



## Ignition curves for deuterium/helium-3 fuel in spherical tokamak reactor

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**Abstract.** In this paper, ignition curve for deuterium/helium-3 fusion reaction is studied. Four fusion reactions are considered. Zero-dimensional model for the power balance equation has been used. The closed ignition curves for  $\rho = \text{constant}$  (ratio of particle to energy confinement time) have been derived. The results of our calculations show that ignited equilibria for deuterium/helium-3 fuel in a spherical tokamak is only possible for  $\rho = 5.5$  and 6. Then, by using the energy confinement scaling and parameters of the spherical tokamak reactor, the plasma stability limits have been obtained in  $n_e$ ,  $T$  plane and, to determine the thermal instability of plasma, the time-dependent transport equations have been solved.

**Keywords.** Fusion reaction; ignition curve; confinement time; spherical tokamak.

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### 1. Introduction

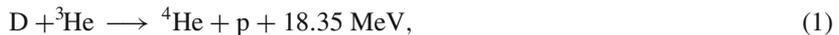
With the present rate of growth in world's population, one of the critical problems in the near future will be the requirement of some new sources of energy. Nuclear fission and fusion are candidates for such new energy sources. Fusion energy with regard to the economic, environmental and safety characteristics is more attractive than an advanced fission breeder reactor. However, many complications should be solved technologically [1,2]. Today's fusion research project is focussed on the deuterium–tritium (D–T) reaction fuel used in a fusion reactor. It has the largest cross-section of all the fusion reactions and it burns at the lowest temperature. In a deuterium–tritium reactor, 80% of the fusion energy is in the form of energetic neutrons that induce radioactivity and neutron damage in the structural materials. It is also necessary to breed large amount of tritium for plasma fueling. These are two principal disadvantages of the D–T cycle. The aneutronic deuterium/helium-3 (D–<sup>3</sup>He) reaction is usually thought of as the solution to these problems. Significant physics and engineering difficulties are the drawbacks of the D–<sup>3</sup>He fuel

cycle. In comparison to D–T fusion, D–<sup>3</sup>He fusion needs higher operating temperature, better confinement of the plasma, and higher plasma beta. The fact that the terrestrial supply of the helium isotope <sup>3</sup>He is severely limited is an extra impediment to competitive D–<sup>3</sup>He fusion, while deuterium, the hydrogen isotope, is plenty in nature. The moon’s surface contains a large amount of <sup>3</sup>He [3,4]. The technologies needed have been essentially demonstrated [4].

In the past few years, a new approach called the spherical tokamak or spherical torus has made remarkable inroads in magnetic fusion research [5–7] in parallel to the ITER project [8]. Spherical tokamak (ST) has attracted the attention of many researchers. In spherical torus experiments, a magnetohydrodynamics stable high beta value (the ratio of the plasma pressure to the toroidal magnetic pressure) of up to 40% has been routinely obtained [9]. Although the confinement scaling for ST is less-confident than the database of tokamaks, it seems that it is as good as the tokamak. The low cost of STs in comparison with the other conventional tokamaks has led to the conclusion that ST reactors are still considered potential candidates for producing thermonuclear burn. The possibility of D–<sup>3</sup>He-fueled ST reactors has already been indicated [9]. In this paper, the D–<sup>3</sup>He fuel cycle in the ST reactor has been studied. In addition to D–<sup>3</sup>He nuclear fusion reactions, the two branches of D–D reactions and D–T reaction were taken into consideration in our calculations. Burn criterion was defined as the product of plasma density and energy confinement time  $\tau_E$ , in which all loss processes can be included realistically to calculate energy confinement time [10]. In this paper, energy confinement time has been applied in the calculation of triple product  $n_e \tau_E T$ . Zero-dimensional power equations have been used for the D–<sup>3</sup>He fuel cycle. In this model, ions and electrons are assumed to have Maxwellian distributions and share the same temperature at all time. The closed ignition curves have been derived by using the global power equation and then ignition curves transform from  $n_e \tau_E T$ ,  $T$  plane to  $n_e$ ,  $T$  plane by using the energy confinement scaling and parameters of the ST reactor.

## 2. Method of calculation

In this work, the following nuclear fusion reactions have been studied:



The plasma composed of two-fuel density D, <sup>3</sup>He (the default value for the effective fuel densities is 1.0, which assumes  $n_D = n_{{}^3\text{He}}$ ) and three ash ions (proton, alpha and tritium).

Whole portions of the energy released in the D–<sup>3</sup>He reaction are in the form of charged particles with their energy deposited in the plasma. In D–T reaction, only 20% of the energy is released to the  $\alpha$ -particle and is available for plasma heating. Protons and alphas are produced primarily by the reaction <sup>3</sup>He(d, p)<sup>4</sup>He. Neutrons are produced directly by the D(d, n)<sup>3</sup>He reaction and indirectly by the D(d, p)T and T(d, n)<sup>4</sup>He reactions.

In this study, the zero-dimensional plasma transport model was used. The particle balance equations have the forms

$$\frac{dn_D}{dt} = -\frac{n_D}{\tau_p} - n_D n_T \langle \sigma v \rangle_{dt} - n_D n^3_{He} \langle \sigma v \rangle_{dh} - 0.5 n_D^2 \langle \sigma \rangle_{dd1} - 0.5 n_D^2 \langle \sigma v \rangle_{dd2} + S \quad (5)$$

$$\frac{dn_T}{dt} = -\frac{n_T}{\tau_p} - n_D n_T \langle \sigma v \rangle_{dt} + 0.5 n_D^2 \langle \sigma \rangle_{dd1} \quad (6)$$

$$\frac{dn^3_{He}}{dt} = -\frac{n^3_{He}}{\tau_p} - n_D n^3_{He} \langle \sigma v \rangle_{dh} + 0.5 n_D^2 \langle \sigma \rangle_{dd2} + S \quad (7)$$

$$\frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_a} + n_D n_T \langle \sigma v \rangle_{dt} + n_D n^3_{He} \langle \sigma v \rangle_{dh} \quad (8)$$

$$\frac{dn_p}{dt} = -\frac{n_p}{\tau_p} + n_D n^3_{He} \langle \sigma v \rangle_{dh} + 0.5 n_D^2 \langle \sigma \rangle_{dd1}. \quad (9)$$

In these equations,  $S$  and  $\langle \sigma v \rangle$  are the refueling rate and the fusion reaction rate, respectively, where the fusion reaction rates  $\langle \sigma v \rangle_{ij}$  represent the following four reactions: dh for  $D(^3\text{He}, p)\alpha$ , dd1 for  $D(D, p)T$ , dd2 for  $D(D, n)^3\text{He}$  and dt for  $D(T, n)\alpha$ . The reaction rate of D–D, D–T and D– $^3\text{He}$  fusion reactions are used in the following form [11]:

$$\langle \sigma v \rangle = \exp\left(\frac{a_1}{T^r} + a_2 + a_3 T + a_4 T^2 + a_5 T^3 + a_6 T^4\right) \quad (\text{cm}^3 \cdot \text{s}^{-1}), \quad (10)$$

where the constant  $a_1, \dots, a_6$  are taken from ref. [11].

The fuel ion density  $n_i = n_D + n^3_{He}$  and the ash density fractions are given by

$$\begin{aligned} f_p &= n_p/n_e, f_\alpha = n_\alpha/n_e, f_t = n_T/n_e \\ f_{\text{tot}} &= n_{\text{tot}}/n_e, f_z = n_z/n_e, \end{aligned} \quad (11)$$

where  $z$  is the impurity and  $e$  stands for the electron.

The quasineutrality condition [12] and total density are

$$n_e = n_D + 2n_\alpha + 2n^3_{He} + n_p + n_T + Zn_z \quad (12)$$

$$n_{\text{tot}} = n_i + n_\alpha + n_p + n_T + n_z + n_e. \quad (13)$$

The density fractions are given by

$$f_i = \frac{2}{3}(1 - 2f_\alpha - Zf_z - f_p - f_t) \quad (14)$$

$$f_{\text{tot}} = \frac{1}{3}(5 - f_\alpha - (2Z - 3)f_z + f_p + f_t). \quad (15)$$

The general global plasma power balance equation for D– $^3\text{He}$  plasmas is given by

$$\frac{dE}{dt} = -\frac{E}{\tau_E} + P_{\text{Fus}} + P_{\text{Ohmic}} + P_{\text{Aux}} - P_{\text{Bre}} - P_{\text{Syn}}. \quad (16)$$

where  $P_{\text{Fus}}$ ,  $P_{\text{Ohmic}}$ ,  $P_{\text{Aux}}$ ,  $P_{\text{Bre}}$  and  $P_{\text{Syn}}$  are the fusion power density, ohmic power density, auxiliary heating power density, bremsstrahlung radiation loss term and synchrotron radiation loss, respectively.

It is useful to express the auxiliary power in multiples of fusion power by

$$P_{\text{Aux}} = f P_{\text{Fus}}. \quad (17)$$

Fusion power density is calculated by considering all fusion reactions, and is given by:

$$\begin{aligned} P_{\text{Fus}} = & n_{\text{D}} n_{\text{T}} \langle \sigma v \rangle_{\text{dt}} E_{\text{dt}} + 0.53 n_{\text{D}}^2 \langle \sigma v \rangle_{\text{dd2}} E_{\text{dd2}} \\ & + 0.47 n_{\text{D}}^2 \langle \sigma v \rangle_{\text{dd1}} E_{\text{dd1}} + n_{\text{D}} n_{\text{He}} \langle \sigma v \rangle_{\text{dh}} E_{\text{dh}}. \end{aligned} \quad (18)$$

Ohmic heating power density is

$$P_{\text{Ohmic}} = \eta j^2, \quad (19)$$

where  $\eta$  is the Spitzer resistivity and  $j$  is the average plasma current density. As ohmic heating at thermonuclear temperatures is a very slow process and there is no loop voltage for steady-state operation, it is negligible for steady state.

The radiation loss  $P_{\text{Bre}}$  is given by

$$P_{\text{Bre}} = A_{\text{b}} Z_{\text{eff}}^2 n_{\text{e}}^2 T^{1/2}. \quad (20)$$

$Z_{\text{eff}}$  is the effective charge of the plasma ions, where

$$Z_{\text{eff}} = \frac{\sum_i Z_i^2 n_i}{n_{\text{e}}} \quad (21)$$

and  $A_{\text{b}}$  is the bremsstrahlung radiation coefficient [13]. The synchrotron radiation loss given by [14] is

$$P_{\text{Syn}} = 6.214 \times 10^{-23} n_{\text{e}} T B^2 \Phi_{\text{T}}, \quad (22)$$

where

$$\Phi_{\text{T}} = \frac{5.198 \times 10^{-3}}{\Lambda^{1/2}} T^{1.5} \left( 1 + \frac{22.61}{AT^{1/2}} \right)^{1/2} (1 - R_{\text{w}})^{1/2} \quad (23)$$

$$\Lambda^{1/2} = 7.78 \times 10^{-9} \left( \frac{n_{\text{e}} a}{B} \right). \quad (24)$$

$A$  is the aspect ratio, that is  $R/a$  and  $R_{\text{w}}$  is the reflection factor of radiation from the vacuum wall. The synchrotron power depends on plasma parameters, i.e., aspect ratio, beta, magnetic field, minor radius of the plasma and density. It is different from the other terms. The density can be eliminated by using the beta relationship [12],

$$\beta = \frac{1.6 \times 10^{-16} (n_{\text{e}} T_{\text{e}} + n_{\text{i}} T_{\text{i}})}{B^2 / 2\mu_0}. \quad (25)$$

Synchrotron radiation is important at high temperature and at low temperatures the bremsstrahlung dominates the losses.

In order to consider the steady-state setting,  $d/dt$  equals zero. For global power density, we get

$$\frac{3}{2} n_{\text{tot}} T_E / \tau_E = P_{\text{Fus}} + P_{\text{Aux}} - P_{\text{Br}} - P_{\text{Syn}}. \quad (26)$$

In the first case, we shall neglect the presence of impurity as well as that of the alpha, tritium and proton ash, i.e., we set  $f_\alpha = f_z = f_t = f_p = 0$ . To consider eqs (18), (20), (22), (25) and (26),  $n_e \tau_E T$  is obtained for ideal steady state that is given by

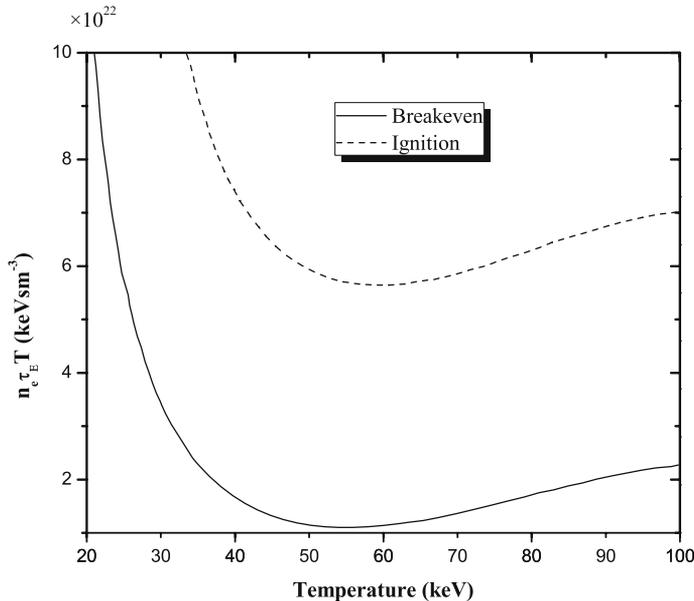
$$n_e \tau_E T = \frac{3}{2} f_{\text{tot}} T^2 / [(1 + f)(E_{\text{dt}} \langle \sigma v \rangle_{\text{dt}} f_i f_t + 0.5 \langle \sigma v \rangle_{\text{dd1}} E_{\text{dd1}} f_i^2 + 0.5 \langle \sigma v \rangle_{\text{dd2}} f_i^2 E_{\text{dd2}}) + \langle \sigma v \rangle_{\text{d3}} f_i^2 E_{\text{dt}} - A_b Z_{\text{eff}} \sqrt{T} - P_{\text{Syn}} n_e^{-2}]. \quad (27)$$

The  $n_e \tau_E T$  vs. temperature dependence is shown on figure 1. It is shown that the minimum temperature required for ideal ignition is about 65 keV for ignition ( $f = 0$ ) and 60 keV for break-even ( $f = 1$ ).

Now, the effect of ash particles on the conditions for ignition are going to be discussed for  $\tau_\alpha \neq 0$ ,  $f_\alpha \neq 0$ ,  $f_p \neq 0$  and  $f_t \neq 0$ . Equation (27) is rewritten, but consider the balance equation for ash particles. For this an equation should be written for calculating the confinement time of ash particles.

Although the mechanisms of particle diffusion differ greatly from that of the energy diffusion, the coupling of these two is strong. The scaling ansatz [15] appears as a good approximation for the ash particles in the plasma core,

$$\frac{\tau_p}{\tau_E} = \rho = \text{constant}. \quad (28)$$



**Figure 1.** Curves  $n_e \tau_E T$  for ideal ignition,  $f = 0$  (solid lines) and ideal break-even,  $f = 1$  (dashed lines) in the deuterium/helium-3 fuel cycle in the ST reactor.

In order to satisfy these conditions, the balance equation for the alpha, proton and tritium particles must be considered,

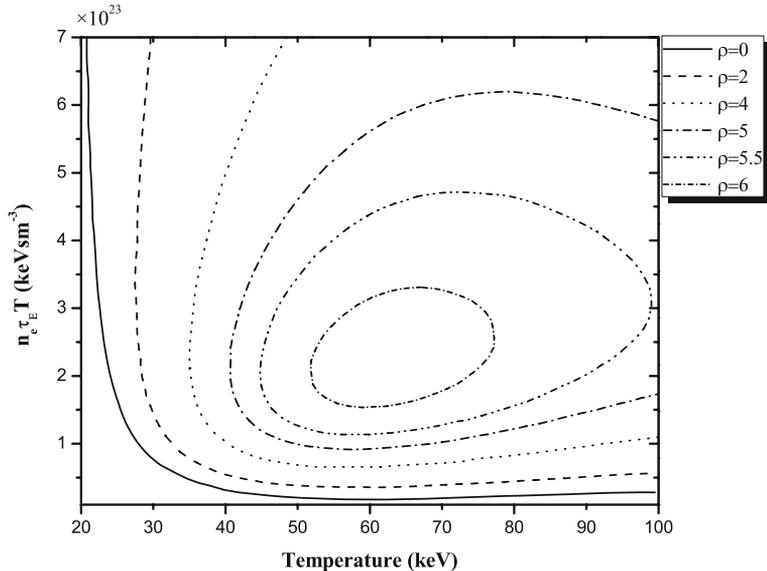
$$\frac{n_\alpha}{\tau_\alpha} = n_D n_{^3\text{He}} \langle \sigma v \rangle_{\text{dh}} + n_D n_t \langle \sigma v \rangle_{\text{dt}} \tag{29}$$

$$\frac{n_p}{\tau_\alpha} = n_D n_{^3\text{He}} \langle \sigma v \rangle_{\text{dh}} + 0.47 n_D^2 \langle \sigma v \rangle_{\text{dd1}} \tag{30}$$

$$\frac{n_t}{\tau_\alpha} = 0.47 n_D^2 \langle \sigma v \rangle_{\text{dd1}} - n_D n_t \langle \sigma v \rangle_{\text{dt}}. \tag{31}$$

In order to obtain the desired ignition curves, one has to solve eqs (27), (29), (30) and (31) with regard to the scaling ansatz eq. (28). After eliminating  $f_\alpha$ ,  $f_p$  and  $f_t$  from the equations, one can again derive an equation for  $n_e \tau_E T$ , which is a function of  $T$  and  $\rho$ . In figure 2, the ignition curves  $\rho = \text{constant}$  numerically for  $f = 0$  is illustrated. For  $\rho = 0$ , our pervious ideal ignition curve is recovered. For  $\rho > 0$ , closed ignition curves are generally obtained [12]. The most notable result of these calculations is that ignited equilibria exist in D-<sup>3</sup>He plasma only for  $\rho = 5.5$  and 6. If  $\rho$  becomes larger, the concentration of ash particles (alpha, proton, tritium) becomes too large and ignition is impossible [16].

The confinement time is one of the most important parameters for designing a tokamak reactor. Scaling expressions for the global energy confinement time are widely used for predicting future devices. They are taken from databases of global plasma parameters collected from different devices. Theoretically, plasma transport is a very tough process and it has not yet been solved completely. The answer to this question must be basically derived from experimental data. The expectation is that the energy confinement time  $\tau_E$



**Figure 2.** Curves  $n_e \tau_E T$  for non-ideal ignited equilibria in the deuterium/helium-3 fuel cycle in the ST reactor.

**Table 1.** Parameters of deuterium/helium-3 ST reactor [9].

Parameter	Unit	Value
Toroidal current $I_P$	MA	128
Magnetic Field $B_t$	T	2.7
Elongation $\kappa$		3
Major radius $R$	m	8.0
Minor radius $a$	m	6.15
Plasma volume $V$	m <sup>3</sup>	$1.792 \times 10^4$
Wall reflection $R_w$		1
Troyon coefficient $g_f$		4.2

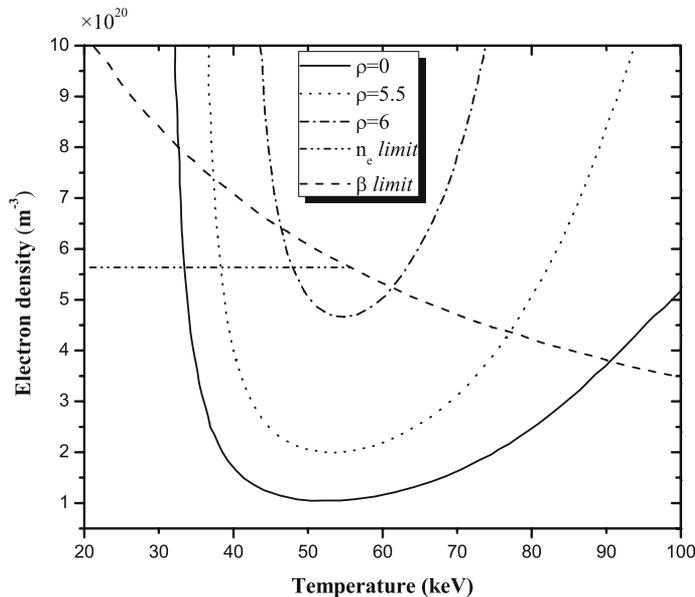
will depend upon design parameters such as IPB98 (y, 2) [17,18],

$$\tau_E = 0.144 H_h B_t^{0.15} I_P^{0.93} \kappa^{0.78} n_e^{0.41} a^{0.58} R^{1.39} M^{0.19} P^{-0.69}, \quad (32)$$

where  $P$ , the net plasma heating in MW is given by

$$P = \frac{3 n_{\text{tot}} T}{2 \tau_E}. \quad (33)$$

$H_h$  is the enhancement factor. The recent NSTX experiments show that the enhancement factor  $H_h > 1.5$  is possible. However, we conservatively set  $H_h = 1.0$  [19] where the units in eq. (32) are, T, MA,  $10^{20} \text{ m}^{-3}$ , m, m, amu, MW.



**Figure 3.** Ignition curve in an electron density,  $T$  plane for  $\rho = 0, 5.5$  and in the deuterium/helium-3 fuel cycle in the ST reactor.

Using the equilibrium equation and parameters of spherical tokamak reactor in table 1 and setting this equation on the relation  $n_e \tau_E T$ , the ignition contours can be translated from the  $n_e \tau_E T$  to density. In figure 3, the ignition curve in  $\rho = 0, 5.5, 6$  in  $n_e, T$  is shown. The normalized beta  $\beta_N$  is given by

$$\beta_N = \beta \frac{a B_t}{I_p}. \tag{34}$$

The following operation limits are mainly implied from the tokamak operation window [20]. The beta limit is expressed as

$$\beta < g_f \frac{I_p}{a B_t}, \tag{35}$$

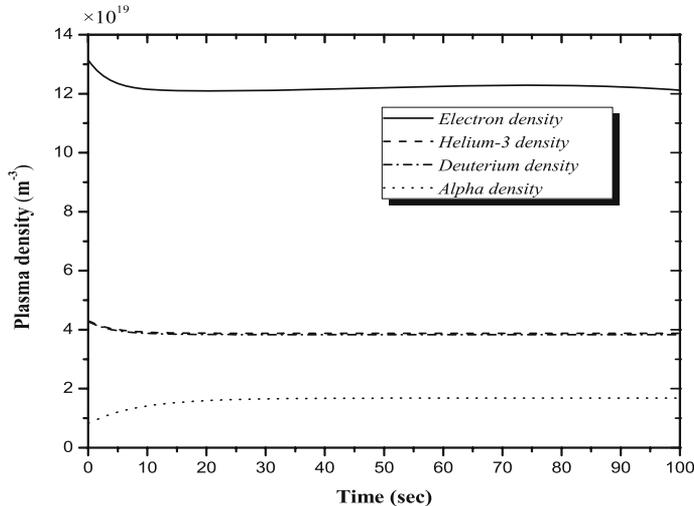
where  $g_f$  is the Troyon coefficient and the Greenwald density limit is [20]

$$n_e = 0.85 \frac{I_p}{\pi a_0^2}. \tag{36}$$

The advantage of representing the ignition curve as  $n_e(T)$  is that the impact of plasma stability limits such as the beta limit or the Greenwald density limit can be seen immediately.

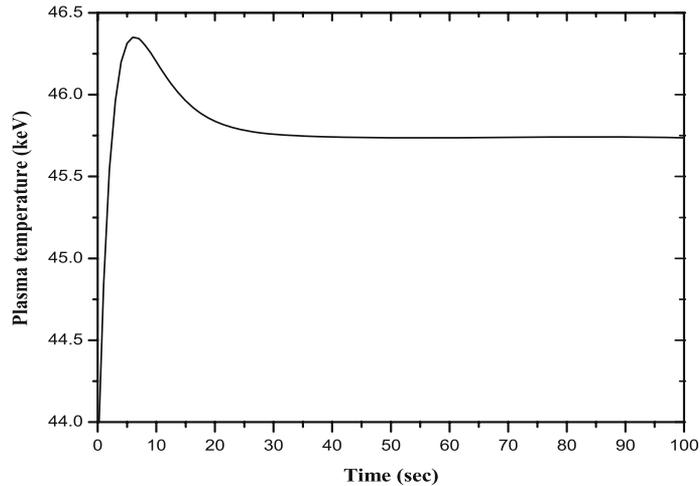
### 3. Time-dependent simulations

In order to determine the thermal instability of the D-<sup>3</sup>He plasma, one has to solve the time-dependent transport equations of plasma. Now, using eqs (5)–(9), particle balance equations and energy balance equation (16)–(24), the dynamic behaviour of a D-<sup>3</sup>He



**Figure 4.** The temporal evolution of plasma density in the deuterium/helium-3 fuel cycle in the ST reactor.

### Ignition curves for deuterium/helium-3 fuel



**Figure 5.** The temporal evolution of plasma temperature in the deuterium/helium-3 fuel cycle in the ST reactor.

spherical tokamak plasma is investigated. The calculation for the time evolution of D-<sup>3</sup>He plasma parameters has been done by solving a set of coupled differential equations numerically. The plasma densities and temperature as functions of time are shown in figures 4 and 5. These figures show the thermal instability of D-<sup>3</sup>He plasma. In figure 4, it is clearly seen that the density of electron leaves the equilibrium point and reaches low density. Figure 5 shows the system leaves the equilibrium point and settles on a higher temperature equilibrium point. These figures show that the system is driven from a low-temperature and high-density unstable zone to a high-temperature and low-density stable zone.

#### 4. Conclusions

In this paper, we have derived closed ignition curves for D-<sup>3</sup>He fusion reaction by solving the energy balance equation and also by considering the anastz scaling. We have shown that closed ignition curves exist only for  $\rho = 5.5$  and 6. Then, using the energy confinement time and spherical tokamak reactor parameters, ignition curves are transformed from  $n_e \tau_E T$ ,  $T$  plane to  $n_e$ ,  $T$ . In these two figures, we have determined the plasma stability limits. By solving the time-dependent transport equations numerically, the thermal instability of D-<sup>3</sup>He plasma has been shown. Yet, the presented calculations include only some simplification i.e., ions and electrons have the same temperature. So the equilibration power between ions and electron is zero in energy balance equation. These issues need to be substantiated in future.

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