6 b_2 a_0 \chi_0 = 0, \tag{36}$$

$$-2D_1 a_1 + 40 b a_1 D_2 + 2\chi_0 b_1 a_0 + 2\beta b a_1^3 + 12\chi_0 a_1 b_2 b = 0, \tag{37}$$

$$-w a_1 + 4\chi_0 a_1 b_1 b + 2\beta a_0 a_1^2 b + 8\chi_0 a_0 b_2 b = 0, \tag{38}$$

$$-2D_1 a_1 b + 16 b^2 a_1 D_2 + 2\chi_0 b_1 a_0 b - \alpha a_1 + \beta a_1^3 b^2 + 4\chi_0 a_1 b_2 b^2 = 0, \tag{39}$$

$$-w a_1 b + \chi_0 a_1 b_1 b^2 - \alpha a_0 + \beta b^2 a_0 a_1^2 + 2\chi_0 a_0 b_2 b^2 = 0, \tag{40}$$

corresponding to eq. (10) and

$$-6 b_2 D_0 + k b_2^2 = 0, \tag{41}$$

$$-2 b_2 w + 2 b_1 b_2 k - 2 b_1 D_0 = 0, \tag{42}$$

$$-w b_1 - h a_1^2 + k b_1^2 + 2 b_0 b_2 k - 8 D_0 b_2 b = 0, \tag{43}$$

$$-2 b_2 b w - 2 h a_0 a_1 + 2 k b_1 b_0 - 2 D_0 b_1 b = 0, \tag{44}$$

$$-w b_1 b - h a_0^2 + k b_0^2 - 2 b_2 D_0 b^2 = 0, \tag{45}$$

corresponding to eq. (12).

After solving the above set of equations, we obtain the values of unknowns a_0, a_1, b_0, b_1 and b in terms of known parameters

$$a_0 = \frac{3\chi_0}{10(D_0\chi_0 + 2kD_2)}\sqrt{\frac{(-24D_2 - 48\frac{\chi_0 D_0}{k})}{\beta}};$$

$$a_1 = \sqrt{\frac{(-24D_2 - 48\frac{\chi_0 D_0}{k})}{\beta}}; \qquad b_1 = \frac{6w}{5k}; \qquad b_2 = \frac{6D_0}{k};$$

$$\alpha = 0; \quad b = -\frac{5k}{24\chi_0}\left[\frac{D_0\chi_0 + 2kD_2}{2D_0\chi_0 + kD_2}\right];$$

$$b_0 = \left[\frac{-5D_0}{4\chi_0}\frac{(D_0\chi_0 + 2kD_2)}{(2D_0\chi_0 + kD_2)} + \frac{h\chi_0}{4k\beta}\frac{(24D_2k + 48\chi_0 D_0\beta)}{(D_0\chi_0 + 2kD_2)}\right];$$

and finally, we obtain the solution of eqs (10) and (12) and it turns out to be

$$\begin{aligned} u(\xi) = {} & \sqrt{\frac{-1}{\beta}\left(24D_2 + 48\frac{\chi_0 D_0}{k}\right)} \\ & \times \left[\frac{3\chi_0 w}{10(D_0\chi_0 + 2kD_2)} - \sqrt{\frac{5k}{24\chi_0}\frac{D_0\chi_0 + 2kD_2}{2D_0\chi_0 + kD_2}}\right. \\ & \left.\times \tanh\left(\frac{5k}{24\chi_0}\frac{D_0\chi_0 + 2kD_2}{2D_0\chi_0 + kD_2}\xi\right)\right], \end{aligned} \tag{46}$$

$$\begin{aligned} v(\xi) = {} & \left[b_0 - \frac{6w}{5k}\sqrt{\frac{5k}{24\chi_0}\frac{D_0\chi_0 + 2kD_2}{2D_0\chi_0 + kD_2}}\tanh\left(\sqrt{\frac{5k}{24\chi_0}\frac{D_0\chi_0 + 2kD_2}{2D_0\chi_0 + kD_2}}\xi\right)\right. \\ & + \frac{6D_0}{k}\left(\frac{5k}{24\chi_0}\frac{D_0\chi_0 + 2kD_2}{2D_0\chi_0 + kD_2}\right)^2 \\ & \left.\times \tanh^2\left(\sqrt{\frac{5k}{24\chi_0}\frac{D_0\chi_0 + 2kD_2}{2D_0\chi_0 + kD_2}}\xi\right)\right], \end{aligned} \tag{47}$$

for $b < 0$. Here w can be determined from either eq. (43) or (45). As the analysis of results becomes difficult for this general case, we resort to use $D_2 = 0$, i.e., the case when long-term interactions are negligible. In this case, the unknown parameters determined from eqs (35)–(45) turn out to be

$$a_0 = \frac{6w}{5}\sqrt{\frac{-3\chi_0}{\beta D_0 k}}; \qquad a_1 = \sqrt{\frac{-48\chi_0 D_0}{\beta k}}; \qquad b = -\frac{5k}{48\chi_0}$$

$$b_0 = -\left(\frac{5D_0k\beta - 96h\chi_0^2}{8k\beta\chi_0}\right); \quad b_1 = \frac{6w}{5k}; \quad \alpha = 0; \quad b_2 = \frac{6D_0}{k};$$

with an expression for w^2 as

$$w^2 = \frac{25kD_0}{36\chi_0}[4D_1 - 5D_0].$$

It may be noted that α again turns out to be zero in the present case. The constraining relations among various parameters now become

$$4D_1\beta - 5D_0\beta(1 + 3k) + 1152h\chi_0^2 = 0, \tag{48}$$

$$25k^2\beta^2 D_0(2D_1 - D_0) + 864\chi_0^2 h(2D_1k\beta + 24h\chi_0^2 - 5D_0k\beta) = 0, \tag{49}$$

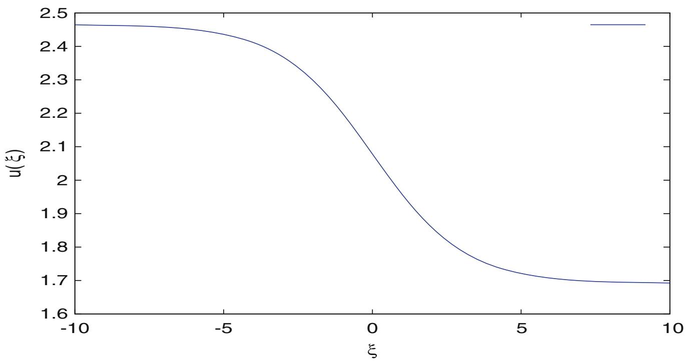

Figure 3. Solution $u(\xi)$ of coupled eqs (10) and (12) (cf. eq. (50)) corresponding to $D_0 = D_1 = 1.0$ and $k < 0$, $\beta < 0$ and $\chi_0 < 0$.

and finally, we obtain the solutions of eqs (10) and (12) in the forms

$$u(\xi) = \frac{6w}{5}\left[\sqrt{\frac{-3\chi_0}{D_0\beta k}} - \sqrt{\frac{5}{48\chi_0 k}}\tanh\left(\sqrt{\frac{5k}{48\chi_0}}\xi\right)\right], \tag{50}$$

$$v(\xi) = \left[\left(\frac{-5D_0k\beta + 96h\chi_0^2}{8k\beta\chi_0}\right) + \frac{3w}{\sqrt{60\chi_0 k}}\tanh\left(\sqrt{\frac{5k}{48\chi_0}}\xi\right) + \frac{5}{8\chi_0}\tanh^2\left(\sqrt{\frac{5k}{48\chi_0}}\xi\right)\right], \tag{51}$$

for

$$b = -\frac{5k}{48\chi_0} < 0.$$

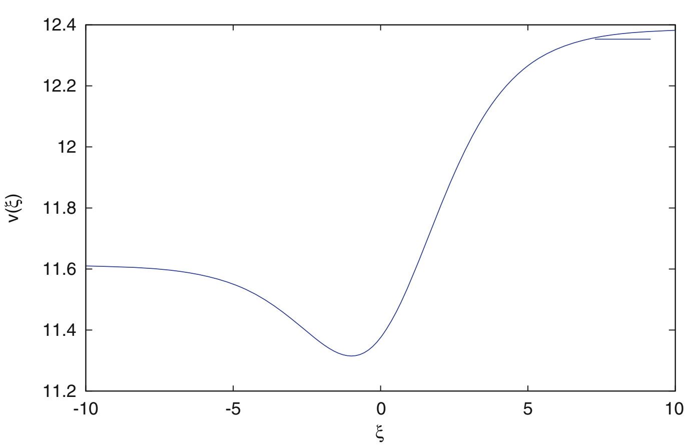

Figure 4. Solution $v(\xi)$ of coupled eqs (10) and (12) (cf. eq. (51)) corresponding to $\beta = D_0 = h = w = 1$ and $k < 0$, $\chi_0 < 0$.

While eq. (50) represents a kink or antikink-type solution, eq. (51) represents a bell-type solitary wave solution in the absence of the second term. However, representative plots of complete solutions (50) and (51) are shown in figures 3 and 4, respectively. These solutions though limit their viability by considering D_2 as zero, however, are rich enough as far as the role of nonlinearity in the model under study is concerned.

5. Discussion and concluding remarks

Several new features of the chemotaxis mechanism, particularly with reference to one species and one stimulant, are investigated here by way of obtaining exact solutions of the constructed nonlinear differential equations. While such solutions of several cases of particular interest have remained a far cry in the past, we have obtained them here using the well-known auxilliary equation method and with certain restrictions on the underlying parameters.

As far as the choice of the source terms $F(n, s)$ and $G(n, s)$ in eqs (3) and (4) through (5) and (6) is concerned, note that α in eq. (5) turns out to be zero for all cases investigated here. This yields the nonlinear and nonlocal nature of $f(n)$. As a mater of fact, the requirement of physically acceptable solutions demands that $\beta < 0$ at least for the choice of the stimulant source term (6a). In order that the solutions for this case are of kink–antikink-type, the condition $b < 0$ in this case requires that $D_0 < 0$ in eq. (28), i.e., diffusion coefficient is negative. Such a result is not surprising. In fact, recent studies by Winkler [23] clearly suggest such possibilities if one studies the role of χ_0 and D_0 together.

It may be mentioned that in earlier works on the study of chemotactic mechanism, either the diffusion of the stimulant is not considered [25] or if at all it is considered [2], then a simplified, often a linear version of the source term [1,2,23,24], is chosen. This in fact limits the scope of applications of the derived results. In the present case, however, our choices are quite general. In addition to diffusion, some generalized nonlinear choices of the source term $G(n, s)$ are also considered. For the case (6a), while kink–antikink-type solutions are obtained, for (6b) however, the equations also admit a bell-type solitary wave solution for certain densities of the stimulant. Clearly, our solutions are going to provide a great freedom for experimentalist as far as the choice of a model in general or of a stimulant in particular is concerned and that too with exact solutions of the underlying equations.

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