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Rational homoclinic solution and rogue wave solution for the coupled long-wave–short-wave system

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Abstract. In this paper, a rational homoclinic solution is obtained via the classical homoclinic solution for the coupled long-wave–short-wave system. Based on the structures of ratinal homoclinic solution, the characteristics of homoclinic solution are discussed which might provide us with useful information on the dynamics of the relevant physical fields.

Keywords. Coupled long-wave–short-wave system; homoclinic breather wave; rogue wave; homoclinic test.

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1. Introduction

For the past five years, rogue waves (also known as freak waves, monster waves, killer waves, rabid-dog waves etc.) have become a hot topic in many research fields [1–4]. Rogue waves were first observed in deep ocean [5]. A wave can be called a rogue wave when its height and steepness is much greater than the average crest, and appears from nowhere and disappears without a trace [6]. Rogue waves are the subject of intense research in fields of oceanography [7,8], optical fibres [9,10], superfluids [11], Bose–Einstein condensates and related fields [12,13]. The first-order rational solution of the self-focussing nonlinear Schrödinger equation (NLS) was first introduced by Peregrine [3] to describe the rogue wave phenomenon. Recently, by using the Darboux dressing technique or Hirotas bilinear method, rogue wave solutions were reported in some complex systems [13–17]. In the present work, we use a homoclinic (heteroclinic) breather wave limit method for getting rogue wave solution to the coupled long-wave–short-wave system.

Now we consider the coupled long-wave–short-wave system

$$\begin{cases} iu_t - 2u_{xx} + 2u(f - w) = 0, \\ iv_t - 2v_{xx} + 2v(f - w) = 0, \\ f_t - 4(uv^* + u^*v)_x = 0, \end{cases} \tag{1}$$

where $u(x,t)$ and $v(x,t)$ are the orthogonal components of the envelope of a rapidly varying complex field (the short wave) representing a transverse wave whose group velocity resonates with the phase velocity of a real field $f(x,t)$ (the long wave) representing a longitudinal wave. The $*$ denotes complex conjugation and ω is an arbitrary real constant. The CLS equations (1) generalize the scalar long-wave–short wave resonance equations derived by Djordjevic and Redekopp [18] for long-wave–short wave interactions when the more generic nonlinear Schrödinger equation breaks down due to a singularity in the coefficient of the cubic nonlinearity. The dispersion of the short wave is balanced by the nonlinear interaction of the long wave, while the self-interaction of the short wave drives the evolution of the long wave. The CLS equation (1) is integrable and has been studied extensively in various aspects [19,20]. Recently, rogue wave and breather solutions were constructed for the uncoupled and coupled long-wave–short-wave system by using the two-soliton method [21,22].

2. Hirota's bilinear form and rogue wave solutions

In this section, we concentrate on heteroclinic breather wave solutions for coupled long-wave–short-wave system by a suitable ansätz approach. Let $b \to b + w$. The system (1) can be written as

$$\begin{cases} iu_t - 2u_{xx} + 2uf = 0, \\ iv_t - 2v_{xx} + 2vf = 0, \\ f_t - 4(uv^* + u^*v)_x = 0. \end{cases} \tag{2}$$

We now make the dependent variable transformation

$$u = \frac{G}{F}, \quad v = \frac{H}{F}, \quad f = A - 2(\ln F)_{xx}, \tag{3}$$

where G, H are complex-valued functions and F is a real-valued function. Then, system (2) can be rewritten as the following coupled bilinear differential equations for F, G and H:

$$\begin{cases} (iD_t - 2D_x^2)(G \cdot F) + 2A(G \cdot F) = 0, \\ (iD_t - 2D_x^2)(H \cdot F) + 2A(H \cdot F) = 0, \\ (D_t D_x - c)(F \cdot F) + 4(G \cdot H^* + G^* \cdot H) = 0, \end{cases} \tag{4}$$

where C is the integration constant. H^* is the conjugate function of H, G^* is the conjugate function of G. Using the extended homoclinic test approach [21] we suppose

$$\begin{cases} G = \mathrm{e}^{-iat}[\mathrm{e}^{-p_1(x-\alpha t)} + b_1 \cos(p(x + \alpha t)) + b_2 \mathrm{e}^{p_1(x-\alpha t)}], \\ H = \mathrm{e}^{-iat}[\mathrm{e}^{-p_1(x-\alpha t)} + b_3 \cos(p(x + \alpha t)) + b_4 \mathrm{e}^{p_1(x-\alpha t)}], \\ F = \mathrm{e}^{-p_1(x-\alpha t)} + b_5 \cos(p(x + \alpha t)) + b_6 \mathrm{e}^{p_1(x-\alpha t)}, \end{cases} \tag{5}$$

where a, α, p, p_1, b_5 and b_6 are real and b_1, b_2, b_3, b_4 are complex. Substituting eq. (5) into eq. (4), we can get a set of algebraic equations of $e^{p_1(x-\alpha t)}$. Then, equating the coefficients of all powers of $e^{jp_1(x-\alpha t)}$, $j = -2, -1, 0, 1, 2$ to zero yields the relations among $A, a, p, p_1, b_1, b_2, b_3, b_4, b_5, b_6$ as follows:

$$a = -2A, \quad c = 8, \quad p_1^2 = \frac{p^2}{3} = \frac{\alpha^3 + 64}{-16\alpha}, \quad b_1 = \frac{3i\alpha + 4\sqrt{3}p}{3i\alpha - 4\sqrt{3}p} b_5,$$

$$b_2 = \left(\frac{3i\alpha + 4\sqrt{3}p}{3i\alpha - 4\sqrt{3}p}\right)^2 b_6, \quad b_5^2 = \frac{-4(-9\alpha^2 + 16p^2)}{3(3\alpha^2 + 16p^2)} b_6, \tag{6}$$

where $\alpha < -4$. Substituting eq. (6) into eq. (3) and taking $b_6 = 1$, we obtain two families of solutions of eq. (2):

$$\begin{cases} u_1 = v_1 = \dfrac{e^{-iat}(2\sqrt{c^2}\cosh(\frac{p\xi}{\sqrt{3}} - \frac{1}{2}\ln(c^2)) + B\cos(p\eta))}{2\cosh(\frac{p\xi}{\sqrt{3}}) + D\cos(p\eta)}, \\ f_1 = -\frac{a}{2} - \dfrac{2p^2[\frac{4}{3} - D^2 - \frac{4D}{3}\cosh(\frac{p\xi}{\sqrt{3}})\cos(p\eta) + \frac{4D}{\sqrt{3}}\sinh(\frac{p\xi}{\sqrt{3}})\sin(p\eta)]}{[2\cosh(\frac{p\xi}{\sqrt{3}}) + D\cos(p\eta)]^2}, \end{cases} \tag{7}$$

$$\begin{cases} u_2 = v_2 = \dfrac{e^{-iat}(2\sqrt{c^2}\cosh(\frac{p\xi}{\sqrt{3}} - \frac{1}{2}\ln(c^2)) - B\cos(p\eta))}{2\cosh(\frac{p\xi}{\sqrt{3}}) - D\cos(p\eta)}, \\ f_2 = -\frac{a}{2} - \dfrac{2p^2[\frac{4}{3} - D^2 + \frac{4D}{3}\cosh(\frac{p\xi}{\sqrt{3}})\cos(p\eta) - \frac{4D}{\sqrt{3}}\sinh(\frac{p\xi}{\sqrt{3}})\sin(p\eta)]}{[2\cosh(\frac{p\xi}{\sqrt{3}}) - D\cos(p\eta)]^2}, \end{cases} \tag{8}$$

where

$$B = CD, \quad C = \frac{3i\alpha + 4\sqrt{3}p}{3i\alpha - 4\sqrt{3}p}, \quad D = \sqrt{\frac{-4(-9\alpha^2 + 16p^2)}{3(3\alpha^2 + 16p^2)}},$$

$$\xi = x - \alpha t, \quad \eta = x + \alpha t. \tag{9}$$

Here we have used $p_1 = p/\sqrt{3}$. Notice that η and ξ are the vertical relations.

The solution represented by (u_1, v_1, f_1) (respectively, (u_2, v_2, f_2)) of eq. (1) is homoclinic breather solution, which is a homoclinic wave to a fixed cycle $(e^{-iat}, e^{-iat}, -a/2)$ when $t \to \infty$ and $(e^{-iat+2i\theta}, e^{-iat+2i\theta}, -a/2)$ when $t \to -\infty$ of eq. (1) and meanwhile contain a periodic wave $\cos(p(x + \alpha t))$ whose amplitude periodically oscillates with the evolution of time. Now we consider the limit behaviours of p as the period $2\pi/p$ of the periodic wave $\cos(p(x + \alpha t))$ goes to infinity, i.e. $p \to 0$. Note

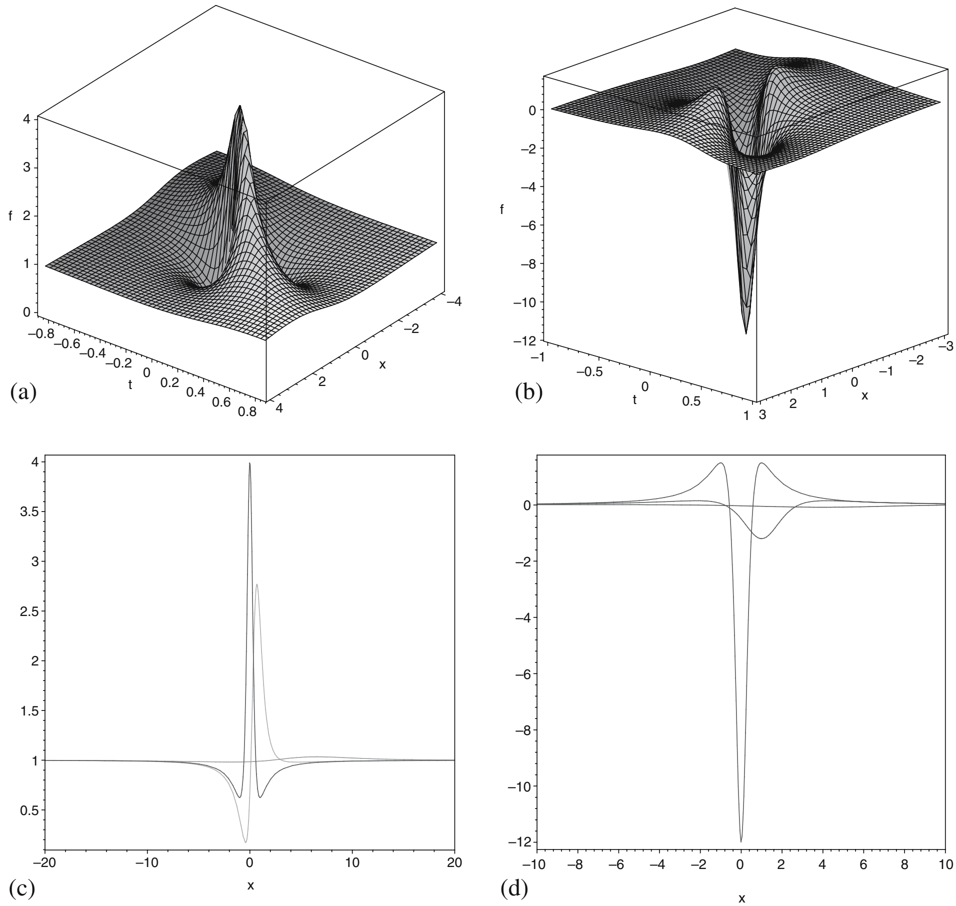

Figure 1a, b. Spatial–temporal structures of rational homoclinic wave: (**a**) u_3 wave and (**b**) f_3 wave with $a = 0, \alpha = -4$. (**c**) u_3 wave for time $t = 0$, 0.2, 2 and (**d**) f_3 wave for time $t = 0$, 0.5, 2.

that (u_2, v_2, f_2) is 0/0 type as $p \to 0$, and by computing, we obtain the following result:

$$\begin{cases} u_3 = v_3 = \dfrac{e^{-iat}(3x^2 - 12xt + 48t^2 - 2 + 3ix + 12it)}{3x^2 - 12xt + 48t^2 + 1}, \\ f_3 = -\dfrac{a}{2} + 12\,\dfrac{3x^2 - 12xt - 24t^2 - 1}{(3x^2 - 12xt + 48t^2 + 1)^2}. \end{cases} \tag{10}$$

It is easy to verify that (u_3, v_3, f_3) is a solution of eq. (1). Moreover, (u_3, v_3, f_3) is also the homoclinic solution homoclinic to the fixed cycle $(e^{-iat}, e^{-iat}, -a/2)$ as $t \to \infty$ for fixed x. This shows (u_3, v_3, f_3) is just a homoclinic rogue wave solution of the coupled long-wave–short-wave system. The maximum amplitude of the rogue wave solution f_3 occurs at point $(t = 0, x = 0)$ and the maximum amplitude of this rogue wave solution is equal to $-\frac{a}{2} + 12$. The minimum amplitude of f_3 occurs at two points $(t = 0, x = \pm 1)$, and the minimum amplitude of this rogue wave solution is equal to $-\frac{a}{2}-\frac{3}{2}$. The maximum amplitude of the rogue wave solution u_3 occurs at point $(t = 0,\ x = 0)$ and the maximum amplitude of this rogue wave solution is equal to 4. The minimum amplitude of u_3 occurs at point $(t = -\sqrt{2}/12, x = \sqrt{2}/3)$, and the minimum amplitude of this rogue wave solution is equal to 0 (figure 1).

3. Conclusion

By applying homoclinic (heteroclinic) breather wave limit method to the coupled long-wave–short-wave system we obtained a family of homoclinic breather solutions and the rational homoclinic solution. Rational homoclinic solution obtained here is just a homoclinic rogue wave solution. In future, we intend to study the interaction between the rational breather wave and the solitary wave.

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