PRAMANA
— journal of
physics

Vol. 86, No. 5
May 2016
pp. 1009–1019

Transformation of bipartite non-maximally entangled states into a tripartite W state in cavity QED

XUE-PING ZANG[1,2], MING YANG[1,*], CHAO-QUN DU[1], MIN WANG[1], SHU-DONG FANG[2] and ZHUO-LIANG CAO[3]

[1]Key Laboratory of Opto-electronic Information Acquisition and Manipulation, Ministry of Education, School of Physics & Material Science, Anhui University, Hefei 230601, People's Republic of China

[2]Department of Mechanical and Electronic Engineering, Chizhou University, Chizhou, Anhui 247000, People's Republic of China

[3]Department of Physics & Electronic Engineering, Hefei Normal University, Hefei 230061, People's Republic of China

*Corresponding author. E-mail: mingyang@ahu.edu.cn

MS received 9 October 2014; revised 10 January 2015; accepted 22 April 2015

DOI: 10.1007/s12043-015-1129-y; ***e*Publication:** 21 November 2015

Abstract. We present two schemes for transforming bipartite non-maximally entangled states into a W state in cavity QED system, by using highly detuned interactions and the resonant interactions between two-level atoms and a single-mode cavity field. A tri-atom W state can be generated by adjusting the interaction times between atoms and the cavity mode. These schemes demonstrate that two bipartite non-maximally entangled states can be merged into a maximally entangled W state. So the scheme can, in some sense, be regarded as an entanglement concentration process. The experimental feasibility of the schemes is also discussed.

Keywords. Quantum state merging; W state; non-maximally entangled state; cavity QED.

PACS Nos 03.67.Bg; 03.65.Ud; 03.67.Hk

1. Introduction

Quantum entanglement, first noted by Einstein, Podolsky, Rosen (EPR) [1], and Schrödinger [2], plays a key role in quantum information sciences [3], such as quantum teleportation [4], high-precision frequency measurement [5], quantum dense coding [6], quantum cryptography [7] and so on.

The preparation of entangled state becomes more and more important in quantum information processing. One knows that bipartite entanglement can be generated through a variety of solutions [8–10]. A maximally entangled state of two qubits is called a Bell state. For atomic qubits, the states $|\phi^{\pm}\rangle = (|ee\rangle \pm |gg\rangle)/\sqrt{2}$, $|\psi^{\pm}\rangle = (|eg\rangle \pm |ge\rangle)/\sqrt{2}$ are Bell states, where $|e\rangle$ and $|g\rangle$ represent the excited and the ground states of the atoms

[11]. For photonic qubits, the two states can be of two different polarizations (horizontal or vertical) or Fock states with zero or one photon [12,13]. However, multipartite entangled states have been shown to have many advantages over the bipartite ones in quantum information and quantum computation and provide even stronger tests of quantum non-locality. The typical representatives of the multipartite entangled state are the Greenberger–Horne–Zeilinger (GHZ) [14] state and the W state [15]. The key distinction between these two states is that when any one of the three qubits is traced out, the remaining two qubits for GHZ state is completely unentangled, but, the entanglement between the remaining two qubits survives for the W state. This characteristic makes the pairwise entanglement in the W state robust against loss of qubit. Recently, there has been a lot of theoretical proposals and experimental schemes for the preparation of three-qubit GHZ and the W states in cavity QED, superconducting quantum interference devices (SQUID) system, optical systems etc. [16–21]. Experimental preparation and the study of the features of these states can give us a better understanding of multipartite entanglement [22] and find their applications in information processing tasks such as teleportation [23].

But the direct preparation of a W state is still a challenging task. So it is of great importance to expand or merge the relatively easily generated bipartite entanglement into a multipartite entanglement. Recently, the W state was prepared by the quantum state expansion techniques. Tashima *et al* demonstrated an optical gate that can increase the size of polarization-entangled W states by accessing only one of the entangled photons [24]. Ikuta *et al* derived the maximum success probability of the corresponding quantum circuits with passive linear optics for postselectively expanding an N-photon W state to an $(N + n)$-photon W state, by accessing only one photon of the initial W state and adding n photons in a Fock state [25]. In addition, Tashima *et al* proposed a scheme for transforming two Einstein–Podolsky–Rosen photon pairs into a three-photon W state using local operations and classical communication (LOCC) [26]. Walther *et al* proposed a scheme based on local positive operator valued measures (POVM) and classical communication that can convert the ideal N-qubit GHZ state to a state arbitrarily close to the ideal N-qubit W state [27]. Liu proposed a scheme to generate W-type states of N atomic ensembles trapped in N single-mode cavities, and these cavities are connected by $N - 1$ optical fibres [28]. Mlynek *et al* showed that maximally entangled W-type states can be generated by the resonant interaction between three two-level superconducting qubits and a single mode of the electromagnetic field, and these four systems dynamically share a single excitation [29]. Very recently, Fatih Ozaydin *et al* showed that it is possible to create a large-scale W state by fusing multiple W states simultaneously in a linear optical system [30,31].

Most of these expanding and merging schemes for the W state are only applicable in optical system. In cavity QED system, Guo *et al* proposed a direct generation scheme for a multiatom W state [18], but it is just a theoretical scheme. Recently, two-atom entanglement has been experimently realized with coherent control of an atomic collision in a detuned cavity [32]. A direct generalization of this scheme to three-atom W state case is reasonable. But, from the discussions in [32] we can see that, a direct generalization of this scheme to the W states of four or more atoms are not possible because different atoms are tagged by velocity, and all of the atoms must be sent at different times. The velocities of all the atoms must be appropriately selected so that all the atoms will reach and cross the cavity axis simultaneously. But this condition can hardly be satisfied for a multiatom system. So it is of great importance to merge or expand W state in cavity QED system, which can release this rigid condition for multiatom systems. So in this paper, we shall present two schemes for transforming bipartite non-maximally entangled states into a W state in cavity QED. The resonant and detuned interactions between atoms and cavity modes are used here, which are feasible within the current technology. Because the initial non-maximally entangled states can be merged into a maximally entangled W state, the scheme can, in some sense, be regarded as an entanglement concentration process.

2. Detuned interaction-based creation of triatom W state via state merging

As depicted in figure 1, there are four identical two-level atoms (1, 2, 3 and 4) and three cavities (A, B and C), and the cavity modes are highly detuned from the atomic transition frequency. Atoms 1, 2 and cavity A are in Alice's lab, atoms 3, 4 and cavity B are in Bob's lab and cavity C is in Charlie's lab. Let atoms 1 and 2 cross cavity A, and atoms 3 and 4 cross cavity B simultaneously. Atoms 2 and 3 will fly through cavity C in Charlie's lab after they leave the cavities A and B.

From the set-up in figure 1, we can see that the main procedure involves only the interactions between two atoms and a cavity mode. In the interaction picture, the large detuned interaction Hamiltonian between two atoms and a cavity mode can be expressed as [9]

$$H^D = \lambda \left[\sum_{j=1}^{2} (|e_j\rangle\langle e_j| aa^\dagger - |g_j\rangle\langle g_j| a^\dagger a) + s_1^+ s_2^- + s_1^- s_2^+ \right], \tag{1}$$

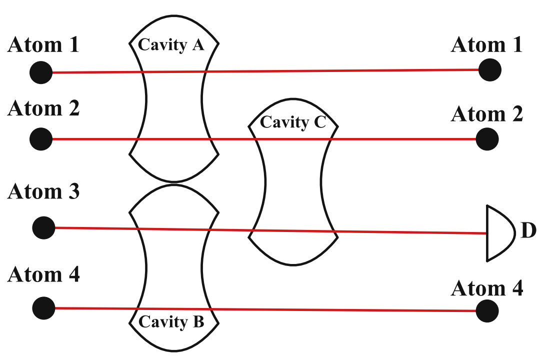

Figure 1. The set-up for the preparation of a three-atom W state. Atoms 1, 2 and cavity A are in Alice's lab, atoms 3, 4 and cavity B are in Bob's lab and cavity C is in Charlie's lab. The atoms 1, 2 fly through the cavity A and 3, 4 through the cavity B to generate entanglement between atoms 1, 2 and 3, 4, respectively. After flying out of the cavities A and B, atoms 2, 3 will enter the third cavity C in Charlie's lab. Conditioned on the detection result on atom 3 after cavity C, the atoms 1, 2 and 4 can be left in a maximally entangled W state. The key point here is to select the interaction times appropriately.

where s_j^+ and s_j^- are atomic operators, $s_j^+ = |e_j\rangle\langle g_j|$, $s_j^- = |g_j\rangle\langle e_j|$ with $|e_j\rangle$ and $|g_j\rangle$ representing the excited and ground states of the jth atom, respectively. $a^\dagger$ and a denote, respectively, the creation and annihilation operators of the cavity mode, g is the coupling constant between each atom and the cavity mode and δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω. In addition, the superscript D denotes the detuned interaction Hamiltonian. In the large detuning limit, $\delta = \omega_0 - \omega \gg g$, $\lambda = g^2/\delta$, and suppose the cavity mode is initially prepared in the vacuum state, the effective Hamiltonian can be reduced to [9]

$$H_e^D = \lambda \left(\sum_{j=1}^{2} (|e_j\rangle\langle e_j| + s_1^+ s_2^- + s_1^- s_2^+ \right). \tag{2}$$

Now, let us describe the detailed scheme. Atoms 1 and 2, initially prepared in the product state $|e_1 g_2\rangle$, are sent to the vacuum cavity A for an interaction time t_A. In the large detuning limit as mentioned above, the two atoms will be left in the non-maximally entangled state after leaving cavity A,

$$|\psi_{12}\rangle = a|e_1 g_2\rangle + b|g_1 e_2\rangle. \tag{3}$$

Atoms 3, 4 and cavity B will undergo the same process as atoms 1, 2 and cavity A with a different interaction time t_B. In the large detuning limit, the two atoms 3, 4 will be left in the non-maximally entangled state after leaving cavity B,

$$|\psi_{34}\rangle = c|e_3 g_4\rangle + d|g_3 e_4\rangle, \tag{4}$$

where a, b, c and d satisfy the normalization relations

$$|a|^2 + |b|^2 = 1, \quad |c|^2 + |d|^2 = 1.$$

Moreover,

$$a = \mathrm{e}^{-i\lambda t_A} \cos \lambda t_A, \quad b = -i\mathrm{e}^{-i\lambda t_A} \sin \lambda t_A,$$

$$c = \mathrm{e}^{-i\lambda t_B} \cos \lambda t_B, \quad d = -i\mathrm{e}^{-i\lambda t_B} \sin \lambda t_B,$$

respectively.

After flying through cavities A and B, atoms 2, 3 will be sent through the cavity C with an interaction time t. The evolution of the total system can be expressed as

$$\begin{aligned} |\psi_{12}\rangle|\psi_{34}\rangle &= (a|e_1 g_2\rangle + b|g_1 e_2\rangle) \otimes (c|e_3 g_4\rangle + d|g_3 e_4\rangle) \\ &\longrightarrow \mathrm{e}^{-i\lambda t}[bc\mathrm{e}^{-i\lambda t}|g_1 e_2 g_4\rangle - ibd \sin \lambda t|g_1 g_2 e_4\rangle \\ &+ ac \cos \lambda t|e_1 g_2 g_4\rangle]|e_3\rangle + [bd\mathrm{e}^{-i\lambda t} \cos \lambda t|g_1 e_2 e_4\rangle \\ &- iac\mathrm{e}^{-i\lambda t} \sin \lambda t|e_1 e_2 g_4\rangle + ad|e_1 g_2 e_4\rangle]|g_3\rangle. \end{aligned} \tag{5}$$

Atom 3 will be detected after flying out of cavity C. Conditioned on the detection result on atom 3, the left three atoms 1, 2, 4 will be in the following two different states:

$$\begin{aligned} |\psi_{124}^e\rangle &= \mathrm{e}^{-i\lambda t}[ac \cos \lambda t|e_1 g_2 g_4\rangle \\ &+ bc\mathrm{e}^{-i\lambda t}|g_1 e_2 g_4\rangle - ibd \sin \lambda t|g_1 g_2 e_4\rangle] \end{aligned} \tag{6a}$$

for result $|e_3\rangle$,

$$|\psi^g_{124}\rangle = bd\mathrm{e}^{-i\lambda t}\cos\lambda t|g_1e_2e_4\rangle + ad|e_1g_2e_4\rangle - iac\mathrm{e}^{-i\lambda t}\sin\lambda t|e_1e_2g_4\rangle \tag{6b}$$

for result $|g_3\rangle$.

According to eqs (6a) and (6b), we find that if we control the interaction times satisfying $\cos\lambda t = \tan\lambda t_A$ and $\sin\lambda t = \cot\lambda t_B$, the states in eqs (6a) and (6b) will become

$$|\psi^e_{124}\rangle = \sin\lambda t_A\cos\lambda t_B\mathrm{e}^{-i\lambda(t+t_A+t_B)}[|e_1g_2g_4\rangle - i\mathrm{e}^{-i\lambda t}|g_1e_2g_4\rangle + i|g_1g_2e_4\rangle] \tag{7a}$$

and

$$|\psi^g_{124}\rangle = \cos\lambda t_A\sin\lambda t_B\mathrm{e}^{-i\lambda(t_A+t_B)}[\mathrm{e}^{-i\lambda t}\cos^2\lambda t|g_1e_2e_4\rangle + i|e_1g_2e_4\rangle + i\mathrm{e}^{-i\lambda t}\sin^2\lambda t|e_1e_2g_4\rangle]. \tag{7b}$$

Obviously, the state in eq. (7a) is a maximally entangled W state, and can be transformed into the standard W state by a simple relative phase modulation on atom 2:

$$|\psi^e_{124}\rangle = \sqrt{3}\sin\lambda t_A\cos\lambda t_B \times[1/\sqrt{3}(|e_1g_2g_4\rangle + |g_1e_2g_4\rangle + |g_1g_2e_4\rangle)]. \tag{8}$$

The state in eq. (7b) is not a maximally entangled W state, but it is still a W-like state, which can be distilled into the standard W state by some entanglement distillation protocol [33].

In our scheme, we shall select the case where atom 3 is detected in $|e_3\rangle$ state. The left three atoms 1, 2, 4 are prepared in a standard W state with a success probability

$$P_1 = 3\sin^2\lambda t_A\cos^2\lambda t_B = \frac{3\sin^2\lambda t\cos^2\lambda t}{(1+\cos^2\lambda t)(1+\sin^2\lambda t)}. \tag{9}$$

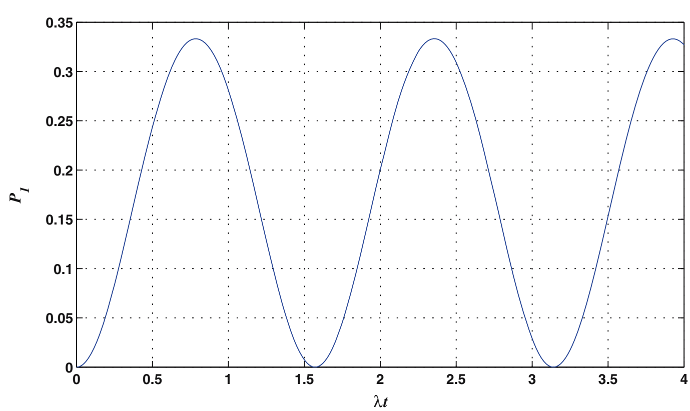

Figure 2. The successful probability P_1 for producing the W state as a function of interaction time λt in the detuned cavity C.

If we select the case where atom 3 is detected in $|g_3\rangle$ state, the three atoms 1, 2, 4 can be prepared in a standard W state too with a different set of conditions for interaction times, but the success probability is equal to that of the $|e_3\rangle$ case.

Figure 2 shows that the maximum success probability for this scheme can reach $P_1^m = 0.33$, and it corresponds to the condition $\lambda t = \pi/4$.

3. Resonant interaction-based creation of triatom W state via state merging

If we use the resonant interaction between the atoms and the cavity mode C in the above scheme, the maximally entangled W state can be generated in a similar way. For simplicity, we suppose the interactions inside cavities A and B are still detuned as in the first scheme (otherwise, we need to detect the states of cavities A, B, which will inevitably increase the complexity of the scheme). Here, all the interaction times must be adjusted to satisfy another set of relations. The resonant interaction between two identical atoms and a single cavity mode can be described by the following effective Hamiltonian [34]:

$$H_e^R = g[(a^\dagger|g_1\rangle\langle e_1| + a|e_1\rangle\langle g_1|) + (a^\dagger|g_2\rangle\langle e_2| + a|e_2\rangle\langle g_2|)]. \tag{10}$$

Suppose that the cavity field C is initially prepared in vacuum state. The evolution of total system can be expressed as

$$\begin{aligned}|\psi_{12}\rangle|\psi_{34}\rangle = {} & (a|e_1g_2\rangle + b|g_1e_2\rangle)\otimes(c|e_3g_4\rangle + d|g_3e_4\rangle)\\ & \longrightarrow bc|g_1g_4\rangle\{[1 + (\cos(\sqrt{6}gt) - 1)/3]|e_2e_3\rangle|0\rangle\\ & -i\sin(\sqrt{6}gt)/\sqrt{6}(|e_2g_3\rangle + |g_2e_3\rangle)|1\rangle\\ & +\sqrt{2}/3[\cos(\sqrt{6}gt) - 1]|g_2g_3\rangle|2\rangle\}\\ & +bd|g_1e_4\rangle\{1/2[\cos(\sqrt{2}gt) + 1]|e_2g_3\rangle|0\rangle\\ & +1/2[\cos(\sqrt{2}gt) - 1]|g_2e_3\rangle|0\rangle - i\sin(\sqrt{2}gt)/\sqrt{2}|g_2g_3\rangle|1\rangle\}\\ & +ac|e_1g_4\rangle\{1/2[\cos(\sqrt{2}gt) - 1]|e_2g_3\rangle|0\rangle\\ & +1/2[\cos(\sqrt{2}gt) + 1]|g_2e_3\rangle|0\rangle\\ & -i\sin(\sqrt{2}gt)/\sqrt{2}|g_2g_3\rangle|1\rangle\} + ad|e_1e_4\rangle|g_2g_3\rangle|0\rangle.\end{aligned} \tag{11}$$

We shall detect the states of cavity C and atom 3, after atom 3 flying out of cavity C. Conditioned on the detection results, atoms 1, 2 and 4 will be left in five different states. Obviously, the states corresponding to the detection results $|1\rangle|e_3\rangle$ and $|2\rangle|g_3\rangle$ do not contribute to our successful result. For detection results $|1\rangle|g_3\rangle$, $|0\rangle|g_3\rangle$ and $|0\rangle|e_3\rangle$, we get the following three states:

$$\begin{aligned}|\psi_{124}^{0g}\rangle = {} & -ac/2[1 - \cos(\sqrt{2}gt)]|e_1e_2g_4\rangle + ad|e_1g_2e_4\rangle\\ & +bd/2[\cos(\sqrt{2}gt) + 1]|g_1e_2e_4\rangle,\end{aligned} \tag{12a}$$

$$\begin{aligned}|\psi_{124}^{0e}\rangle = {} & ac/2[\cos(\sqrt{2}gt) + 1]|e_1g_2g_4\rangle\\ & -bd/2[1 - \cos(\sqrt{2}gt)]|g_1g_2e_4\rangle\\ & +bc[1 + (\cos(\sqrt{6}gt) - 1)/3]|g_1e_2g_4\rangle\end{aligned} \tag{12b}$$

and

$$|\psi_{124}^{1g}\rangle = -iac\sin(\sqrt{2}gt)/\sqrt{2}|e_1g_2g_4\rangle - ibc\sin(\sqrt{6}gt)/\sqrt{6}|g_1e_2g_4\rangle$$
$$-ibd\sin(\sqrt{2}gt)/\sqrt{2}|g_1g_2e_4\rangle, \quad (12c)$$

respectively.

According to eqs (12a), (12b) and (12c), we find that if we control the interaction times satisfying $\tan\lambda t_A = (\sqrt{3}\sin(\sqrt{2}gt))/\sin(\sqrt{6}gt) = \cot\lambda t_B$, the states in eqs (12a), (12b) and (12c) will become

$$|\psi_{124}^{0g}\rangle = \cos\lambda t_A\cos\lambda t_B\mathrm{e}^{-i\lambda(t_A+t_B)}(-(1-\cos\sqrt{2}gt)/2|e_1e_2g_4\rangle$$
$$-i\tan\lambda t_B|e_1g_2e_4\rangle - (1+\cos\sqrt{2}gt)/2|g_1e_2e_4\rangle), \quad (13a)$$

$$|\psi_{124}^{0e}\rangle = \cos\lambda t_A\cos\lambda t_B\mathrm{e}^{-i\lambda(t_A+t_B)}$$
$$\times(-i(2+\cos(\sqrt{6}gt))/3\tan\lambda t_A|g_1e_2g_4\rangle$$
$$+(1-\cos\sqrt{2}gt)/2|g_1g_2e_4\rangle + (1+\cos\sqrt{2}gt)/2|e_1g_2g_4\rangle), \quad (13b)$$

and

$$|\psi_{124}^{1g}\rangle = \cos\lambda t_A\cos\lambda t_B\sin\sqrt{2}gt/\sqrt{2}\mathrm{e}^{-i\lambda(t_A+t_B)}$$
$$\times(-i|e_1g_2g_4\rangle - |g_1e_2g_4\rangle + i|g_1g_2e_4\rangle). \quad (13c)$$

Obviously, the state in eq. (13c) is a maximally entangled W state, and it can be transformed into the standard W state by a simple phase modulation on atom 2:

$$|\psi_{124}^{1g}\rangle = \sqrt{3}\cos\lambda t_A\cos\lambda t_B\sin\sqrt{2}gt/\sqrt{2}$$
$$\times[1/\sqrt{3}(|e_1g_2g_4\rangle + |g_1e_2g_4\rangle + |g_1g_2e_4\rangle)]. \quad (14)$$

The states in eqs (13a) and (13b) are not maximally entangled W states, but they are still W-like states, which can be distilled into the standard W state by some entanglement distillation protocol [33].

In this scheme, we shall select the case where cavity C and atom 3 are detected in $|1\rangle|g_3\rangle$ state. So the left three atoms 1, 2, 4 are prepared in a standard W state with a success probability

$$P_2 = 3\cos^2\lambda t_A\sin^2\lambda t_B\sin^2\sqrt{2}gt/2$$
$$= \frac{9\sin^4\sqrt{2}gt\sin^2\sqrt{6}gt}{2(\sin^2\sqrt{6}gt + 3\sin^2\sqrt{2}gt)^2}. \quad (15)$$

If we select the other two cases $|0\rangle|g_3\rangle$, $|0\rangle|e_3\rangle$, the three atoms 1, 2, 4 can be prepared in a standard W state too with different sets of conditions for interaction times.

Figure 3 shows that the maximum success probability for this scheme can reach $P_2^m = 0.28$, and it corresponds the condition $gt = 3.24s$.

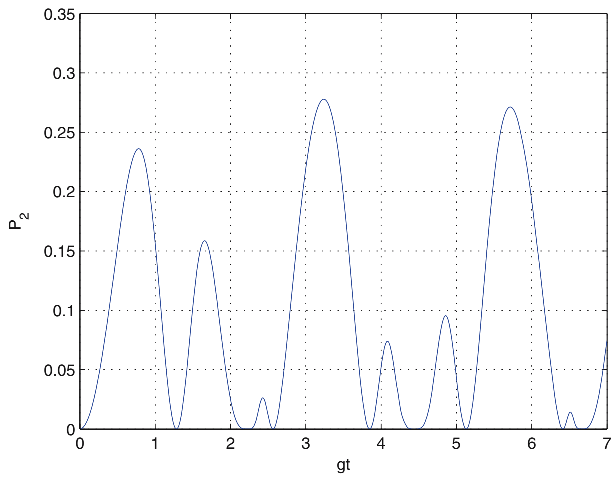

Figure 3. The successful probability P_2 for producing the W state as a function of interaction time gt in the resonant cavity C.

Now, it is necessary to have a look at the experimental feasibility of our schemes. From the analysis in ref. [9], the radiative time of the Rydberg atoms with principal quantum numbers 50 and 51 is $T_a = 3 \times 10^{-2}$ s, and the interaction time is of the order of $T_i = 2 \times 10^{-4}$ s. In the detuned scheme, the cavity mode is only virtually excited. So we only need to compare the interaction time with the radiative time of atoms. The atomic radiative time is much longer than the interaction time, and so the detuned scheme is feasible.

As for the resonant scheme, we need to take the cavity decay time into account, i.e., the interaction time must be much shorter than both the cavity decay time and the atomic radiative time. From refs [34–36], the decay time of a normal cavity can reach $T_c = 1.0 \times 10^{-3}$ s. The atom–cavity coupling strength is $g = 2\pi \times 25$ kHz. In this scheme, we get that the atom–cavity interaction time is $t = 3.24/g = 2.1 \times 10^{-5}$ s. So we can evaluate that the order of the time for the whole procedure is about 10^{-4} s, which is shorter than both the cavity decay time and the atomic radiative time, and so the resonant scheme is feasible.

Although the two schemes are feasible in the time sense, we still need to have a comparison of these two schemes. Because $P_2^m < P_1^m$, the first scheme is more efficient than the second one in the success probability respect. Moreover, the experimental complexity is the key point. The cavity decay time in the resonant scheme is obviously shorter than the radiative time of atoms. The detection of cavity states is needed in the resonant scheme, and to modulate a detuned interaction is much easier than a resonant one. Thus, we can say that the detuned scheme is more feasible than the resonant one.

Till now we have considered the ideal cases, and we are now going to discuss practical aspects and their effects on our scheme. All the calculations of the first scheme are based on the simplified effective Hamiltonian in eq. (2). But the condition for this effective Hamiltonian is $\delta \gg g$. Whether some choice of δ, such as $\delta = 10g$, satisfies this condition must be checked by solving the initial Schrödinger equation numerically. Fortunately, this numerical simulation has been done in [32], and from figure 2 of [32] we get that the choice $\delta = 10g$ only affects the basic dynamics in the order of 2%–5%. That is to say, we can choose $\delta = 10g$, and then the effective Hamiltonian in eq. (2) is a very good

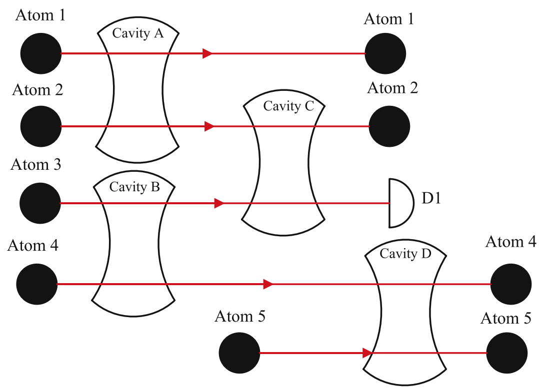

Figure 4. The set-up for expanding a three-atom W state to a four-atom W state.

approximation of the exact Hamiltonian. In additon, we have assumed that the cavity mode affects both atoms in the same way, but the cavity field strength will depend on the position of the two atoms and generally be different for both. What will happen if the coupling constants between the two atoms and the cavity mode are different? To simplify the discussion, we assume that $g_1 = g$ and $g_2 = g(1+k)$, $k \in (-0.2, 0.2)$. After a careful calculation, for $k = 0.2$, the fidelity of the output state for this case with respect to that of the identical coupling constants case can reach 0.968. That is to say, the differences in coupling constant only slightly affect the scheme.

Although the scheme proposed here is for three-atom W state case, it can be generalized to N-atom W state case. To clearly show this point, we take an example of expanding a three-atom W state as depicted in figure 4. Atoms 1, 2 and cavity A are in Alice's lab, atoms 3, 4 and cavity B are in Bob's lab, cavity C is in Charlie's lab and cavity D and atom 5 are in David's lab. Atoms 1, 2 will be sent through cavity A in Alice's lab and 3, 4 through cavity B in Bob's lab to generate the entanglement between atoms 1, 2 and 3, 4, respectively. After exiting cavities A and B, atoms 2, 3 will enter the third cavity C in Charlie's lab, and atoms 4, 5 will enter the fourth cavity D in David's lab. By adjusting the atomic velocities, atoms 1, 2, and 4, 5 can be left in a maximally entangled W state conditioned on the detection results on atom 3 after cavity C. The detailed calculations are similar to the first scheme.

4. Summary

In summary, we have presented two schemes for transforming bipartite non-maximally entangled states into a W state in cavity QED. The first scheme is mainly based on the large detuned interactions between the atoms and cavity modes, and only atomic detection is needed. The second scheme needs the resonant interaction between the atoms and a cavity mode, and the detections of atomic states and cavity mode state are needed. Obviously, the detuned scheme is more feasible than the resonant one. In addition, from the analysis of our schemes, we can see that all the atoms involved in the transforming schemes can be distributed among different labs. That is to say, the output entangled atoms of our schemes are naturally distributed among different labs, which may have potential applications in quantum communication.

Acknowledgements

This work is supported by National Natural Science Foundation of China (NSFC) under Grants Nos 11274010, 11204002, 11374085, 61370090 and 11204061, the Specialized Research Fund for the Doctoral Programme of Higher Education (20113401110002, 20123401120003), the Key Project of Chinese Ministry of Education (No. 210092), Anhui Provincial Natural Science Foundation under Grant Nos 1208085QA14, 1308085QA18, and 1408085MA16, the Key Programme of the Education Department of Anhui Province under Grants Nos KJ2013A195, KJ2012A244, KJ2012A206, the '211' Project of Anhui University, the Talent Foundation of Anhui University under Grant No. 33190019, and the personnel department of Anhui province.

References

[1] A Einstein, B Podolsky and N Rosen, *Phys. Rev.* **47**, 777 (1935)
[2] E Schrödinger, *Naturwissenschaften* **23**, 807 (1935)
[3] R Horodecki, P Horodecki, M Horodecki and K Horodecki, *Rev. Mod. Phys.* **81**, 865 (2009)
[4] C H Bennett, G Brassard, C Crépeau, R Jozsa, A Peres and W K Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993)
[5] J J Bollinger, W M Itano, D J Wineland and D J Heinzen, *Phys. Rev. A* **54**, R4649 (1996)
[6] K Mattle, H Weinfurter, P G Kwiat and A Zeilinger, *Phys. Rev. Lett.* **76**, 4656 (1996)
[7] C H Bennett, G Brassard and N D Mermin, *Phys. Rev. Lett.* **68**, 557 (1992)
[8] Q A Turchette, C S Wood, B E King, C J Myatt, D Leibfried, W M Itano, C Moroe and D J Wineland, *Phys. Rev. Lett.* **81**, 3631 (1998)
[9] S B Zheng and G C Guo, *Phys. Rev. Lett.* **85**, 2392 (2000)
[10] L B Chen and W Yang, *Laser Phys. Lett.* **11**, 105201 (2014)
[11] J M Raimond, M Brune and S Haroche, *Rev. Mod. Phys.* **73**, 565 (2001)
[12] R Lo Franco, G Compagno, A Messina and A Napoli, *Phys. Rev. A* **72**, 053806 (2005)
[13] R Lo Franco, G Compagno, A Messina and A Napoli, *Phys. Rev. A* **76**, 011804(R) (2007)
[14] D M Greenberger, M A Horne and A Zeilinger, in: *Bell's theorem, quantum theory, and conceptions of the universe* edited by M Kafatos (Kluwer Academics, Dordrecht, The Netherlands, 1989) pp. 73–76
[15] W Dür, G Vidal and J I Cirac, *Phys. Rev. A* **62**, 062314 (2000)
[16] M Eibl, N Kiesel, M Bourennane, C Kurtsiefer and H Weinfurte, *Phys. Rev. Lett.* **92**, 077901 (2004)
[17] G C Guo and Y S Zhang, *Phys. Rev. A* **65**, 054302 (2002)
[18] G P Guo, C F Li, J Li and G C Guo, *Phys. Rev. A* **65**, 042102 (2002)
[19] Z J Deng, M Feng and K L Gao, *Phys. Rev. A* **74**, 064303 (2006)
[20] X L Zhang, K L Gao and M Feng, *Phys. Rev. A* **74**, 024303 (2006)
[21] G Y Xiang, Y S Zhang, J Li and G C Guo, *J. Opt. B: Quantum Semiclass. Opt.* **5**, 208 (2003)
[22] A Miyake, *Phys. Rev. A* **69**, 012101 (2004)
[23] J Joo, Y J Park, S Oh and J Kim, *New J. Phys.* **5**, 136 (2003)
[24] T Tashima, T Kitano, S K Ozdemir, T Yamamoto, M Koashi and N Imoto, *Phys. Rev. Lett.* **105**, 210503 (2010)
[25] R Ikuta, T Tashima, T Yamamoto, M Koashi and N Imoto, *Phys. Rev. A* **83**, 012314 (2011)

[26] T Tashima, T Wakatsuki, S Kaya Özdemir, T Yamamoto, M Koashi and N Imoto, *Phys. Rev. Lett.* **102**, 130502 (2009)
[27] P Walther, K J Resch and A Zeilinger, *Phys. Rev. Lett.* **94**, 240501 (2005)
[28] Y M Liu, *Sci. China* **56**, 2122 (2013)
[29] J A Mlynek, A A Abdumalikov Jr, J M Fink, L Steffen, M Baur, C Lang, A F van Loo and A Wallraff, *Phys. Rev. A* **86**, 053838 (2012)
[30] S Bugu, C Yesilyurt and F Ozaydin, *Phys. Rev. A* **87**, 032331 (2013)
[31] F Ozaydin, S Bugu, C Yesilyurt, A A Altintas, M Tame, Özdemir and Şahin Kaya, *Phys. Rev. A* **89**, 042311 (2014)
[32] S Osnaghi, P Bertet, A Auffeves, P Maioli, M Brune, J M Raimond and S Haroche, *Phys. Rev. Lett.* **87**, 037902 (2001)
[33] Z L Cao and M Yang, *J. Phys. B* **36**, 4245 (2003)
[34] S B Zheng, *Phys. Rev. A* **69**, 064302 (2004)
[35] M Yang, W Song and Z L Cao, *Phys. Rev. A* **71**, 034312 (2005)
[36] A Rauschenbeutel, P Bertet, S Osnaghi, G Nogues, M Brune, J M Raimond and S Haroche, *Phys. Rev. A* **64**, 050301 (2001)