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Evolution of holographic dark energy with interaction term $Q \propto H\rho_{\rm de}$ and generalized second law

P PRASEETHA* and TITUS K MATHEW

Department of Physics, Cochin University of Science and Technology, Cochin 682 022, India
*Corresponding author. E-mail: praseethapankunni@cusat.ac.in

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Abstract. A flat FLRW Universe with dark matter and dark energy, which are interacting with each other, is considered. The dark energy is represented by the holographic dark energy model and the interaction term is taken as proportional to the dark energy density. We have studied the cosmological evolution and analysed the validity of the generalized second law of thermodynamics (GSL) under thermal equilibrium conditions and non-equilibrium conditions. We have found that the GSL is completely valid at the apparent horizon but violated at the event horizon under thermal equilibrium condition. Under thermal non-equilibrium condition, for the GSL to be valid, we found out that the temperature of the dark energy must be greater than the temperature of the apparent horizon if the dark energy behaves as a quintessence fluid.

Keywords. Dark energy; generalized second law; cosmological evolution; thermodynamics.

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1. Introduction

The accelerated expansion of the Universe is proved by the observations of type-Ia Supernovae [1,2]. Observations of cosmic microwave background radiation (CMB) [3] and investigation of large-scale structures [4] have confirmed this discovery. The exotic form of energy characterized by the negative pressure causing the accelerated expansion is termed as dark energy. Various models were introduced to study the exact nature and evolution of dark energy which include cosmological constant model [5], dynamical dark energy models such as quintessence model [6,7], phantom dark energy model [8,9], holographic model [10–14] etc. Despite the wide acceptability in explaining dark energy, cosmological constant model is unable to explain the cosmic coincidence of dark matter and dark energy densities and also faces the problem of fine tuning. Among the numerous models, holographic dark energy model is one which is said to give a more satisfactory explanation of the nature of dark energy. Holographic model of dark energy was formulated based on the holographic principle [15], which says that the degrees of freedom of a system is proportional not to its volume, but to its surface area enclosing the volume. The principle implies that the total energy inside a region of size L, should not exceed the mass of a black hole of the same size. From effective quantum field theory, an effective IR cut-off can saturate the length scale, so that the dark energy density can be written in the form,

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}, \tag{1}$$

where c is a dimensionless numerical factor and M_p is the Planck mass. The size L can be taken as the size of particle horizon, Hubble horizon, future event horizon etc. The first two options will not give an accelerated expansion of the Universe while the third is faced with the causality problem [16]. A better candidate proposed in recent literature is the Ricci scalar as the IR cut-off, which gives the holographic Ricci dark energy model. A modified form of the holographic Ricci dark energy model was proposed and studied further in refs [17–19]. In this paper, we have considered that the holographic dark energy interacts with the dark matter.

To study the status of the GSL, one needs to obtain the entropies of both the horizon and the fluid inside the horizon. It was Gibbons and Hawking who first studied the thermodynamics of cosmological horizon in the de-Sitter model [20]. Their study was motivated by the result of Bekenstein and Hawking that showed the entropy of the black hole is proportional to the area of its event horizon. The entropy of the fluid within the Universe can be calculated by using the Gibb's rule. Having known the entropies of both the horizon and the fluid within it, one can formulate the GSL as the entropy of the horizon plus that of the fluid inside the horizon will never decrease as the Universe expands [21–24].

Various studies are done to find out the validity of GSL both at the apparent horizon and event horizon in various cosmological models with varying components of fluids. Most of the results in this respect agree that the GSL is valid at the apparent horizon but is violated at the event horizon [25]. In this paper, we are curbing the validity of the GSL in the interacting holographic dark energy scenario following our works in [26,27]. The paper is organized as follows: in §2, we describe our model and its cosmological evolution. In §3, we describe the validity of the GSL under thermal equilibrium conditions and in §4, we describe the validity of GSL under thermal non-equilibrium conditions, followed by conclusions in §5.

2. Interacting holographic Ricci dark energy model

Flat FLRW Universe is described by the Friedmann equation

$$3H^2 = \rho_m + \rho_{de}, \tag{2}$$

where we standardize the equation with $8\pi G = 1$; H is the Hubble parameter, ρ_{de} is the energy density of the holographic dark energy and ρ_m is the energy density of the dark matter. The dark energy is being represented by holographic dark energy with Ricci scalar as the IR cut-off, which is given as

$$\rho_{de} = \frac{2(\dot{H} + \frac{3}{2}\alpha H^2)}{\Delta}, \tag{3}$$

where $\dot{H}$ denotes the derivative of H with respect to the cosmic time t and $\Delta = \alpha - \beta$ where α and β are parameters of the model. The conservation equations describing the interaction between the dark energy and the dark matter in this model becomes

$$\dot{\rho}_{\rm de} + 3H(\rho_{\rm de} + p_{\rm de}) = -Q, \tag{4}$$

$$\dot{\rho}_{\rm m} + 3H(\rho_{\rm m}) = Q, \tag{5}$$

which together satisfies the standard conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{6}$$

where Q is the interaction term. $Q > 0$ implies the energy transfer from dark energy to dark matter and $Q < 0$ implies the energy transfer from dark matter to dark energy, while the dot represents the derivative with respect to time. Dark matter is pressureless. The conservation equations imply that the interaction term should be a function of a quantity with units of inverse of time (the first and natural choice can be the Hubble factor H) multiplied with the energy density. Thus, Q mainly can have any of the following phenomenological forms: $Q = 3bH(\rho_{r\rm de} + \rho_{\rm m})$, $Q = 3bH\rho_{\rm m}$ and $Q = 3bH\rho_{\rm de}$ [18,28]. For our present study we consider the form, $Q = 3bH\rho_{\rm de}$.

Using the equation for dark energy density, eq. (3), the Friedmann equation (2) can be recast into a second-order differential equation with respect to the variable $x = \log a$ as

$$\frac{{\rm d}^2h^2}{{\rm d}x^2} + 3(\beta + b + 1)\frac{{\rm d}h^2}{{\rm d}x} + 9(\alpha b + \beta)h^2 = 0, \tag{7}$$

where $h = H/H_0$, H_0 is the present value of Hubble parameter. The analytical solution of this equation is obtained as [26],

$$h^2 = f_1{\rm e}^{(u_1/2)x} + f_2{\rm e}^{(u_2/2)x}, \tag{8}$$

where the constants are evaluated using the initial conditions, $h^2|_{x=0} = 1$ and $\frac{{\rm d}h^2}{{\rm d}x}|_{x=0} = 3\Omega\Delta_{\rm de0} - 3\alpha$, as

$$f_1 = \frac{[3 - 6\alpha + 3b + 3\beta - \sqrt{-36(\alpha b + \beta) + 9(1 + b + \beta)^2} + 6(\alpha - \beta)\Omega_{\rm de0}]}{[2b - 2\sqrt{-36(\alpha b + \beta) + 9(1 + b + \beta)^2}]}, \tag{9}$$

$$f_2 = 1 - f_1$$

and

$$\begin{aligned} u_1 &= -3 - 3b - 3\beta - \sqrt{9(1 + b + \beta)^2 - 36(\alpha b + \beta)}, \\ u_2 &= -3 - 3b - 3\beta + \sqrt{9(1 + b + \beta)^2 - 36(\alpha b + \beta)}. \end{aligned} \tag{10}$$

Using the Friedmann equation, the dark energy density parameter can be obtained as

$$\Omega_{\rm de} = f_1{\rm e}^{(u_1/2)x} + f_2{\rm e}^{(u_2/2)x} - \Omega_{\rm m0}{\rm e}^{-3x} \tag{11}$$

and the equation of state parameter is obtained as

$$\omega_{\rm de} = -1 - \left(\frac{f_1\frac{u_1}{2}{\rm e}^{(u_1/2)x} + f_2\frac{u_2}{2}{\rm e}^{(u_2/2)x} + 3\Omega_{\rm m0}{\rm e}^{-3x}}{3\left(f_1{\rm e}^{(u_1/2)x} + f_2{\rm e}^{(u_2/2)x} - \Omega_{\rm m0}{\rm e}^{-3x}\right)}\right). \tag{12}$$

The evolution of the equation of state parameter for the holographic dark energy is plotted against red-shift z for the model parameters $(\alpha, \beta) = (1.01, -0.01)$, and the plot is shown as an example in figure 1. The equation of state is evolving from zero in the past phase of the Universe to negative values in the later phase. The present value of the equation of state parameter ω_{de0} for the model parameters $(\alpha, \beta) = (1.01, -0.01)$ is -0.97 and this is very close to the WMAP value $\omega_{\text{de0}} = -0.93$ [29]. A similar kind of work is shown in ref. [25], where the authors have studied the interacting holographic dark energy with event horizon as the IR cut-off. They found that as the Universe evolves, the equation of state parameter increases from values below -1 in the past and stabilizes near to -1 in the future for interaction coupling constant, $b^2 = 0.08$, while the equation of state parameter of the present model decreases from zero in the past and stabilizes at values greater than -1, for parameter $\beta > 0$ and tends to values less than -1, for $\beta < 0$. This shows that in the present model there is a transition from deceleration to acceleration as the Universe evolves. This can be further verified by studying the deceleration parameter.

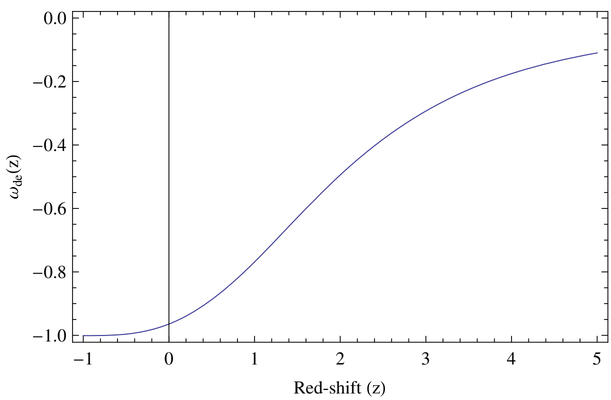

Figure 1. The evolution of the equation of state parameter for $(\alpha, \beta) = (1.01, -0.01)$ with coupling constant $b = 0.009$.

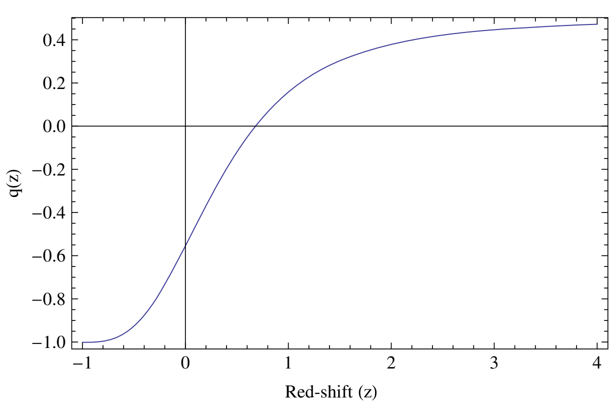

Figure 2. The study of deceleration parameter for $(\alpha, \beta) = (1.01, -0.01)$ with coupling constant $b = 0.009$.

The deceleration parameter q can be obtained as [26]

$$q = -1 - \frac{1}{2}\left[\frac{\frac{u_1}{2} f_1 e^{(u_1/2)x} + \frac{u_2}{2} f_2 e^{(u_2/2)x}}{f_1 e^{(u_1/2)x} + f_2 e^{(u_2/2)x}}\right]. \tag{13}$$

The evolution of the deceleration parameter is studied by plotting q against the red-shift z and the plot is shown in figure 2. The present value of q for $(\alpha, \beta) = (1.01, -0.01)$ from the plot is $q_0 = -0.57$, which is in the range of observational value [29]. The transition red-shift, the red-shift at which the Universe enters the accelerating expansion phase is found to be $z_{\mathrm{T}} = 0.70$ which is in compliance with the observed data [29].

3. GSL at thermal equilibrium

This section studies the validity of the generalized second law of thermodynamics of the holographic dark energy, in a flat FLRW Universe, under thermal equilibrium condition. The GSL restricts the sum of the change in entropy of the dark sectors added with the change in entropy of the cosmological horizon to be greater than zero [30–32]. That is,

$$\dot{S} + \dot{S}_{\mathrm{h}} \geq 0, \tag{14}$$

where $\dot{S}$ denotes the change in entropy of dark sectors with respect to cosmic time and $\dot{S}_{\mathrm{h}}$ is the change in horizon entropy with respect to cosmic time. Thermal equilibrium condition is that state where the temperatures of the dark sectors and the horizon is said to be the same due to the mutual interactions between the dark sectors. One can argue that, as both the entities inside the horizon themselves are the integral parts of the Universe, it is likely to have an interaction between them, and consequently the dark sectors will soon reach a state with common temperature. We are taking both the apparent horizon and event horizon as separate cases for our study which are dealt with in the following subsections.

3.1 *Validity of GSL at apparent horizon*

Validity of the GSL at the apparent horizon implies that the sum of the changes in the entropy of the dark energy together with that of the dark matter and the apparent horizon must increase with time. Karami *et al* [33,34] have also studied the same case for a different dark energy form. Consider the expression for the apparent horizon distance for a flat FLRW Universe,

$$r_{\mathrm{h}} = H^{-1}. \tag{15}$$

The temperature of the apparent horizon $T_{\mathrm{h}} = 1/2\pi r_{\mathrm{h}}$ [35] is given by

$$T_{\mathrm{h}} = \frac{H}{2\pi}. \tag{16}$$

The temperature of the dark entities T must be equal to that of the horizon temperature T_{h}, under local thermal equilibrium [36]. Otherwise the FLRW space geometry will be deformed due to the energy flow between the horizon and the fluid within it [37,38]. The area of the horizon is given by $A = 4\pi r_{\mathrm{h}}^2$ and the entropy of the horizon can be written in terms of its area as $S = A/4G$. Hence the entropy is obtained as

$$S_{\mathrm{h}} = \frac{8\pi^2}{H^2}. \tag{17}$$

The entropy of the dark sectors can be obtained using the Gibb's equation,

$$T\mathrm{d}S = \mathrm{d}E + P\mathrm{d}V, \tag{18}$$

where $V = \frac{4}{3}\pi r_{\mathrm{h}}^3$ is the volume, $E = \frac{4}{3}\pi r_{\mathrm{h}}^3(\rho_{\mathrm{de}} + \rho_{\mathrm{m}})$ and P is the pressure of the dark component. However, dark matter is pressureless. Using the above equations and the conservation equation given in the previous section, the total entropy variation, comprising of the dark sectors and the apparent horizon, is obtained as

$$S' = \frac{16\pi^2}{H^2} + \frac{16\pi^2}{H^2}\left(1 + \frac{3}{2}(1 + \omega_{\mathrm{de}}\Omega_{\mathrm{de}})\right) q, \tag{19}$$

where the variation is with respect to $x = \log a$, ω_{de} is the equation of state parameter of the dark energy, Ω_{de} is the dark energy density parameter and q is the deceleration parameter given by the expression, $q = \frac{1}{2}(1 + 3\omega_{\mathrm{de}}\Omega_{\mathrm{de}})$. In accordance with the above equation, for S' the condition for the entropy to be positive becomes

$$q \geq \frac{-1}{1 + \frac{3}{2}(1 + \omega_{\mathrm{de}}\Omega_{\mathrm{de}})}, \tag{20}$$

which will approximately lead to

$$H > \frac{1}{t} \tag{21}$$

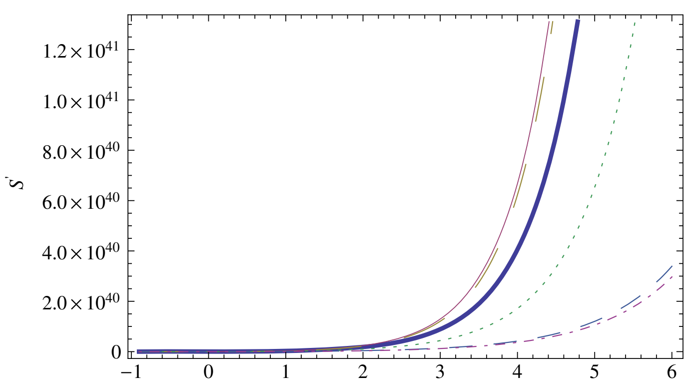

Figure 3. The behaviour of S' for $(\alpha, \beta) = (1.01, -0.01)$ (thick continuous line), $(\alpha, \beta) = (1.2, -0.1)$ (thin continuous line), $(\alpha, \beta) = (4/3, -0.1)$ (large dashed line), $(\alpha, \beta) = (1.2, 0.1)$ (dotted line), $(\alpha, \beta) = (1.2, 0.3)$ (small dashed line) and $(\alpha, \beta) = (4/3, 0.3)$ (dotted-dashed line), with the interaction coupling constant $b = 0.009$ inside the apparent horizon under thermal equilibrium conditions.

provided $\frac{3}{2}(1+\omega_{\rm de}\Omega_{\rm de})$ is relatively smaller than 1 (which in fact is true). The above bound on H is a reasonable one, where t is the cosmic time. The equation for S' can be further modified using the expression for q as

$$S' = \frac{36\pi^2}{H^2}(1+\omega_{\rm de}\Omega_{\rm de})^2. \tag{22}$$

From the above equation, it is clear that the rate of change of the total entropy is always positive in an expanding Universe, and thus the GSL is satisfied. We have numerically plotted S' with x in figure 3. The behaviour of the plots verifies that S' is positive for all parameter values. Thus, the GSL is completely satisfied inside the apparent horizon under thermal equilibrium conditions for all model parameters.

3.2 *Validity of GSL at the event horizon*

In this section, we analyse the validity of GSL under thermal equilibrium condition inside the event horizon. The distance of the event horizon is, in general, given by

$$R_{\rm h} = \frac{1}{1+z}\int_z \frac{{\rm d}z}{H}. \tag{23}$$

The distance of the event horizon for this model using the above expression is obtained as

$$R_{\rm h} = -4.2265\times 10^{17}\left(I\sqrt{(1+z)^{u_2/2}}\;2F_1\left[P, 0.5, 1+P, \frac{-f_1}{f_2}(1+z)^{u_2/2}\right]\right), \tag{24}$$

for positive and negative values of β; where I and P are constants having different values for different model parameters. The respective I, P values are 0.751155, 0.62582 for $(\alpha,\beta)=(1.2, 0.1)$, 0.780009, 0.626971 for $(\alpha,\beta)=(4/3, 0.1)$, 0.837601, 0.736071 for $(\alpha,\beta)=(1.2, 0.3)$, 0.68156, 0.58306 for $(\alpha,\beta)=(1.01, -0.01)$, 0.682953, 0.556458 for $(\alpha,\beta)=(1.2, -0.1)$ and 0.703692, 0.557082 for $(\alpha,\beta)=(4/3, -0.1)$.

Let the temperature of the event horizon be $T = 1/2\pi R_{\rm h}$ and area of the event horizon be $A = 4\pi R_{\rm h}^2$. Then the total entropy variation, using the aforesaid Gibb's equation for the dark sectors with respect to $x = \log a$ can be obtained as [39]

$$T(S'_{\rm de}+S'_{\rm m}) = H^{-1}(\rho_{\rm de}+\rho_{\rm m}+p_{\rm de})4\pi R_{\rm h}^2(\dot{R}_{\rm h}-HR_{\rm h}). \tag{25}$$

After substituting for the temperature and using the equation $\dot{R}_{\rm h} = HR_{\rm h}-1$, eq. (25) changes to

$$S'_{\rm de}+S'_{\rm m} = -H^{-1}(\rho_{\rm de}+\rho_{\rm m}+p_{\rm de})8\pi^2R_{\rm h}^3. \tag{26}$$

When the entropy of the horizon is added to the above equation then the total entropy change becomes

$$S' = H^{-1}\left[16\pi^2R_{\rm h}\left(\dot{R}_{\rm h}-\frac{R_{\rm h}^2}{2}(\rho_{\rm de}+\rho_{\rm m}+p_{\rm de})\right)\right]. \tag{27}$$

Using the equation $\dot{H} = -\frac{1}{2}(\rho_{\rm de}+\rho_{\rm m}+p_{\rm de})$, eq. (27) is modified to

$$S' = H^{-1}[16\pi^2R_{\rm h}(\dot{R}_{\rm h}-\dot{H}R_{\rm h}^2)]. \tag{28}$$

As $H > 0$ and $R_{\mathrm{h}} > 0$, the condition for the validity of the GSL can be deduced from eq. (28) as

$$\dot{R}_{\mathrm{h}} \geq \frac{1}{2}(\rho + p)R_{\mathrm{h}}^2. \tag{29}$$

Thus, from eq. (29), it is clear that the GSL is satisfied in this context if $(\rho + p) > 0$, which is equivalent to what is called the dominant energy condition [24]. This shows that the GSL is satisfied at the event horizon if the dominant energy condition is satisfied, which in turn implies that the effective equation of state is greater than -1. Substituting for $\dot{R}_{\mathrm{h}}$ in terms of Hubble parameter, also for the density and pressure of the fluid components, the above equation becomes

$$HR_{\mathrm{h}} - 1 - \frac{3}{2}(1 + \omega_{\mathrm{de}}\Omega_{\mathrm{de}})H^2R_{\mathrm{h}}^2 \geq 0. \tag{30}$$

Using the derived expressions for H, R_{h}, ω_{de} and Ω_{de}, we have numerically checked the above inequality for different model parameters α and β and the results are shown in figures 4 and 5.

From the plots, we can infer that the GSL is only partially satisfied irrespective of the sign of the model parameter β inside the event horizon under thermal equilibrium conditions. Thus, it can be concluded that event horizon is not a good candidate for the thermodynamic boundary. In ref. [25] Wang *et al* have considered the interacting holographic dark energy model with event horizon as the IR cut-off and they concluded that the second law of thermodynamics is not valid inside the event horizon while it is always valid inside the apparent horizon. Our result also agrees with this result in ref. [25].

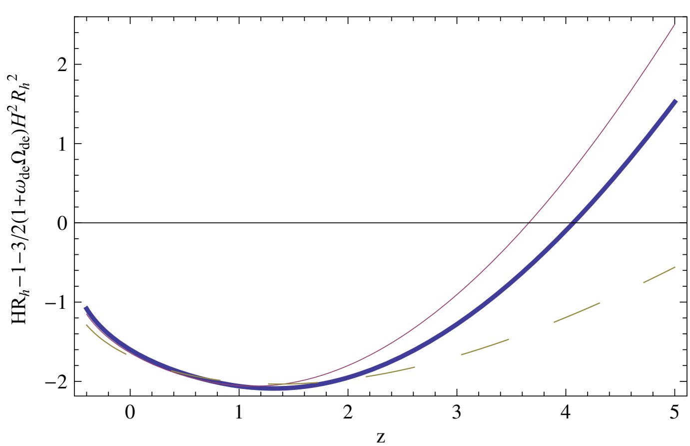

Figure 4. The behaviour of S' for $(\alpha, \beta) = (1.2, 0.1)$ (thick continuous line), $(\alpha, \beta) = (4/3, 0.1)$ (thin continuous line), $(\alpha, \beta) = (1.2, 0.3)$ (dashed line), with the interaction coupling constant $b = 0.009$ inside the event horizon under thermal equilibrium conditions.

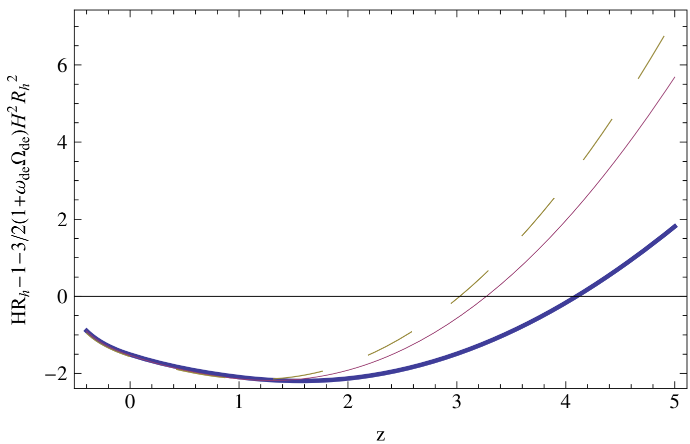

Figure 5. The behaviour of S' for $(\alpha, \beta) = (1.01, -0.01)$ (thick continuous line), $(\alpha, \beta) = (1.2, -0.1)$ (thin continuous line), $(\alpha, \beta) = (4/3, -0.1)$ (dashed line), with the interaction coupling constant $b = 0.009$ inside the event horizon under thermal equilibrium conditions.

4. GSL with thermal non-equilibrium condition

In this section we analyse the validity of the GSL under thermal non-equilibrium condition, i.e., there is no energy transfer between the dark sectors so that they have different temperatures, $T_{\rm de} \neq T_{\rm m} \neq T_{\rm h}$ where $T_{\rm de}$ is the temperature of the dark energy, $T_{\rm m}$ is the temperature of the dark matter and $T_{\rm h}$ is the temperature of the horizon. In this case, we consider apparent horizon as the physical boundary of the Universe and study the conditions for which the GSL is valid.

Using the Gibb's equation, the rate of change of total entropy comprising dark energy, dark matter and apparent horizon is obtained as

$$S' = -H^{-1}\left(12\pi\left(1+\frac{H'}{H}\right)\left(\frac{\Omega_{\rm de}(1+\omega_{\rm de})}{T_{\rm de}}+\frac{\Omega_{\rm m}}{T_{\rm m}}\right)+\frac{8\pi}{T_{\rm h}}\frac{H'}{H}\right). \tag{31}$$

Let us assume that the contribution from matter is negligible so that $\Omega_{\rm m} \sim 0$. Then eq. (31) becomes

$$S' = -H^{-1}\left(12\pi\left(1+\frac{H'}{H}\right)\left(\frac{\Omega_{\rm de}(1+\omega_{\rm de})}{T_{\rm de}}\right)+\frac{8\pi}{T_{\rm h}}\frac{H'}{H}\right). \tag{32}$$

For the GSL to be valid $S' \geq 0$. Equation (32) thus reduces to the condition given as

$$\left(12\pi\left(1+\frac{H'}{H}\right)\left(\frac{\Omega_{\rm de}(1+\omega_{\rm de})}{T_{\rm de}}\right)\right) \leq -\left(\frac{8\pi}{T_{\rm h}}\frac{H'}{H}\right). \tag{33}$$

In terms of deceleration parameter q the above condition can be reduced to [39]

$$\frac{T_{\rm de}}{T_{\rm h}} \geq -\frac{3}{2}\Omega_{\rm de}(1+\omega_{\rm de})\frac{q}{1+q}, \tag{34}$$

where in general $q = -1-(H'/H)$ [19]. From the above equation it can be assessed that, as q is always negative for an accelerating Universe and if $(1+q) > 0$ (corresponding to the quintessence phase), the temperature of dark energy $T_{\rm de}$ will be greater than the horizon temperature $T_{\rm h}$, in a dark energy-dominated Universe. This is in line with the conclusion made in ref. [40], that dark energy temperature is greater than that of the horizon by considering a more general form for dark energy.

The conclusion made in the previous paragraph can be further verified by calculating the entropy of both the dark energy and the apparent horizon. The entropy of the dark energy can be calculated using the standard relation, $S = ((\rho + P)V)/T$, by taking the dark energy temperature as $T_{\rm de} = kT_{\rm h}$ where $k > 1$. The entropy of dark energy can then be obtained as

$$S_{\rm de} = \frac{8\pi^2}{kH^2}\Omega_{\rm de}(1+\omega_{\rm de}). \tag{35}$$

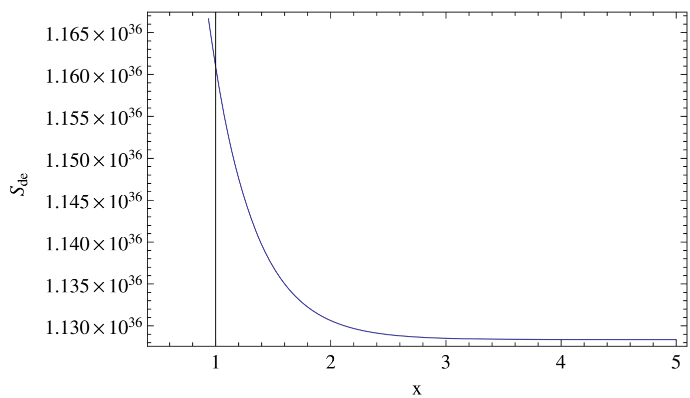

Figure 6. The behaviour of $S_{\rm de}$ against x for $(\alpha, \beta) = (1.2, 0.1)$ with $k = 1.25$ inside the apparent horizon under thermal non-equilibrium conditions.

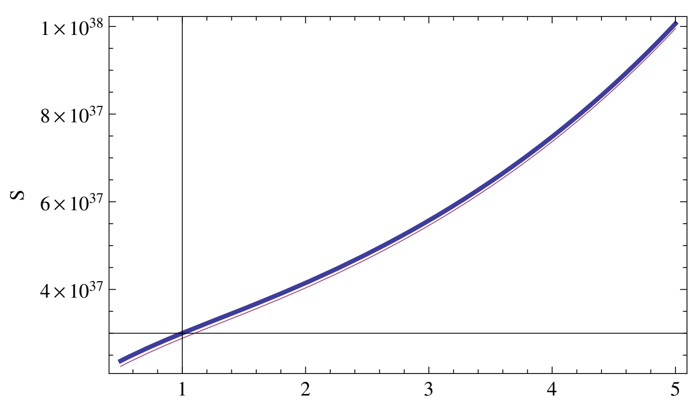

Figure 7. The behaviour of $S_{\rm h}$ (thin line) along with $S_{\rm total}$ (thick line) with x for $(\alpha, \beta) = (1.2, 0.1)$ inside the apparent horizon under thermal non-equilibrium conditions.

The total entropy, given by the sum of that of the dark energy and the horizon from (17), is obtained as

$$S = \frac{8\pi^2}{kH^2}\Omega_{\rm de}(1+\omega_{\rm de}) + \frac{8\pi^2}{H^2}. \tag{36}$$

The dark energy entropy and the total entropy are plotted against $x = \log a$ and is shown in figures 6 and 7.

From figure 6, it can be seen that the entropy of the dark energy decreases with respect to an increase in x, while from figure 7 it is clear that the entropy of the horizon increases with respect to x. Hence it can be inferred that even if the entropy of the dark energy decreases, the loss is covered by the increase in the horizon entropy, and as an effect the total entropy increases which in turn means that the GSL is satisfied. This again supports our earlier conclusion that the temperature of dark energy is greater than that of the horizon, a constraint implied from the validity of GSL.

5. Conclusion

This study was done in a flat FLRW Universe with holographic dark energy using Ricci scalar as its IR cut-off and interaction with the dark matter. We have concentrated on the validity of the GSL both at the apparent horizon and the event horizon of the Universe. The study was carried out under both thermal equilibrium and thermal non-equilibrium conditions.

Under the thermal equilibrium condition, the GSL was found to be completely satisfied inside the apparent horizon always, while it was found to be partially satisfying inside the event horizon for all model parameters. Thus it can be summarized that the apparent horizon is a better candidate as thermodynamic boundary when compared to the event horizon under thermal equilibrium. A similar result is obtained in refs [41,42] regarding the validity of the GSL at apparent horizon.

Under thermal non-equilibrium condition, the temperatures of the dark sectors and the apparent horizon are not the same. Considering a dark energy-dominated Universe, we have found that for the GSL to be valid, the temperature of dark energy must be greater than that of the horizon, provided the dark energy is in the quintessence phase. We have also calculated the evolution of dark energy entropy and the total entropy and found that the dark energy decreases while the total entropy increases, so that the decrease in the dark energy entropy is compensated by the increase in the horizon entropy. Our study in effect implies that the apparent horizon can be considered as a viable thermodynamic boundary but the event horizon cannot. Similar conclusion was obtained by others in varied contexts [33,40,43].

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