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Decoherence of quantum excitation of even/odd coherent states in thermal environment

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Abstract. In this paper, we study the decoherence of quantum excitation (photon-added) even/odd coherent states, $((\hat{a}^\dagger)^m |\alpha_\pm\rangle)$, in a thermal environment by investigating the variation of negative part of the Wigner quasidistribution function vs. the rescaled time. For this purpose, at first we obtain the time-dependent Wigner function corresponding to the mentioned states in the framework of standard master equation. Then, the time evolution of the Wigner function associated with photon-added even/odd coherent states, as well as the number of added photons m are analysed. It is shown that, in both states, the negative part of the Wigner function decreases with time. By deriving the threshold value of the rescaled time for single photon-added even/odd coherent states, it is also found that, if the rescaled time exceeds the threshold value, the associated Wigner function becomes positive, i.e., the decoherence occurs completely.

Keywords. Photon-added even/odd coherent state; decoherence; Wigner function; nonclassical state.

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1. Introduction

The Wigner function is a quasidistribution function, because it may be negative in some regions of phase space. This negativity is a good indicator for the nonclassical behaviour of optical fields. Naturally, the negative part of the Wigner function is diminished due to the interaction of the quantum state with its surrounding environment. The negative volume of the Wigner function decreases with time, so that there exists generally a particular threshold time for any quantized field state for which the transition from quantum to classical mechanics will occur, i.e., decoherence will occur. Due to this, the Wigner function is an appropriate candidate for describing decoherence phenomenon.

On the other hand, the nonclassical optical fields have great roles in quantum information processing [1]. In this regard, using the photon addition and subtraction are interesting means to obtain nonclassical states from classical states like coherent state [2–9]. For instance, excitation (photon addition) on coherent state which is first introduced by Agarwal and Tara [2] can be easily obtained by the repeated application of creation operator ($a^\dagger$) on the coherent state. From the point of view of applicability, in addition to the manifestation of nonclassical properties, persistence of these properties also possesses particular importance. However, basically the interaction of any nonclassical radiation field with its surrounding environment also leads to the decay of nonclassical behaviour of the state. This decoherence property can be due to thermal environment (thermal channel) or due to the decay of photons to the environment (photon-loss channel). The evaluation of the time evolution of Wigner function helps one to study decoherence in both channels.

Such studies have been done by considering several quantum states of the radiation field. For instance, Dodonov *et al* [10] have studied the time evolution of the quantitative measures describing the decoherence of different initial (coherent, cat, squeezed and number) states. Furthermore, Kim and Buzek [11] discussed the influence of interaction with thermal bath on nonclassical properties of Schrödinger cat states. Dodonov *et al* have discussed the decoherence of even/odd superpositions of displaced number states, by obtaining the decoherence time by deriving the explicit form of the corresponding time-dependent Wigner function [12]. Agarwal and Biswas [13] studied the decoherence of single photon-subtracted squeezed vacuum state with the help of two different models (amplitude decay model and phase diffusion model) by finding the time evolution of the associated Wigner function. Dodonov *et al* [14] investigated the evolution of Wigner functions of arbitrary initial quantum states of field modes in a one-dimensional ideal cavity, specially the case of even and odd coherent states ('Schrödinger cat states') as the initial states. The decoherence of even and odd superpositions of displaced number states were studied using the standard master equation, describing phase insensitive attenuators and amplifiers [12]. The decoherence of superpositions of displaced coherent states $\sum_{k=1}^{N} C_k \hat{D}(\alpha_k)|g\rangle$ (where $|g\rangle$ is an arbitrary 'fiducial' state and $\hat{D}(\alpha_k)$ is the unitary displacement operator) was discussed within the framework of the standard master equation for a quantum damped or amplified harmonic oscillator interacting with a phase-insensitive (thermal) reservoir in [15]. It should be noticed that although in [15] the Wigner functions of even and odd photon-added coherent states are obtained, our approach to obtain the decoherence time (by evaluating the time-dependent Wigner function) is rather different. In addition, we discuss the decoherence properties in its different aspects by presenting complete numerical results. The details of decoherence of quantum excitation (photon-added) of even/odd coherent states in a thermal environment have been evaluated by investigating the variation of negative part of the Wigner quasidistribution function vs. the rescaled time. Serafini *et al* [16] discussed the evolution of cat-like states in general Gaussian noisy channels by considering superpositions of coherent and squeezed coherent states coupled to an arbitrarily squeezed bath.

In addition, the nonclassical nature and decoherence of photon-added coherent states were studied in [17,18]. The authors have explored the negative volume of the Wigner function of their states and found that it decreases with time. Recently, the time evolution of Wigner function of the photon-added coherent state of Agarwal and Tara [2] by using the entangled state representation approach is studied [19]. Indeed, the authors presented a new approach for deriving time evolution of the Wigner functions when a quantum system interacts with an environment (in photon-loss channel), where during the time, quantum decoherence, damping and/or amplification may happen. In this same line, Wang *et al* [20] analysed the decoherence of photon-subtracted, squeezed coherent state in thermal channel by the above-mentioned procedure.

Other interesting types of nonclassical states have been introduced. For instance, we may refer to symmetric/antisymmetric superposition of coherent states which have been called as even/odd coherent states [21]. As another generalization, we may refer to repeated applications of the creation operator on the even/odd coherent states [22]. These excited states which are intermediate between the number states and the even/odd coherent states, are called photon-added even/odd coherent states (PAE(O)CSs) [23]. It is observed that, these states have remarkable nonclassical properties in comparison with coherent state [22,23]. PAE(O)CSs are defined as

$$|\alpha_\pm, m\rangle = \mathcal{N}'_\pm \left((\hat{a}^\dagger)^m|\alpha_\pm\rangle\right) = \mathcal{N}_\pm(\hat{a}^{\dagger^m}|\alpha\rangle \pm \hat{a}^{\dagger^m}|-\alpha\rangle), \tag{1}$$

where $\hat{a}^\dagger$ is the creation operator and m is a non-negative integer. $|\alpha_+\rangle$ and $|\alpha_-\rangle$ are respectively the even and odd coherent states: $|\alpha_\pm\rangle = \mathcal{N}''_\pm(|\alpha\rangle \pm |-\alpha\rangle)$. $\mathcal{N}'_\pm$, $\mathcal{N}_\pm$ and $\mathcal{N}''_\pm$ are appropriate normalization factors. In particular, $\mathcal{N}_\pm$ which is used in the rest of the paper can be determined as follows:

$$\mathcal{N}_\pm = \sqrt{\frac{\mathrm{e}^{|\alpha|^2}}{2m!\left(L_m(-|\alpha|^2)\mathrm{e}^{|\alpha|^2} \pm L_m(|\alpha|^2)\mathrm{e}^{-|\alpha|^2}\right)}}. \tag{2}$$

As far as we know, the decoherence of states in (1) has not been discussed upto now in thermal environment. Therefore, in this contribution, we shall investigate the decoherence properties of these two classes of states in thermal environment. So, the main goal of this paper is to achieve an expression for the time-dependent Wigner function corresponding to PAE(O)CSs in (1). Then, as will be observed, as time passes the negativity of the Wigner function disappears. Finally, this function behaves similar to the Gaussian distribution which is always positive. Furthermore, we deduce a formula for the rescaled threshold time (γt_c) in terms of the mean photon number, in which the negative region of the Wigner function is fully lost and indeed, the complete decoherence occurs only after this critical time.

2. Decoherence of the PAE(O)CSs in thermal channel

In order to study the decoherence of PAE(O)CSs, as usual we evaluate the time evolution of the Wigner function. This study is mainly based on the standard master equation (the time evolution of density operator), which is described as [12]

$$\frac{\partial\hat{\rho}}{\partial t} = \left(\frac{\gamma}{N_1 - N_2}\right)N_1\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right) + \left(\frac{\gamma}{N_1 - N_2}\right)N_2\left(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger\right), \tag{3}$$

where N_1 and N_2 are respectively the number of atoms of the environment in the ground and excited states and γ has the role of the decay coefficient. It is shown that, the master equation can be equivalently transformed to a Fokker–Planck-like equation as follows [24]:

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial z}(\gamma z W) + \frac{\partial}{\partial z^*}\left(\gamma z^* W\right) + D\frac{\partial^2 W}{\partial z \partial z^*}, \tag{4}$$

where D is defined as

$$D = \frac{\gamma}{\sigma}, \quad \sigma = \frac{N_1 - N_2}{N_1 + N_2}. \tag{5}$$

Recall that, while Fokker–Planck equation deals with the evolution of the Glauber–Sudarshan P function, the above partial differential equation deals with the Wigner function. Generally, when a field state like the PAE(O)CSs interacts, particularly with thermal environment, the corresponding master equation is given by [25]

$$\begin{aligned} \frac{\partial \hat{\rho}}{\partial t} &= \gamma(\bar{n}+1)\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right) \\ &\quad + \gamma\bar{n}\left(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger\right), \end{aligned} \tag{6}$$

where $\bar{n}$ denotes the average thermal photon number of the thermal reservoir. Therefore, the time evolution of the associated Wigner function is also governed by eq. (4) by noticing that, $N_1 = \bar{n}+1$ and $N_2 = \bar{n}$. Using the thermal field dynamics and thermal entangled state representation, the form of the time-dependent Wigner function is deduced as [26]

$$\begin{aligned} W(z, \gamma t) &= \frac{2}{\pi T(2\bar{n}+1)} \int \mathrm{d}^2 z' W\left(z', 0\right) \\ &\quad \times \exp\left(-\frac{2}{T(2\bar{n}+1)}\left|z - z' \mathrm{e}^{-\gamma t}\right|^2\right), \end{aligned} \tag{7}$$

where $z = q + ip$ is a complex number, $T = 1 - \mathrm{e}^{-2\gamma t}$ and $W\left(z', 0\right)$ is the initial Wigner function of the system. To apply the above general procedure by considering our PAE(O)CSs in (1), as the first step, the Wigner functions of such states are required. These required functions are obtained as [22]

$$\begin{aligned} W_\pm(z', 0, m) &= \frac{2m!(-1)^m \mathcal{N}_\pm^2 \mathrm{e}^{2|z|^2 - |\alpha|^2}}{\pi} \\ &\quad \times \left\{L_m\left(|2z-\alpha|^2\right)\exp\left(-|2z-\alpha|^2\right)\right. \\ &\quad + L_m\left(|2z+\alpha|^2\right)\exp\left(-|2z+\alpha|^2\right) \\ &\quad \pm \left[L_m\left[(2z^*+\alpha^*)(2z-\alpha)\right]\right. \\ &\quad \times \exp\left[-(2z^*+\alpha^*)(2z-\alpha)\right]\Big] \\ &\quad \pm \left[L_m\left[(2z^*-\alpha^*)(2z+\alpha)\right]\right. \\ &\quad \left.\times \exp\left[-(2z^*-\alpha^*)(2z+\alpha)\right]\Big]\right\}. \end{aligned} \tag{8}$$

In this and all other relations which will be obtained in the rest of the paper, up sign refers to PAECSs and down sign refers to PAOCSs, and $L_m(x)$ is the Laguerre polynomial of order m. Clearly, by inserting (8) into eq. (7), the obtained expression includes four integral statements. To solve the four evolved integrals, the following method is used. For the first and second terms of eq. (8), after applying the change of variables $z'-(\alpha/2)=z_1$ and $z'+(\alpha/2)=z_2$ respectively, the integrals can be solved by using the following integral formula [12]:

$$\int \mathrm{d}^2u\ \exp\left(-g\,|u|^2+\xi u+\eta u^*\right)L_m\left(a\,|u|^2\right)$$
$$=\frac{\pi\,(g-a)^m}{g^{m+1}}\exp\left(\frac{\xi\eta}{g}\right)L_m\left(\frac{a\xi\eta}{g\,(g-a)}\right). \tag{9}$$

Similarly, the related third and fourth terms of integrals from eq. (8) can be solved by changing the variables as $z'+(\alpha/2)=z_3$ and $z'-(\alpha/2)=z_4$ respectively and by using the formula (see Appendix)

$$\int \mathrm{d}^2v\ \exp\left(-g\,|v|^2+\xi(\alpha^*,v^*)v+\eta(\alpha,v)v^*\right)L_m\left(a(|v|^2\mp\alpha v^*)\right)$$
$$=\frac{\pi\,(g-a)^m}{g^{m+1}}\exp\left(\frac{\xi\eta}{g}\right)L_m\left(\frac{a\xi\eta\left(1\mp\frac{\alpha g}{\eta}\right)}{g\,(g-a)}\right). \tag{10}$$

After straightforward but lengthy calculations, we achieve the time-dependent Wigner function for PAE(O)CSs as follows:

$$\begin{aligned}
W_\pm(z,\gamma t) = {} & \frac{Y^m m!\mathrm{e}^{-|\alpha|^2}\mathcal{N}_\pm^2}{\pi X^{m+1}} \\
& \times\left\{\exp\left(\frac{-\left|\mathrm{e}^{-\gamma t}\alpha-z\right|^2+X|\alpha|^2}{X}\right)\right. \\
& \times L_m\left(-\frac{\left|2\mathrm{e}^{-\gamma t}z+\tau\alpha\right|^2}{4XY}\right) \\
& +\exp\left(\frac{-\left|\mathrm{e}^{-\gamma t}\alpha+z\right|^2+X|\alpha|^2}{X}\right) \\
& \times L_m\left(-\frac{\left|2\mathrm{e}^{-\gamma t}z-\tau\alpha\right|^2}{4XY}\right) \\
& \pm\left[\exp\left(\frac{-|z|^2+\mathrm{e}^{-\gamma t}\alpha z^*-\mathrm{e}^{-\gamma t}\alpha^* z-\frac{\tau}{2}|\alpha|^2}{X}\right)\right. \\
& \left.\times L_m\left[\frac{(Y\alpha+\mathrm{e}^{-\gamma t}z)(Y\alpha^*-\mathrm{e}^{-\gamma t}z^*)}{XY}\right]\right] \\
& \pm\left[\exp\left(\frac{-|z|^2-\mathrm{e}^{-\gamma t}\alpha z^*+\mathrm{e}^{-\gamma t}\alpha^* z-\frac{\tau}{2}|\alpha|^2}{X}\right)\right. \\
& \left.\left.\times L_m\left[\frac{(Y\alpha-\mathrm{e}^{-\gamma t}z)(Y\alpha^*+\mathrm{e}^{-\gamma t}z^*)}{XY}\right]\right]\right\},
\end{aligned} \tag{11}$$

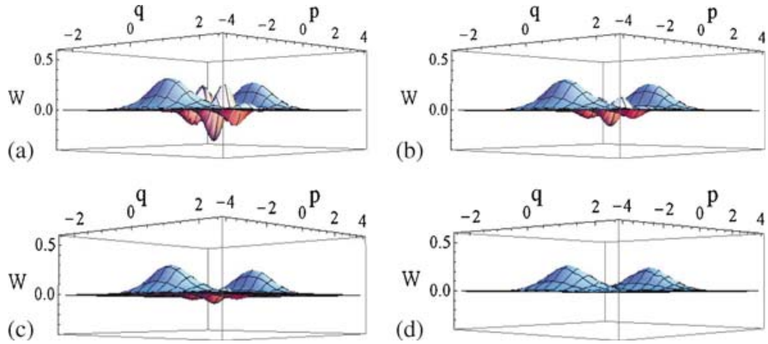

Figure 1. The time evolution of Wigner function of PAECSs in phase space with $\bar{n} = 1$, $m = 1$ and $\alpha = 2i$ for (**a**) $\gamma t = 0.01$, (**b**) $\gamma t = 0.02$, (**c**) $\gamma t = 0.03$ and (**d**) $\gamma t = 0.06$.

where the parameters X, Y and τ are defined as below:

$$X = \frac{1}{2}\left(T(2\bar{n}+1) + \mathrm{e}^{-2\gamma t}\right), \quad Y = \frac{\tau}{2},$$
$$\tau = T(2\bar{n}+1) - \mathrm{e}^{-2\gamma t}. \tag{12}$$

We have plotted the Wigner functions of PAECSs and PAOCSs in phase space with $\bar{n} = 1$, $\alpha = 2i$ and $m = 1$ for different values of rescaled time in figures 1 and 2 respectively. In general, as is observed, the volume of the negative part of the Wigner function decreases as γt increases. Furthermore, from figures 3 and 4 which are drawn with the parameters $\bar{n} = 1$, $\alpha = 2i$ and $\gamma t = 0.01$, we can see that, as m increases the negative regions in phase space of the Wigner function gradually diminish. It ought to be emphasized that, (i) our further calculations, not shown here, by selecting other values of $\bar{n}$, α, γt confirm this result and (ii) PAECSs and PAOCSs possess nonclassical feature for $m = 8$, even though in figure 3(d) it is not very clear. In fact, by presenting the curves in this figure we just want to show that as m increases, the negative part of the WF reduces.

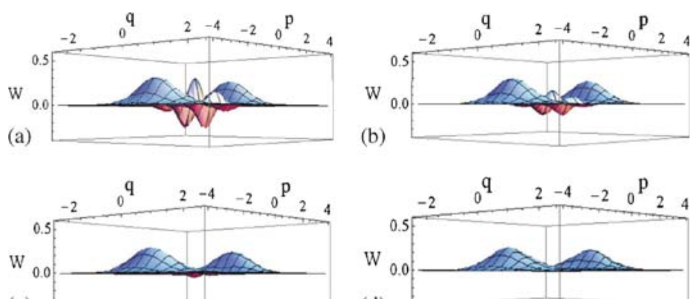

Figure 2. The time evolution of Wigner function of PAOCSs in phase space with $\bar{n} = 1$, $m = 1$ and $\alpha = 2i$ for (**a**) $\gamma t = 0.01$, (**b**) $\gamma t = 0.02$, (**c**) $\gamma t = 0.03$ and (**d**) $\gamma t = 0.06$.

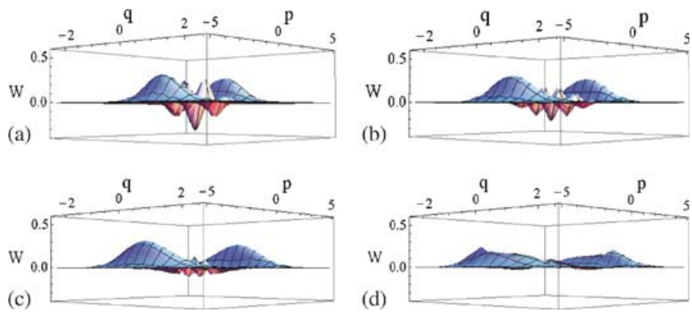

Figure 3. The Wigner function of PAECSs in phase space with $\bar{n} = 1$, $\gamma t = 0.01$ and $\alpha = 2i$ for (**a**) $m = 1$, (**b**) $m = 3$, (**c**) $m = 6$ and (**d**) $m = 8$.

3. The rescaled threshold time

In §2 we have illustrated that, as time goes by, the negativity of Wigner function will be finally lost. Now, we intend to evaluate the threshold value of rescaled time γt_c for the single PAECSs. It is remarkable to notice that, the Wigner function of single PAECSs at the origin of the phase space has its maximum negative value. At the origin $z = 0$, from eq. (11) and using $L_1(x) = 1 - x$ one has

$$\begin{aligned} W_+(0, \gamma t) = & \frac{4(T(2\bar{n}+1) - e^{-2\gamma t})e^{-|\alpha|^2}\mathcal{N}_+^2}{\pi(T(2\bar{n}+1) + e^{-2\gamma t})^2} \\ & \times \left\{ \exp\left(\frac{(T(2\bar{n}+1) - e^{-2\gamma t})|\alpha|^2}{(T(2\bar{n}+1) + e^{-2\gamma t})} \right) \right. \\ & + \left[\left(\frac{(T(2\bar{n}+1) - e^{-2\gamma t})|\alpha|^2}{T(2\bar{n}+1) + e^{-2\gamma t}} \right) \right. \\ & \left. \times \exp\left(\frac{(T(2\bar{n}+1) - e^{-2\gamma t})|\alpha|^2}{T(2\bar{n}+1) + e^{-2\gamma t}} \right) \right] \end{aligned}$$

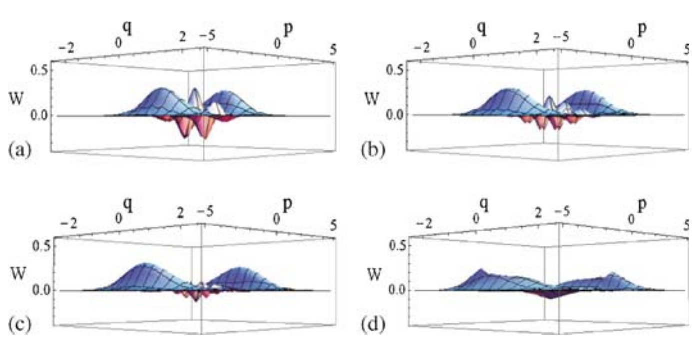

Figure 4. The Wigner function of PAOCSs in phase space with $\bar{n} = 1$, $\gamma t = 0.01$ and $\alpha = 2i$ for (**a**) $m = 1$, (**b**) $m = 3$, (**c**) $m = 6$ and (**d**) $m = 8$.

$$-\left[\left(\frac{(T(2\bar{n}+1)-e^{-2\gamma t})|\alpha|^2}{T(2\bar{n}+1)+e^{-2\gamma t}}\right)\right.$$
$$\left.\times\exp\left(\frac{-(T(2\bar{n}+1)-e^{-2\gamma t})|\alpha|^2}{T(2\bar{n}+1)+e^{-2\gamma t}}\right)\right]$$
$$\left.+\exp\left(\frac{-(T(2\bar{n}+1)-e^{-2\gamma t})|\alpha|^2}{(T(2\bar{n}+1)+e^{-2\gamma t})}\right)\right\}. \tag{13}$$

We can see that if

$$\exp\left(\frac{\tau|\alpha|^2}{T(2\bar{n}+1)+e^{-2\gamma t}}\right)-\exp\left(\frac{-\tau|\alpha|^2}{T(2\bar{n}+1)+e^{-2\gamma t}}\right)>0, \tag{14}$$

then, the Wigner function for single PAECSs has always positive value. Simplifying the last expression leads one to the following condition for the occurrence of complete decoherency:

$$\gamma t>\gamma t_c=\frac{1}{2}\ln\left(\frac{2+2\bar{n}}{1+2\bar{n}}\right). \tag{15}$$

Our further numerical calculations showed that the same value of γt_c which was obtained in eq. (15) works well for single PAOCS, too. Thus, when γt exceeds the rescaled threshold time (γt_c), Wigner function gets always positive values and decoherence occurred. In figure 5, the rescaled threshold time for single PAE(O)CSs in terms of $\bar{n}$ is plotted. An observation of this figure confirms that, by increasing the average thermal photon number, the rescaled time will be decreased. In figure 6 where we have plotted the Wigner function of PAECSs at different rescaled times with $\bar{n}=1$ and $m=1$ for different values of α vs. real z, it is clear that the negative part of Wigner function disappears after its rescaled threshold time ($\gamma t_c=0.1438$) and it becomes completely positive (this critical value may be easily obtained from eq. (15)). One can conclude that, γt_c is only dependent on $\bar{n}$; by this we intend to emphasize that it is independent of α. Similar observations can be seen in figure 7, where we have examined our presented formalism for PAOCS.

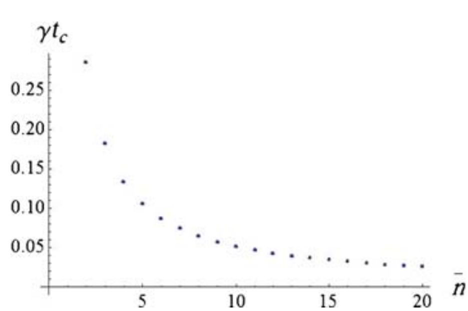

Figure 5. The rescaled threshold time γt_c vs. $\bar{n}$ for PAE(O)CSs with $m=1$.

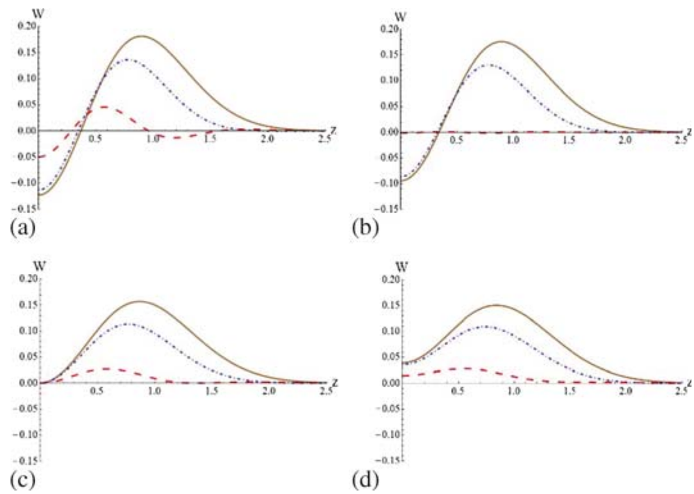

Figure 6. The Wigner function of PAECSs vs. z with $\bar{n} = 1$, $m = 1$ and $\alpha = 0.08$ (solid line), $\alpha = 0.5i$ (dot–dashed line), $\alpha = i$ (dashed line) for (**a**) $\gamma t = 0.09$, (**b**) $\gamma t = 0.1$, (**c**) $\gamma t = 0.144$ and (**d**) $\gamma t = 0.17$.

The negative volume of the Wigner function is also a useful parameter to determine the nonclassical nature of any quantum state. It is defined as follows [27]:

$$\delta = \int \mathrm{d}q\mathrm{d}p \left[|W(q, p)| - W(q, p) \right]. \tag{16}$$

If $\delta = 0$, the Wigner function is positive in the entire phase space and if $\delta > 0$ it has negative values in some regions of phase space. Thus, the positivity of δ can be used as

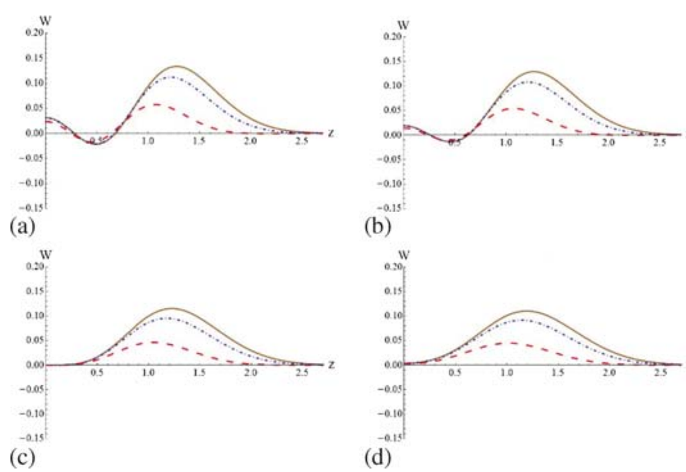

Figure 7. The Wigner function of PAOCSs vs. z with $\bar{n} = 1$, $m = 1$ and $\alpha = 0.08$ (solid line), $\alpha = 0.5i$ (dot–dashed line), $\alpha = i$ (dashed line) for (**a**) $\gamma t = 0.09$, (**b**) $\gamma t = 0.1$, (**c**) $\gamma t = 0.144$ and (**d**) $\gamma t = 0.17$.

the sufficient condition for nonclassical nature of quantum state. We evaluate the negative volume of time-dependent Wigner function of PAE(O)CSs by numerical integration as the analytical solution of eq. (16) is a very hard task, if not impossible.

In figures 8 and 9, we plotted the δ quantity for PAECSs and PAOCSs, respectively as a function of γt for different values of m with $\alpha = 0.8i$ and $\bar{n} = 1$. It is observed that, the negative volume of Wigner function decreases as γt increases so that it finally tends to zero. Also, after performing numerical variation of the δ parameter corresponding to PAECSs and PAOCSs in terms of the rescaled time for other values of m ($5 \leq m \leq 10$)

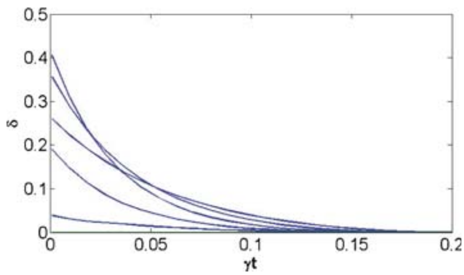

Figure 8. The evolution of negative volume of the Wigner function for PAECSs vs. γt with $\alpha = 0.8i$ and $\bar{n} = 1$ for different values of m. From top to bottom, $m = 3, 2, 1, 4, 0$ respectively.

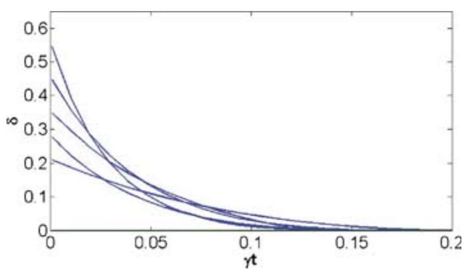

Figure 9. The evolution of negative volume of the Wigner function for PAOCSs vs. γt with $\alpha = 0.8i$ and $\bar{n} = 1$ for different values of m. From top to bottom, $m = 4, 2, 1, 3, 0$ respectively.

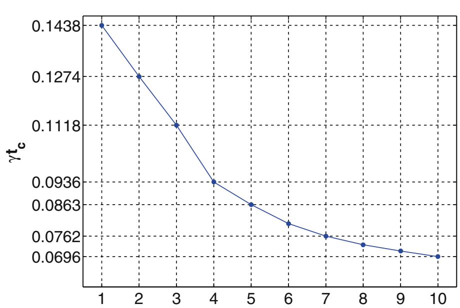

Figure 10. The rescaled threshold time of PAECSs as a function of m with $\alpha = 0.8i, \bar{n} = 1$.

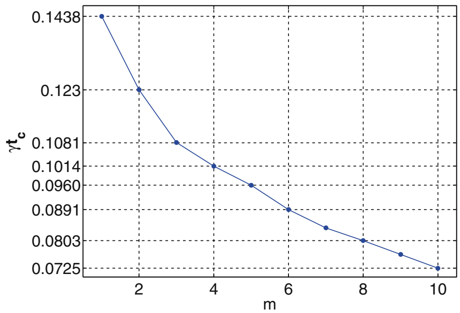

Figure 11. The rescaled threshold time of PAOCSs as a function of m with $\alpha = 0.8i, \bar{n} = 1$.

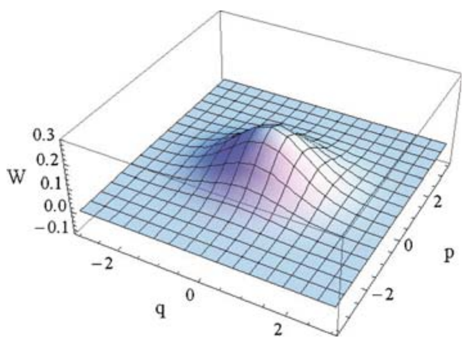

Figure 12. The Wigner function of PAE(O)CSs in the phase space with $\bar{n} = 1$ after a long time.

(like figures 8 and 9 for $1 \leq m \leq 4$), we could determine the rescaled threshold time γt_c for both classes of the considered states and any value of $m \leq 10$. Thus, we plotted the rescaled threshold time for PAECSs and PAOCSs as a function of discrete values of m in figures 10 and 11, respectively. From these two figures (figures 10, 11) one can observe that for both states, either PAECSs or PAOCSs, larger values of m corresponds to smaller values of rescaled threshold time γt_c.

4. The variation of the Wigner function after a long time

After a long time, i.e., $\gamma t \to \infty$, one simply obtains $T \to 1$, $X = Y = \frac{1}{2}(2\bar{n} + 1)$. In such a limited condition, the Wigner function of PAE(O)CSs in eq. (11) gets the following form:

$$W_{\pm}(z, \infty) = \frac{2}{\pi (2\bar{n}+1) \left(L_m\left(-|\alpha|^2\right) \mathrm{e}^{|\alpha|^2} \pm L_m(|\alpha|^2)\, \mathrm{e}^{-|\alpha|^2}\right)} \times \left\{ \exp\left(\frac{-2|z|^2 + (2\bar{n}+1)|\alpha|^2}{(2\bar{n}+1)}\right) L_m\left(-|\alpha|^2\right) \pm \exp\left(\frac{-2|z|^2 - (2\bar{n}+1)|\alpha|^2}{(2\bar{n}+1)}\right) L_m\left(|\alpha|^2\right) \right\}. \tag{17}$$

In other words, in the interaction with the environment after a long time, based on eq. (17), the Wigner function becomes similar to a (always positive) Gaussian distribution which is a completely classical distribution. We have shown this fact in figure 12, for the chosen parameter: $\bar{n} = 1$ (as may be easily verified from the above equation, $W_{+}(z, \infty) = W_{-}(z, \infty)$).

5. Summary and conclusion

In this paper, due to the importance of stability of nonclassical features of optical fields, we have studied the decoherence of PAE(O)CSs in thermal environment. To achieve this goal, we have used the standard master equation, by which the time-dependent Wigner functions of the considered states are obtained. It is observed that, the negative part of the Wigner function decreases due to the long-lasting interaction with environment. In particular, for single PAE(O)CSs ($m = 1$), we have derived a relation for the evaluation of the rescaled threshold time. It is shown that, this critical time depends on the average photon number of the thermal environment. Indeed, at higher temperature or by increasing the average thermal photon number, the negative part of the Wigner function rapidly disappeared and consequently, decoherence happened. Decoherence properties of PAE(O)CSs in photon-loss channel have been analysed in [28].

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Appendix A

Proof of eq. (10). If we denote the LHS of eq. (10) by I_m and use the well-known formula for the Laguerre polynomials ($|w| < 1$) [29]

$$\sum_{m=0}^{\infty} L_m(x) w^m = \frac{1}{1-w} \exp\left(\frac{xw}{w-1}\right), \tag{A.1}$$

then, we have

$$\sum_{m=0}^{\infty} I_m w^m = \frac{1}{1-w} \times \int \mathrm{d}^2 v \exp\left(\frac{g+(a-g)w}{w-1}\,|v|^2 + \xi v + \frac{\eta(w-1) \mp a\alpha w}{w-1} v^*\right). \tag{A.2}$$

Letting $v = x + iy$, the above equation can be rewritten as

$$\begin{aligned} I(w) &= \frac{1}{1-w}\int \mathrm{d}x\,\mathrm{d}y \exp\left[G(x^2+y^2)+\xi(x+iy)+\eta'(x-iy)\right] \\ &= \frac{1}{1-w}\int \mathrm{d}x\,\mathrm{d}y \exp\left[G(x^2+y^2)+(\xi+\eta')x+(i\xi-i\eta')y\right], \end{aligned} \tag{A.3}$$

where we have set

$$G = \frac{g+(a-g)w}{w-1}, \quad \eta' = \frac{\eta(w-1)\mp a\alpha w}{w-1}. \tag{A.4}$$

Consequently, by defining $c \equiv \xi + \eta'$, $d \equiv i(\xi - \eta')$,

$$\begin{aligned} I(w) &= \frac{1}{1-w}\int \mathrm{d}x\,\mathrm{d}y\ \exp\left[G\left(x+\frac{c}{2G}\right)^2 - \frac{c^2}{4G}\right] \\ &\quad \times \exp\left[G\left(y+\frac{d}{2G}\right)^2 - \frac{d^2}{4G}\right]. \end{aligned} \tag{A.5}$$

According to the Gaussian integral $\int \mathrm{d}x' \exp[-a'(x'+b')^2] = \sqrt{\pi/a'}$,

$$I(w) = \frac{\pi}{(a-g)w+g}\exp\left[\frac{\xi\eta(w-1\mp\frac{a\alpha w}{\eta})(1-w)}{((a-g)w+g)\,(w-1)}\right]. \tag{A.6}$$

After replacing $[w(g-a)/g] \to y$ and using eq. (A.1), we can arrive at

$$I(y) \equiv \sum_{m=0}^{\infty} I_m y^m = \frac{\pi}{g}\exp\left(\frac{\xi\eta}{g}\right)\sum_{m=0}^{\infty} L_m\left(\frac{\xi\eta(a\mp\frac{a\alpha g}{\eta})}{g(g-a)}\right)y^m. \tag{A.7}$$

Now, if one replaces $y \to [w(g-a)/g]$, then the result reads as

$$I(w) \equiv \sum_{m-0}^{\infty} I_m w^m = \frac{\pi}{g}\exp\left(\frac{\xi\eta}{g}\right)\sum_{m=0}^{\infty} L_m\left(\frac{\xi\eta(a\mp\frac{a\alpha g}{\eta})}{g(g-a)}\right)\frac{(g-a)^m}{g^m}w^m. \tag{A.8}$$

Consequently, by simplifying (A.6) one can readily arrive at eq. (10) of this paper.

References

[1] D Bouwmeester, A Ekert and A Zeilinger, *The physics of quantum information* (Springer, Berlin, 2000)
[2] G S Agarwal and K Tara, *Phys. Rev. A* **43**, 492 (1991)
[3] A Zavatta, S Viciani and M Bellini, *Science* **306**, 660 (2004)
[4] Z Wang, H C Yuan and H Y Fan, *J. Opt. Soc. Am. B* **28**, 1964 (2011)
[5] A Zavatta, V Parigi and M Bellini, *Phys. Rev. A* **75**, 052106 (2007)
[6] L Y Hu and H Y Fan, *J. Opt. Soc. Am. B* **25**, 1955 (2008)
[7] P Marek, H Jeong and M S Kim, *Phys. Rev. A* **78**, 063811 (2008)

[8] J Lee, J Kim and H Nha, *J. Opt. Soc. Am. B* **26**, 1363 (2009)
[9] O Safaeian and M K Tavassoly, *J. Phys. A: Math. Theor.* **44**, 225301 (2011)
[10] V V Dodonov, S S Mizrahi and A L de Souza Silva, *J. Opt. B: Quantum Semiclass. Opt.* **2**, 271 (2000)
[11] M S Kim and V Bužek, *Phys. Rev. A* **46**, 4239 (1992)
[12] V V Dodonov and L A de Souza, *J. Opt. B: Quantum Semiclass. Opt.* **7**, S490 (2005)
[13] A Biswas and G S Agarwal, *Phys. Rev. A* **75**, 032104 (2007)
[14] V V Dodonov, M A Andreata and S S Mizrahi, *J. Opt. B: Quantum Semiclass. Opt.* **7**, S468 (2005)
[15] V V Dodonov and L A de Souza, *J. Phys. A: Math. Theor.* **40**, 13955 (2007)
V V Dodonov and L A de Souza. *J. Russ. Laser Res.* **28**, 453 (2007)
[16] A Serafini, S De Siena, F Illuminati and M G A Paris, *J. Opt. B: Quantum Semiclass. Opt.* **6**, S591 (2004)
[17] S B Li, X B Zou and G C Guo, *Phys. Rev. A* **75**, 045801 (2007)
[18] S B Li, J Liu, X B Zou *et al*, *J. Opt. Soc. Am. B* **25**, 54 (2008)
[19] Li-Yun Hu and Hong-Yi Fan, *Phys. Scr.* **79**, 035004 (2009)
[20] Z Wang, X G Meng and H Y Fan, *J. Opt. Soc. Am. B* **29**, 397 (2012)
[21] V V Dodonov, I A Malkin and V I Man'ko, *Physica* **72**, 597 (1974)
[22] Z Z Xin, Y B Duan, W Zhang *et al*, *J. Phys. B: At. Mol. Opt. Phys.* **29**, 2597 (1996)
[23] V V Dodonov, Ya A Korennoy, V I Man'ko *et al*, *Quantum Semiclass. Opt.* **8**, 413 (1996)
[24] D F Walls and G J Milburn, *Quantum optics* (Springer, Berlin, 1995) 108
[25] C Gardiner and P Zoller, *Quantum noise* (Springer, Berlin), 2000) 15
[26] H Y Fan and L Y Hu, *Opt. Commun.* **282**, 4379 (2009)
[27] A Kenfack and K Zyczkowski, *J. Opt. B: Quantum semiclass. Opt.* **6**, 396 (2004)
[28] A Mohammadbeigi and M K Tavassoly, *J. Phys. B: At. Mol. Opt. Phys.* **47**, 165502 (2014)
[29] I S Gradshteyn and I M Ryzhik, *Table of integrals, series, and products* edited by A Jeffrey and D Zwillinger (Academic Press is an imprint of Elsevier, 2007) 1002