



# Spin polarization of electrons in a magnetic impurity doped semiconductor quantum dot – The effect of electron–phonon interaction

DEEPANJALI MISRA\* and SUKANTA KUMAR TRIPATHY

National Institute of Science and Technology, Palur Hills, Berhampur 761 008, India

\*Corresponding author. E-mail: deepanjali.misra@gmail.com

MS received 25 February 2014; revised 10 January 2015; accepted 12 January 2015

DOI: 10.1007/s12043-015-1032-6; ePublication: 5 November 2015

**Abstract.** A theoretical model is presented in this paper for degree of spin polarization in a light emitting diode (LED) whose epitaxial region contains quantum dots doped with magnetic impurity. The model is then used to investigate the effect of electron–phonon interaction on degree of spin polarization at different temperatures and magnetic fields. It is found that magnetic impurity increases the degree of spin polarization irrespective of temperature, while the electron–phonon interaction decreases the degree of spin polarization. Results are found to be in better agreement with experiments.

**Keywords.** Impurity-doped semiconductor quantum dot; impurity concentration; electron–phonon interaction; spin polarization.

PACS No. 85.25.Qc

## 1. Introduction

Ordered spin structures in both real space and reciprocal space are becoming increasingly more important in the context of spintronics [1], where functionalities are introduced in electronic devices that are based on the electron spin. Optically, spin is oriented by the transfer of angular momenta of circularly polarized photons. Electrically, this is done by using magnetic electrode. The spin of electrons in semiconductors strongly couple with electric and magnetic fields due to the spin-orbit and exchange interactions. A single magnetic impurity embedded into a self-assembled quantum dot permits efficient selective optical control and manipulation of individual spin states and define a single qubit for the magnetic impurity [2]. Awschalom has shown in [3] that (Ga,Mn)As and (In,Mn)As show even hole spin injection at zero applied field and high temperature. Again, as the optical properties, band structure and spin polarization of electrons in GaMnAs are not available yet [4], we investigated theoretically the spin polarization of electrons under the effect of

electron–phonon interaction when (GaMn)As quantum dot is placed in the epitaxial region of LED. In this paper, we analyse the effect of magnetic impurity, temperature, magnetic field and electron–phonon interaction (e–ph interaction) on degree of spin polarization of electron in LED having a magnetic impurity-doped quantum dot layer in the epitaxial region, using a full quantum model [5] for generating polarized light in the presence of magnetic field at high temperature. Effect of electron–phonon interaction is particularly studied for different concentrations of magnetic impurity at high temperature following Langevin approach [6].

## 2. Model

The total Hamiltonian for a system of GaAs LED containing magnetic impurity-doped quantum dot in the epitaxial layer (figure 1) is given by

$$H_T = H_c + H_p + H_d + H_m + H_{e-ph} + H_{imp}, \quad (1)$$

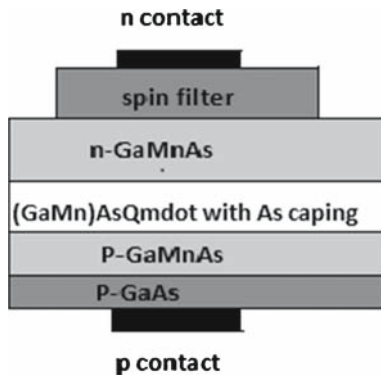
where  $H_c$  is the Hamiltonian for free carriers and is given as [5]

$$H_c = \sum_k \left( \sum_{\mu} \epsilon_{c,k,\mu} c_{k,\mu}^{\dagger} c_{k,\mu} + \sum_{\mu'} \epsilon_{v,-k,\mu'} d_{-k,\mu'}^{\dagger} d_{-k,\mu'} \right), \quad (2)$$

where  $c_{k\mu}$  and  $d_{-k\mu'}$  are annihilation operators for electron with momentum  $k$  and spin  $\mu$  and hole with momentum  $-k$  spin  $\mu'$  respectively.  $\epsilon_{ck\mu}$  and  $\epsilon_{d-k\mu'}$  are the conduction and valence band energy respectively.  $H_p$  is the Hamiltonian due to multiphotonic process and is represented by the Hamiltonian [5]

$$H_p = \sum_{l\mu\mu'} \hbar\nu_l a_{l\mu\mu'}^{\dagger} a_{l\mu\mu'}. \quad (3)$$

Here  $\nu_l$  is the frequency of photon in the  $l$ th mode for spin transition  $\mu$  to  $\mu'$  and  $a_{l\mu\mu'}$  is the photonic annihilation operator.



**Figure 1.** Schematic diagram of a magnetic impurity-doped quantum dot in the epitaxial region of GaAs LED.

The dipole interaction is described by [5]

$$H_d = \sum_{lk\mu\mu'} \hbar \left( g_{lk\mu\mu'} d_{-k\mu'}^\dagger c_{k\mu}^\dagger a_{l\mu\mu'} + g_{lk\mu\mu'}^* d_{-k\mu'} c_{k\mu} a_{l\mu\mu'}^\dagger \right), \quad (4)$$

where  $g_{lk\mu\mu'}$  is the dipole coupling constant. The effect of the many-body interaction is included in the renormalized parameters of  $\epsilon_{ck\mu}$ ,  $\epsilon_{v-k\mu'}$  and  $g_{lk\mu\mu'}$  using a mean-field approximation.

The interaction term here is in accordance with [7]. This interaction energy is such that it conserves energy. Here  $d_{-k,\mu'} c_{k\mu} a_{l,\mu,\mu'}^\dagger$  annihilates an electron–hole pair and creates a photon and transfers one photon of energy from the semiconductor to the field. Similarly,  $d_{-k,\mu'}^\dagger c_{k\mu}^\dagger a_{l,\mu,\mu'}$  creates an electron–hole pair annihilating one photon. We have not included  $d_{-k,\mu'}^\dagger c_{k\mu}^\dagger a_{l,\mu,\mu'}^\dagger$  and its adjoint as they do not conserve energy.

The effect of magnetic field on the device is described by Zeeman Hamiltonian given by [5]

$$H_m = \mu_B B_z \sum_k \left( \sum_{\mu\nu} G_e S_{c\mu\nu} c_{k\mu}^\dagger c_{k\nu} + \sum_{\mu\nu} G_h S_{v\mu'\nu'} d_{-k\mu'}^\dagger d_{-k\nu'} \right). \quad (5)$$

Here  $\mu_B$  is the Bohr magnetron,  $G_{e(h)}$  is electron (hole) Landau  $g$ -factor,  $S_{c\mu\nu}$  and  $S_{v\mu'\nu'}$  are spin matrices of electrons and holes and  $B_z$  is the magnetic field strength.

In addition to the above interactions, we consider electron–phonon interaction, which is represented by the Hamiltonian [8]

$$H_{e-ph} = Q_{k\mu} c_{k\mu}^\dagger c_{k\mu} (f_{k\mu} + f_{k\mu}^\dagger) + Q'_{k\mu} e_{k\mu}^\dagger e_{k\mu} (f_{k\mu} + f_{k\mu}^\dagger), \quad (6)$$

where  $e_{k\mu}^\dagger$  and  $e_{k\mu}$  are annihilation and creation operators for impurity electrons.  $Q_{k\mu}$  and  $Q'_{k\mu}$  are the coefficient matrices for electron–phonon interaction.  $f_{k\mu}$  and  $f_{k\mu}^\dagger$  are the creation and annihilation operators for phonon given by

$$f_{k\mu} = \frac{q_{k\mu} n_k}{\hbar(\gamma_q - i\omega_q)}.$$

Here  $q_{k\mu}$  is the interaction parameter. The Hamiltonian for magnetic impurity is [9]

$$H_{\text{imp}} = x E_{k\mu}^e e_{k\mu}^\dagger e_{k\mu} + x V_e^c \left( c_{k\mu}^\dagger e_{k\mu} + e_{k\mu}^\dagger c_{k\mu} \right) + x V_e^d \left( d_{-k\mu'}^\dagger e_{k\mu}^\dagger + e_{k\mu} d_{-k\mu'} \right),$$

where  $x$  is the fraction of impurity,  $E_{k\mu}^e$  is the single-particle energy for impurity electron,  $V_e^c$  and  $V_e^d$  are the interaction energy coefficients for the host electron and hole with impurity electron. Using Langevin rate equations for dipole operator  $\sigma_k^{\mu\mu'}$  and photon annihilation operator ( $A_{l\mu\mu'} = a_{l\mu\mu'} e^{i\nu_l t}$ ), we get

$$\begin{aligned} & \sigma_k^{\mu\mu'} \\ &= \frac{i \sum_l g_{lk\mu\mu'} (n_{ck}^\mu + n_{d-k}^{\mu'} - 1) A_{l\mu\mu'} - \frac{i}{\hbar} x V_e^d (n_{d-k}^{\mu'} + n_{ck}^\mu) + F_{\sigma k}^{\mu\mu'}}{\gamma + \frac{i}{\hbar} [\mu_B B_z (G_e S_{c\mu\nu} + G_h S_{v\mu'\nu'}) + (\epsilon_{ck\mu} + \epsilon_{v-k\mu'} - \hbar\nu_l)] + (Q_{k\mu} + Q'_{k\mu}) W_{\mu k} + x (E_{k\mu}^e + V_e^c)}, \end{aligned}$$

where

$$D_{lk\mu\mu'} = \frac{1}{\gamma + \frac{i}{\hbar} [\mu_B B_z (G_e S_{c\mu\nu} + G_h S_{v\mu'\nu'}) + (\epsilon_{ck\mu} + \epsilon_{vk\mu'} - \hbar\nu_l) + (Q_{k\mu} + Q'_{k\mu}) W_{\mu k} + x(E_{k\mu}^e + V_e^c)]}$$

whereas  $n_{ck}^\mu$  and  $n_{-dk}^{\mu'}$  are the number of electrons and holes in the conduction and valence bands respectively,  $F_{\sigma k}^{\mu\mu'}$  is the fluctuation term for carriers and  $W_{\mu k}$  is related to electron-phonon interaction parameter as

$$W_{\mu k} = \frac{q_{k\mu} n_k}{\hbar} \frac{2\gamma_q}{\gamma_q^2 + \omega_q^2}$$

$$\frac{dA_{l\mu\mu'}}{dt} = \sum_l - \left[ \frac{k_l^0}{2} + i \left( \nu_l + \frac{Q_{k\mu} n_{ck}^\mu + Q'_{k\mu} n_{ek}^{\mu'}}{\hbar a_{l\mu\mu'}} \right) \right] A_{l\mu\mu'}$$

$$+ \sum_{l'} G_{ll'}^{\mu\mu'} A_{l\mu\mu'} + F_{\sigma l}^{\mu\mu'} + G'_{ll'}^{\mu\mu'} + F_l,$$

where  $k_l^0$  is the field decay constant,  $F_l$  is the fluctuation term for the field and the polarized gain matrices are  $F_{\sigma l}^{\mu\mu'}$ ,  $G_{ll'}^{\mu\mu'}$  and  $G'_{ll'}^{\mu\mu'}$  [10].

$$G'_{ll'}^{\mu\mu'} = \frac{g_{lk\mu\mu'}^*}{\hbar} x(n_{d-k}^{\mu'} + n_{ck}^\mu) V_e^d.$$

$D_{lk\mu\mu'}$  is introduced due to the addition of impurity. Solving the above equations for adiabatic approximation, the photon number in steady state can be shown to be

$$n_{lk\mu\mu'} = \frac{k_l^0 n(\nu_l) + R_{\text{spl}}^{\mu\mu'}}{k_l^0 + R_{\text{absl}}^{\mu\mu'} - R_{\text{spl}}^{\mu\mu'}}, \tag{7}$$

where  $R_{\text{absl}}^{\mu\mu'}$  and  $R_{\text{spl}}^{\mu\mu'}$  are the rate of stimulated absorption and spontaneous emission respectively. Degree of spin polarization is defined as  $P = (I^+ - I^-) / (I^+ + I^-)$  where  $I^\pm$  is proportional to photon number  $n_{l\mu\mu'}^\pm$ . In the low injection limit  $k_l^0 \gg (R_{\text{absl}}^{\mu\mu'} - R_{\text{spl}}^{\mu\mu'})$ . Considering the transition from spin states  $(-1/2, -3/2)$  to  $(-1/2, -1/2)$  to be right circularly polarized and the transition  $(-1/2, 1/2)$  to  $(-1/2, 3/2)$  left circularly polarized, the degree of polarization can be shown to be

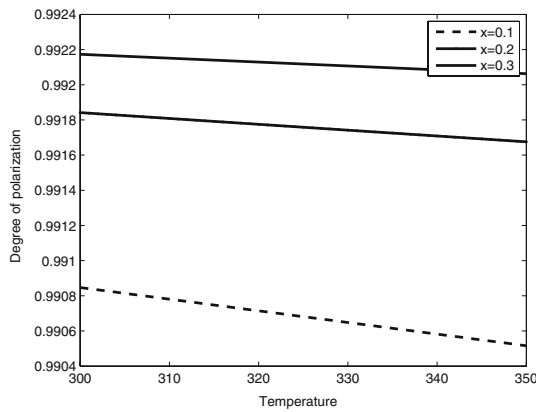
$$P = \frac{\sum_l \left[ R_{\text{spl}}^{-\frac{1}{2}-\frac{3}{2}} + R_{\text{spl}}^{\frac{1}{2}-\frac{1}{2}} - R_{\text{spl}}^{-\frac{1}{2}\frac{1}{2}} - R_{\text{spl}}^{\frac{1}{2}\frac{3}{2}} \right]}{\sum_l \left[ \left( R_{\text{spl}}^{-\frac{1}{2}-\frac{3}{2}} + R_{\text{spl}}^{\frac{1}{2}-\frac{1}{2}} + R_{\text{spl}}^{-\frac{1}{2}\frac{1}{2}} + R_{\text{spl}}^{\frac{1}{2}\frac{3}{2}} \right) + 4n_0 \bar{\nu}_l \right]}, \tag{8}$$

where

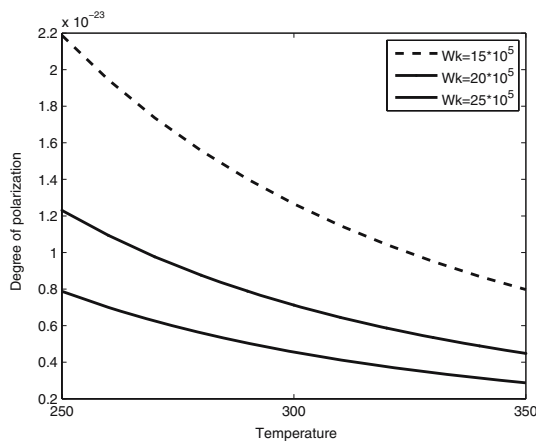
$$R_{\text{spl}}^{\mu\mu'} = \frac{2 \sum_k \gamma |g_{lk\mu\mu'}|^2 n_{ck}^\mu n_{-dk}^{\mu'} x n_{ek}^\mu}{\gamma^2 + \frac{1}{\hbar^2} \{ \mu_B B_z (G_e S_{c\mu\nu} + G_h S_{v\mu'\nu'}) + (\epsilon_{ck\mu} + \epsilon_{vk\mu'} + \hbar\nu_l) + (Q_{k\mu} + Q'_{k\mu}) W_{\mu k} + x(E_{k\mu}^e + V_e^c) \}^2}$$

### 3. Analysis of the result

We use eq. (8) and expression for  $R_{\text{spl}}^{\mu\mu'}$  to investigate the effect of temperature, magnetic field and electron–phonon interaction on degree of spin polarization. In figure 2, we have shown the variation of degree of spin polarization with temperature, for different values of impurity concentration of quantum dots, keeping electron–phonon interaction constant and in figure 3 the same variation is shown with constant impurity concentration and different e–ph interaction. From these figures it is found that the degree of spin polarization increases with the increase in magnetic impurity concentration, while it decreases with the increase in e–ph interaction. This may be due to increase in concentration of spin-polarized electrons due to the presence of magnetic impurity which gets affected by

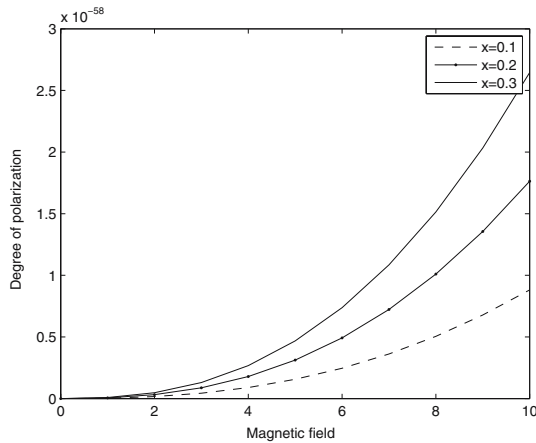


**Figure 2.** Variation of the degree of polarization with temperature for different concentrations of impurity  $x$  at magnetic field 8 T when the electron–phonon interaction parameter is constant.

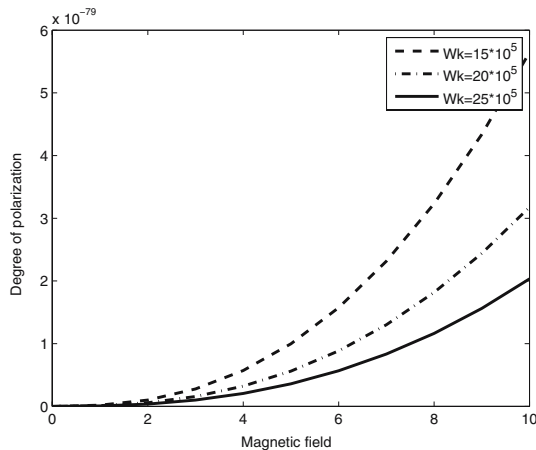


**Figure 3.** Variation of the degree of polarization with temperature for different electron–phonon interaction parameters when magnetic impurity concentration is constant.

the applied external magnetic field [11]. In figures 4 and 5, the variation of degree of spin polarization with magnetic field are simulated. Here also it is found that the peak value of degree of spin polarization increases with increase in impurity concentration. Further, it is found that as the strength of magnetic field increases, the degree of polarization increases monotonously which is in agreement with the experimental results in [5,12]. The variation of degree of spin polarization with e–ph interaction is simulated at different impurity concentrations and is shown in figure 6. A decrease of the degree of polarization with increase in electron–phonon interaction is observed in this case. This result may be accounted for the decrease of the spin relaxation time which is responsible for the

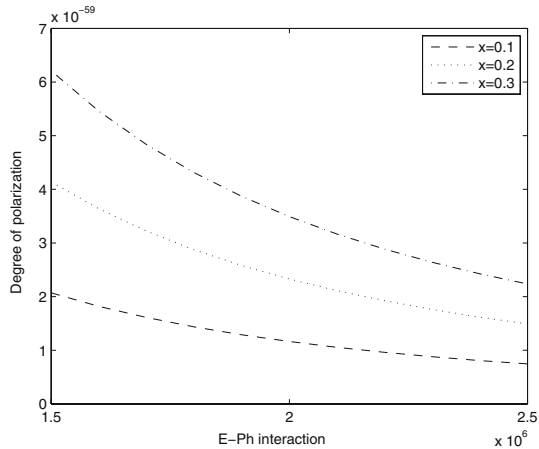


**Figure 4.** Variation of the degree of polarization with magnetic field for different concentrations of impurity at temperature 300 K when the electron–phonon interaction parameter is constant.



**Figure 5.** Variation of the degree of polarization with magnetic field for different values of electron–phonon interaction parameter keeping the concentration of magnetic impurity constant at 300 K.

## Effect of electron–phonon interaction



**Figure 6.** Variation of the degree of polarization with electron–phonon interaction parameter for different concentrations of impurity at constant temperature (300 K) and magnetic field (8 T).

decrease in the degree of spin polarization. But when impurity concentration is increased, there is a sudden rise in the saturation value of the degree of spin polarization. The results in this investigation seem to be important particularly in spintronic devices for controlling electron spin polarization. Nevertheless, this also helps one to understand the effect of some fundamental interactions, on electron spin polarization in a LED.

## References

- [1] S A Wolf, D D Awschalom, R A Buhrman, J M Daughton, S von Molnar, M L Roukes, A Y Chtchelkanova and D M Treger, *Science* **294**, 1488 (2001)
- [2] Alexander O Govorov and Alexander V Kalameitsev, *Phys. Rev. B* **71**, 035338 (2005)
- [3] David D Awschalom *et al*, Optoelectronic manipulation of spin in semiconductors, in: *Spin electronics* (Springer, 2004)
- [4] Y Ohno, D K Young, B Beschoten, F Matsukura, H Ohno and D D Awschalom, *Nature* **402**, 790 (1999)
- [5] M C de Oliveira and He Bi Sun, *Phys. Rev. B* **69**, 85322 (2004)
- [6] H Fujisaki and A Shimizu, *Phys. Rev. A* **57(4)**, 3074 (1998)
- [7] W W Chow, S W Koch and M Sargent III, Many-body gain, in: *Semiconductor laser physics* (Springer, 1994)
- [8] M Radulaski, *Numerical simulations of electron–phonon interaction in quantum dots*, Diploma thesis (University of Belgrade, 2011)
- [9] S K Tripathy and Mihir Hota, *McMillan Adv. Res. Ser.* **1**, 589 (2009)
- [10] Sukanta Kumar Tripathy and Deepanjali Misra, *Optik* **124**, 2709 (2013)
- [11] R C Myers, A C Gossard and D D Awschalom, *Phys. Rev. B* **69**, 1305 (2004)
- [12] W Loffler, N Hopcke, H Kalt, S F Li, M Grun and M Hetterich, *Appl. Phys. Lett.* **96**, 052113 (2010)