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Economic scheme for remote preparation of an arbitrary five-qubit Brown-type state

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Abstract. A scheme for remotely preparing an arbitrary five-qubit Brown state by using three three-qubit GHZ states as the quantum channel is proposed. It is shown that, after the sender performs two different three-qubit projective measurements, the receiver should introduce two auxiliary qubits and employ suitable C-NOT gates, Toffoli gate and unitary operations on his qubits, the original state can be recovered with unit probability. Compared with the previous scheme, the advantage of the present scheme is that the entanglement resource can be reduced.

Keywords. Remote state preparation; five-qubit Brown-type state; three-qubit projective measurement; unit successful probability; entanglement resource

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1. Introduction

Quantum entanglement plays a very critical role in quantum mechanics. Multipartite entanglement is also a very important physical resource in quantum information processing. So far, multipartite entanglement has been well studied theoretically and experimentally (e.g. [1–9]). A few years ago, Verstraete *et al* [10] have shown that the four-qubit entangled state may be divided into nine families of states. Recently, Brown *et al* [11] have constructed a kind of five-qubit entangled state (called Brown state) through a numerical optimization procedure. The Brown state can be expressed as [11]

$$|B\rangle = \frac{1}{2}(|000\rangle|\phi^-\rangle + |010\rangle|\psi^-\rangle + |100\rangle|\phi^+\rangle + |111\rangle|\psi^+\rangle), \tag{1}$$

where $|\phi^\pm\rangle$ and $|\psi^\pm\rangle$ are the standard Bell states given by

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \tag{2}$$

The Brown state has been shown to be useful for quantum information, such as quantum teleportation [12], quantum state sharing [13], superdense coding [13], quantum secure communication [14,15], quantum information splitting [16,17], etc.

In the last decade, Lo [18], Pati [19] and Bennett *et al* [20] presented a new quantum communication scheme that uses classical communication and a previously shared entangled resource to remotely prepare a quantum state. This communication scheme is called remote state preparation (RSP). Since then, RSP has attracted much attention, various theoretical schemes for the generalization of RSP have been proposed and experimental implementation of RSP has been presented [21–37]. In recent years, some researchers [38–42] have proposed various theoretical schemes for RSP of four-qubit entangled state. More recently, Ma *et al* [43] investigated the remote preparation of a five-qubit Brown-type state. In their deterministic scheme, a three-qubit GHZ state and two four-qubit GHZ states were employed as the quantum channel.

In the present paper, we proposed a new scheme for remote preparation of a five-qubit Brown-type state by using two sets of three-qupit measuring basis with three three-qubit GHZ states as the quantum channel. To present our scheme more clearly, here we consider only maximally entangled channel. In our scheme, after the sender performed the projective measurements on her qubits, according to the result of the sender's measurements, the receiver should introduce two auxiliary qubits and then perform suitable C-NOT gates on his qubits, the original state can be recovered with unit successful probability. Compared with the previous scheme for the RSP of a five-qubit Brown-type state [43], the advantage of our scheme is that the entanglement resource can be reduced.

2. RSP of a five-qubit Brown-type state

Now, let us propose an economic scheme for remotely preparing an arbitrary five-qubit Brown-type state. Suppose that the sender Alice wishes to help the receiver Bob to remotely prepare five-qubit Brown-type state

$$
\begin{aligned}
|\psi\rangle = {} & x_0\,|00000\rangle + x_1 e^{i\delta_1}\,|00011\rangle + x_2 e^{i\delta_2}\,|01001\rangle + x_3 e^{i\delta_3}\,|01010\rangle \\
& + x_4 e^{i\delta_4}\,|10000\rangle + x_5 e^{i\delta_5}\,|10011\rangle + x_6 e^{i\delta_6}\,|11101\rangle + x_7 e^{i\delta_7}\,|11110\rangle\,, \qquad (3)
\end{aligned}
$$

where x_j and $\delta_j (j = 0, 1, \ldots, 7)$ are real, $\delta_0 = 0$ and $\sum_{j=0}^{7} x_j^2 = 1$. Assume that Alice knows x_j and $\delta_j (j = 0, 1, \ldots, 7)$, but Bob does not know them at all. We also suppose that the states shared by Alice and Bob as the quantum channel are three GHZ states

$$
\begin{aligned}
|\varphi_1\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_1A_2B_1}\,, \\
|\varphi_2\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_3A_4B_2}\,, \\
|\varphi_3\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_5A_6B_3}\,, \qquad (4)
\end{aligned}
$$

where the qubits A_1, A_2, A_3, A_4, A_5 and A_6 belong to Alice and B_1, B_2, B_3 to Bob.

In order to complete the RSP, Alice should construct her measuring bases. The first measuring basis chosen by Alice is a set of mutual orthogonal basis vectors (MOBVs) $\{|\mu_k\rangle\}(k = 0, 1, \ldots, 7)$ which is given by

$$\begin{aligned}(|\mu_0\rangle, |\mu_1\rangle, |\mu_2\rangle, |\mu_3\rangle, |\mu_4\rangle, |\mu_5\rangle, |\mu_6\rangle, |\mu_7\rangle)^T \\ = F(|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle)^T,\end{aligned} \tag{5}$$

where

$$F = \begin{pmatrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ x_1 & -x_0 & x_3 & -x_2 & x_5 & -x_4 & x_7 & -x_6 \\ x_2 & -x_3 & -x_0 & x_1 & -x_6 & x_7 & x_4 & -x_5 \\ x_3 & x_2 & -x_1 & -x_0 & x_7 & x_6 & -x_5 & -x_4 \\ x_4 & -x_5 & x_6 & -x_7 & -x_0 & x_1 & -x_2 & x_3 \\ x_5 & x_4 & -x_7 & -x_6 & -x_1 & -x_0 & x_3 & x_2 \\ x_6 & -x_7 & -x_4 & x_5 & x_2 & -x_3 & -x_0 & x_1 \\ x_7 & x_6 & x_5 & x_4 & -x_3 & -x_2 & -x_1 & -x_0 \end{pmatrix}. \tag{6}$$

The second measuring basis chosen by Alice is a set of MOBVs $\{|\lambda_j\rangle\}$, which is given by

$$\begin{aligned}(|\lambda_0\rangle, |\lambda_1\rangle, |\lambda_2\rangle, |\lambda_3\rangle, |\lambda_4\rangle, |\lambda_5\rangle, |\lambda_6\rangle, |\lambda_7\rangle)^T \\ = H(|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle)^T,\end{aligned} \tag{7}$$

where

$$H = \begin{pmatrix} 1 & r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 \\ 1 & -r_1 & r_2 & -r_3 & r_4 & -r_5 & r_6 & -r_7 \\ 1 & -r_1 & -r_2 & r_3 & -r_4 & r_5 & r_6 & -r_7 \\ 1 & r_1 & -r_2 & -r_3 & r_4 & r_5 & -r_6 & -r_7 \\ 1 & -r_1 & r_2 & -r_3 & -r_4 & r_5 & -r_6 & r_7 \\ 1 & r_1 & -r_2 & -r_3 & -r_4 & -r_5 & r_6 & r_7 \\ 1 & -r_1 & -r_2 & r_3 & r_4 & -r_5 & -r_6 & r_7 \\ 1 & r_1 & r_2 & r_3 & -r_4 & -r_5 & -r_6 & -r_7 \end{pmatrix}, \tag{8}$$

where $r_j = \mathrm{e}^{-i\delta_j}(j = 1, 2, \ldots, 7)$.

Now let Alice first perform three-projective measurement on the qubits A_1, A_3, A_5 by using the basis $\{|\mu_k\rangle\}(k = 0, 1, \ldots, 7)$. Next, in accord with the result of measurement, Alice should employ suitable unitary operations on her qubits A_2, A_4 and A_6. After that, Alice measures these qubits under the basis $\{|\lambda_j\rangle\}(j = 0, 1, \ldots, 7)$ and then informs Bob of her results of the measurement by the classical channel. For example, without loss of generality, assume that Alice's first measurement outcome is $|\mu_1\rangle_{A_1A_3A_5}$, and the qubits A_2, A_4, A_6, B_1, B_2 and B_3 will be collapsed into the state

$$\begin{aligned}|\zeta\rangle = \frac{1}{2\sqrt{2}}(&x_1|000000\rangle - x_0|000011\rangle + x_3|001100\rangle \\ &-x_2|001111\rangle + x_5|110000\rangle - x_4|110011\rangle \\ &+x_7|111100\rangle - x_6|111111\rangle)_{A_2B_1A_4B_2A_6B_3}.\end{aligned} \tag{9}$$

Then Alice performs a local unitary operation $(-i\sigma_y)$ on the qubit A_6. After that, Alice measures the qubits A_2, A_4 and A_6 under the basis $\{|\lambda_j\rangle\}(j = 0, 1, \ldots, 7)$, and informs

Bob of her results of the measurement by the classical channel. Without loss of generality, assume that the outcome of Alice's measurement is $|\lambda_3\rangle_{A_2A_4A_6}$, and the qubits B_1, B_2, B_3 will be collapsed into the state

$$\left|\zeta'\right\rangle = \frac{1}{2\sqrt{2}}(x_0\left|001\right\rangle + x_1 e^{i\delta_1}\left|000\right\rangle + x_2 e^{i\delta_2}\left|011\right\rangle + x_3 e^{i\delta_3}\left|010\right\rangle$$
$$+ x_4 e^{i\delta_4}\left|101\right\rangle + x_5 e^{i\delta_5}\left|100\right\rangle + x_6 e^{i\delta_6}\left|111\right\rangle + x_7 e^{i\delta_7}\left|110\right\rangle)_{B_1B_2B_3}. \quad (10)$$

According to Alice's public announcements, Bob can perform local unitary operation σ_x on the qubit B_3, and thus the state (10) will be transformed into

$$\left|\zeta''\right\rangle = \frac{1}{2\sqrt{2}}(x_0\left|000\right\rangle + x_1 e^{i\delta_1}\left|001\right\rangle + x_2 e^{i\delta_2}\left|010\right\rangle + x_3 e^{i\delta_3}\left|011\right\rangle$$
$$+ x_4 e^{i\delta_4}\left|100\right\rangle + x_5 e^{i\delta_5}\left|101\right\rangle + x_6 e^{i\delta_6}\left|110\right\rangle + x_7 e^{i\delta_7}\left|111\right\rangle)_{B_1B_2B_3}. \quad (11)$$

One can see that the state (11) contains the complete information of the original state $|\psi\rangle$. In order to complete the RSP, Bob introduces auxiliary qubits B_4 and B_5 with the initial state $|0\rangle_{B_4}$ and $|0\rangle_{B_5}$, and the state (11) will be described as

$$\left|\zeta'''\right\rangle = \frac{1}{2\sqrt{2}}(x_0\left|000\right\rangle + x_1 e^{i\delta_1}\left|001\right\rangle + x_2 e^{i\delta_2}\left|010\right\rangle$$
$$+ x_3 e^{i\delta_3}\left|011\right\rangle + x_4 e^{i\delta_4}\left|100\right\rangle + x_5 e^{i\delta_5}\left|101\right\rangle$$
$$+ x_6 e^{i\delta_6}\left|110\right\rangle + x_7 e^{i\delta_7}\left|111\right\rangle)_{B_1B_2B_3} \otimes \left|0\right\rangle_{B_4}\left|0\right\rangle_{B_5}. \quad (12)$$

Then Bob performs four C-NOT gates $C_{B_3-B_4}$, $C_{B_3-B_5}$, C_{B5-B3}, $C_{B_2-B_5}$ and a Toffoli gate T_{B_1,B_2-B_3} on the qubits B_1, B_2, B_3, B_4 and B_5, where C_{i-j} denotes i as the control qubit and j as target one, and $T_{p,q-s}$ denotes p, q as the control qubits and s as the target one. After that, the state (12) can be transformed into the original state $|\psi\rangle$ and RSP succeeds in this case. If Alice's first measurement outcomes are the other seven cases in the basis $\{|\mu_k\rangle\}$ $(k = 0, 1, \ldots, 7)$, she should choose suitable unitary operations on the qubits A_2, A_4, A_6, and then measure these qubits in the basis $\{|\lambda_j\rangle\}(j = 0, 1, \ldots, 7)$. In accord with Alice's public announcement, Bob can employ appropriate unitary transformation on the qubits B_1, B_2 and B_3 and the state $|\zeta''\rangle$ (see eq. (12)) can always be obtained. Next, using the same approach mentioned above, i.e., Bob introduces auxiliary qubits B_4 and B_5 with the initial states $|0\rangle_{B_4}$ and $|0\rangle_{B_5}$, and then performs four C-NOT gates and a Toffoli gate as above on his qubits successively, the desired state $|\psi\rangle$ can be reconstructed. It is easily found that, for all the 64 measurement outcomes of Alice, the receiver Bob can reconstruct the original state $|\psi\rangle$ with unit probability, and the requital classical communication cost is six bits in this scheme.

3. Conclusion

In conclusion, we have proposed a novel scheme for remotely preparing a five-qubit Brown-type state with complex coefficients. In the scheme, three three-qubit GHZ states have been used as the quantum channel. The sender first performs a three-qubit projective measurement on her three qubits, then according to the result of measurement, she should employ appropriate unitary operations on her other three qubits. After that, Alice carries out another three-qubit measuring basis to measure her other three qubits. Next, in accord with the sender's measurement results, the receiver can employ suitable unitary operations on his qubits, and a three-qubit entangled state which contains full information of the sender's original state can be obtained. To complete the RSP, the receiver should introduce two auxiliary qubits and perform four C-NOT gates and a Toffoli gate on his qubits, the original state can be reconstructed. In this scheme, the total success probability of the RSP is 1. Compared with the previous scheme [43], the advantage of our scheme is that the entanglement resource can be reduced. In ref. [43], exact RSP of an arbitrary five-qubit Brown-type state needs 5.5 ebits [44] and in our scheme only 4.5 ebits are required. Apparently, our entanglement cost is less than that in ref. [43]. In this sense, we emphasize that our scheme is an optimal one. Furthermore, so far there have been several theories and experiments implementing the optical C-NOT gate [45–49] and Toffoli gate [50–52]. Hence, we claim our scheme is feasible according to current technologies, and further expect that it can be demonstrated via linear optics systems in the near future.

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