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Anisotropic cosmological models with bulk viscosity and particle creation in Saez–Ballester theory of gravitation

S CHANDEL and SHRI RAM*

Department of Mathematical Sciences, Indian Institute of Technology, Banaras Hindu University, Varanasi 221 005, India

*Corresponding author. E-mail: srmathitbhu@rediffmail.com

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Abstract. The paper deals with the study of particle creation and bulk viscosity in the evolution of spatially homogeneous and anisotropic Bianchi type-V cosmological models in the framework of Saez–Ballester theory of gravitation. Particle creation and bulk viscosity are considered as separate irreversible processes. The energy–momentum tensor is modified to accommodate the viscous pressure and creation pressure which is associated with the creation of matter out of gravitational field. A special law of variation of Hubble parameter is applied to obtain exact solutions of field equations in two types of cosmologies, one with power-law expansion and the other with exponential expansion. Cosmological model with power-law expansion has a Big-Bang singularity at time $t = 0$, whereas the model with exponential expansion has no finite singularity. We study bulk viscosity and particle creation in each model in four different cases. The bulk viscosity coefficient is obtained for full causal, Eckart's and truncated theories. All physical parameters are calculated and thoroughly discussed in both models.

Keywords. Bianchi type-V model; cosmology; bulk viscosity; particle creation.

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1. Introduction

In recent years, there has been a lot of interest in alternative theories of gravitation [1–3]. The aim of modern cosmology is to study the past history, the present state and future evolution of the Universe. Particle creation processes are supposed to play important roles in the early evolution of the Universe. Phenomenologically, particle creation has been described in terms of effective bulk viscosity coefficients [4–7]. Prigogine *et al* [8] have introduced a phenomenological model of particle creation by considering thermodynamics of open system in the framework of cosmology. They have suggested that matter creation takes place out of gravitational energy in irreversible process of non-equilibrium dynamics. This type of creation basically corresponds to irreversible energy flow from gravitational field to the created matter constituent. The rate of particle production can also be described by the quantum field theory in curved space-time [9]. Parker [10] has suggested the idea of particle creation in the expanding Universe. Hoyle and Narlikar [11] have suggested the idea of continuous creation of matter which was subsequently modified in the form of quasisteady-state cosmology [12].

Bulk viscosity is another irreversible process which contributes to entropy production in the Universe. It has been suggested that dissipative process in the early stages of the expansion of the Universe may well account for the present degree of isotropy and the ratio of the number of photon to baryons. Eckart [13] developed the first relativistic theory of non-equilibrium thermodynamics to study the effect of bulk viscosity. Most of the works with bulk viscosity have been studied using standard Eckart's theory of irreversible processes. However, it was pointed out later that the Eckart's theory has serious drawbacks concerning casuality and stability.

Several researchers have also explored the idea of cosmological models with bulk viscosity and particle creation.The evolution of Bianchi cosmologies with bulk viscosity and particle creation has been studied by Krori and Mukherjee [14]. The effect of bulk viscosity on the evolution of FRW models was studied by Desikan [15]. Johri and Pandey [16] discussed a new class of FRW models with matter creation and analysed behaviours of the models. Singh and Beesham [17] and Singh *et al* [18] studied cosmological models having bulk viscosity and particle creation within the framework of Brans–Dicke theory. Grøn [19] and Maartens [20] presented exhaustive reviews on cosmological models with non-causal and causal thermodynamics. Singh and Kale [21] studied anisotropic bulk viscous cosmological models with particle creation in general relativity. They have also studied the role of particle creation and bulk viscosity in the evolution of homogeneous and anisotropic model of the Universe represented by Bianchi type-I metric in Brans–Dicke theory [22]. Chaubey [23] has found the solution for Bianchi type-V bulk viscous cosmological models with particle creation in Brans–Dicke theory of gravitation.

Motivated by the above-mentioned works, in this paper, we study the particle creation and bulk viscosity in spatially homogeneous and anisotropic Bianchi type-V metric in Saez–Ballester theory of gravitation. We find exact solutions of the field equations which are obtained by applying a special law of variation of Hubble's parameter yielding constant value of the deceleration parameter. Two types of cosmological models are presented, one with power-law expansion and the other one with exponential expansion. The bulk viscosity coefficient is calculated for full causal, Eckart's and truncated theories in both the models.

2. Basic equations

The effective energy–momentum tensor T_{ij} of the standard Einstein's field equations in the presence of particle creation and bulk viscosity, which includes the creation pressure term p_c and the bulk viscous stress Π, is given as

$$T_{ij} = (\rho + p + p_c + \Pi)\, u_i u_j - (p + p_c + \Pi) g_{ij}, \tag{1}$$

where ρ is the energy density, p is the pressure and u^i is the four-velocity vector of the fluid satisfying $u^i u_i = 1$.

The particle number density flow vector $N^i(=\eta u^i)$ and the entropy flux vector $S^i(=\nu\eta u^i)$ in the second law of thermodynamics suggests the following equations:

$$N^i_{;i} = \dot{\eta} + 3\eta H = \Gamma, \tag{2}$$

$$S^i_{;i} = \eta\dot{\nu} + \nu\Gamma \geq 0, \tag{3}$$

where η is the particle number density, ν is the entropy per particle, H is the Hubble parameter and Γ is the source term which will be positive if there is production of particles and it is negative when there is annihilation of particles. In the absence of particle creation or annihilation, it is zero. Here a dot denotes differentiation with respect to time t and comma and semicolon denote ordinary and covariant derivatives respectively.

The Gibbs equation for an open thermodynamical system may be written as

$$\eta T\dot{\nu} = \dot{\rho} - (\rho + p)\frac{\dot{\eta}}{\eta}, \tag{4}$$

where T is the temperature of the cosmic fluid. Here the expression for entropy per particle is given by

$$\dot{\nu} = -\frac{3Hp_{\mathrm{c}}}{\eta T} - \frac{3H\Pi}{\eta T} - \frac{(\rho + p)}{\eta^2 T}\Gamma. \tag{5}$$

For an open adiabatic system in cosmology, the supplementary pressure p_{c}, due to the creation of matter, assumes the following form [8]:

$$p_{\mathrm{c}} = -\frac{(\rho + p)}{3H\eta}\Gamma = -\frac{(\rho + p)}{3H}\left(3H + \frac{\dot{\eta}}{\eta}\right). \tag{6}$$

Equation (6) predicts the amount of pressure arising from particle creation. From (5) and (6), the expression for entropy per particle is given by

$$\dot{\nu} = -\frac{3H\Pi}{\eta T}. \tag{7}$$

From eqs (4) and (7), we obtain

$$\frac{\dot{\eta}}{\eta} = \frac{\dot{\rho} + 3H\Pi}{\rho + p}. \tag{8}$$

Using the equation of state $p = \gamma\rho, 0 \leq \gamma \leq 1$ in (8), the particle number density can be obtained as

$$\eta^{1+\gamma} = \eta_0\rho\exp\left(\int 3H\Pi\rho^{-1}\mathrm{d}t\right), \tag{9}$$

where η_0 is an integration constant. The conventional bulk viscous effect in Bianchi Universe can be modelled within the framework of non-equilibrium thermodynamics. The transport equation for the bulk viscous pressure Π takes the form [20]

$$\Pi + \tau\dot{\Pi} = -3\xi H - \frac{\epsilon\tau\Pi}{2}\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right), \tag{10}$$

where ξ is the bulk viscous coefficient and τ is the relaxation time associated with the dissipative effect.

We can now study the behaviour of bulk viscosity in full causal theory ($\epsilon = 1$), Eckart's theory ($\tau = 0$) and truncated theory ($\epsilon = 0$). In truncated theory the evolution equation (10) reduces to

$$\Pi + \tau \dot{\Pi} = -3\xi H. \tag{11}$$

In this theory, to ensure that the viscous signals do not exceed the speed of light, we consider the following relation:

$$\tau = \frac{\xi}{\rho}. \tag{12}$$

In full causal theory, the equation of state for pressure and temperature [21,24] are taken to be barotropic, i.e., $p = \gamma\rho$ and $T = T(\rho)$. Then

$$T \propto \exp\int \frac{\mathrm{d}p(\rho)}{\rho + p(\rho)}. \tag{13}$$

This will reduce to the following equation:

$$T = T_0 \rho^{\gamma/(1+\gamma)}, \tag{14}$$

where T_0 is a constant. Using (12) and (14), the evolution equation (10) reduces to

$$\Pi + \frac{\xi}{\rho}\dot{\Pi} = -3H\xi - \frac{\xi\Pi}{2\rho}\left[3H - \frac{(1+2\gamma)\dot{\rho}}{(1+\gamma)\rho}\right]. \tag{15}$$

In Eckart's non-causal theory, the evolution equation (10) will reduce to

$$\Pi = -3\xi H. \tag{16}$$

3. The metric and field equations

We now solve the field equations for the anisotropic Bianchi type-V metric in the presence of particle creation and bulk viscous fluid within the framework of Saez–Ballester [3] theory of gravitation. The diagonal form of Bianchi type-V metric is considered in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2\mathrm{d}x^2 - \mathrm{e}^{2mx}[B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2], \tag{17}$$

where A, B and C are cosmic scale factors and m is the positive constant parameter.

The field equations in the scalar–tensor theory, proposed by Saez and Ballester are given by

$$G_{ij} - \omega\phi^r\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij}, \tag{18}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ and $8\pi G = c = 1$. All symbols have their meanings. The scalar field ϕ satisfies the equation

$$2\phi^r\phi_{;i}^{;i} + r\phi^{r-1}\phi_{,k}\phi^{,k} = 0. \tag{19}$$

Here r is an arbitrary constant and ω is a dimensionless coupling constant.

In co-moving coordinate system $u^i = (0, 0, 0, 1)$, the field equations (18) and (19) for the metric (17) lead to the following equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -(p + p_\mathrm{c} + \Pi) + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{20}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -(p + p_\mathrm{c} + \Pi) + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{21}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -(p + p_\mathrm{c} + \Pi) + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{22}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} = \rho - \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{23}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \tag{24}$$

$$\frac{\ddot{\phi}}{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{r}{2\phi}\dot{\phi}^2 = 0. \tag{25}$$

By combining (20)–(25), we obtain the continuity equation as

$$\dot{\rho} + 3(\rho + p)H = -3(p_\mathrm{c} + \Pi)H. \tag{26}$$

We recall that for Bianchi type-V metric (17) the average scale factor 'a' and the spatial volume V are given by

$$V = a^3 = ABC. \tag{27}$$

The generalized mean Hubble parameter H is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{28}$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$, $H_3 = \dot{C}/C$ are the directional Hubble's parameters in the direction of x, y and z respectively. The expressions for dynamical scalars such as expansion (θ), shear scalar (σ) and anisotropic parameter (A_m) are given by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{29}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6}, \tag{30}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{31}$$

where $\Delta H_i = H_i - H$, $(i = 1, 2, 3)$.

The deceleration parameter q is given as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{32}$$

The sign of q indicates whether the model inflates or not. The positive sign corresponds to the standard decelerating Universe whereas the negative sign indicates inflation.

We now obtain exact solutions of the field equations (20)–(25). Integrating eq. (24) and observing the integration constant into function B or C, we obtain

$$A^2 = BC. \tag{33}$$

We follow Saha and Rikhvitsky [25] and Shri Ram *et al* [26] to obtain exact solutions of field equations. Subtracting (20) from (21), (21) from (22) and (22) from (20) and integrating we find that

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{\mathrm{d}t}{a^3}\right), \tag{34}$$

$$\frac{B}{C} = d_2 \exp\left(k_2 \int \frac{\mathrm{d}t}{a^3}\right), \tag{35}$$

$$\frac{C}{A} = d_3 \exp\left(k_3 \int \frac{\mathrm{d}t}{a^3}\right), \tag{36}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants. Combining eqs (33)–(36), the quadrature form of metric functions can be written as

$$A(t) = a, \tag{37}$$

$$B(t) = B_0 a \exp\left(\frac{X}{3} \int \frac{\mathrm{d}t}{a^3}\right), \tag{38}$$

$$C(t) = {B_0}^{-1} a \exp\left(-\frac{X}{3} \int \frac{\mathrm{d}t}{a^3}\right), \tag{39}$$

where B_0 and X are integration constants.

It deserves mention that once we get the value of average scale factor a in (37)–(39), we can find the metric functions A, B and C. In order to find a, let us assume that the Hubble parameter H is related to the average scale factor a by

$$H = la^{-n}, \tag{40}$$

where $l > 0$ and $n \geq 0$ are constants. Such type of relation has already been considered by Berman [27], Berman and Gomide [28] for solving field equations in FRW models. Such a relation gives a constant value of the deceleration parameter. From (28) and (40), we obtain

$$\dot{a} = la^{-n+1}, \tag{41}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1}. \tag{42}$$

From eqs (32), (41) and (42), we find that

$$q = n - 1. \tag{43}$$

We see that, under the law of variation of H in (40), the deceleration parameter q is constant. From (41), we obtain the expression for the average scale factor as

$$a = (nlt)^{1/n}, \quad n \neq 0, \tag{44}$$

$$a = a_0 \exp(lt), \quad n = 0, \tag{45}$$

where a_0 and l are constants of integration.

4. Solution of field equations

We derive cosmological model corresponding to the cosmic scale functions A, B and C by using the power-law and exponential forms of the average scale factor a appearing in (44) and (45) separately.

4.1 *Model I*

Using (44) into (37)–(39) and integrating, we obtain

$$A = (nlt)^{1/n}, \tag{46}$$

$$B = B_0(nlt)^{1/n}\exp\left[M(nlt)^{(n-3)/n}\right], \tag{47}$$

$$C = {B_0}^{-1}(nlt)^{1/n}\exp\left[-M(nlt)^{(n-3)/n}\right], \tag{48}$$

where the constant M is given by

$$M = \frac{X}{l(n-3)}, \quad n \neq 3. \tag{49}$$

Equation (25) has the general solution

$$\phi = \phi_0(nlt)^{2(n-3)/n(r+2)}, \tag{50}$$

where

$$\phi_0 = \left[\frac{h(r+2)}{2l(n-3)}\right]^{2/(r+2)}$$

is a constant.

The expansion scalar, shear scalar, directional Hubble's parameter, generalized Hubble's parameter, anisotropic parameter and the volume scalar are given by

$$\theta = \theta_0(nlt)^{-1}, \tag{51}$$

$$\sigma^2 = {\sigma_0}^2(nlt)^{-6/n}, \tag{52}$$

$$H_1 = l(nlt)^{-1}, \tag{53}$$

$$H_2 = l(nlt)^{-1} + N(nlt)^{-3/n}, \tag{54}$$

$$H_3 = l(nlt)^{-1} - N(nlt)^{-3/n}, \tag{55}$$

$$H = l(nlt)^{-1}, \tag{56}$$

$$A_m = A_0(nlt)^{2(n-3)/n}, \tag{57}$$

$$V = (nlt)^{3/n}, \tag{58}$$

where $\theta_0, \sigma_0, N, A_0$ are constants. From (23), the value of the energy density can be found as

$$\rho = \rho_1(nlt)^{-2} - \rho_2(nlt)^{-6/n} - \rho_3(nlt)^{-2/n}, \tag{59}$$

where ρ_1, ρ_2, ρ_3 are constants.

If we assume the baratropic equation of state $p = \gamma\rho$, the expression for the pressure is

$$p = \gamma\left[\rho_1(nlt)^{-2} - \rho_2(nlt)^{-6/n} - \rho_3(nlt)^{-2/n}\right]. \tag{60}$$

We observe that the scale factors A, B and C are zero at the initial time $t = 0$. The expansion scalar, shear scalar, Hubble parameter and the three-directional Hubble's parameter are all infinite at $t = 0$. The spatial volume is zero at this epoch. The mean anisotropy parameter is infinite at $t = 0$ for $n < 3$. The energy density and pressure tend to infinity at this epoch. All these values of different physical parameters show that the Universe starts evolving with zero volume and expand with cosmic time. Thus, the model has a point singularity at $t = 0$. The spatial volume tends to infinity at $t \to \infty$. The expansion scalar, shear scalar, energy density, pressure and mean anisotropic parameter will all become zero for large time. All these indicate that the Universe is expanding with increase of cosmic time but the rate of expansion and shear scalar decrease to zero and finally tend to isotropic. This model approaches isotropic during late time of its evolution as $\lim \sigma/\theta = 0$ for $t \to \infty$.

Now in the following subsections, we study the behaviour of particle creation and bulk viscosity of this model in four different physical laws.

4.1.1 *Bulk viscosity energy–density law.* We assume that the bulk viscous stress Π is associated with energy density ρ by the following relationship:

$$\Pi = \Pi_0 \rho^{\alpha}, \tag{61}$$

where Π_0 is a constant. This relationship is motivated by the relation

$$\xi = \xi_0 \rho^{\alpha}, \tag{62}$$

where $\xi_0 \geq 0$, $\alpha \geq 0$ [29]. Further, it is suggested that the case $\alpha = 1$ corresponds to a radiative fluid [30]. The expression for bulk viscous stress Π and creation pressure p_c can therefore be written in the form

$$\Pi = \Pi_0 \left[\rho_1 (nlt)^{-2} - \rho_2 (nlt)^{-6/n} - \rho_3 (nlt)^{-2/n}\right]^{\alpha}, \tag{63}$$

$$p_c = F_1(t) - \Pi_0 \rho^{\alpha}, \tag{64}$$

where

$$F_1(t) = p_1 (nlt)^{-2} + p_2 (nlt)^{-6/n} + p_3 (nlt)^{-2/n}, \tag{65}$$

p_1, p_2, p_3 being constants.

The graphical behaviours of Π and p_c given in (63) and (64) are shown in figure 1. Now, the bulk viscosity coefficient in different theories: Full causal theory, Eckart's theory and truncated theory, are given in table 1.

4.1.2 *Uniform particle number density* ($\dot{\eta} = 0$). We consider the case when particle number density is uniform during the evolution of the Universe. This assumption leads to the particle production term Γ and creation pressure p_c as

$$\Gamma = 3H\eta, \tag{66}$$

$$p_c = -(1 + \gamma)\rho. \tag{67}$$

The value of bulk viscous stress can be obtained as

$$\Pi = \Pi_1 (nlt)^{-2} - \Pi_2 (nlt)^{-6/n} - \Pi_3 (nlt)^{-2/n}, \tag{68}$$

where Π_1, Π_2 and Π_3 are constants.

The graphical representation of bulk viscous stress Π and particle creation pressure p_c are given in figure 2 for different parameters.

The coefficients of bulk viscosity in different theories are given in table 2.

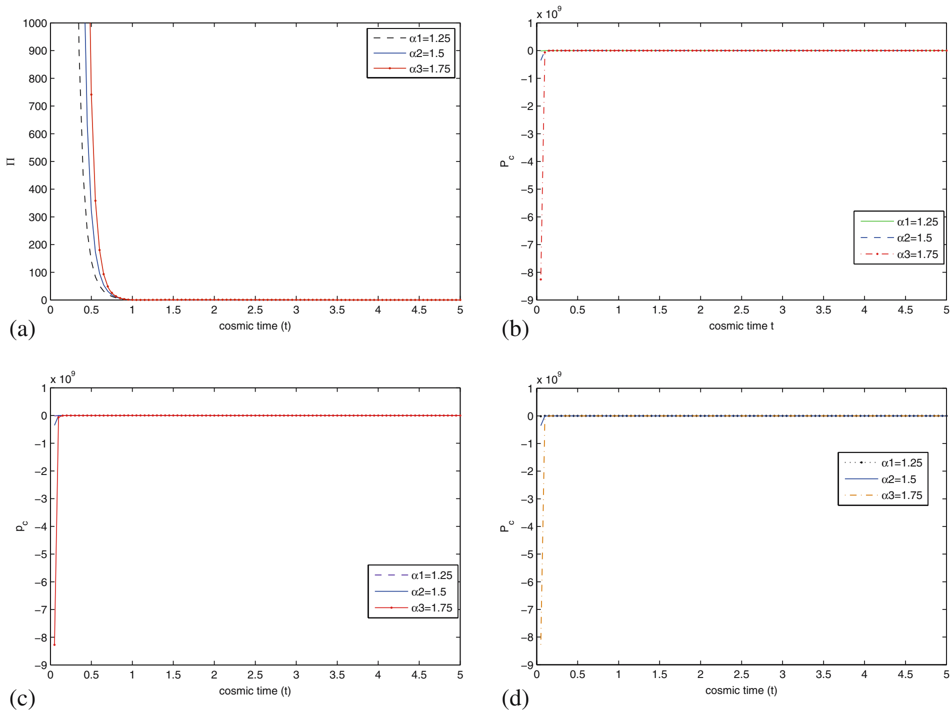
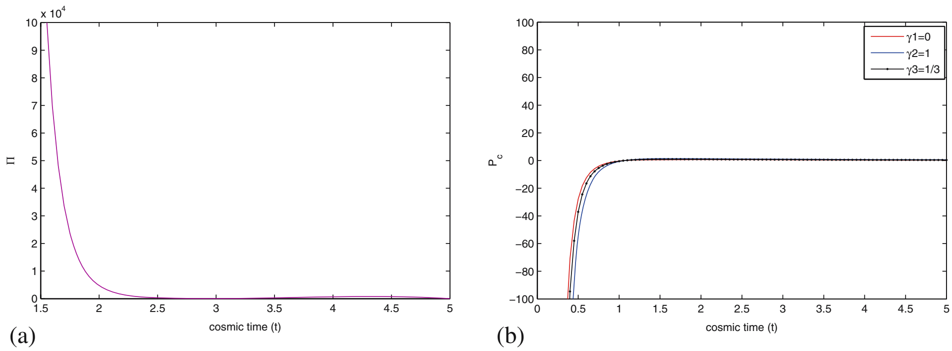


Figure 1. (**a**) The variation of bulk viscous stress vs. time t for different values of α and $n = 1.5$ and (**b**), (**c**) and (**d**) represent particle creation pressure for different values of α and for $\gamma = 0$, $\gamma = 1$ and $\gamma = \frac{1}{3}$ and $n = 1.5$ respectively.

Table 1. The expression for the coefficient of bulk viscosity in different theories in case of bulk viscosity energy–density law.

Full causal theory	$\xi = \rho^{\alpha+2}[F_2(t)\rho^2 + F_3(t) + F_4(t)\rho^{\alpha+1} + F_5(t)\rho^{\alpha-1}]^{-1}$, where $F_2(t) = \xi_1(nlt)^{-1}$, $F_3(t) = \xi_2(nlt)^{-3} - \xi_3(nlt)^{-(6/n)-1} - \xi_4(nlt)^{-(2/n)-1}$, $F_4(t) = \xi_5(nlt)^{-1}$, $F_5(t) = \xi_6(nlt)^{-3} + \xi_7(nlt)^{-(6/n)-1} + \xi_8(nlt)^{-(2/n)-1}$, $\xi_1, \ldots, \xi_8$ are constants
Eckart's theory	$\xi = F_6(t)\rho^{\alpha}$, where $F_6(t) = \xi_9 t$, ξ_9 is constant
Truncated theory	$\xi = F_7(t) + F_8(t)\rho^{\alpha-1}$, where $F_7(t) = \xi_{10}(nlt)^{-3} - \xi_{11}(nlt)^{-(6/n)-1} - \xi_{12}(nlt)^{-(2/n)-1}$, $F_8(t) = \xi_{13}(nlt)^{-(6/n)-1} + \xi_{14}(nlt)^{-(2/n)-1} - \xi_{15}(nlt)^{-3}$, $\xi_{10}, \ldots, \xi_{15}$ are constants



(a) (b)

Figure 2. (**a**) Variation of Π with time for $n = 1.5$ and (**b**) variation of p_c with time for $\gamma = 0, 1, \frac{1}{3}$ and $n = 1.5$ respectively.

4.1.3 *Ideal gas.* In the case of an ideal gas $\Gamma = 0$ and $p_c = 0$. Then eq. (2) becomes

$$\dot{\eta} + 3\eta H = 0. \tag{69}$$

Equation (69), on integration gives

$$\eta = \eta_1 t^{-3/n}, \tag{70}$$

where η_1 is an integrating constant. Equation (69) is the expression for particle creation density. This model has only bulk viscosity and bulk viscous stress is obtained as

$$\Pi = \Pi_4(nlt)^{-2} + \Pi_5(nlt)^{-6/n} + \Pi_6(nlt)^{-2/n}, \tag{71}$$

where Π_4, Π_5 and Π_6 are constants. Figure 3 shows the graphical behaviour of bulk viscous stress for ideal gas in model I.

The expression for bulk viscous coefficient assumes the forms which are shown in table 3.

Table 2. The expression for the coefficient of bulk viscosity in different theories in case of uniform particle number density.

Full causal theory	$\xi = -\Pi\rho^2[F_9(t)\rho^2 + F_{10}(t)\rho - F_{11}(t)\Pi]^{-1}$, where $F_9(t) = 3l(nlt)^{-1}$, $F_{10}(t) = \xi_{16}(nlt)^{-3} + \xi_{17}(nlt)^{-(6/n)-1} + \xi_{18}(nlt)^{-(2/n)-1}$, $F_{11}(t) = \xi_{19}(nlt)^{-3} + \xi_{20}(nlt)^{-(6/n)-1} + \xi_{21}(nlt)^{-(2/n)-1}$, $\xi_{16}, \ldots, \xi_{21}$ are constants
Eckart's theory	$\xi = F_{12}(t)$, where $F_{12}(t) = \xi_{22}(nlt)^{-(6/n)+1} + \xi_{23}(nlt)^{-(2/n)+1} - \xi_{24}(nlt)^{-1}$, ξ_{22}, ..., ξ_{24} are constants
Truncated theory	$\xi = -\Pi\rho[F_{13}(t)]^{-1}$, where $F_{13}(t) = \xi_{25}(nlt)^{-3} + \xi_{26}(nlt)^{-(6/n)} - \xi_{27}(nlt)^{-(2/n)}$, $\xi_{25}, \ldots, \xi_{27}$ are constants

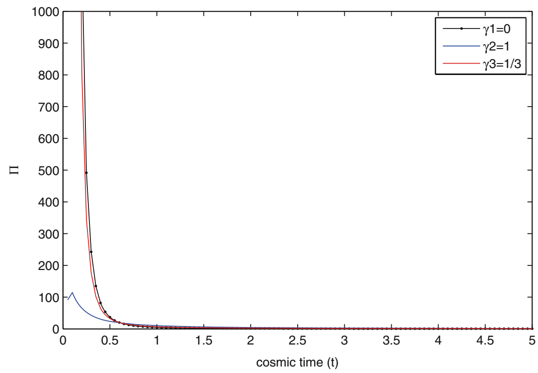


Figure 3. Variation of Π with time for $\gamma = 0, 1, \frac{1}{3}$ and $n = 1.5$.

4.1.4 *Creation with second-order correction in H.* Triginer and Pavon [31] suggested a generalization of the conservation of total particle number in standard cosmology by considering the Taylor expansion of $(\dot{\eta}/\eta) = f(H)$ upto second order in H as

$$\frac{\dot{\eta}}{\eta} = -3H + dH^2, \tag{72}$$

where d is a constant. The particle number density η also satisfies the balance equation

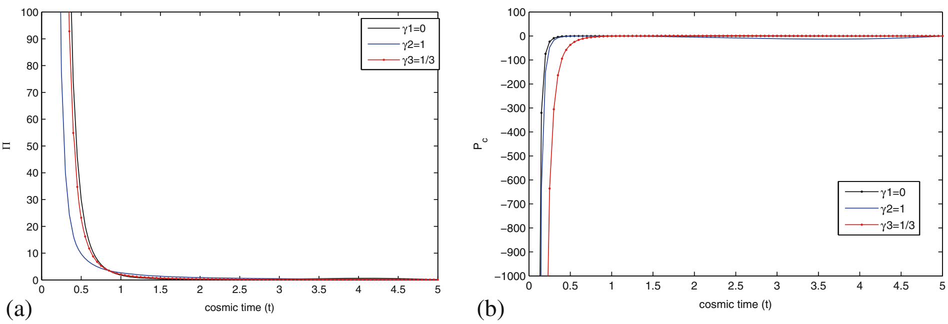
$$\dot{\eta} + 3\eta H = \Gamma. \tag{73}$$

So, from the above equation

$$\Gamma = d\eta H^2. \tag{74}$$

Table 3. The expression for the coefficient of bulk viscosity in different theories in case of ideal gas.

Full causal theory	$\xi = F_{14}(t)\rho^2[F_{15}(t)\rho + F_{16}(t)\rho^2 + F_{17}(t)F_{18}(t)]^{-1}$, where $F_{14}(t) = \xi_{28}(nlt)^{-2} + \xi_{29}(nlt)^{-6/n} - \xi_{30}(nlt)^{-2/n}$, $F_{15}(t) = \xi_{31}(nlt)^{-3} + \xi_{32}(nlt)^{-(6/n)-1} - \xi_{33}(nlt)^{-(2/n)-1}$, $F_{16}(t) = 3l(nlt)^{-1}$, $F_{17}(t) = \xi_{34}(nlt)^{-2} + \xi_{35}(nlt)^{-(6/n)} + \xi_{36}(nlt)^{-2/n}$, $F_{18}(t) = \xi_{37}(nlt)^{-3} - \xi_{38}(nlt)^{-(2/n)-1} - \xi_{39}(nlt)^{-(6/n)-1}$, $\xi_{28}, \ldots, \xi_{39}$ are constants
Eckart's theory	$\xi = F_{19}(t)$, where $F_{19}(t) = \xi_{40}(nlt)^{-1} + \xi_{41}(nlt)^{-(6/n)+1} - \xi_{42}(nlt)^{-(2/n)+1}$, $\xi_{40}, \ldots, \xi_{42}$ are constants
Truncated theory	$\xi = F_{20}(t)\rho[F_{21}(t)]^{-1}$, where $F_{20}(t) = \xi_{43}(nlt)^{-2} + \xi_{44}(nlt)^{-6/n} - \xi_{45}(nlt)^{-2/n}$, $F_{21}(t) = \xi_{46}(nlt)^{-3} + \xi_{47}(nlt)^{-(6/n)-1} - \xi_{48}(nlt)^{-(2/n)-1}$, $\xi_{43}, \ldots, \xi_{48}$ are constants



(a) (b)

Figure 4. Variation of (**a**) Π and (**b**) p_{c} with time for $\gamma = 0, 1, \frac{1}{3}$ and $n = 1.5$ respectively.

Equation (74) suggests that for $d > 0$, $d = 0$ and $d < 0$, respectively, there is creation, no creation and annihilation of particles. Using eq. (72), the expression for creation pressure is as follows:

$$p_{\mathrm{c}} = -\frac{(1+\gamma)d}{3} H\rho. \tag{75}$$

Table 4. The expression for the coefficient of bulk viscosity in different theories in case of creation with second-order correction in H.

Full causal theory	$\xi = F_{23}(t)\rho^2[F_{24}(t)\rho + F_{25}(t)\rho^2 + F_{26}(t)F_{27}(t)]^{-1}$ where $F_{23}(t) = -\xi_{49}(nlt)^{-2} - \xi_{50}(nlt)^{-3} - \xi_{51}(nlt)^{-2/n} + \xi_{52}(nlt)^{-(2/n)-1} + \xi_{53}(nlt)^{-6/n} + \xi_{54}(nlt)^{-(6/n)-1}$, $F_{24}(t) = \xi_{55}(nlt)^{-3} - \xi_{56}(nlt)^{-4} - \xi_{57}(nlt)^{-(2/n)-1} + \xi_{58}(nlt)^{-(2/n)-2} + \xi_{59}(nlt)^{-(6/n)-1} + \xi_{60}(nlt)^{-(6/n)-2}$, $F_{25}(t) = 3l(nlt)^{-1}$, $F_{26}(t) = \xi_{61}(nlt)^{-2} + \xi_{62}(nlt)^{-3} + \xi_{63}(nlt)^{-2/n} - \xi_{64}(nlt)^{-(2/n)-1} - \xi_{65}(nlt)^{-6/n} - \xi_{66}(nlt)^{-(6/n)-1}$, $F_{27}(t) = \xi_{67}(nlt)^{-3} - \xi_{68}(nlt)^{-(2/n)-1} - \xi_{69}(nlt)^{-(6/n)-1}$, $\xi_{49}, \ldots, \xi_{69}$ are constants
Eckart's theory	$\xi = F_{28}(t)$, where $F_{28}(t) = -\xi_{70}(nlt)^{-1} - \xi_{71}(nlt)^{-2} + \xi_{72}(nlt)^{-2/n} - \xi_{73}(nlt)^{-(2/n)+1} + \xi_{74}(nlt)^{-6/n} + \xi_{75}(nlt)^{-(6/n)+1}$, $\xi_{70}, \ldots, \xi_{75}$ are constants
Truncated theory	$\xi = F_{29}(t)\rho[F_{30}(t)]^{-1}$, where $F_{29}(t) = -\xi_{76}(nlt)^{-2} - \xi_{77}(nlt)^{-3} - \xi_{78}(nlt)^{-2/n} + \xi_{79}(nlt)^{-(2/n)-1} + \xi_{80}(nlt)^{-6/n} + \xi_{81}(nlt)^{-(6/n)-1}$, $F_{30}(t) = \xi_{82}(nlt)^{-3} - \xi_{83}(nlt)^{-4} - \xi_{84}(nlt)^{-(2/n)-1} + \xi_{85}(nlt)^{-(2/n)-2} + \xi_{86}(nlt)^{-(6/n)-1} + \xi_{87}(nlt)^{-(6/n)-2}$, $\xi_{76}, \ldots, \xi_{87}$ are constants

Hence the expression for creation pressure and bulk viscous stress can be obtained as

$$p_c = F_{22}(t), \tag{76}$$

where

$$F_{22} = p_4(nlt)^{-(6/n)-1} + p_5(nlt)^{-(2/n)-1} - p_6(nlt)^{-3}, \tag{77}$$

p_4, p_5 and p_6 being constants.

$$\Pi = \Pi_7(nlt)^{-2} + \Pi_8(nlt)^{-3} + \Pi_9(nlt)^{-2/n} - \Pi_{10}(nlt)^{-(2/n)-1} - \Pi_{11}(nlt)^{-(6/n)} - \Pi_{12}(nlt)^{-(6/n)-1}, \tag{78}$$

where $\Pi_7, \ldots, \Pi_{12}$ are constants. Figure 4 shows the behaviour of Π and p_c respectively.

The expression for the coefficient for bulk viscosity can be seen in table 4.

4.2 *Model II*

We derive in this section an exponentially expanding cosmological model with particle creation and bulk viscosity. Substituting the value of a given by (45) into eqs (37)–(39) and integrating, we obtain

$$A = a_0 \exp(lt), \tag{79}$$

$$B = B_1 a_0 \exp\big[lt - P\exp(-lt)\big], \tag{80}$$

$$C = {B_1}^{-1} a_0 \exp\big[lt + P\exp(-lt)\big], \tag{81}$$

where B_1 and P are constants. Using this value of a, the general solution of eq. (25) is given by

$$\phi = \phi_1 \exp(-\phi_2 t), \tag{82}$$

where

$$\phi_1 = \left\{\frac{h(r+2)}{-6la_0^3}\right\}^{2/(r+2)},$$

$$\phi_2 = \frac{6l}{r+2},$$

and h is a constant.

For the cosmological model with scale factors given by (79)–(81), the dynamical parameters are given by

$$\theta = 3l, \tag{83}$$

$$\sigma^2 = {\sigma_1}^2 \exp(-2lt), \tag{84}$$

$$H = l, \tag{85}$$

$$V = V_0 \exp(3lt), \tag{86}$$

$$H_1 = l, \tag{87}$$

$$H_2 = l + Q \exp(-lt), \tag{88}$$

$$H_3 = l - Q \exp(-lt), \tag{89}$$

$$A_m = A_1 \exp(-2lt), \tag{90}$$

where σ_1, V_0, Q and A_1 are constants.

If we assume the equation of state $p = \gamma\rho$, then energy density and pressure are calculated as

$$\rho = \rho_4 - \rho_5 \exp(-2lt), \tag{91}$$

$$p = \gamma \left[\rho_4 - \rho_5 \exp(-2lt)\right], \tag{92}$$

where ρ_4 and ρ_5 are constants.

We observe that all the three scale factors A, B, C, the spatial volume V, expansion scalar θ, shear scalar σ, anisotropy parameter A_m, the energy density ρ are constants at the initial time. Therefore, the cosmological model has no singularity at $t = 0$. These results show that the Universe starts evolving with constant volume and expands with uniform exponential expansion and volume grows exponentially with time. We see that shear scalar is decreasing function of time and ultimately tends to zero as $t \to \infty$. The mean anisotropy parameter tends to zero as time tends to infinity. The directional Hubble parameters become constant and uniform as time tends to infinity. We also observe that the energy density as well as pressure become constant for large time. As $q = -1$, the Universe is accelerating. As $\lim \sigma/\theta = 0$ for $t \to \infty$, the model tends to isotropy for large time.

In the following subsections, we also study the behaviour of particle creation and bulk viscosity of this model using four different laws.

4.2.1 *Bulk viscosity energy–density law.* In this case, again, a relation between viscous pressure and energy density is considered as in eq. (61). Then the expression for bulk viscous stress can be obtained as

$$\Pi = \Pi_0[\rho_4 - \rho_5 \exp(-2lt)]^\alpha. \tag{93}$$

The expression for creation pressure is given as

$$p_c = G_1(t) - \Pi_0 \rho^\alpha, \tag{94}$$

where $G_1(t) = p_7 + p_8 \exp(-2lt)$, p_7 and p_8 being constants. Figure 5 indicates behaviours of Π and p_c.

The bulk viscosity coefficients in different theories are given in table 5.

4.2.2 *Uniform particle number density* ($\dot{\eta} = 0$). Considering uniform particle number density ($\dot{\eta} = 0$), the expression for the bulk viscous stress and creation pressure are obtained as

$$\Pi = -\Pi_{13} \exp(-2lt), \tag{95}$$

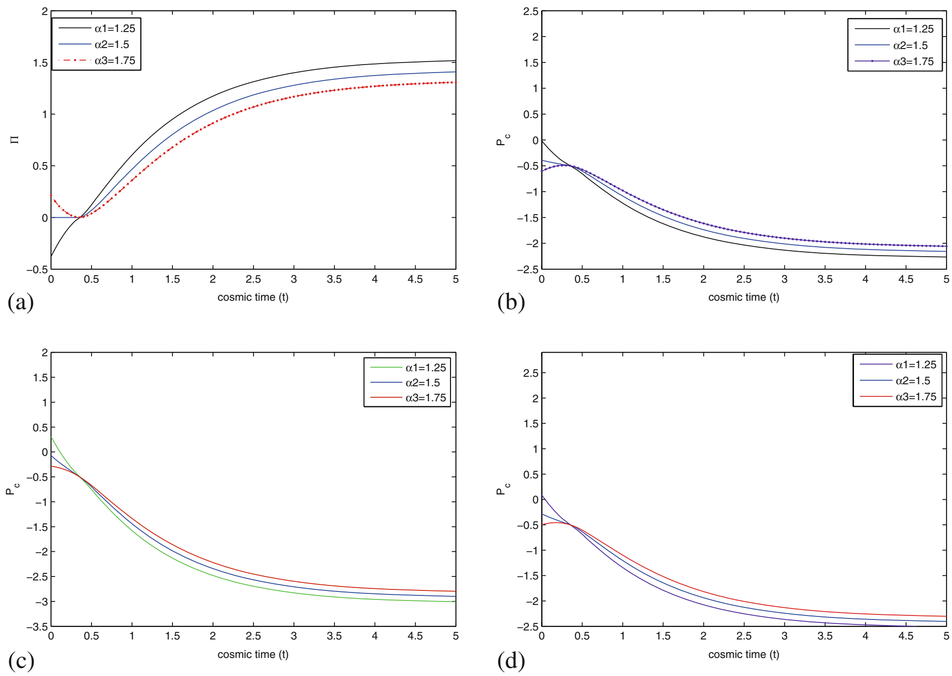


Figure 5. (**a**) The variation of bulk viscous stress vs. time t for different values of α; (**b**), (**c**) and (**d**) represent particle creation pressure for different values of α and for $\gamma = 0$, 1 and $\frac{1}{3}$ respectively.

Table 5. The expression for the coefficient of bulk viscosity in different theories in case of bulk viscosity energy–density law.

Full causal theory	$\xi = -2\Pi_0\rho^{\alpha+2}[G_2(t)\rho^{\alpha} + G_3(t)\rho^{\alpha+1} + G_4(t)\rho^2 - G_5]^{-1}$, where $G_2(t) = \xi_{88}\exp(-2lt)$, $G_3(t) = 3\Pi_0 l_0 =$ constant, $G_4(t) = 6l =$ constant, $G_5(t) = \xi_{89}\exp(-2lt)$, ξ_{88}, ξ_{89} are constants
Eckart's theory	$\xi = -\xi_{90}\rho^{\alpha}$, where ξ_{90} is constant
Truncated theory	$\xi = -\Pi_0\rho^{\alpha+1}[G_6(t) + G_7(t)\rho^{\alpha-1}]^{-1}$, where $G_6(t) = \xi_{91} - \xi_{92}\exp(-2lt)$, $G_7(t) = \xi_{93}\exp(-2lt)$, $\xi_{91}, \ldots, \xi_{93}$ are constants

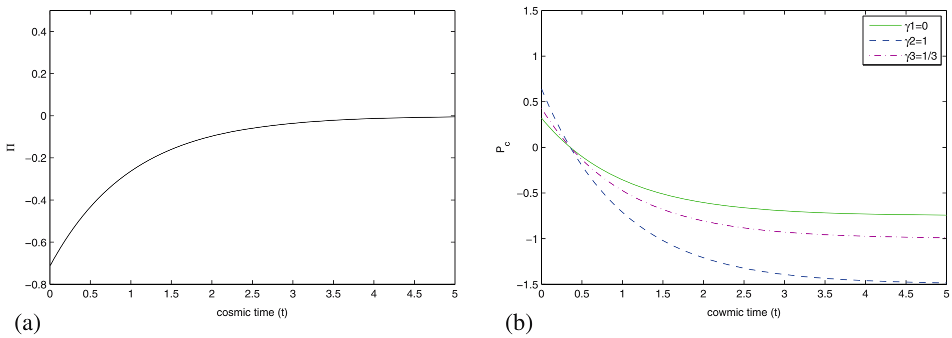


Figure 6. (**a**) The variation of Π with time and (**b**) variation of p_c with time for $\gamma = 0, 1, \frac{1}{3}$ respectively.

$$p_c = -(1+\gamma)\rho. \tag{96}$$

The graphical behavior of bulk viscous stress and particle creation pressure can be seen through figure 6. The bulk viscosity coefficients for different theories are shown in table 6.

4.2.3 *Ideal gas.* The value of particle number density and bulk viscous stress in this case can be obtained as

$$\eta = \eta_2 \exp(-3lt), \tag{97}$$

$$\Pi = -\Pi_{14} + \Pi_{15} \exp(-2lt), \tag{98}$$

where η_2, Π_{14} and Π_{15} are constants. Nature of Π in this case can be observed in figure 7.

Here, the bulk viscosity coefficient assumes the forms shown in table 7.

Table 6. The expression for the coefficient of bulk viscosity in different theories in case of uniform particle number density.

Full causal theory	$\xi = G_8(t)\rho^2[G_9(t)]^{-1}$, where $G_8(t) = \xi_{94}\exp(-2lt)$, $G_9(t) = \xi_{95}\exp(-2lt) + \xi_{96}\exp(-4lt)$, $\xi_{94}, \ldots, \xi_{96}$ are constants
Eckart's theory	$\xi = G_{10}(t)$, where $G_{10}(t) = \xi_{97}\exp(-2lt)$, ξ_{97} is constants
Truncated theory	$\xi = G_{11}(t)\rho[G_{12}(t)]^{-1}$, where $G_{11}(t) = \xi_{98}\exp(-2lt)$, $G_{12}(t) = \xi_{99} - \xi_{100}\exp(-2lt)$, $\xi_{98}, \ldots, \xi_{100}$ are constants

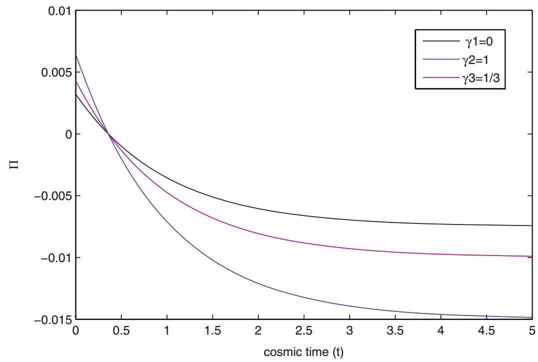


Figure 7. Variation of Π with time for $\gamma = 0$, 1 and 1/3.

4.2.4 *Creation with second-order correction in H.* Also in this case, the expression for creation pressure and bulk viscous stress can be obtained as

$$p_c = G_{18}(t), \tag{99}$$

where

$$G_{18}(t) = -p_9 + p_{10}\exp(-2lt),$$

p_9 and p_{10} are constants.

$$\Pi = \Pi_{16} - \Pi_{17}\exp(-2lt), \tag{100}$$

where Π_{16} and Π_{17} are constants. The behavior of bulk viscous stress and particle creation pressure can be observed through figure 8.

The expression for bulk viscous coefficients in all the three cases can be seen in table 8.

Table 7. The expression for the coefficient of bulk viscosity in different theories in case of ideal gas.

Full causal theory	$\xi = G_{13}(t)\rho^2[G_{14}(t)]^{-1}$, where $G_{13}(t) = \xi_{101} - \xi_{102}\exp(-2lt)$, $G_{14}(t) = \xi_{103} + \xi_{104}\exp(-2lt) - \xi_{105}\exp(-4lt)$, $\xi_{101}, \ldots, \xi_{105}$ are constants
Eckart's theory	$\xi = G_{15}(t)$, where $G_{15}(t) = \xi_{106} - \xi_{107}\exp(-2lt)$, ξ_{106}, ξ_{107} are constants
Truncated theory	$\xi = G_{16}(t)\rho[G_{17}(t)]^{-1}$, where $G_{16}(t) = \xi_{108} - \xi_{109}\exp(-2lt)$, $G_{17}(t) = \xi_{110} - \xi_{111}\exp(-2lt)$, $\xi_{108}, \ldots, \xi_{111}$ are constants

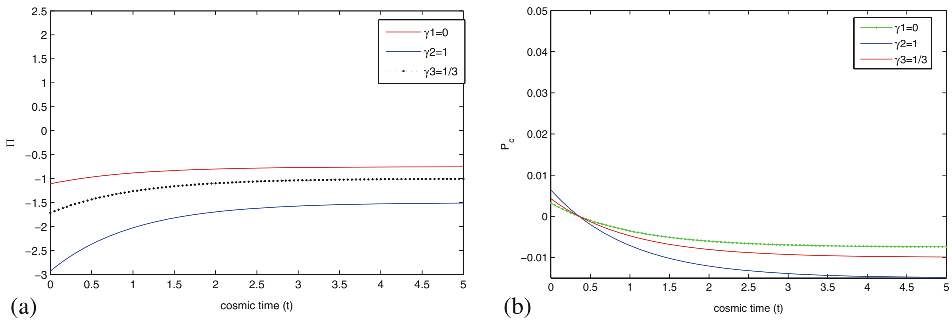


Figure 8. The variation of (**a**) Π and (**b**) p_c with time respectively for $\gamma = 0, 1, \frac{1}{3}$.

Table 8. The expression for the coefficient of bulk viscosity in different theories in case of creation with second-order correction in H.

Full causal theory	$\xi = G_{19}(t)\rho^2[G_{20}(t)]^{-1}$, where $G_{19}(t) = -\xi_{112} + \xi_{113}\exp(-2lt)$, $G_{20}(t) = \xi_{114} + \xi_{115}\exp(-2lt) - \xi_{116}\exp(-4lt)$, $\xi_{112}, \ldots, \xi_{116}$ are constants
Eckart's theory	$\xi = G_{21}(t)$, where $G_{21}(t) = -\xi_{117} + \xi_{118}\exp(-2lt)$, ξ_{117}, ξ_{118} are constants
Truncated theory	$\xi = G_{22}(t)\rho[G_{23}(t) + G_{24}(t)\rho]^{-1}$, where $G_{19}(t) = -\xi_{119} + \xi_{120}\exp(-2lt)$, $G_{20}(t) = \xi_{121}\exp(-2lt)$, $G_{21}(t) = 3l = $ constant, $\xi_{119}, \ldots, \xi_{121}$ are constants

5. Conclusion

In this paper, we have considered field equations for a spatially homogeneous and anisotropic Bianchi type-V space–time in the presence of bulk viscosity and particle creation within the framework of Saez–Ballester theory of gravitation. Two types of solutions of the average scale factor, one is of power-law type and other one of exponential form, are obtained by using a special law of Hubble's parameter which yields a constant value of deceleration parameter. We have presented models in two types of cosmologies corresponding to singular and non-singular cosmological models. We have studied separately the bulk viscosity and particle creation in each model by considering four different cases. The bulk viscosity coefficients are obtained for full causal, Eckart's and truncated theories in all the cases. We have also discussed physical and dynamical properties of both the models using graphs. Results presented in this paper are consistent with the observed feature of the Universe.

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