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On the stabilization of modulus in Randall–Sundrum model by $R\Phi^2$ interaction

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Abstract. A solution to the problem of modulus stabilization is to couple a massless bulk scalar field non-minimally to five-dimensional curvature. We present an exact treatment of the stabilization condition. Our results show that the square of effective mass of this scalar field is necessarily negative. We also find the existence of a closely spaced maximum near the minimum of the effective potential.

Keywords. Field theories in higher dimensions; Randall–Sundrum model; modulus stabilization.

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1. Introduction

To explain the large hierarchy between the weak scale and the Planck scale, many theories such as supersymmetry and higher-dimensional theories have been proposed. One of these attempts, Randall–Sundrum I [1], explains this hierarchy in terms of a small extra dimension. This proposal involves a 'Planck brane' and a 'TeV brane' and the space between the branes is a slice of anti-de Sitter space. By solving the five-dimensional Einstein equations, one obtains the metric for this space as

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu - r^2 d\varphi^2, \tag{1}$$

where

$$\sigma = kr|\varphi| \qquad \text{and} \qquad \eta_{\mu\nu} = \text{diag}[-1, 1, 1, 1], \tag{2}$$

$-\pi < \varphi < \pi$ is the extra-dimensional coordinate, r is the compactification radius and k is a parameter which is assumed to be of order 5d Planck scale, M.

The problem of stability of this extra dimension was addressed by Goldberger and Wise (GW) in ref. [2]. Their solution involved a massive bulk scalar field with the usual kinetic term in the bulk and quartic interactions localized on the two branes. Since then, many studies have appeared on this subject [3–12]. The studies of refs [3,4] consider models for the stabilization of the modulus containing a bulk scalar field interacting with the space-time curvature R.

Grzadkowski and Gunion [3] considered a class of generalizations of the Randall–Sundrum model containing a bulk scalar field Φ, interacting with the curvature R through the general coupling $Rf(\Phi)$. They showed that by choosing a non-trivial background for the bulk scalar field it is possible to neglect the effect of the metric back-reaction, and they obtained the general form of the scalar potential $V(\Phi)$.

Granda and Oliveros [4] considered the case of a massless scalar field but with non-minimal interaction with the curvature R. In this work, by a suitable choice of the parameter, one can neglect the effect of back-reaction of the scalar field on background geometry. Their work essentially corresponds to the work of ref. [3], but with $V(\Phi) = 0$. However, the discussion in refs [4,5] are related to infinitely large quartic coupling.

In this work, we present an exact treatment of the Granda–Oliveros model. An exact analysis of the GW mechanism is discussed in ref. [6]. The plan of this paper as follows: In §2, we describe the model, obtain the effective potential, express the extremization condition for this effective potential and obtain the value of the stabilized modulus. We also show that the square of the effective mass of the bulk scalar field is negative. In §3, we study the stability of the modulus r. In the limit of infinite quartic coupling our results are in agreement with previous results [5]. We also investigate the case where the quartic coupling is finite but very large, and finally in §4, we present our conclusions.

2. Effective potential

The action of the model is of the form:

$$S = S_{\text{gravity}} + S_{\text{vis}} + S_{\text{hid}} + S_{\Phi}, \tag{3}$$

where

$$S_{\text{gravity}} = \int \mathrm{d}^4x \int_{-\pi}^{\pi} \sqrt{G}[2M^3R - \Lambda], \tag{4}$$

$$S_{\text{vis}} = \int \mathrm{d}^4x \int_{-\pi}^{\pi} \sqrt{-g_s}[L_s - V_s], \quad S_{\text{hid}} = \int \mathrm{d}^4x \int_{-\pi}^{\pi} \sqrt{-g_p}[L_p - V_p], \tag{5}$$

$$\begin{aligned} S_{\Phi} = & \int \mathrm{d}x^4 \int_{-\pi}^{\pi} \mathrm{d}\phi \sqrt{G}(G^{MN}\partial_M\Phi\partial_N\Phi - \xi R\Phi^2) \\ & - \int \mathrm{d}x^4 \sqrt{-g_s}\lambda_s(\Phi^2 - v_s^2)^2 - \int \mathrm{d}x^4 \sqrt{-g_p}\lambda_p(\Phi^2 - v_p^2)^2, \end{aligned} \tag{6}$$

where Λ is the five-dimensional cosmological constant, V_s, V_p are the visible and hidden brane tensions, $G = \det[G_{MN}]$, R is the bulk curvature for the metric (1) and is given by

$$R = \frac{20\sigma' - 8\sigma''}{r^2}, \tag{7}$$

where $\sigma' = \partial_\phi\sigma$, $\sigma'' = 2kr[\delta(\phi) - \delta(\phi - \pi)]$.

The ϕ-dependent vacuum expectation value $\Phi(\phi)$ is obtained from the equation of motion

$$\partial_\phi(e^{-4\sigma}\partial_\phi\Phi) = \xi R r^2 e^{-4\sigma}\Phi + 4e^{-4\sigma}\lambda_s r\Phi(\Phi^2 - v_s^2)\delta(\phi-\pi) + 4e^{-4\sigma}\lambda_p r\Phi(\Phi^2 - v_p^2)\delta(\phi). \tag{8}$$

Away from the boundaries ($\phi = 0, \pi$) the solution is

$$\Phi(\phi) = Ae^{(\nu+2)\sigma} + Be^{(-\nu+2)\sigma}, \tag{9}$$

where $\nu = \sqrt{4+20\xi}$. If we insert this solution in eq. (6) and integrate over ϕ, we obtain the effective four-dimensional potential, $V_\Phi(r)$, for the modulus r, which is given by

$$V_\Phi(r) = k(\nu+a)A^2(e^{2\nu kr\pi}-1) + k(\nu-a)B^2(1-e^{-2\nu kr\pi}) + \lambda_s e^{-4kr\pi}(\Phi^2(\pi)-v_s^2)^2 + \lambda_p(\Phi^2(0)-v_p^2)^2. \tag{10}$$

Here $a = 2+8\xi$. The coefficients A and B are determined by imposing appropriate boundary conditions on the 3-branes. We obtain these boundary conditions by inserting eq. (9) into the equations of motion and matching the delta functions. The results are

$$k[(a+\nu)A + (a-\nu)B] - 2\lambda_p\Phi(0)[\Phi^2(0)-v_p^2] = 0 \tag{11}$$

and

$$ke^{2kr\pi}[(a+\nu)e^{\nu kr\pi}A + (a-\nu)e^{-\nu kr\pi}B] + 2\lambda_s\Phi(\pi)[\Phi^2(\pi)-v_s^2] = 0. \tag{12}$$

In a previous work [5], we considered the limit of $\lambda_p \to \infty$, $\lambda_s \to \infty$. In this limit $\Phi(0) = v_p$ and $\Phi(\pi) = v_s$. In order to investigate the case of finite quartic coupling, we must calculate the first and second derivatives of the effective potential. By using eqs (11), (12) and after a lengthy calculation we get

$$\frac{dV_\Phi(r)}{dr} = -4k^2\pi[(a+\nu)e^{2\nu kr\pi}A^2 + (a-\nu)e^{-2\nu kr\pi}B^2 + (2a-\nu^2)AB] - 4k\pi e^{-4kr\pi}(\Phi^2(\pi)-v_s^2)^2. \tag{13}$$

From eq. (13) we obtain a simple form for the second derivative of the potential which is given by

$$\frac{dV_\Phi^2(r)}{dr^2} = 4k^2\pi\nu\left[(2+\nu)A\frac{dB}{dr} + (\nu-2)B\frac{dA}{dr}\right]. \tag{14}$$

In obtaining the above result we used the extremization condition $(dV_\Phi(r)/dr) = 0$.

If we denote $\Phi(\phi = 0) = Q_p(r)$ and $\Phi(\phi = \pi) = Q_s(r)$, then from eq. (9) we can express the coefficients A and B as

$$A = \frac{Q_s(r)e^{-2\sigma} - Q_p(r)e^{-\nu\sigma}}{2\sinh(\nu\sigma)}, \tag{15}$$

$$B = \frac{Q_p(r)e^{\nu\sigma} - Q_s(r)e^{-2\sigma}}{2\sinh(\nu\sigma)}. \tag{16}$$

By substituting these expressions in eqs (11), (12), we get

$$\frac{\nu}{2\sinh(\nu\sigma)}\left(\mathrm{e}^{-2\sigma}-\left(\frac{a+\nu}{2\nu}\mathrm{e}^{-\nu\sigma}+\frac{\nu-a}{2\nu}\mathrm{e}^{\nu\sigma}\right)\right) = \frac{2\lambda_p}{k}\frac{Q_p}{Q_s}(Q_p^2-v_p^2), \tag{17}$$

$$\frac{\nu}{2\sinh(\nu\sigma)}\left(\frac{Q_p}{Q_s}-\left(\frac{a+\nu}{2\nu}\mathrm{e}^{(\nu-2)\sigma}+\frac{\nu-a}{2\nu}\mathrm{e}^{-(\nu+2)\sigma}\right)\right) = \frac{2\lambda_s}{k}(Q_s^2-v_s^2)\mathrm{e}^{-2\sigma}. \tag{18}$$

By inserting eqs (15), (16), (18) into eq. (13) under extremization condition we get (for $\lambda_s \neq 0$)

$$\frac{k}{\lambda_s Q_s^2}\left[x-\frac{4\xi}{\nu}(\mathrm{e}^{(\nu-2)\sigma}-\mathrm{e}^{(\nu+2)\sigma})\right]^2+x^2=\tilde{C}^2, \tag{19}$$

where

$$x = \frac{Q_p}{Q_s}-\frac{2+\nu}{2\nu}\mathrm{e}^{(\nu-2)\sigma}-\frac{\nu-2}{2\nu}\mathrm{e}^{-(\nu+2)\sigma},$$

$$\tilde{C} = \left[\frac{2+\nu}{2\nu}\mathrm{e}^{(\nu-2)\sigma}+\frac{\nu-2}{2\nu}\mathrm{e}^{-(\nu+2)\sigma}\right]C \tag{20}$$

and

$$C=\sqrt{1-\frac{4[(a+\nu)\mathrm{e}^{2(\nu-2)\sigma}-\mathrm{e}^{-4\sigma}(2a-\nu^2)+(a-\nu)\mathrm{e}^{-2(\nu+2)\sigma}]}{[(2+\nu)\mathrm{e}^{(\nu-2)\sigma}+(\nu-2)\mathrm{e}^{-(\nu+2)\sigma}]^2}}. \tag{21}$$

It is easy to obtain the variable x from the quadratic eq. (19) and by some manipulation we obtain

$$kr = \frac{1}{\pi(2-\nu)}\ln\left(\left(\frac{2+\nu}{2\nu}+\frac{\nu-2}{2\nu}\mathrm{e}^{-2\nu\sigma}\right)\times\left(1+\frac{kb\pm\sqrt{k\lambda_s Q_s^2(C^2-b^2)+\lambda_s^2Q_s^4C^2}}{k+\lambda_s Q_s^2}\right)\left(\frac{Q_s(r)}{Q_p(r)}\right)\right), \tag{22}$$

where

$$b=\frac{8\xi(\mathrm{e}^{(\nu-2)\sigma}-\mathrm{e}^{-(\nu+2)\sigma})}{(2+\nu)\mathrm{e}^{(\nu-2)\sigma}+(\nu-2)\mathrm{e}^{-(\nu+2)\sigma}}. \tag{23}$$

Expression for kr in eq. (22) is valid for any value of the quartic coupling constant.

We note that in the large kr limit $C \sim (\sqrt{-12\xi}/(\nu+2))$. Therefore, in order to have a meaningful result, the coupling constant ξ must be negative. This in turn implies that the square of the effective mass of the bulk scalar field is negative.

To compare this result with the corresponding result for GW mechanism, we note that from ref. [6]

$$C_{\mathrm{GW}} \sim \sqrt{\frac{\nu_{\mathrm{GW}}-2}{\nu_{\mathrm{GW}}+2}}, \quad \nu_{\mathrm{GW}}=\sqrt{4+\frac{m^2}{k^2}}. \tag{24}$$

Hence in the GW mechanism, the effective mass squared of the bulk scalar field is strictly positive.

Utilizing the above results, we calculate the second derivative of the effective potential which is given by

$$\begin{aligned}\frac{\mathrm{d}V_\Phi^2(r)}{\mathrm{d}r^2} &= -\frac{4k\pi\nu \mathrm{e}^{-2\sigma}}{\sinh(\nu\sigma)}\Bigg[(\lambda_p(Q_p^2 - v_p^2) - 4k\xi)Q_p Q_s' + (\lambda_s(Q_s^2 - v_s^2) \\ &\quad + 4k\xi)Q_s Q_p' + 2\pi\lambda_p\lambda_s(Q_s^2 - v_s^2)(Q_p^2 - v_p^2)Q_p Q_s \\ &\quad + \frac{4k^2\nu\pi\xi}{\sinh(\nu\sigma)}(\mathrm{e}^{-2\sigma}Q_s^2 - \mathrm{e}^{2\sigma}Q_p^2) + 16k^2\pi\xi(1+2\xi)Q_p Q_s\Bigg]. \end{aligned} \tag{25}$$

Here 'prime' denotes derivative with respect to r. In order to have a negligible back-reaction of the scalar field on the background geometry, we require $v_s, v_p \ll M^{3/2}$ and $\xi \ll 1$. Hence we can neglect the stress tensor for the scalar field in comparison to the stress tensor induced by the bulk cosmological constant [4].

3. Stability of the modulus

To investigate the stability of the modulus r we consider two different cases.

Case I. $\lambda_p \to \infty, \lambda_s \to \infty$.

In the discussion of this case for the GW mechanism of ref. [6], the value of second derivative of the effective potential for $Q_s = v_s$, $Q_p = v_p$, $Q_s' = 0$ and $Q_p' = 0$, was identically zero, hence they had to resort to an asymptotic analysis. But for our case eq. (25) has a more complex structure than its GW counterpart. So a direct analysis is possible.

In this limit from eq. (22), we obtain (in the large kr limit)

$$kr = \frac{1}{\pi(\nu-2)}\ln\left[\left(\frac{v_p}{v_s}\right)\frac{2\nu}{\nu+2\pm\sqrt{-12\xi}}\right]. \tag{26}$$

Moreover, by using eq. (26), the second derivative becomes

$$\frac{\mathrm{d}V_\Phi^2(r)}{\mathrm{d}r^2} = \frac{16k^3\pi^2\nu\xi v_p v_s\sqrt{-3\xi}\,\mathrm{e}^{-2\sigma}}{\sinh(\nu\sigma)}[\sqrt{-3\xi}\pm 2]. \tag{27}$$

In order to have meaningful results for the modulus, v_p and v_s must have similar signs. Hence, for

$$kr_- = \frac{1}{\pi(\nu-2)}\ln\left[\left(\frac{v_p}{v_s}\right)\frac{2\nu}{\nu+2-\sqrt{-12\xi}}\right] \tag{28}$$

$(\mathrm{d}V_\Phi^2(r)/\mathrm{d}r^2) > 0$. That means kr_- corresponds to the value of the stable modulus. This result agrees with our first-order calculations reported in ref. [5]. For the configuration $v_p = 0.135$, $v_s = 1$, $k = 4$ and $\xi = -0.01$, the value of $kr_- = 12.2$.

Case II. λ_p and λ_s are finite but very large.

In this case, from eq. (17) we find that the value of Q_p is lower than v_p and in the limit of $\lambda_p \to \infty$ approaches v_p. Similarly, if $v_p/v_s > 1$ then from eq. (18) we find that the value of Q_s is higher than v_s and in the limit of $\lambda_s \to \infty$ approaches v_s. Hence it is appropriate to consider a $1/\lambda$ expansion of boundary scalar field. From eqs (17), (18) we get

$$Q_p(r) = v_p + \frac{k}{\lambda_p v_p} \frac{\nu e^{-2\sigma}}{4\sinh(\nu\sigma)} \times \left(\frac{v_s}{v_p} - \left(\frac{2+\nu}{2\nu} e^{(2-\nu)\sigma} + \frac{\nu-2}{2\nu} e^{(\nu+2)\sigma} \right) \right), \tag{29}$$

$$Q_s(r) = v_s + \frac{k}{\lambda_s v_s} \frac{\nu e^{2\sigma}}{4\sinh(\nu\sigma)} \times \left(\frac{v_p}{v_s} - \left(\frac{2+\nu}{2\nu} e^{(\nu-2)\sigma} + \frac{\nu-2}{2\nu} e^{-(\nu+2)\sigma} \right) \right). \tag{30}$$

Now by using eqs (29), (30) we obtain a modified expression for the modulus (in the large kr limit)

$$kr = \frac{1}{\pi(\nu-2)} \ln\left(\frac{2\nu}{2+\nu} \frac{n}{1 \pm \frac{\sqrt{-12\xi}}{\nu+2}(1-\frac{q}{2})} \times \left(1 - \frac{t(\nu-2)}{4} + \frac{q(\nu+2)}{4} - \frac{q\nu n}{2} e^{(2-\nu)k\pi r} \right) \right), \tag{31}$$

where

$$n = \frac{v_p}{v_s}, \quad t = \frac{k}{\lambda_p v_p^2}, \quad q = \frac{k}{\lambda_s v_s^2}. \tag{32}$$

4. Conclusions

We have utilized a massless bulk scalar field with non-minimal coupling to five-dimensional Ricci scalar to stabilize the size of extra dimension in the Randall–Sundrum model. We have assumed the value of the coupling $\xi \ll 1$. Hence there is no need to consider the back-reaction of the scalar field on the background geometry.

So, we have presented an alternative formulation for stabilizing the modulus. In this framework the large value of kr is due to the small value of the coupling constant ξ while in the Goldberger–Wise [2] mechanism the large value of kr is due to a small bulk scalar mass.

For finite quartic couplings we have obtained analytical expression for the size of the stabilized modulus. Our analysis shows that the value of coupling ξ must be negative. We have made a $1/\lambda$ expansion in the large kr limit and obtained an analytical expression for this case. The parameters in this case are ξ, n, q and t. Similar to GW case [6], our study also reveals the existence of a very closely spaced maximum along with the minimum. It remains a problem to investigate the physical consequences of this result.

The issue of the stability of Randall–Sundrum brane-world with a tachyonic scalar has been dealt with in refs [9,10]. In our model, the presence of an effective negative mass term in the five-dimensional Lagrangian is due to the negative value of the parameter ξ. However, this will not induce an instability provided the tachyonic modes do not appear in the four-dimensional effective theory [10].

It will be interesting to study the parameter space of the model.

Instead of brane potential with quartic coupling, it is also possible to consider brane potential of the quadratic form. We plan to report on these issues in future.

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