PRAMANA
— journal of
physics

Vol. 86, No. 4
April 2016
pp. 777–781

Simulation of a quantum NOT gate for a single qutrit system

M ÁVILA* and J RUEDA-PAZ

Centro Universitario UAEM Valle de Chalco, UAEMex, María Isabel, Valle de Chalco, Estado de México, CP 56615, México

*Corresponding author. E-mail: vlkmanuel@uaemex.mx

MS received 30 August 2014; revised 23 March 2015; accepted 30 March 2015
DOI: 10.1007/s12043-015-1116-3; ***e*Publication:** 4 November 2015

Abstract. A three-level system based an a three-level atom interacting with a detuned cavity is considered. Because of the fact that the three-level atom defines a total normalized state composed of superposition of three different single-level states, it is assumed that such a system implements a qutrit. In order to achieve a quantum NOT gate for a single qutrit, the respective Schrödinger equation is solved numerically within a two-photon rotating wave approximation. For small values of one-photon detuning, there appear decoherence effects. Meanwhile, for large values of one-photon detuning, an ideal quantum NOT gate for a single qutrit is achieved. An expression for the execution time of the quantum NOT gate for a single qutrit as a function of the one-photon detuning is found.

Keywords. Three-level system; qutrit; three-level transitions; one-qutrit quantum gate.

PACS Nos 03.67.−a; 03.67.Ac

Any quantum computation can be reduced to one-qubit and two-qubit logic gates [1–4]. To implement one-qubit and two-qubit quantum gates, a three-level atom interacting with a detuned cavity has been employed [5]. In such a case, the two-photon transitions have the advantage that they involve long-lived ground states of the atom. Furthermore, the excited state does not participate in the transition, thus minimizing the effects of decoherence associated with the finite lifetime of the excited state [6–8]. On the other hand, in order to search for further possibilities, an extension to a general d-system or qudits arises. In particular, a three-level system or qutrit has attracted the attention [9]. The qutrit has proved to be more efficient than qubit systems for executing some tasks. For instance, for quantum key distribution it provides better security than that obtained by employing two-level encoding protocols [10,11]. In fact, it has been shown that the best security is achieved by using a qutrit [12].

A qutrit state $|\psi\rangle$ can be defined as follows: $|\psi\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle$, where $\{|0\rangle, |1\rangle, |2\rangle\}$ is an orthonormal set of states and $|c_0|^2 + |c_1|^2 + |c_2|^2 = 1$. There are several experimental implementations for a qutrit. Some of the techniques employed for implementing a qutrit are the following: the orbital angular momentum [13], the energy–time correlation of photon pairs [14] and the two-photon polarization [15,16]. In the present work, we introduce a three-level system based on a three-level atom interacting with a detuned cavity as shown in figure 1. Such a system works on the basis of a two-photon transition where one expects long-lived ground states of the atom. Given that the three-level atom can be described through a superposition of three single level states, we can assume that such a system defines a one-qutrit state. Once the above is done, we simulate a qutrit NOT quantum gate. The NOT quantum gate in the context of a qutrit can be defined as follows:

$$\mathrm{NOT}|0\rangle \longrightarrow A|1\rangle + B|2\rangle,$$

$$\mathrm{NOT}(A|1\rangle + B|2\rangle) \longrightarrow |0\rangle,$$

where $|A|^2 + |B|^2 = 1$. The simulation of the NOT quantum gate starts with the initial conditions $c_1(t = 0) = 1$ and $c_2(t = 0) = c_3(t = 0) = 0$ in the definition of a qutrit $|\psi\rangle = c_1|0\rangle + c_2|1\rangle + c_3|2\rangle$. Then, by solving the Schrödinger equation associated with the three-level atom interacting with the detuned cavity for $|\psi(t)\rangle$, we prove that after an elapsed time T_f, the coefficients satisfy $c_1(t = T_f) = 0$; $|c_2(t = T_f)|^2 + |c_3(t = T_f)|^2 = 1$.

The above cited three-level atom passes through a bimodal cavity in such a way that the modes with annihilation operators a and b interact with the $|0\rangle \leftrightarrow |1\rangle$ and $|0\rangle \leftrightarrow |2\rangle$ transitions respectively. The Hamiltonian of the rotating wave approximation approach must be [5]

$$H = \hbar[\omega_{01}|0\rangle\langle 0| + \omega_{21}|2\rangle\langle 2| + \omega_a a^\dagger a + \omega_b b^\dagger b + \{g_1|0\rangle\langle 1|a + g_2|0\rangle\langle 2|b + \mathrm{h.c.}\}], \quad (1)$$

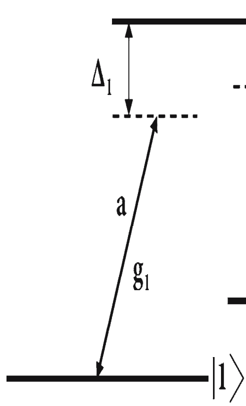

Figure 1. Three-level system with levels $\{|0\rangle, |1\rangle, |2\rangle\}$ interacting with the cavity annihilation operators a and b with atom–cavity couplings g_1 and g_2 respectively. The respective one-photon detunings are $\{\Delta_1, \Delta_2\}$.

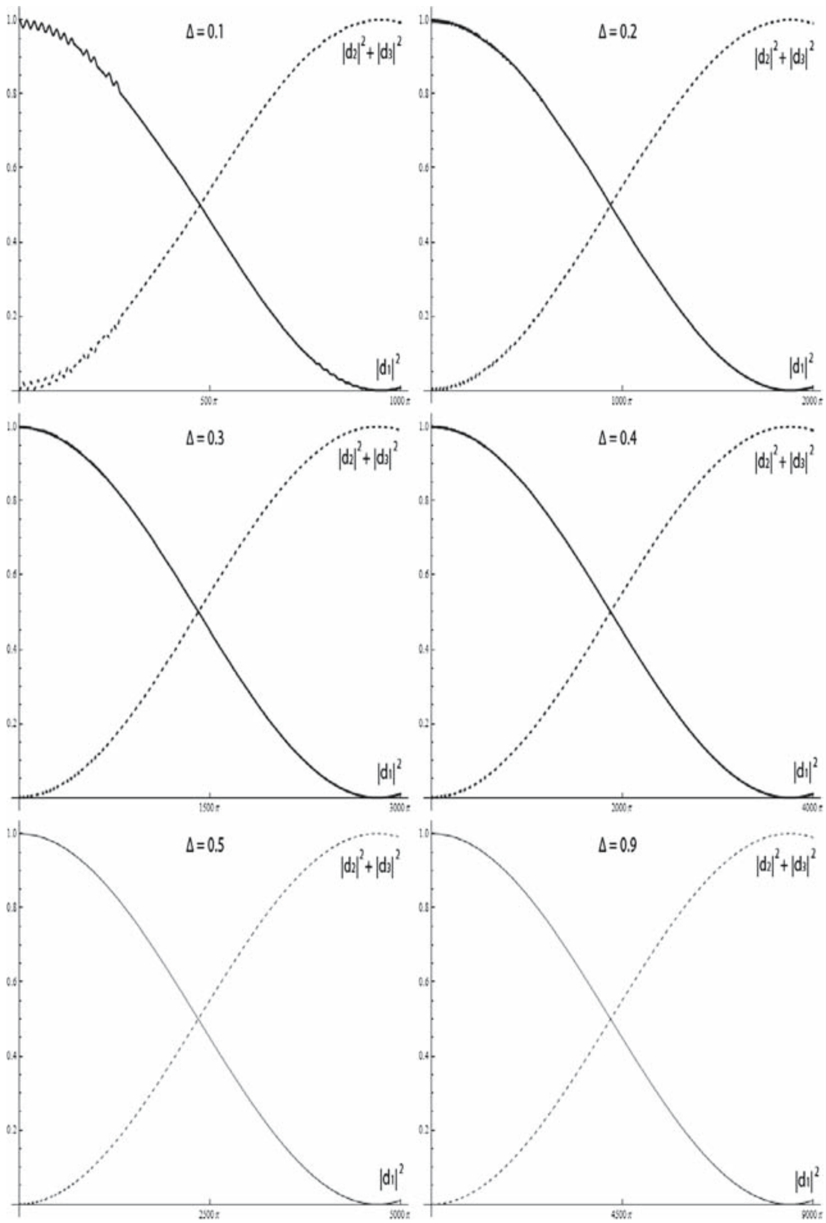

Figure 2. Quantum NOT gate, where after an elapsed time T_f, the qutrit transits from the initial state $|\psi(t = 0)\rangle = |0\rangle$ ($d_1(t = 0) = 1$; $d_2(t = 0) = d_3(t = 0) = 0$) to the final state $|\psi(t = T_f)\rangle = d_2(t = T_f)|1\rangle + d_3(t = T_f)|2\rangle$ ($|c_2(t = T_f)|^2 + |c_3(t = T_f)|^2 = 1$). The real-valued atom–cavity couplings of the modes a and b are $g_1 = g_2 = 1/137$. The plots are done for several values of one-photon detunings $\Delta_1 = \Delta_2 = \Delta$ in units of Hz.

where ω_{i1} $(i = 0, 2)$ is the atomic transition frequency, ω_j $(j = a, b)$ is the frequency of the cavity modes a and b, and g_k $(k = 1, 2)$ is the real-valued atom–cavity coupling constant. The interaction Hamiltonian in the interaction picture can be written as

$$H = \hbar(g_1|0\rangle\langle 1|a\mathrm{e}^{i\Delta_1 t} + g_2|0\rangle\langle 2|b\mathrm{e}^{i\Delta_2 t} + \mathrm{h.c.}), \tag{2}$$

where $\Delta_i = \omega_{01,21} - \omega_i$ $(i = 1, 2)$ is the one-photon detuning of the cavity modes. By assuming [16a] that the initial number of photons in the a and b modes are $n = 1$ and $\mu = 1$, the state vector of the atom–cavity system can be expanded as [5]

$$|\psi(t)\rangle = c_1|0\rangle|1, 1\rangle + c_2|1\rangle|0, 1\rangle + c_3|2\rangle|0, 2\rangle. \tag{3}$$

The equation for the probability amplitudes $\{c_1, c_2, c_3\}$ can be obtained from the Schrödinger equation as

$$\begin{aligned} \dot{d}_1 &= -ig_1 d_2, \\ \dot{d}_2 &= -i[g_1 d_1 + g_2\sqrt{2}d_3] - i\Delta_1 d_2, \\ \dot{d}_3 &= -ig_2\sqrt{2}d_2 - i(\Delta_1 - \Delta_2)d_3, \end{aligned} \tag{4}$$

where

$$c_1 \to d_1, \qquad c_2\mathrm{e}^{-i\Delta_1 t} \to d_2, \qquad c_3\mathrm{e}^{-i(\Delta_1-\Delta_2)t} \to d_3. \tag{5}$$

The system of coupled differential equations given by eq. (4) are solved numerically through the Runge–Kutta method of fifth-order (which reduces the numerical error and optimize the result) [17,18] for several values of one-photon detunings $\{\Delta_1, \Delta_2\}$. We fix the values of the atom–cavity coupling constants as follows: $g_1 = g_2 = 1/137$. Also, we have taken $\Delta_1 = \Delta_2 = \Delta$. As can be observed from figure 2, the system of coupled differential equations of eq. (4) reproduces the quantum NOT gate for a single qutrit. That is, initially the system is $|\psi(t = 0)\rangle = |0\rangle$ where $c_1(t = 0) = 1$; $c_2(t = 0) = c_3(t = 0) = 0$, and after an elapsed time T_f, the system evolves towards the qutrit state $|\psi(t = T_f)\rangle = c_2(t = T_f)|1\rangle + c_3(t = T_f)|2\rangle$ such that $|c_2(t = T_f)|^2 + |c_3(t = T_f)|^2 = 1$.

In order to reproduce the quantum NOT gate for a single qutrit as was defined above, we have numerically solved the Schrödinger equation given by eq. (4) for different values of one-photon detuning Δ. We have assumed that only a single photon is exchanged for each of the modes a and b respectively. As shown by figure 2, eq. (4) successfully reproduces the quantum NOT gate for a single qutrit. Furthermore, the time of execution of such a gate is $t = T_f$. From figure 2, we can see that the execution time of the quantum NOT gate for a single qutrit obeys approximately the equation $T_f = \pi\Delta \times 10^4$ s. For instance, for $\Delta = 0.3$ Hz one has $T_f = 3000\pi$. Due to decoherence, for small values of one-photon detuning Δ, there appear small fluctuations in the coefficients. However, for large values of Δ, one obtains an ideal quantum NOT gate for a single qutrit.

Acknowledgements

We thank the SNI-Conacyt grant.

References

[1] M A Nielsen and I L Chuang, *Quantum computation and quantum information* (Cambridge University Press, Cambridge, 2000)
[2] D P DiVincenzo, *Phys. Rev. A* **51**, 1015 (1995); *Science* **270**, 255 (1995)
[3] S Lloyd, *Phys. Rev. Lett.* **75**, 346 (1995)
[4] A Barenco, C H Bennet, R Cleve, D P DiVincenzo, N Margolus, P Shor, T Sleator, J A Smolin and H Weinfurter, *Phys. Rev. A* **52**, 3457 (1995)
[5] A Biswas and G S Agarwal, *Phys. Rev. A* **69**, 062306 (2004)
[6] A Kuhn, M Heinrich and G Rempe, *Phys. Rev. Lett.* **89**, 067901 (2002)
[7] A S Parkins, P Marte, P Zoller and H J Kimble, *Phys. Rev. Lett.* **71**, 3095 (1993)
[8] A S Parkins, P Marte, P Zoller, O Carnal and H J Kimble, *Phys. Rev. A* **51**, 1578 (1995)
[9] Y I Bogdanov, M V Chekhova, S P Kulik, G A Maslennikov, A A Zhukov, C H Oh and M K Tey, *Phys. Rev. Lett.* **93**, 230503 (2004)
[10] H Bechmann-Pasquinucci and A Peres, *Phys. Rev. Lett.* **85**, 3313 (2000)
[11] D Kaszlikowski *et al*, *Phys. Rev. A* **67**, 012310 (2003)
[12] R W Spekkens and T Rudolph, *Phys. Rev. A* **65**, 012310 (2001)
[13] A Mair *et al*, *Nature* **412**, 313 (2001)
[14] R T Thew *et al*, *Phys. Rev. Lett.* **93**, 010503 (2004)
[15] J C Howell, A Lamas-Linares and D Bouwmeester, *Phys. Rev. Lett.* **88**, 030401 (2002)
[16] K Tsujino *et al*, *Phys. Rev. Lett.* **92**, 153602 (2004)
[16a] If the modes a and b are excessively populated by photons, the system can become very unstable indicating the strong presence of decoherence that will deteriorate the execution of the NOT gate.
[17] R W Hamming, *Numerical methods for scientists and engineers* (Dover Publications, USA, 1962)
[18] S C Chapra, *Numerical methods for engineers* (McGraw Hill, New York, 2010)