Ionization of Rydberg atoms by the kicks of half-cycle pulses

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Abstract. We present a quantum mechanical model to study the ionization of quasione-dimensional Rydberg atoms interacting with half-cycle pulses (HCPs) and use it to demonstrate the inadequacy of semiclassical approaches to calculate ionization probabilities of such atoms subject to the impact of more than one HCP. For a single-kicked atom both models correctly reproduce the experimentally observed 's-curve' as can be seen by plotting the ionization probability P as a function of momentum transfer q_1. We demonstrate that for a twice-kicked atom, the semiclassical model yields numbers for P which are not physically realizable. For fixed values of momentum transfers q_1 and q_2, in a twice-kicked atom, the ionization probability as a function of time delay between the kicks exhibits periodic decay and revival. The results of the semiclassical approach appear to agree with the quantum mechanical values at the times of revival of P, else these show considerable deviation. We attempt to provide a physical explanation for the limitation of the semiclassical approach.

Keywords. Rydberg atom; half-cycle pulses; ionization; quantum mechanical model.

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1. Introduction

There is increasing interest in studying physical processes that result from the impact of half-cycle pulses (HCPs) (unipolar electromagnetic field pulses) on weakly bound Rydberg electrons [1]. If a pulse is of much shorter duration than the Kepler period (t_k) of the orbiting electron, then the pulse will kick the electron to change its momentum in the field direction. The corresponding change in electron's energy ΔE may be less than E_B (the binding energy of the Rydberg electron) or it may be greater than or equal to E_B. For $\Delta E < E_B$, the impact of the HCP will generate the so-called electron wave packet which represents a non-stationary quantum state formed by coherent superposition of non-degenerate stationary states. Because of interference, such superposition can form a transient localized state which subsequently undergoes collapse and revival. This novel dynamical behaviour of Rydberg wave packets provides a bridge between quantum

and classical physics [2]. A second time-delayed HCP or more generally a train of HCPs can be judiciously used to convert a short-lived localized state into a long-lived one or even be stabilized [3]. On the other hand, if $\Delta E \geq E_B$, the impact of a single HCP will be able to ionize the atom. A Rydberg atom may also be ionized by the impact of two time-delayed HCPs. Here the first pulse might have insufficient amplitude to ionize the atom. Instead it merely redistributes the Rydberg population to create a dynamically evolving wave packet. The atom is then ionized by the impact of the second HCP. The efficiency of the second kick in ionizing the atom, in addition to its amplitude, depends crucially on its time of application [4]. For ionization produced by the kicks of k time-delayed HCPs, the first $(k - 1)$ kicks create a wave packet which is then acted upon by the kth HCP to finally ionize the atom. Understandably, here the dynamics of the created wave packet is controlled by the action of $(k - 2)$ time-delayed HCPs.

Classical and semiclassical approaches are often used to envisage theoretical studies on the ionization of Rydberg atoms driven by HCPs [5,6]. The semiclassical method is developed by taking recourse to the use of stationary-phase approximation. There have also been attempts to study the problem within the framework of quantum mechanical models [7,8]. To calculate ionization probabilities by using these models, one needs to implement purely numerical routines for solving the time-dependent Schrödinger equation. The direct numerical solution of Schrödinger equation with explicit time dependence often requires large computational effort. This appears to call for a straightforward analytic framework which is fully quantum mechanical. In this work, we derive such a model to study the ionization of quasione-dimensional (Q1D) Rydberg atoms by the impact of HCPs. Our main objective in this work is to attain some added realism for the calculation of ionization probabilities of Rydberg atoms subject to the impact of HCPs and thereby demonstrate the inadequacy of the semiclassical approach to study similar problems. In §2 we introduce the model and construct expressions for the ionization probabilities. We devote §3 to compare the numerical results for ionization probabilities of our quantum mechanical model with the corresponding results of the semiclassical method. The results of both models show that the ionization probability $P^{(1)}$ for a single-kicked atom as a function of the momentum transfer q_1 exhibits the experimentally confirmed s-like behaviour [4,9]. Moreover, the numerical values of $P^{(1)}$ obtained from both approaches are in close agreement with each other. This is, however, not true for ionization induced by two time-delayed HCPs.

For twice-kicked atoms, in addition to the momentum transfers, the ionization probability $P^{(2)}$ depends on the time delay between the applications of the HCPs. For a chosen value of the time delay, the probability $P^{(2)}$ as a function of the momentum q_2 transferred by the second kick to the wave packet also exhibits a similar s-like behaviour as observed in the case of a single-kicked atom. As expected, the quantum mechanical values of $P^{(2)}$ tends to unity for large values of q_2. But unfortunately, results obtained by the semiclassical model exhibit unphysical behaviour in that $P^{(2)}$ becomes greater than one for large values of q_2. For fixed values of momentum transfers we present results for $P^{(2)}$ as a function of time delay and demonstrate its oscillatory nature as found in the experiment of Arbò *et al* [10]. Here the results of semiclassical and quantum mechanical calculations agree only qualitatively. In §4, we make some concluding remarks and try to realize why the semiclassical approach is inapplicable for studying ionization of Rydberg atoms by the impact of a large number of HCPs.

2. Quantum mechanical and semiclassical ionization probabilities

Experimentally, a Q1D atom, which mimics many properties of the 3D atom, was created [11] by photoexcitation of the extreme Stark states of a high-n manifold in a weak dc electric field. The electronic orbits corresponding to these states lie on a line on one side of the atomic nucleus. In atomic units ($e = m = \hbar = 1$), the Hamiltonian operator for the Q1D Rydberg atom is given by

$$H = -\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \frac{1}{x}, \quad x > 0. \tag{1}$$

The eigenvalues of this operator are $E_n = -1/2n^2$, $n = 1, 2, \ldots$ with the corresponding eigenfunctions defined by

$$\psi_n(x) = \frac{2x}{\sqrt{n^3}}\mathrm{e}^{-x/n}{}_1F_1\left(-n+1; 2; \frac{2x}{n}\right), \tag{2}$$

where ${}_1F_1(\cdot)$ stands for the confluent hypergeometric function. From (2), we observe that the bound state eigenfunctions of the Q1D atom can be obtained from the corresponding eigenfunctions [12] of the 3D atom using the angular momentum $l = 0$. It will, therefore, be reasonable to assume that the continuum eigenfunctions of the Q1D atom can be obtained from the corresponding continuum eigenfunctions of the 3D atom [13] by restricting to $l = 0$. We thus write the continuum eigenfunction of the atom as

$$\psi(k, x) = 2x\sqrt{\frac{k}{1 - \mathrm{e}^{-2\pi/k}}}\mathrm{e}^{-ikx}{}_1F_1\left(\frac{i}{k} + 1; 2; 2ikx\right), \tag{3}$$

where k stands for the modulus of wave vector of the continuum electron. The eigenfunction in (3) is momentum-scale normalized.

In the regime where the duration of HCP is much shorter than the classical round-trip time of the Rydberg electron, the ionization can be described by using the impulse approximation. In this approximation, the HCP transfers a momentum kick q_1 given by [9]

$$q_1 = -\int_{-\infty}^{\infty} F_{\mathrm{HCP}}(t)\mathrm{d}t, \tag{4}$$

where $F_{\mathrm{HCP}}(t)$ is the time-varying electric field of the HCP. If we consider that the atom is ionized by the impact of a single kick, the ionization probability can be written as [14]

$$P_{k,n}^{(1)}(q_1) = \int_0^{\infty} |T_{kn}(q_1)|^2 \mathrm{d}k, \tag{5}$$

where $T_{kn}(q_1) = \int_0^{\infty} \psi^*(k, x)\mathrm{e}^{iq_1 x}\psi_n(x)\mathrm{d}x$ represents the transition matrix from a bound state $\psi_n(x)$ to the continuum state $\psi(k, x)$. For an atom ionized by the impact of two time-delayed HCPs, the result similar to that in (5) reads as

$$P_{k,n}^{(2)}(q_1, q_2, t) = \int_0^{\infty} \left|\sum_{n'} T_{kn'}(q_2)\mathrm{e}^{-iE_{n'}t}T_{n'n}(q_1)\right|^2 \mathrm{d}k. \tag{6}$$

Here q_1 and q_2 represent the momentum transfers from the first and second kicks respectively, and t is the time delay between the pulses. From (6), it is clear that the first

kick raises the bound atomic electron to intermediate excited states $|n'\rangle$ from which it is raised to the energy continuum by the impact of the time-delayed second kick. The bound–bound transition matrix or T-matrix like $T_{n'n}(q_1)$ can be written as [12]

$$T_{n'n}(q_1) = \langle n'|e^{iq_1 x}|n\rangle$$

$$= \frac{-z(\lambda - 2/n)^n(\lambda - 2/n')^{n'}}{\sqrt{nn'}\lambda^{n+n'}}\left\{\left[\frac{n-1}{\lambda - 2/n} + \frac{n'-1}{\lambda - 2/n'} - \frac{n+n'}{\lambda}\right]{}_2F_1(-n+1, -n'+1; 2; z)\right.$$

$$\left. - \frac{(n-1)(n'-1)}{2}\left[\frac{1}{\lambda - 2/n} + \frac{1}{\lambda - 2/n'}\right]z\,{}_2F_1(-n+2, -n'+2; 3; z)\right\}, \qquad (7)$$

where

$$\lambda = \left(\frac{1}{n} + \frac{1}{n'} - iq_1\right) \qquad (8)$$

and

$$z = -\frac{4n'n}{(n-n')^2 + q_1^2 n^2 n'^2}. \qquad (9)$$

The Gaussian hypergeometric functions ${}_2F_1(\cdot)$ which appear in (7) are polynomials and can, therefore, be evaluated without any truncation error. For use in (5) and (6), we found the bound–continuum T-matrix in the generic form

$$T_{kn}(q) = -\frac{1}{\sqrt{n^5(1 - e^{-2\pi/k})}}$$

$$\times \left[4i\sqrt{k}\left(\frac{1}{n} - ik - iq\right)^{-n-(i/k)}\left(\frac{1}{n} + ik - iq\right)^{-2+(i/k)}\left(-\frac{1}{n} - ik - iq\right)^{n-2}\right.$$

$$\times \left\{(n-1)a\,{}_2F_1\left(-n+1, -\frac{i}{k} + 1; 2; z_1\right)\right.$$

$$+ \left(1 - \frac{i}{k}\right)b\,{}_2F_1\left(-n+1, -\frac{i}{k} + 1; 2; z_1\right)$$

$$- \frac{1}{k(nk + nq + i)}\left((nk+i)ab\,{}_2F_1\left(-n+1, -\frac{i}{k} + 1; 2; z_1\right)\right)$$

$$\left.\left. + \frac{4(1+ik)(n-1)n^2q}{ab}\,{}_2F_1\left(-n+2, -\frac{i}{k} + 2; 3; z_1\right)\right\}\right], \qquad (10)$$

where

$$a = nk - nq - i, \quad b = nk + nq - i$$

and

$$z_1 = \frac{4ikn}{n^2q^2 - n^2k^2 + 1 + 2ikn}. \qquad (11)$$

In the semiclassical approximation, the bound–bound transition matrix is given by [15]

$$T_{n'n}^{sc}(q_1) = \frac{x_n^{n'}}{\sqrt{2\pi|q_1|n^3n'^3}}\exp(iS_n^{n'}), \qquad (12)$$

where

$$S_n^{n'} = 2(n + n') \tan^{-1} \left(\frac{q_1 n n'}{n + n'} \right) + 2(n' - n) \tan^{-1} \left(\frac{q_1 n n'}{n - n'} \right) - \frac{\pi q_1}{4|q_1|}. \quad (13)$$

In (12) $x_n^{n'}$ stands for the coordinate at which the electron changes its energy from E_n to $E_{n'}$ and is given by

$$x_n^{n'} = \frac{2q_1^2}{Q_{n'n}} \quad (14)$$

with

$$Q_{n'n} = (E_{n'} - E_n)^2 - q_1^2 (E_n + E_{n'}) + \frac{q_1^4}{4}. \quad (15)$$

The superscript 'sc' on $T_{n'n}$ in (12) has been used to indicate that it is merely a semiclassical transition matrix. Similarly, the bound–continuum T-matrix (in generic form) is written as [16]

$$T_{kn}^{\text{sc}}(q) = \sqrt{\frac{2k}{\pi}} \frac{q^2}{|q|} \frac{\exp[iS_n^k]}{n^{3/2} \left[\left(\frac{k^2}{2} + \frac{1}{2n^2} \right)^2 - \left(\frac{k^2}{2} - \frac{1}{2n^2} \right) q^2 + q^4/4 \right]}, \quad (16)$$

where

$$S_n^k = 2n \tan^{-1} \left(\frac{q}{n \left(\frac{k^2}{2} + \frac{1}{2n^2} - \frac{q^2}{2} \right)} \right) + \frac{1}{k} \ln \left(\frac{1 + n^2(k - q)^2}{1 + n^2(k + q)^2} \right). \quad (17)$$

The transition matrices (7) and (10) in conjunction with (5) and (6) give quantum mechanical ionization probabilities for the single- and twice-kicked atoms. The corres ponding semiclassical results can be found by taking recourse to the use of (12) and (16).

3. Results for ionization probabilities

Based on (5) and (6), we have calculated the ionization probabilities of Rydberg atoms for ejection of an electron of principal quantum number $n = 25$. In performing the numerical integration over the momentum of the continuum electron, we noted that the integrand assumes vanishingly small values for $k \geq 1$. Thus, for all practical purposes, the infinite integrals in (5) and (6) could be replaced by integrals of finite intervals so as to permit straightforward numerical evaluation. In order to compare the results for quantum mechanical and semiclassical calculations, we plot in figure 1, the values for ionization probabilities $P_{k,n}^{(1)}(q_1)$ of the single-kicked atom as a function of q_1. The solid curve - represents the results of our quantum mechanical model, while the dotted curve represents the corresponding numbers obtained by the semiclassical approach. Both curves exhibit the familiar s-like behaviour [4,9]. This behaviour arises due to the fact that the interaction of HCP with the Rydberg electron is different from the interaction with the optical pulses. For example, optical pulses interact with the electron only when the electron is near the
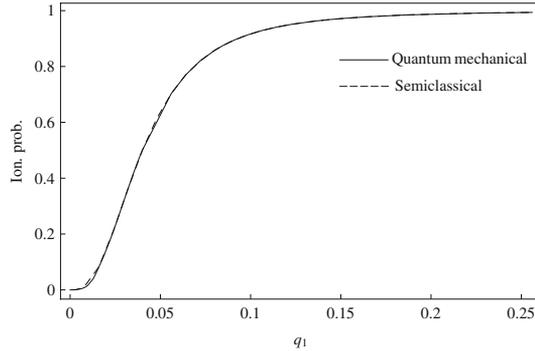
Figure 1. Ionization probability of the single-kicked atom as a function of momentum transfer q_1. The solid curve represent the values of $P_{k,n}^{(1)}(q_1)$ obtained from the quantum mechanical model, while the dotted curve represents the corresponding ionization probabilities obtained from the semiclassical approach.

inner turning point and moves with very high speed. In contrast, a HCP can interact with the electron wherever it may be in the course of its roundtrip journey around the nucleus. This specific feature of the interaction between a HCP and the orbiting electron is responsible for the observed s-like behaviour of the ionization probabilities [17]. Interestingly, looking at the figure, we see that for the single-kicked atom, both semiclassical and quantum mechanical models give identical results which are not discernible on the plot used by us. Ideally, one would, however, expect deviations between quantum mechanical and semiclassical results at low values of q_1. We have verified that for $q_1 < 0.016$, the values of ionization probabilities obtained from the semiclassical calculation are slightly augmented compared to the corresponding quantum mechanical results. Otherwise the dotted curve blends smoothly with the solid curve. We shall now present similar results for the twice-kicked atom with a view to demonstrate that in this case, the semiclassical model gives unphysical values for the ionization probabilities for large values of momentum transfer.

Looking at (6), we observe that the ionization of atom by the impact of two time-delayed HCPs can be viewed as a two-step process in which the first kick raises the Rydberg electron to some excited states, the superposition of which might form a wave packet. The time-delayed second pulse then acts on the wave packet to impart a momentum q_2 to ionize the atom. Thus, in computing the numbers for $P_{k,n}^{(2)}(q_1, q_2, t)$, the first problem that arises is to decide on the number of excited states to be included in forming the wave packet. In this context, we note that Wetzels *et al* [9] considered seven excited states to form a similar wave packet to study ionization of Rydberg atom due to ejection of an electron from principal quantum number $n = 40$. A simple hydrogenic formula can be used to show that the spacing of energy levels increases as n decreases. Here we are concerned with studies in ionization due to the removal of an electron from the state $n = 25$. Understandably, in this case we form the wave packet by including excited states which have wider spacing than those considered in ref. [9]. Thus, for $n = 25$, a pulse of a given bandwidth will coherently excite fewer number of states than those of Wetzels *et al* [9]. Consequently, in the present case we could evaluate the sum in (6) by including

only five excited states. However, we have taken into account seven excited states to form the wave packet. With this specific restriction on the summation over n', we have chosen to work with $q_1 = 1/n^2$.

We calculated numbers for $P^{(2)}_{k,n}(q_1, q_2, \tau)$ at different values of q_2 for the scaled time delays $\tau = 2$ and 3.5 respectively. The scaled time is defined as $\tau = t/t_k$ with the Kepler period $t_k = 2\pi n^3$. At $\tau = 2$, the wave packet is near the inner turning point while at $\tau = 3.5$, it is near the outer turning point [18]. In figure 2a, we display the plots of $P^{(2)}_{k,n}(q_1, q_2, \tau)$ as a function of q_2 for $\tau = 2$ and 3.5. The solid curves (curve (a) for $\tau = 2$ and curve (b) for $\tau = 3.5$) give the variation of $P^{(2)}_{k,n}(q_1, q_2, \tau)$ computed by using our quantum mechanical model. The corresponding results for the semiclassical calculation are shown by dotted curves. For $q_2 < 0.04$, the curves for $\tau = 2$ lie above the curves for $\tau = 3.5$. On the other hand, for all values of $q_2 > 0.04$, the curves for $\tau = 3.5$ lie above the curve for $\tau = 2$. This shows that for $q_2 < 0.04$, the atom is more likely to be ionized if the second kick is applied when the electron is near the inner turning point, while for $q_2 > 0.04$, the atom is more likely to be ionized if the second kick is applied when the electron is near the outer turning point. In both cases ($\tau = 2$ and 3.5), the solid and dotted curves lie quite close to each other for small values of q_2. But unexpectedly, the solid and dotted curves exhibit deviations as q_2 becomes large. For $\tau = 3.5$, separation between the two curves starts at $q_2 \approx 0.05$. Similar separation begins at a rather large value of q_2, namely, $q_2 \approx 0.1$ for $\tau = 2$. It appears that initially deviations between the curves are not very large. Despite that, such deviations finally lead us to an awkward physical situation. For example, the values of $P^{(2)}_{k,n}(q_1, q_2, \tau)$ obtained from the semiclassical model become greater than unity after $q_2 = 0.24$ for $\tau = 2$ and after $q_2 = 0.08$ for $\tau = 3.5$. This is shown clearly in figure 2b. One may be interested to know why the semiclassical values of the ionization probabilities exhibit this unphysical behaviour.

The expression for $T_{kn}(q)$ has been constructed by using exact hydrogenic and Coulomb functions. Thus, the quantum mechanical transition matrix will satisfy the
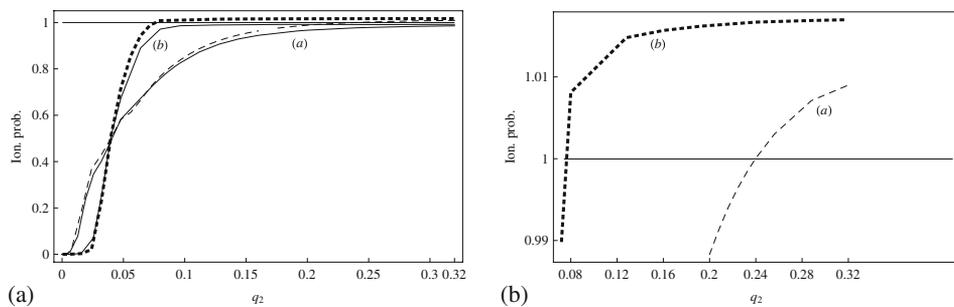
Figure 2. (**a**) Ionization probability $P^{(2)}_{k,n}(q_1, q_2, \tau)$ of the twice-kicked atom as a function of q_2 for two different values of the scaled time τ. The solid lines are for values of $P^{(2)}_{k,n}(q_1, q_2, \tau)$ obtained from the quantum mechanical calculation. Curve a is for $\tau = 2$ when the classical electron is near the inner turning point, while curve b is for $\tau = 3.5$ when the same electron is near the outer turning point. The corresponding dotted lines give the variation of $P^{(2)}_{k,n}(q_1, q_2, \tau)$ obtained from the semiclassical approach. (**b**) Same as figure 2a but for only large values of q_2.

unitarity condition. On the contrary, the semiclassical transition matrix $T_{kn}^{sc}(q)$ calculated on the basis of stationary-phase approximation violates this condition. The physical constraint arising due to violation of unitarity condition is expected to affect the semiclassical results for both $P_{k,n}^{(1)}(q_1)$ and $P_{k,n}^{(2)}(q_1, q_2, t)$, the ionization probabilities of the single- and twice-kicked atoms respectively. It appears that the semiclassical and quantum mechanical calculations give almost identical results of $P_{k,n}^{(1)}(q_1)$ for all values of q_1 but the semiclassical results for $P_{k,n}^{(2)}(q_1, q_2, t)$ become unphysical for large q_2 values. This may be due to the fact that $P_{k,n}^{(1)}(q_1)$ is characterized by a single transition matrix, while the expression for $P_{k,n}^{(2)}(q_1, q_2, t)$ involves the summation over a number of transition matrices, the small error in each of which gives rise to a cumulative effect to produce an unphysical result. Keeping this in view we venture to suggest that violation of the unitarity condition by the semiclassical transition matrices may be the dynamical origin for the observed unphysical results for $P_{k,n}^{(2)}(q_1, q_2, t)$. In the recent past, Requate *et al* [19] studied the range of validity of the stationary phase method in studying ionization of atoms in laser fields and found that although this approximation is, in general, valid for high laser intensities, the final expression of the ionization rate may occasionally involve unwanted singularities.

In figures 3 and 4, we display $P_{k,n}^{(2)}(q_1, q_2, \tau)$ for the initial state $n = 25$ as a function of τ for $q_1 = \pm 0.1/n$ and $q_2 = \pm 9q_1$. As before, the solid curves give the results of quantum mechanical calculation while the dashed curves represent the corresponding results obtained by the semiclassical approach. The absolute value of q_1 chosen here is larger than that used in presenting the plots of figure 2. Despite that we have considered seven excited states to form the wave packet. One may reasonably ask: Is it justified? In this context, we note that the bandwidth of a pulse determines the range of states which is coherently excited to form the wave packet which receives a time-delayed kick for the atom to be ionized and bandwidths, in general, have very little dependence on energies of the pulse [20]. Our chosen values of q_1 and q_2 are completely arbitrary. We could take any other set of values with the only restriction that $|q_2| \gg |q_1|$ because the second kick ionizes the atom by interacting with the wave packet formed by the impact of the first kick. In figure 3, we plot the results for $q_1 = 0.1/n$ (initial momentum directed towards the
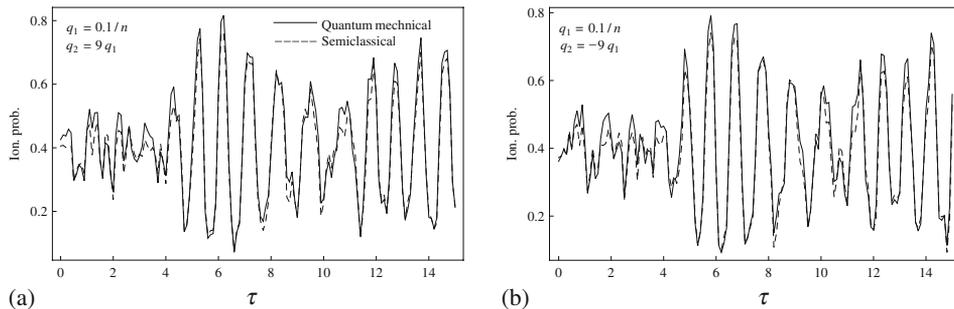
Figure 3. Ionization probability $P_{k,n}^{(2)}(q_1, q_2, \tau)$ from the initial state $n = 25$ as a function of the scaled time τ for (**a**) $q_1 = 0.1/n$, $q_2 = 9q_1$ and (**b**) $q_1 = 0.1/n$, $q_2 = -9q_1$. The solid line represents the quantum mechanical values while the dotted line represents the values of semiclassical calculation.
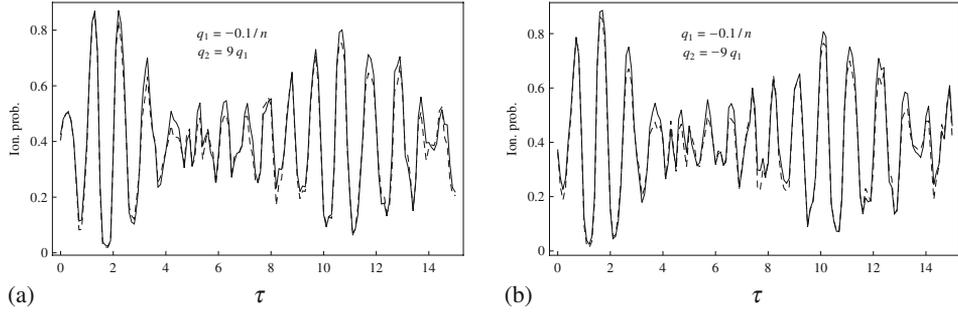
Figure 4. Same as figure 3 but (**a**) $q_1 = -0.1/n$, $q_2 = 9q_1$ and (**b**) $q_1 = -0.1/n$, $q_2 = -9q_1$.

nucleus) and figure 4 gives similar results for $q_1 = -0.1/n$ (initial momentum directed away from the nucleus). There appears a very fundamental difference between the curves with positive and negative values of q_1.

When the initial wave packet, which is subsequently ionized by the impact of the second kick, is produced by kicking the electron towards the nucleus, $P_{k,n}^{(2)}(q_1, q_2, \tau)$ oscillates with small values at small τ and as τ increases the probability oscillates with large values (figure 3). Figure 3a shows the plot of ionization probability with τ for $q_1 = 0.1/n$, $q_2 = 9q_1$ and figure 3b gives the same plot for $q_1 = 0.1/n$, $q_2 = -9q_1$. Both solid and dashed curves exhibit this particular nature of oscillation with respect to decay and revival of the ionization probability. Looking closely into the figures we see that the deviations between solid and dashed curves are appreciable only for τ values at which the ionization probabilities assume small amplitudes. Thus, it appears that the quantum and semiclassical models tend to predict somewhat different results for the decaying of ionization probabilities.

As opposed to this, when the sign of the initial kick is reversed, $P_{k,n}^{(2)}(q_1, q_2, \tau)$ oscillates with large amplitudes at small τ and as τ increases, the probability oscillates with small values (figure 4). As before, figure 4a shows the variation of $P_{k,n}^{(2)}(q_1, q_2, \tau)$ with τ for $q_1 = -0.1/n$, $q_2 = 9q_1$ and figure 4b gives the same variation but for $q_1 = -0.1/n$, $q_2 = -9q_1$. In this case, we also find that deviations between the solid and dashed curves are appreciable only when the ionization probabilities have minimum amplitudes. However, irrespective of the sign of q_1, the decay (revival) and revival (decay) of the probabilities appear to be a periodic phenomenon [17].

In figure 3a, signs of both q_1 and q_2 are positive but in figure 3b q_1 is positive and q_2 is negative. The curves in these figures show that their oscillations are out of phase. For example, at $\tau = 6.5$ the curve in figure 3a has a minimum while at this value of τ the curve in figure 3b shows a maximum. Similarly, we also see that the curves of figures 4a and 4b are out of phase.

4. Concluding remarks

Based on a crucial observation on the bound-state wave function of the Q1D Rydberg atom, we introduce a corresponding continuum wave function and thereby derive a fully

quantum mechanical model to study ionization of the atom subject to the impact of uni-polar HCPs. The numerical results presented for the ionization probabilities for the single- and twice-kicked atoms are in agreement with the experimental findings of Jones *et al* [4] and Arbò *et al* [10]. The model presented by us is fully quantum mechanical and provides a real improvement over the semiclassical approach. The semiclassical model becomes inapplicable in studying ionization of Rydberg atoms by the impact of more than one HCP. But our quantum mechanical model does not have any difficulty to deal with ionization processes induced by a number of time-delayed HCPs. For simplicity of presentation, we have demonstrated this for the ionization of a Q1D Rydberg atom by the impact of two delayed HCPs.

The model derived by us works quite satisfactorily for hydrogenic systems. However, it will be highly illuminating to extend the model for non-hydrogenic systems [21] like the alkali-metal atoms because most of the experiments on ionization have been carried out with these atoms [1]. In this context we note that the energy spectra of alkali-metal atoms can be described quite accurately using hydrogenic formula but with the principal quantum number n replaced by an effective quantum number ν such that $\nu = n - \mu$. Here μ stands for the so-called quantum defect [22]. The value of the quantum defect determines not only the energy of the state but also the spatial character of the wave function. The hydrogenic wave functions are expressed in terms of the confluent hypergeometric or regular Whittaker function. As opposed to this, the non-hydrogenic wave functions require the use of irregular Whittaker functions [23]. One can try to find the expression for ionization probability as that of ours by taking recourse to the use of non-hydrogenic bound and continuum wave functions of $l = 0$ with appropriately chosen values of μ.

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