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Behaviour of tunnelling transition rate of argon atom exposed to strong low-frequency elliptical laser field

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Abstract. We considered the tunnelling ionization of an electron under the influence of a monochromatic laser beam with the elliptical polarization. Arbitrary values of ellipticity were observed. The influence of ponderomotive potential and Stark shift on the ionization rate was discussed. A brief description of the dependence of the ponderomotive potential and the Keldysh parameter on the field intensity and ellipticity is given.

Keywords. Tunnel ionization; transition rate.

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1. Introduction

Nonlinear ionization of atoms in the intense light fields has attracted considerable attention from theorists and experimentalists over a long period of time and it is well documented in [1–3]. Tunnelling is one of the most prominent and important processes in the strong laser field ionization of atoms and molecules. Tunnel ionization takes place when the electric field changes the atomic potential and electron sees a very small modified potential and can tunnel through it. Keldysh [4] made the first attempt to perform a general analysis of ionization of atoms by laser radiation as a transition between the initial state in the discrete spectrum and the final state in the continuum of a quantum system in the field of an electromagnetic wave. This investigation showed that the specific features of ionization are determined by the Keldysh parameter, $\gamma = \omega\sqrt{2E_{\rm i}}/F$, where $E_{\rm i}$ is the ionization potential of the considered quantum system, F and ω are the strength and the frequency of the field in the electromagnetic wave, respectively. In the tunnel regime the dimensionless Keldysh parameter is $\gamma \ll 1$. The atomic ionization dynamics in a strong field can successfully be described by the Ammosov–Delone–Krainov (ADK) theory [5] and the strong field approximation (SFA) calculation [4,6,7]. According to the simple strong field ionization models (the ADK), the ionization rate depends mostly on the ionization potential. Because of that, we focus here on transition rate behaviour for different laser field polarizations when the ionization potentials are modified by the ponderomotive potential and Stark shift [8,9].

An interesting aspect of tunnel ionization concerns the role of laser field polarization and the intent of this work was to compare the transition rate for linearly, circularly and elliptically polarized laser fields. The transition rates in linearly, circularly and elliptically polarized laser field are discussed in [10,11] and [12] respectively. Circularly polarized laser pulse gives larger ionization rates than the linearly polarized light which is consistent with the perturbation theory [13].

2. Theory

In this paper we used the ADK model to describe the tunnel ionization of atoms and atomic ions in an alternating electrical field. The atomic units were used throughout the paper, $e = m = \hbar = 1$. There are two laser field polarizations, linear and circular, as limiting cases of elliptical polarization. For the linearly polarized laser field, according to ADK theory, the atomic ionization rate is given by the following formula [11]:

$$W_{\text{lin}} = \sqrt{\frac{3n^{*3}F}{\pi Z^3}} \frac{F}{8\pi Z} \left(\frac{4\text{e}Z^3}{Fn^{*4}}\right)^{n^{*2}} \text{e}^{\left(\frac{-Z}{3n^* F E_{\text{i}}} - \frac{p^2\gamma^3}{3\omega}\right)}, \tag{1}$$

where $n^* = Z/\sqrt{(2E_{\text{i}})}$ is the effective principal quantum number, Z is the charge, F is the laser field strength and E_{i} is the ionization potential.

As discussed earlier, the ionization rate depends on the ionization potential and hence it is modified by taking into account the ponderomotive potential and the Stark effect. If a quantum system is found in a state with energy E_{i} and is perturbed by an external monochromatic field of amplitude F and frequency ω, then the shift of the ionization energy is $\alpha F^2/4$, the so-called Stark shift. This field also causes the oscillating movement of electron. The ponderomotive potential, i.e. the time-average kinetic energy of the electron in the transverse oscillatory motion of velocity v_{osc}, is then readily calculated as $U_{\text{pl}} = \overline{v_{\text{osc}}^2/2} = F^2/4\omega^2$ [14] for linearly polarized laser field. The bar denotes the time-average over one cycle of the field, $T = 2\pi/\omega$.

The corrected ionization potential can be written as

$$E_{\text{i}} = K_{U_{\text{pl}}S_t}\omega - \left(\frac{F^2}{4\omega^2} + \frac{(\alpha F^2)}{4}\right), \tag{2}$$

where α is the static polarizability of the atom. The experimental data for polarizability of atoms and ions can be found in [15]. $K_{U_{\text{pl}}S_t}$ is the minimum number of photons necessary for the ionization of atom when ponderomotive potential and the Stark effect are incorporated in the ionization rate.

Substituting eq. (2) into eq. (1) and taking into account the nonzero initial momentum of the ejected photoelectrons, we should rewrite the atomic tunnelling ionization rate as

$$W_{\text{lin}}^{p,U_{\text{pl}},S_t} = \sqrt{\frac{3n^{*3}F}{\pi Z^3}} \frac{F}{8\pi Z} \left(\frac{4\text{e}Z^3}{Fn^{*4}}\right)^{n^{*2}} \text{e}^{\left(\frac{-Z}{3n^* F\left(K_{U_{\text{pl}}S_t}\omega - \frac{p^2}{2} - \left(\frac{F}{2\omega}\right)^2 - \frac{\alpha F^2}{4}\right)} - \frac{p^2\gamma^3}{3\omega}\right)}. \tag{3}$$

$W_{\text{lin}}^{p,U_{\text{pl}},S_t}$ indicates that the initial momentum, ponderomotive potential and Stark shift are included in the expression for transition rate. Here $p = \frac{1}{2}(\sqrt{(F\eta - 1} + \frac{1}{\eta\sqrt{F\eta - 1}})$

is the electron momentum [1], expressed using the field strength F and the parabolic coordinate η.

For the circularly polarized light, according to ADK theory, the ionization rate differs from the rate for the linearly polarized light by the factor $\sqrt{3n^{*3}F/\pi Z^3}$ and has a simpler form:

$$W_{\mathrm{cir}}^{p,U_{\mathrm{pc}},S_t} = \frac{F}{8\pi Z}\left(\frac{4\mathrm{e}Z^3}{Fn^{*4}}\right)^{n^{*2}} \mathrm{e}^{\left(\frac{-Z}{3n^* F\left(K_{U_{\mathrm{pc}}S_t}\omega - \frac{p^2}{2} - \frac{1}{2}\left(\frac{F}{\omega}\right)^2 - \frac{\alpha F^2}{4}\right)} - \frac{p^2\gamma^3}{3\omega}\right)}. \tag{4}$$

$W_{\mathrm{cir}}^{p,U_{\mathrm{pc}},S_t}$ is the transition rate for circular polarization of the laser field with incorporated nonzero initial momentum of the photoelectron, the ponderomotive potential and the Stark shift. Ponderomotive potential for the circular polarized laser filed is given as $U_{\mathrm{pc}} = F^2/2\omega^2$ [14].

Finally, we consider the general case of a monochromatic wave with elliptical polarization $F(t) = F(\overrightarrow{e_x}\cos\omega t \pm \epsilon\overrightarrow{e_y}\sin\omega t)$ with the polarization vector $\epsilon = \overrightarrow{e_x}\cos(\xi/2) + i\overrightarrow{e_y}\sin(\xi/2)$. The ellipticity parameter ξ describes all degrees of elliptical polarization and when varied in the range $\pi/2 \le \xi \le \pi/2$ ($\xi = 0$ and $\xi = \pi/2$ correspond to linear and circular polarization, respectively), i.e. the value of ϵ is in the interval $0 \le \epsilon \le 1$, for $\epsilon = 0$ the wave is linearly, and for $\epsilon = \pm 1$, circularly polarized. In this case the atomic ionization rate is [16]

$$W_{\mathrm{elip}} = \left(\frac{\epsilon(1+\epsilon)}{2}\right)^{-1/2} a\left(\frac{1-\epsilon}{3\epsilon}\frac{(2E_{\mathrm{i}})^{3/2}}{F}\right)\frac{F}{8\pi Z}\left(\frac{4\mathrm{e}Z^3}{Fn^{*4}}\right)^{n^{*2}} \mathrm{e}^{\left(\frac{-Z}{3n^* F E_{\mathrm{i}}}\right)}, \tag{5}$$

where $a(x) = \mathrm{e}^{-x}J_0(x)$. Here $J_0(x)$ is the Bessel function of the imaginary argument and $a(x)$ is a monotonically decreasing function: $a(0) = 1$, $a(x) \sim (2\pi x)^{-1/2}$ for $x \gg 1$.

For the elliptical polarization of the laser field, the ponderomotive potential has the following form [17]:

$$U_{\mathrm{pe}} = U_{\mathrm{pc}}\frac{1-\epsilon^2}{1+\epsilon^2} = \frac{F^2}{2\omega^2}\frac{1-\epsilon^2}{1+\epsilon^2}. \tag{6}$$

From eqs (5) and (6) it follows that

$$W_{\mathrm{elip}} = \left(\frac{3}{\pi}\frac{1}{1-\epsilon^2}\frac{F}{(2E_{\mathrm{i}})^{3/2}}\right)^{1/2}\frac{F}{8\pi Z}\left(\frac{4\mathrm{e}Z^3}{Fn^{*4}}\right)^{n^{*2}} \mathrm{e}^{\left(\frac{-Z}{3n^* F E_{\mathrm{i}}}\right)}. \tag{7}$$

Repeating the same procedure, we corrected the ionization potential and obtained:

$$W_{\mathrm{elip}}^{p,U_{\mathrm{pe}},S_t} = \left(\frac{3}{\pi}\frac{1}{1-\epsilon^2}\frac{F}{(2E_{\mathrm{i}})^{3/2}}\right)^{1/2}\frac{F}{8\pi Z}\left(\frac{4\mathrm{e}Z^3}{Fn^{*4}}\right)^{n^{*2}} \times \mathrm{e}^{\left(\frac{-Z}{3n^* F\left(K_{U_{\mathrm{pe}}S_t}\omega - \frac{p^2}{2} - \frac{1}{2}\left(\frac{F}{\omega}\right)^2\frac{1-\epsilon^2}{1+\epsilon^2} - \frac{\alpha F^2}{4}\right)} - \frac{p^2\gamma^3}{3\omega}\right)}. \tag{8}$$

$W_{\mathrm{elip}}^{p,U_{\mathrm{pe}},S_t}$ denotes the tunnelling ionization rate for elliptical polarization of the laser field with corrected ionization potential. We emphasize that the Stark shift has the same form for all three polarizations.

3. Analysis

In this paper, we analysed the tunnelling ionization rate of the single ionized argon, Ar, using single-colour linearly, circularly and elliptically polarized laser pulses obtained by CO_2 laser, $\omega = 0.004298$ a.u.

Nonlinear ionization is a complex process which depends on three parameters – the laser frequency ω, the electric field strength F and the ionization potential $E_{\rm i}$. For fixed laser frequency ω, we observed how the ionization rate depends on the intensity of laser radiation and the ionization potential. The Keldysh parameter approaches zero, $\gamma = 0.01$ where tunnel ionization is assumed to dominate. The polarizability is $\alpha = 11.1$.

We performed a calculation to test how the correction of the ground ionization potential changes the transition rate for different polarizations of laser field. To do this we assumed that Ar atoms were exposed to $\lambda = 10.6$ μm laser pulse with the laser field intensities in the range 10^{12} W cm^{-2}–10^{14} W cm^{-2} and the ellipticity $\epsilon = 0.83$.

In figure 1 (2D and 3D graphs), we plotted values of the atomic ionization rates $W_{\rm elip}^{p,U_{\rm pe},S_t}$ (see eq. (8)) by including momentum, ponderomotive potential and Stark shift respectively over a range of laser intensities 10^{12} W cm^{-2} $< I <$ 10^{14} W cm^{-2} for elliptical polarization of the laser field.

It is seen from figures 1a and 1b, that for an elliptically polarized laser field, the transition rate $W_{\rm elip}^{p,U_{\rm pe},S_t}$ (see eq. (8)) monotonously increases as a function of laser filed intensity until it reaches the maximum value and then monotonically decreases and asymptotically approaches the axis. Figures 1a and 1b also show that the maximum value of the atomic ionization rates $W_{\rm elip}$ decreases by including the initial momentum of the outgoing photoelectron, its ponderomotive potential and the Stark effect.

To estimate the influence of ionization rate corrections we compared the numerical results of the ionization rate obtained by using formulas for elliptically polarized field (eq. (8)). The maximum values of the transition rates and the corresponding laser filed intensities are shown in table 1.

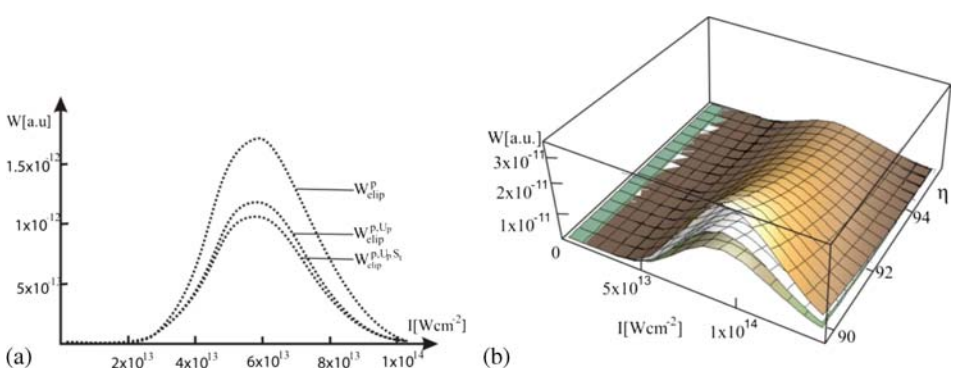

Figure 1. (**a**) The tunnelling ionization rates $W_{\rm elip}^{p}$, $W_{\rm elip}^{p,U_{\rm pe}}$ and $W_{\rm elip}^{p,U_{\rm pe},S_t}$ by taking into account the initial momentum of the ejected photoelectron, ponderomotive potential and Stark shift respectively, for the fixed parabolic coordinate at $\eta = 100$. (**b**) 3D graph which shows the dependence of $W_{\rm elip}^{p}$, $W_{\rm elip}^{p,U_{\rm pe}}$ and $W_{\rm elip}^{p,U_{\rm pe},S_t}$ as a function of the parabolic coordinate $\eta(90, 95)$. Field intensity $I = 10^{12}$–10^{14} W cm^{-2}.

Table 1. Maximum values of the atomic tunnelling ionization rates $W^{p}_{\text{elip,max}}$, $W^{p,U_{\text{pe}}}_{\text{elip,max}}$, $W^{p,U_{\text{pe}},S_t}_{\text{elip,max}}$ and the corresponding laser field intensities.

The laser field intensity	The transition rates for elliptically polarized laser field (a.u.)		
	$W^{p}_{\text{max,elip}}$	$W^{p,U_{\text{pe}}}_{\text{max,elip}}$	$W^{p,U_{\text{pe}},S_t}_{\text{max,elip}}$
	1.83237×10^{-12}	1.25514×10^{-12}	1.03943×10^{-12}
$I\ (\text{W cm}^{-2})$	5.6×10^{13}	5.6×10^{13}	5.5×10^{13}

As can be seen from table 1, inclusion of initial momentum, ponderomotive potential and Stark shift moves the maximum of the corresponding ionization rate through the lower intensities of the laser field. Figure 1b (3D graph) demonstrates $W^{p,U_{\text{pe}},S_t}_{\text{elip}}$ as a function of the parabolic coordinate $\eta(90, 95)$.

Similar behaviour shows curves for the transition rate for linear and circular polarization field. Now we focussed on ionization rate for elliptically polarized laser field $W^{p,U_{\text{pe}},S_t}_{\text{elip}}$. To estimate the influence of ellipticity ϵ we plotted the ionization rate as a function of field intensity, I, and ellipticity ϵ. The result is presented in figure 2.

Figures 2a and 2b (2D and 3D graphs) show how the atomic tunnelling ionization rate, $W^{p,U_{\text{pe}},S_t}_{\text{elip}}$ depends on field intensity, I for different values of the ellipticity ϵ. Note that $W^{p,U_{\text{pe}},S_t}_{\text{elip}}$ increases as ϵ increases for the same laser field intensity. Figure 2b shows various ionization rates for the elliptically polarized laser field as a function of the field amplitude up to $F = 10^{12}$ a.u., for different values of ϵ. In table 2, we gave maximum values of tunnelling ionization rates for each ϵ.

In addition, we used eq. (6) to calculate values of the ponderomotive potential for elliptically polarized laser field. We analysed the ponderomotive potential of the system as a function of ellipticity ϵ, for the fixed laser field intensity, $I = 10^{14}$ W cm^{-2} and results are shown in figure 3 (2D and 3D graphs).

It is evident that as ellipticity ϵ increases the ponderomotive potential decreases.

The comparative review of the atomic tunnelling photoionization rates for all field polarizations is given in figure 4. The transition rate is usually a theoretically analysed value,

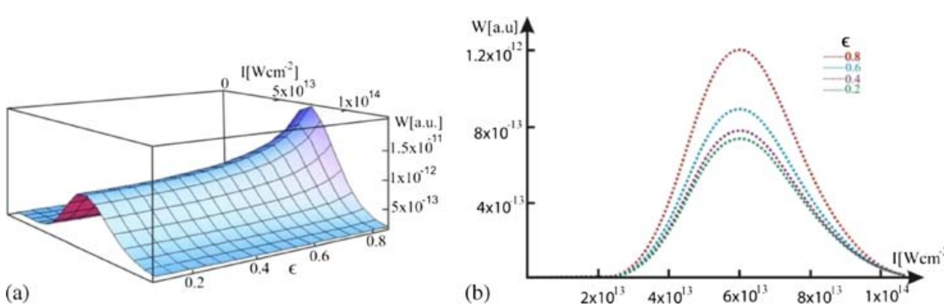

Figure 2. The dependence of $W^{p,U_p,S_t}_{\text{elip}}$ on ellipticity $\epsilon(0.1, 0.9)$. **(a)** 3D graph, **(b)** 2D graph. For both graphs the range of laser field intensity $I = 10^{12}$–10^{14} W cm^{-2}.

Table 2. The maximum value of $W_{\mathrm{elip}}^{p,U_{\mathrm{pe}},S_t}$ as a function of laser field intensity, $I = 10^{12}$–10^{14} W cm^{-2} for different values of $\epsilon(0.1, 0.9)$.

Maximum value of transition rate (a.u.)	Ellipticity ϵ	Laser field intensity $\left(\mathrm{W\,cm^{-2}}\right)$
7.14506×10^{-13}	02	5.6×10^{13}
7.73839×10^{-13}	04	
8.75088×10^{-13}	06	
1.16678×10^{-12}	08	

because the ion yield and cross-sections are generally measured in experiments. Fewer experimental results are available for argon and the other noble gases and our theoretically obtained results can be compared with them [18,19]. We can see that the graphs given in figure 4, where the transition rate is given as a function of electron energy, are convenient for this purpose. The intensity axis can be transformed into units of energy. In a limited case, the energy shift of the continuum is equal to the ponderomotive energy, or the cycle averaged kinetic energy of a free electron in an oscillating electric field, $\Delta E_\infty = U_{\mathrm{p}}$. For a peak intensity, I, in W cm^{-2} and wavelength, λ, in μm, U_{p} can be estimated in electron volts (eV) by the relation $U_{\mathrm{p}} = 9.33 \times 10^{-14} I\lambda^2$ [18]. Now we can make direct comparison between the ionization rate and the cross-section [13,18].

Figure 4a shows that the atomic tunnelling ionization rates W_{lin}, W_{cir}, W_{elip} increase monotonically from zero and reach a slowly rising plateau around the electron energy $E = 840$ eV. The appearance of the ‘plateau’ is due to the fact that maximum number of photoelectrons is ejected under the observed conditions. Here we neglected the initial photoelectron momentum and corrections of the ionization potential (see eq. (7)). Figures 4b, 4c and 4d show the tunnelling rate with corrected ionization potential. It is obvious that all curves have the Gaussian form and that incorporation of the ponderomotive potential and Stark shift leads to a decrease of the transition rate intensity for each case of laser polarization. The presence of the characteristic asymmetric slope through higher electron energies can be observed. It is clear that inclusion of the additional parameters into formulas for the transition rate is important. Also we can see that the transition

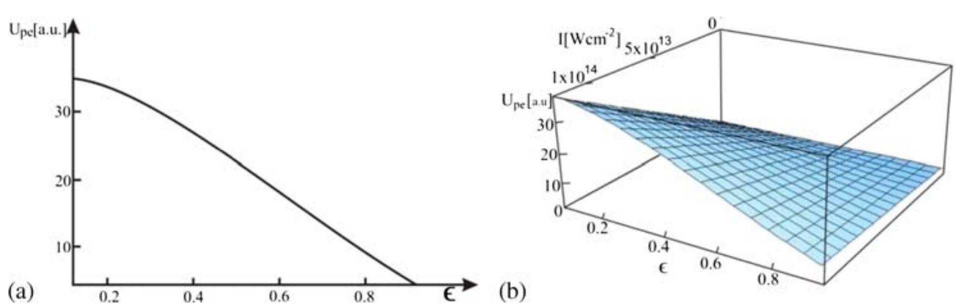

Figure 3. The ponderomotive potential, U_{pe} (eq. (6)) as a function of ellipticity $\epsilon(0.1, 0.9)$ (**a**) for fixed laser field intensity, $I = 10^{14}$ W cm^{-2}, **(b)** over the range $I = 10^{12}$–10^{14} W cm^{-2}.

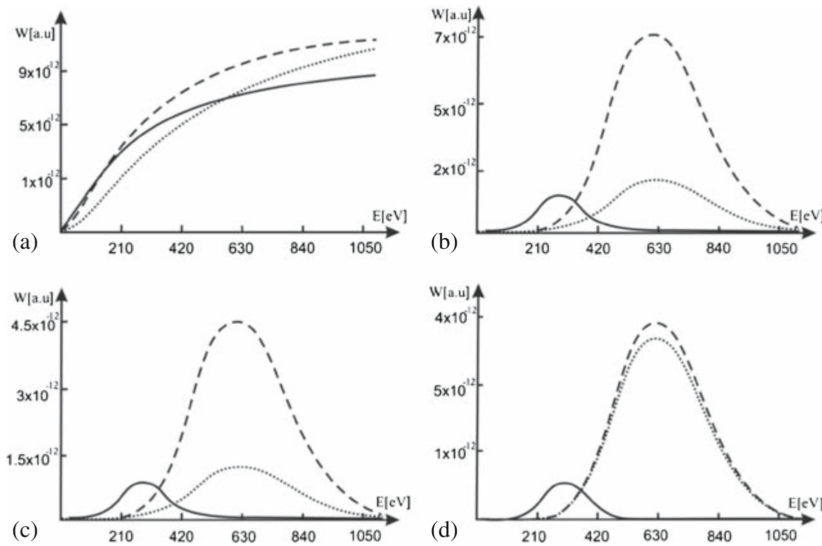

Figure 4. Graphs of the atomic ionization rate as a function of energy: **(a)** With ground ionization potential: W_{lin}, W_{cir}, W_{elip}, **(b)** with initial momentum of the photoelectron: W_{lin}^{p}, W_{cir}^{p}, W_{elip}^{p}, **(c)** with ponderomotive potential included: $W_{\text{lin}}^{p,U_{\text{pl}}}$, $W_{\text{cir}}^{p,U_{\text{pi}}}$, $W_{\text{elip}}^{p,U_{\text{pe}}}$, **(d)** by including ponderomotive potential and linear Stark shift: $W_{\text{lin}}^{p,U_{\text{pl}},S_t}$, $W_{\text{cir}}^{p,U_{\text{pc}},S_t}$, $W_{\text{elip}}^{p,U_{\text{pc}},S_t}$. For all graphs solid curves denote linear polarization, dashed curves denote circular polarization and dotted curves denote elliptical polarization.

rates have higher values for circularly polarized field and lowest for linearly polarized ones. The value for elliptically polarized field depends upon ellipticity ϵ.

In figure 5 (3D graph) which corresponds to figure 4d, we displayed transition rates obtained from all polarizations of laser field and with all corrections (see eqs (3), (4) and (8)), $W_{\text{lin}}^{p,U_{\text{pl}},S_t}$, $W_{\text{cir}}^{p,U_{\text{pc}},S_t}$ and $W_{\text{elip}}^{p,U_{\text{pe}},S_t}$ as a function of laser field intensity and the parabolic coordinate η.

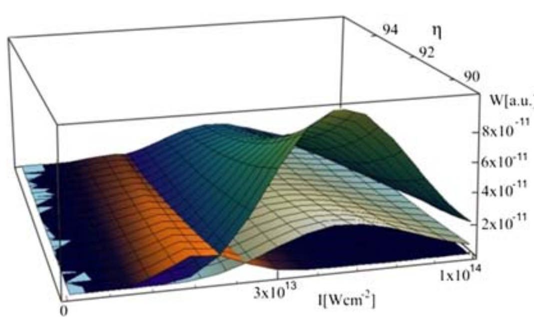

Figure 5. 3D graph for $W_{\text{lin}}^{p,U_{\text{pl}},S_t}$, $W_{\text{cir}}^{p,U_{\text{pc}},S_t}$ and $W_{\text{elip}}^{p,U_{\text{pe}},S_t}$ over the range $I = 10^{12}$–10^{14} W cm^{-2} and parabolic coordinate η(90,95).

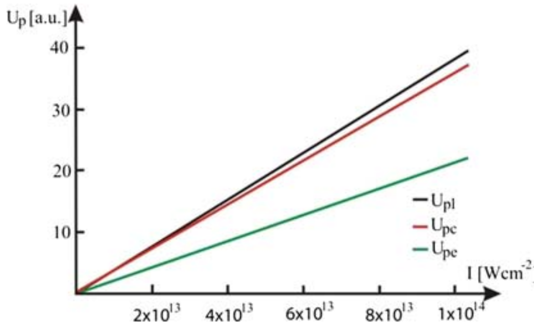

Figure 6. The comparative review of ponderomotive potentials for linear, circular and elliptical laser field polarization over the laser field intensities $I = 10^{12}$–10^{14} W cm^{-2} and $\epsilon = 0.5$.

Next we gave a comparative review of ponderomotive potentials for linear, circular and elliptical polarizations over the same laser field intensities:

The comparisons shown in figure 6 suggest that the ponderomotive potential has the largest value for linearly polarized laser field and hence has the largest influence on the corresponding transition rate. The value of ponderomotive potential for elliptically polarized laser field depends upon the ellipticity ϵ.

Finally, we gave a brief note about Keldysh parameter. As we already mentioned, the Keldysh parameter can be expressed as $\gamma = \omega\sqrt{2E_{\text{i}}}/F$ but it can also be expressed as a function of the ponderomotive potential $\gamma_{\text{l}} = \sqrt{E}_{\text{i}}/2U_{\text{pi}}$ for linear, $\gamma_{\text{c}} = \sqrt{E}_{\text{i}}/U_{\text{pc}}$ for circular and $\gamma_{\text{e}} = \sqrt{E}_{\text{i}}/\left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)U_{\text{pc}}$ for elliptical polarization of the laser field. For ground-state ionization potential of argon, $E_{\text{i}} = 0.5791$ a.u. we obtained the graph shown in figure 7.

From figure 7 it can be seen that for the same values of E_{i} but for different polarizations of laser field Keldysh parameter γ has different values.

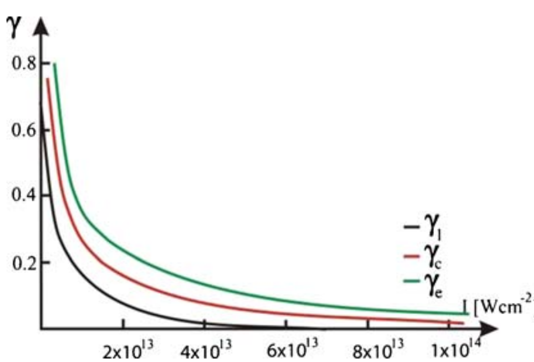

Figure 7. The Keldysh parameters γ_{l}, γ_{c} and γ_{e} as functions of the ponderomotive potentials for different polarizations of laser field over the field intensities, $I = 10^{12}$–10^{14} W cm^{-2}.

4. Conclusion

We considered the tunnelling ionization of argon atoms, Ar, under the influence of an elliptical monochromatic laser field. Also the comparative review of the tunnelling transition rates obtained by linearly, circularly and elliptically polarized CO_2 laser pulses was presented. The influence of the initial momentum of ejected photoelectrons, ponderomotive potentials and the Stark shift was discussed. Our results showed that the incorporation of this effect decreases the values of transition rates. Also it was shown that the transition rate for elliptical polarization laser field depends on the value of parabolic coordinate η as well as the ellipticity ϵ. When the parabolic coordinate increases, the transition rate decreases and when the ellipticity increases, the transition rate increases.

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