



The dust-acoustic mode in two-temperature electron plasmas with charging effects

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MS received 9 December 2014; revised 10 March 2015; accepted 11 March 2015

DOI: 10.1007/s12043-015-1101-x; ePublication: 18 November 2015

Abstract. Dust charging in an unmagnetized collisionless dusty plasma with two-temperature electrons was investigated based on the orbital motion limited theory, where the two-temperature electrons and ions are modelled by the Maxwellian distributions. Then by taking into account the effects of two-temperature electron and the associated charging fluctuations, the dispersion peculiarities of dust-acoustic waves are studied based on dust fluid dynamics. The present results show that the effect will introduce a dissipation on the mode, and the dispersion and the dissipation depend on the temperature ratio and number density ratio of hot and cold electrons.

Keywords. Dusty plasmas; dust charging; Maxwellian distribution; two-temperature electron.

PACS Nos 52.27.Lw; 52.30.Ex; 52.35.Dm

1. Introduction

Dusty plasmas, which are fully or partially ionized low-temperature gases within neutral molecules, charged dust grain, electrons and ions, are quite natural in astrophysical environment [1,2], space [3–5], such as planetary rings, cometary tails, interstellar media and the lower part of the Earth's ionosphere, as well as in laboratory low-temperature plasmas [6–8]. Nowadays, dusty plasmas are exciting interdisciplinary research fields [9]. The dust grains are usually charged due to different kinds of physical processes, such as the collection of background plasma electrons and ions by dust grains, photoelectron emission, secondary electron emission, thermionic emission, etc. [10,11]. This is the main factor that distinguishes the dusty plasmas from multiple-component plasma. The presence of dust component in a plasma not only modifies the wave spectrum, but also introduces new modes and mechanisms of damping, dissipation and instability [9,10,12,13]. Dust particles are extremely massive compared to ions. So the frequency of dusty plasma is several orders of magnitude smaller than the frequency of ion plasma. This leads to the appearance of a new very low-frequency mode, the dust acoustic wave (DAW), which was first predicted theoretically by Rao *et al* [14]. Lately, DAWs have been extensively studied

on the basis of fluid and Vlasov models or by incorporating dust charge fluctuations [15]. Now, it is well recognized that the properties and charging processes of the dust grains were determined by the surrounding plasma environment. Using kappa and generalized (r, q) distribution functions for both electrons and ions, Rubab and Murtaza [16] deduced the charging current and modified dispersion relations of dust-acoustic waves. The results show that the wave dispersions and dissipation rates rely on the parameters κ of kappa distribution and (r, q) of generalized (r, q) distribution. Besides, the effect of the type of ions on the charging of dust grains in plasma has been investigated. Kim and Merlino [17] experimentally investigated the effect of negative ions on the charging of dust particles in plasma. They indicated that for electrons, negative ions and positive ions of comparable temperatures, the charge (or surface potential) of the dust can be positive if the mass of the positive ion is smaller than that of the negative ion. Maharaj *et al* [18] studied the influence of non-thermal ions on linear DAWs in an unmagnetized plasma consisting of ions that have non-thermal velocity distribution, Boltzmann-distribution electrons and streaming dust particles.

It is well known that two-temperature electrons are found to occur both in space [19–21] and laboratory environment [22–24], and there exist a large variety of wave modes, the spatio-temporal scales of which are quite broad. One of the most interesting and important wave modes is the electron-acoustic waves [25,26], in which the pressure of the hot electrons provides the restoring force, while the cold electron component provides the inertia. Due to their potential roles in interpreting the broadband electrostatic noise observed in various regions of the Earth's magnetosphere [19,20], the electron-acoustic waves and the relevant physics of two-temperature electron plasmas have received a great deal of attention.

As mentioned above, the presence of dust in the plasma modifies the wave spectrum and also introduces new modes and mechanisms for damping, dissipation and instabilities for plasma wave modes. Recently, the effect of dust grains in two-temperature electron plasma has been extensively discussed. Chutov *et al* [27] adopted two-temperature electron dusty plasma model to study the dusty sheaths, and found that the peculiarity of the sheaths depends on temperature and density ratio of both electron components. El-Labany *et al* [28] studied ion-acoustic solitary waves in dusty plasma with two-temperature electrons. El-Labany *et al* [29] studied dust-acoustic solitary wave in a magnetized dusty plasma consisting of hot dusty fluid, nonisothermal ions and two-temperature isothermal electrons with charge fluctuations. They found that the amplitude and width of dust-acoustic solitary wave are modified due to the presence of two types of isothermal electrons. Filippov and Pal [30] investigated the charging dynamics of dust particles in a plasma with two-temperature distribution of electrons and cold ions. It is found that the charge of micron-sized particles could reach ultrahigh values up to 10^6 electron charges, which were two to three orders of magnitude higher than the charge of particles of the same size in discharges of extensively investigated types.

In this paper, we study dispersion peculiarities of DAWs in an unmagnetized collisionless dusty plasma by taking into account the effects of two-temperature electrons and the associated dust-charge fluctuation. In our model, the velocity distribution functions of two-temperature electrons and ions are Maxwellian. The paper is organized as follows: In §2, the closed dust fluid dynamics model equations are proposed by considering the effects of two-temperature electrons and the associated dust-charge fluctuation. In §3, the dispersion equations for DAWs are presented, and the influences of the ratio of temperature

and number density of cold to hot electrons on the dispersion and dissipation are analysed. The discussion and conclusion are in §4.

2. Mathematical model

Consider an unmagnetized dusty plasma whose constituents are cool and hot electrons, singly positively charged ions and micron-sized negatively charged dust grains. For the dust component, the linearized fluid continuity and momentum equations have the form

$$\partial_t n_{d1} + n_{d0} \nabla \cdot \vec{v}_{d1} = 0, \quad (1)$$

$$\partial_t \vec{v}_{d1} = -\frac{q_{d0}}{m_d} \nabla \varphi_1, \quad (2)$$

and the Poisson equation

$$\nabla^2 \varphi_1 = -\left[4\pi e (n_{i1} - n_{ec1} - n_{eh1}) + 4\pi q_{d0} n_{d1} + 4\pi q_{d1} n_{d0}\right], \quad (3)$$

where n_{d1} , n_{ec1} , n_{eh1} and n_{i1} are the perturbed dust, cold electron, hot electron and ion number density, \vec{v}_{d1} , φ_1 are the perturbed dust fluid velocity and the electrostatic potential, n_{d0} and q_{d0} are the equilibrium number density and charge of dust, q_{d1} is the dust charge fluctuations which will be determined by dust charging dynamics, respectively. At equilibrium, the quasineutral conditions can be written as $n_{i0} = n_{ec0} + n_{eh0} + (q_{d0}/e)n_{d0}$.

The cool electron, hot electron and ion number density in the electrostatic DAWs potential φ_1 are

$$\begin{aligned} n_{ec(h)} &= n_{ec(h)0} \exp\left(\frac{e\varphi_1}{T_{ec(h)}}\right), \\ n_i &= n_{i0} \exp\left(-\frac{e\varphi_1}{T_i}\right), \end{aligned} \quad (4)$$

where $n_{ec(h)0}$, n_{i0} , $T_{ec(h)}$ and T_i are the equilibrium number density and temperature of cold (hot) electron and ions, respectively. The dust charging equation is [10,12]

$$\frac{dq_d}{dt} = I_j, \quad (5)$$

where q_d is the mean charges of the dust grains and I_j is the collection current of j th species ($j = ec, eh, i$) and determined by the integration of the corresponding cross-section with velocity distribution

$$I_j = \sum_j \int e_j v_j \sigma_j^d f_j(v_j) d^3 v_j. \quad (6)$$

Here e_j and $f_j(v_j)$ are the corresponding charge and distribution functions of the species. The charging cross-section of a dust grain σ_j^d is [10,12]

$$\sigma_j^d = \pi r_d^2 \left(1 - \frac{2q_j \phi_d}{m_j v_j^2}\right), \quad (7)$$

where m_j is the mass of the plasma species j .

It is well known that when the dust plasma is close to thermal equilibrium, the distribution function f_j approximates a Maxwellian distribution

$$f_j(v_j) = n_j \left(\frac{m_j}{2\pi T_j} \right)^{3/2} \exp\left(-\frac{v_j^2}{v_{Tj}^2}\right). \quad (8)$$

By substituting eqs (8) and (7) into eq. (6), we get the collection currents of both hot and cold electrons and ions for a negatively charged dust grain of radius r_d ,

$$I_{ec(h)} = -4\pi r_d^2 e n_{ec(h)} \left(\frac{T_{ec(h)}}{2\pi m_e} \right)^{1/2} \exp\left(\frac{e\phi_d}{T_{ec(h)}}\right) \quad (9)$$

and

$$I_i = 4\pi r_d^2 e n_i \left(\frac{T_i}{2\pi m_i} \right)^{1/2} \left(1 - \frac{e\phi_d}{T_i} \right), \quad (10)$$

where we introduce ϕ_d , the surface potential of the dust grain. In equilibrium, we have $I_{i0} + I_{eh0} + I_{ec0} = 0$, which will give the charge (surface potential) of the dust grain.

By letting $\phi_d = \phi_{d0} + \phi_{d1} \equiv (q_{d0} + q_{d1})/r_d$, and assuming $n_{eh0}/n_{ec0} = \alpha$, $T_{eh}/T_{ec} = \beta$, we obtain the equilibrium and fluctuating collection currents

$$\begin{aligned} I_{ec0} &= -4\pi r_d^2 e n_{ec0} \left(\frac{T_{ec}}{2\pi m_e} \right)^{1/2} \left(1 + \frac{e\phi_{d0}}{T_{ec}} \right), \\ I_{ec1} &= -|I_{ec0}| \left(\frac{e\phi_{d1}}{T_{ec} + e\phi_{d0}} + \frac{n_{ec1}}{n_{ec0}} \right), \end{aligned} \quad (11)$$

for cold electrons, and

$$\begin{aligned} I_{eh0} &\approx \alpha\beta^{1/2} I_{ec0}, \\ I_{eh1} &= -\alpha\beta^{1/2} |I_{ec0}| \left(\frac{e\phi_{d1}}{T_{eh} + e\phi_{d0}} + \frac{n_{eh1}}{n_{eh0}} \right), \end{aligned} \quad (12)$$

for hot electrons, as well as

$$\begin{aligned} I_{i0} &= 4\pi r_d^2 e n_{i0} \left(\frac{T_i}{2\pi m_i} \right)^{1/2} \left(1 - \frac{e\phi_{d0}}{T_i} \right), \\ I_{i1} &= -|I_{i0}| \left(\frac{e\phi_{d1}}{T_i - e\phi_{d0}} - \frac{n_{i1}}{n_{i0}} \right), \end{aligned} \quad (13)$$

for ions, respectively. From these currents, we get the dust charging equation as

$$(\partial_t + \nu_0) q_{d1} = |I_{ec0}| \left[\left(1 + \alpha\beta^{1/2} \right) \frac{n_{i1}}{n_{i0}} - \alpha\beta^{1/2} \frac{n_{eh1}}{n_{eh0}} - \frac{n_{ec1}}{n_{ec0}} \right], \quad (14)$$

where ν_0 is the charging relaxation rate originating from the variation in the effective collision cross-section due to the charge perturbation at the grain surface as experienced by unperturbed particle and is given by

$$\nu_0 = \frac{|I_{ec0}| e}{r_d} \left[\frac{1}{T_{ec} + e\phi_{d0}} + \frac{\alpha\beta^{1/2}}{T_{eh} + e\phi_{d0}} + \frac{1 + \alpha\beta^{1/2}}{T_i - e\phi_{d0}} \right]. \quad (15)$$

Substituting the values of cool electron, hot electron and ion number densities from eq. (4) into the Fourier component of eq. (14), we obtained the following form for the fluctuating charges:

$$\tilde{q}_{d1} = -\frac{i |I_{ec0}| e \left[\frac{1+\alpha\beta^{1/2}}{T_i} + \frac{\alpha\beta^{1/2}}{T_{eh}} + \frac{1}{T_{ec}} \right] \tilde{\varphi}_1}{\omega + i\nu_0}. \quad (16)$$

Now, eqs (1), (2), (3), (4) and (16) model a closed form for the dust fluid dynamics by taking into account the effects of two-temperature electrons and the associated charge fluctuation.

3. Dispersion relation and dissipation rate of DAWs

Generally, the dispersion equation of plasma electrostatic excitations is obtained by Fourier analysing the Vlasov, Poisson and Maxwell equations. Following [10,31], we have

$$\varepsilon(\omega, k) = 1 + \chi_{eh} + \chi_{ec} + \chi_i + \chi_d = 0, \quad (17)$$

where χ_j are the susceptibilities of the j th plasma particles. In the extremely low DAWs phase regime, $\omega/k \ll v_{tc}, v_{ti}, v_{th}$, then the susceptibilities of cold, hot electrons and ions can be expressed as

$$\chi_{eh} + \chi_{ec} + \chi_i \simeq \frac{1}{k^2 \lambda_{dec}^2} + \frac{1}{k^2 \lambda_{deh}^2} + \frac{1}{k^2 \lambda_{di}^2}, \quad (18)$$

where $\lambda_{dec(h)} \equiv \sqrt{T_{ec(h)}/4\pi n_{ec(h)} e^2}$ and $\lambda_{di} \equiv \sqrt{T_i/4\pi n_i e^2}$ are the cold (hot) electron and ion Debye radius, and using eqs (1), (2) and (16) in eq. (3), one obtains the dust susceptibility as

$$\chi_d \simeq -\frac{\omega_{pd}^2}{\omega^2} + \frac{k_{dch}^2}{k^2} \frac{\omega_c}{\nu_0 - i\omega}, \quad (19)$$

where

$$k_{dch}^2 = 4\pi n_{d0} q_{d0}^2 \left[\frac{1 + \alpha\beta^{1/2}}{T_i} + \frac{\alpha\beta^{1/2}}{T_{eh}} + \frac{1}{T_{ec}} \right] \quad (20)$$

is the charging dynamical wavenumber, $\omega_c = |I_{ec0}| e/q_{d0}^2$ is the dust-charging frequency. Finally, the dispersion equation for DAWs in the two-temperature electron dust plasma with the charging effect can be written as

$$\varepsilon(\omega, k) = 1 + \frac{1}{k^2 \lambda_{Deff}^2} - \frac{\omega_{pd}^2}{\omega^2} + \frac{k_{dch}^2}{k^2} \frac{\omega_c}{\nu_0 - i\omega} = 0, \quad (21)$$

where $\lambda_{Deff}^{-2} = \lambda_{deh}^{-2} + \lambda_{dec}^{-2} + \lambda_{di}^{-2}$ is the dusty plasma Debye radius [10,31].

Naturally, the dielectric response function can be separated into the real and imaginary parts, i.e.,

$$\varepsilon(\omega, k) = \text{Re } \varepsilon(\omega, k) + \text{Im } \varepsilon(\omega, k), \quad (22)$$

where $\text{Re } \varepsilon(\omega, k) = 0$ determines the dispersion relations of the DAWs. In order to obtain the damping or instabilities of DAWs, we present the wave frequency in the form $\omega =$

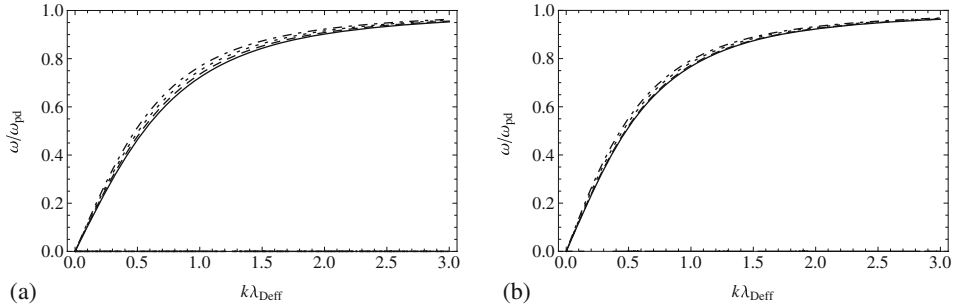


Figure 1. The dispersion relation of DAWs, (a) with different temperature ratios of hot and cool electrons (β), that is, $\beta = 500$ (solid line), 200 (dashed line), 100 (dotted line) and 50 (dot-dashed line); (b) with different number density ratios of hot and cold electrons (α), namely, $\alpha = 4/5$ (solid line), 1/2 (dashed line), 2/7 (dotted line), 1/5 (dot-dashed line).

$\omega_r + i\gamma$, where $\omega_r(k)$ is the dispersion relation and $\gamma = -\text{Im} \varepsilon(\omega, k)/[\partial \text{Re} \varepsilon(\omega, k)/\partial \omega]$ is imaginary part of the frequency for wave instability ($\gamma > 0$) and damping or dissipation ($\gamma < 0$) in our designations. In the present model, we have

$$\gamma = -\frac{\omega_c \omega}{\frac{k^2}{k_{dch}^2} \frac{\omega_{pd}^2 (v_0^2 + \omega^2)}{\omega^3} - \frac{2\omega_c v_0 \omega}{v_0^2 + \omega^2}}. \quad (23)$$

It is shown obviously that when the dust-charging frequency $\omega_c = 0$, then $\gamma = 0$, which means that this imaginary frequency is the result of the fluctuating charge. Then we can call it a dissipation rate rather than growth rate of instability.

We first examine the dispersion behaviour of DAWs. The ω vs. k plots are shown in figure 1, where figure 1a shows different hot and cold electron temperature ratios, i.e. $\beta = 500, 200, 100$ and 50; and figure 1b shows different number density ratios of hot and cold electrons, i.e. $\alpha = 4/5, 1/2, 2/7$ and 1/5. Next, we examine the dissipation behaviour of DAWs under the same parameters of figure 1. The γ vs. k plots are shown in figure 2, where figure 2a with different value of β and figure 2b with different value

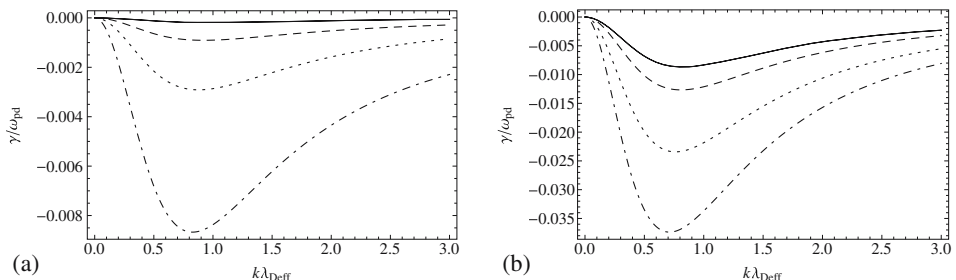


Figure 2. The dissipation rate of DAWs, (a) with different temperature ratios of hot and cold electrons (β), that is, $\beta = 500$ (solid line), 200 (dashed line), 100 (dotted line) and 50 (dot-dashed line); (b) with different number density ratios of hot and cold electrons (α), namely, $\alpha = 4/5$ (solid line), 1/2 (dashed line), 2/7 (dotted line), 1/5 (dot-dashed line).

of α . As shown in figures 1 and 2, by decreasing both β and α , the phase of the DAWs increases, whereas the dissipation rate decreases.

4. Results and discussion

The dispersion characteristics of the DAWs in dust plasmas with two-temperature electrons, singly ionized positive ions and negatively charged micron-size dust grain are investigated in the present paper. In our model, the electrons are assumed to be Maxwellian, and the charge fluctuation effects are taken into account. First, the equilibrium and the fluctuating collection currents carried by two-temperature electrons and ion currents are calculated by OML theory. Then, by considering the effects of two-temperature electrons and the associated charge fluctuation, the dispersion equation of DAWs is given and analysed. Traditionally, the effect of electrons on the dust-acoustic mode is insignificant. They merely modify the Debye shielding of the dust grains and the wave phase of the dust-acoustic mode, if the fluctuating charge of the dust by the background plasma electrons and ions are not taken into account. Nevertheless, the present results show that the effects will introduce a dissipation on the mode. The process of dust charge fluctuation is accompanied by inelastic particle collisions between dust grains and electrons. Accordingly, this inelastic collision results in the dissipation effect for dust-acoustic waves [12,32]. On the other hand, the dispersion and the dissipation depend on the temperature ratio and the number density ratio of hot and cold electrons. As shown in the figures, by decreasing β and α , the phase of the DAWs increases, whereas the dissipation rate decreases. We can infer that the main source for the dissipation is the attachment of the hot electron components on the dusty grains, as it is more mobile when compared to cold electrons and ions.

Acknowledgements

The project is supported by the National Natural Science Foundation of China (No. 11178002), the International S&T Cooperation Program of China (No. S2014ZR0016), the Natural Science Foundation of Jiangxi (No. 20151BAB212010) and the Science Foundation for Youths of Jiangxi Education Committee (No. GJJ14224).

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