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Charged anisotropic star on paraboloidal space-time

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Abstract. The charged anisotropic star on paraboloidal space-time is reported by choosing a particular form of radial pressure and electric field intensity. The non-singular solution of Einstein–Maxwell system of equation has been derived and it is shown that the model satisfies all the physical plausibility conditions. It is observed that in the absence of electric field intensity, the model reduces to a particular case of uncharged Sharma and Ratanpal model. It is also observed that the parameter used in the electric field intensity directly affects mass of the star.

Keywords. General relativity; exact solutions; anisotropic star; charged star.

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1. Introduction

Theoretical investigation of Ruderman [1] leads to the conclusion that matter may be anisotropic in densities of the order 10^{15} g cm^{-3}. The impact of anisotropy on stellar configuration may be found in the pioneering works of Bower and Liang [2] and Herrera and Santos [3]. Anisotropy may occur due to the existence of type-3A superfluids [1,2,4], or phase transition [5]. Consenza *et al* [6] developed a procedure to obtain anisotropic solutions from isotropic solutions of Einstein's field equations. Tikekar and Thomas [7] found exact solutions of Einstein's field equations for anisotropic fluid sphere on pseudo-spheroidal space-time. The key feature of their model is the high variation of density from the centre to the boundary of stellar configuration. The class of exact anisotropic solutions on spherically symmetric space-time was obtained by Mak and Harko [8]. Karmakar *et al* [9] analysed the role of pressure anisotropy for the Vaidya–Tikekar [10] model. Paul *et al* [11] developed anisotropic stellar model for strange star. A core–envelope model describing superdense stars with anisotropic fluid distribution was obtained by Thomas and Ratanpal [12], Thomas *et al* [13] and Tikekar and Thomas [14]. Hence the study of anisotropic fluid distribution is important in general theory of relativity.

The study of Einstein–Maxwell system was carried out by several authors. Patel and Kopper [15] obtained charged analog of the Vaidya–Tikekar [10] solution. The study of analytic models of quark stars was carrried out by Komathiraj and Maharaj [16], and they found a class of solutions of Einstein–Maxwell system. Charged anisotropic matter with linear equation of state was extensively studied by Thirukkanesh and Maharaj [17].

Hence, both anisotropy and electromagnatic field are important in relativistic astrophysics. In this paper, charged anisotropic model of a stellar configuration has been studied on the background of paraboloidal space-time. Section 2 describes the field equations for charged static stellar configuration on paraboloidal space-time and their solution. Section 3 describes the physical plausibility condition and §4 contains the discussion.

2. Field equations and solution

The interior of the stellar configuration is described by the static spherically symmetric paraboloidal space-time metric

$$\mathrm{d}s^2 = \mathrm{e}^{\nu(r)}\mathrm{d}t^2 - \left(1+\frac{r^2}{R^2}\right)\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right), \tag{1}$$

with the energy–momentum tensor for anisotropic charged fluid,

$$T_{ij} = \mathrm{diag}\left(\rho + E^2,\ p_\mathrm{r} - E^2,\ p_\mathrm{t} + E^2,\ p_\mathrm{t} + E^2\right), \tag{2}$$

where ρ is the energy density, p_r is the radial pressure, p_t is the tangential pressure and E is the electric field intensity. These quantities are measured relative to the comoving fluid velocity $u^i = \mathrm{e}^{-\nu}\delta^i_0$. For the space-time metric (1) and energy–momentum tensor (2), the Einstein–Maxwell system takes the form

$$\rho + E^2 = \frac{3 + (r^2/R^2)}{R^2\left(1 + (r^2/R^2)\right)^2}, \tag{3}$$

$$p_\mathrm{r} - E^2 = \frac{\nu'}{r\left(1 + (r^2/R^2)\right)} - \frac{1}{R^2\left(1 + (r^2/R^2)\right)}, \tag{4}$$

$$p_\mathrm{t} + E^2 = \frac{1}{1 + (r^2/R^2)}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} + \frac{\nu'}{2r}\right] - \frac{\nu' r}{2R^2\left(1 + (r^2/R^2)\right)^2} - \frac{1}{R^2\left(1 + (r^2/R^2)\right)^2}, \tag{5}$$

$$\sigma = \frac{\left(r^2 E\right)'}{r^2\sqrt{1 + (r^2/R^2)}}, \tag{6}$$

where σ is the proper charge density and prime denotes differentiation with respect to r. In field eqs (3)–(6), velocity of light c is taken as 1, also $(8\pi G/c^4) = 1$.

The anisotropic parameter Δ is defined as

$$\Delta = p_\mathrm{t} - p_\mathrm{r}. \tag{7}$$

To solve the system (3)–(6), radial pressure is assumed to be of the form

$$p_{\mathrm{r}} = \frac{p_0\left(1-(r^2/R^2)\right)}{R^2\left(1+(r^2/R^2)\right)^2}, \tag{8}$$

where $p_0 > 0$ is the model parameter and p_0/R^2 is the central pressure. At the boundary of the star $r = R$, p_{r} must vanish, which gives $r = R$ as the radius of the star. This form of radial pressure is prescribed by Sharma and Ratanpal [18] to describe anisotropic stellar model admitting a quadratic equation of state on paraboloidal space-time. Equations (8) and (4) give

$$\nu' = \frac{p_0 r\left(1-(r^2/R^2)\right)}{R^2\left(1+(r^2/R^2)\right)} + \frac{r}{R^2} - E^2 r\left(1+\frac{r^2}{R^2}\right). \tag{9}$$

We assume electric field intensity of the form

$$E^2 = \frac{k(r^2/R^2)}{R^2\left(1+(r^2/R^2)\right)^2}, \tag{10}$$

where $k \geq 0$ is a model parameter. From eq. (10) it is clear that E decreases in radially outward direction. Equations (9) and (10) lead to

$$\nu' = \frac{(2p_0+k)\,r}{R^2\left(1+(r^2/R^2)\right)} + (1-p_0-k)\,\frac{r}{R^2}, \tag{11}$$

and hence

$$\nu = \log\left[C\left(1+\frac{r^2}{R^2}\right)^{(2p_0+k)/2}\right] + \left(\frac{1-p_0-k}{2}\right)\frac{r^2}{R^2}, \tag{12}$$

where C is the constant of integration. Therefore, space-time metric (1) is written as

$$\begin{aligned} \mathrm{d}s^2 = {} & C\left(1+\frac{r^2}{R^2}\right)^{(2p_0+k)/2} \mathrm{e}^{(1-p_0-k)/2(r^2/R^2)}\mathrm{d}t^2 \\ & -\left(1+\frac{r^2}{R^2}\right)\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \end{aligned} \tag{13}$$

The space-time metric (13) should continuously match with Reissner–Nordstrom space-time metric

$$\mathrm{d}s^2 = \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}t^2 - \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2+\sin^2\theta\right)\mathrm{d}\phi^2, \tag{14}$$

at the boundary of the star $r = R$, where $(p_{\mathrm{r}})\,(r = R) = 0$. This matching conditions gives

$$M = \frac{k+2R^2}{8R} \tag{15}$$

and

$$C = \frac{\mathrm{e}^{(p_0+k-1)/2}}{(2p_0+2+k)/2}, \tag{16}$$

where M is the mass enclosing the spherical body of radius R. Hence the electric field intensity parameter k directly affects the mass of the star. Equations (4), (5), (7) and (11) give anisotropic parameter Δ as

$$\Delta = \frac{(r^2/R^2)\left[X_1 + Y_1(r^2/R^2) + Z_1(r^4/R^4)\right]}{4R^2\left(1+(r^2/R^2)\right)^3}, \tag{17}$$

which vanishes at $r = 0$, where $X_1 = p_0^2 - 8p_0 - 12k + 3$, $Y_1 = -2p_0^2 - 2p_0k + 2p_0 - 8k + 4$, $Z_1 = 1 + p_0^2 + k^2 - 2p_0 - 2k + 2p_0k$. Equations (3) and (10) give

$$\rho = \frac{3 + (1-k)\,(r^2/R^2)}{R^2\left(1+(r^2/R^2)\right)^2} \tag{18}$$

and from eqs (7), (8) and (17), we get expression of p_t as

$$p_\mathrm{t} = \frac{4p_0 + X_1(r^2/R^2) + Y_1(r^4/R^4) + Z_1(r^6/R^6)}{4R^2\left(1+(r^2/R^2)\right)^3}. \tag{19}$$

Hence eqs (18), (8), (19), (10) and (17) describe matter density, radial pressure, tangential pressure, electric field intensity and measure of anisotropy respectively.

3. Physical plausibility conditions

Following Delgaty and Lake [19], we impose the following conditions on the sytem to make the model physically acceptable:

(i) $\rho(r),\ p_\mathrm{r}(r),\ p_\mathrm{t}(r) \geq 0$, for $0 \leq r \leq R$.
(ii) $\rho - p_\mathrm{r} - 2p_\mathrm{t} \geq 0$, for $0 \leq r \leq R$.
(iii) $(\mathrm{d}\rho/\mathrm{d}r),\ (\mathrm{d}p_\mathrm{r}/\mathrm{d}r),\ (\mathrm{d}p_\mathrm{t}/\mathrm{d}r) < 0$, for $0 \leq r \leq R$.
(iv) $0 \leq (\mathrm{d}p_\mathrm{r}/\mathrm{d}\rho) \leq 1$; $0 \leq (\mathrm{d}p_\mathrm{t}/\mathrm{d}\rho) \leq 1$, for $0 \leq r \leq R$.

From eq. (18), $\rho(r=0) = (3/R^2) > 0$ and $\rho(r=R) = [(4-k)/4R^2]$. Therefore, $\rho > 0$ for $0 \leq r \leq R$ if $k \leq 4$, i.e.

$$0 \leq k \leq 4. \tag{20}$$

From eq. (8) $p_\mathrm{r}(r=0) = (p_0/R^2) > 0$ as $p_0 > 0$ and $p_\mathrm{r}(r=R) = 0$. Hence $p_\mathrm{r} \geq 0$ for $0 \leq r \leq R$. It is required that $p_\mathrm{t} \geq 0$ for $0 \leq r \leq R$ and further to get the simple bounds on p_0 and k, we assume $p_\mathrm{r} = p_\mathrm{t}$ at $r = R$. From (19), $p_\mathrm{t}(r=0) = (p_0/R^2) > 0$ as $p_0 > 0$, and $p_\mathrm{t}(r=R) = [(k^2 - 22k + 8 - 8p_0)/32R^2]$, at $r = R$, $p_\mathrm{t} = p_\mathrm{r} = 0$ if

$$p_0 = \frac{k^2 - 22k + 8}{8}, \tag{21}$$

but k should be chosen such that p_0 is positive, which restricts the value of k as

$$k < 0.3699. \tag{22}$$

Hence,

$$0 \leq k < 0.3699, \quad p_0 = \frac{k^2 - 22k + 8}{8}, \tag{23}$$

which is the condition for positivity of p_t.

Hence condition (i) is satisfied throughout the star. For the values of k and p_0 specified in (23), programatically it has been verified that condition (ii), i.e. energy condition is satisfied throughout the star. From eq. (18),

$$\frac{d\rho}{dr} = -\frac{2r}{R^4}\frac{\left[(5+k) + (1-k)(r^2/R^2)\right]}{\left(1 + (r^2/R^2)\right)^3}, \tag{24}$$

and from eq. (24), $(d\rho/dr)\,(r=0) = 0$ and $(d\rho/dr)\,(r=R) = -(3/2R^3) < 0$. Hence ρ is decreasing throughout the star. From eq. (8)

$$\frac{dp_r}{dr} = \frac{-2p_0 r\left(3 - (r^2/R^2)\right)}{R^4\left(1 + (r^2/R^2)\right)^3}. \tag{25}$$

Now, $(dp_r/dr)\,(r=0) = 0$ and $(dp_r/dr)\,(r=R) = (-p_0/2R^3) < 0$ as $p_0 > 0$. Hence p_r is decreasing throughout the star. From eq. (19)

$$\frac{dp_t}{dr} = \frac{r\left[X_2 + Y_2(r^2/R^2) + Z_2(r^4/R^4)\right]}{2R^4\left(1 + (r^2/R^2)\right)^4}, \tag{26}$$

where $X_2 = p_0^2 - 20p_0 - 12k + 3$, $Y_2 = -6p_0^2 + 12p_0 - 4p_0k + 8k + 2$ and $Z_2 = 5p_0^2 - 4p_0 + 8p_0k + 3k^2 + 2k - 1$. Now $(dp_t/dr)\,(r=0) = 0$ and $(dp_t/dr)\,(r=R) = [(-12p_0 + 3k^2 - 2k + 4p_0k + 4)/32R^3)]$. Substitute p_0 from eq. (21) in (dp_t/dr)

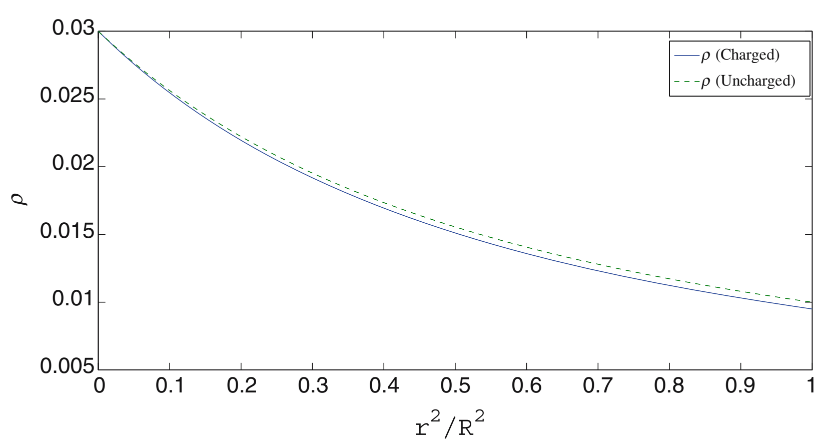

Figure 1. Variation of density (ρ) (charged and uncharged) against r^2/R^2.

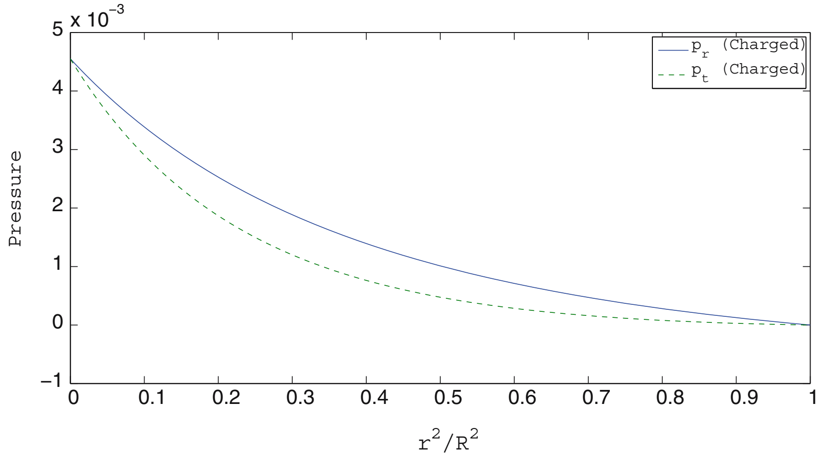

Figure 2. Variation of radial pressure (p_{r}) and tangential pressure (p_{t}) (charged) against r^2/R^2.

$(r = R)$, $(\mathrm{d}p_{\mathrm{t}}/\mathrm{d}r)\,(r = R) < 0$ if $4k^3 - 76k^2 + 280k - 64 < 0$. This further restricts value of k as $k < 0.2446$.

Hence, if

$$0 \leq k < 0.2446, \qquad p_0 = \frac{k^2 - 22k + 8}{8}, \tag{27}$$

then $\mathrm{d}\rho/\mathrm{d}r$, $\mathrm{d}p_{\mathrm{r}}/\mathrm{d}r$ and $\mathrm{d}p_{\mathrm{t}}/\mathrm{d}r$ are decreasing in radially outward direction between $0 \leq r \leq R$. From eqs (24) and (25) we have

$$\frac{\mathrm{d}p_{\mathrm{r}}}{\mathrm{d}\rho} = \frac{p_0\left(3 - (r^2/R^2)\right)}{\left[(5+k) + (1-k)(r^2/R^2)\right]}. \tag{28}$$

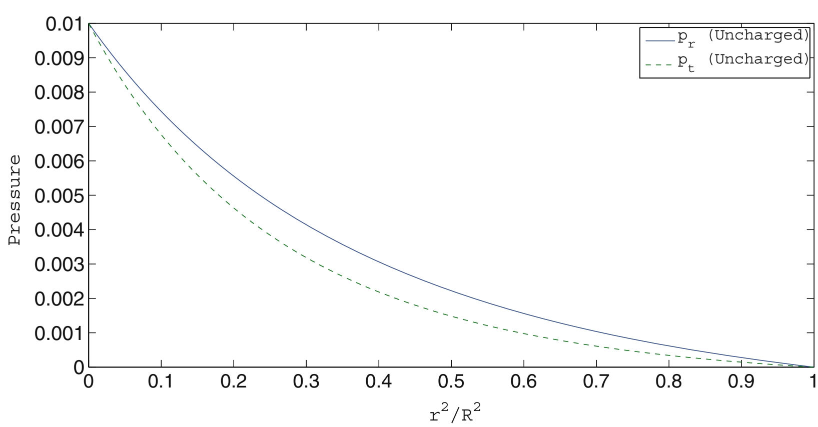

Figure 3. Variation of radial pressure (p_{r}) and tangential pressure (p_{t}) (uncharged) against r^2/R^2.

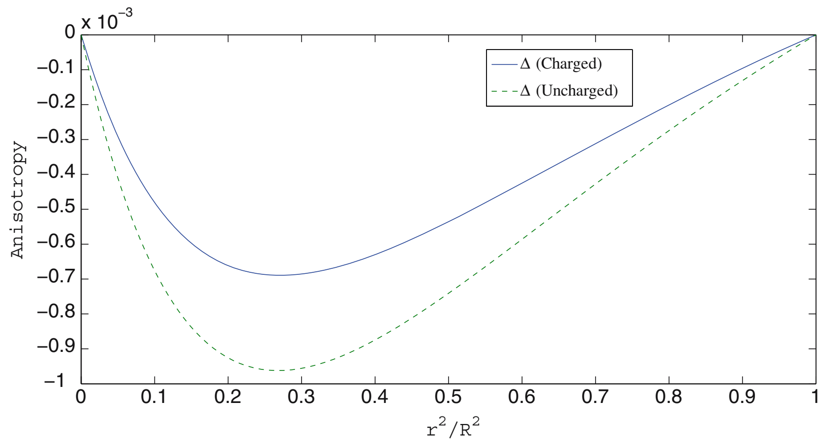

Figure 4. Variation of anisotropy (Δ) (charged and uncharged) against r^2/R^2.

At the centre of the star $(\mathrm{d}p_\mathrm{r}/\mathrm{d}\rho)\,(r=0) < 1$ if $k < 24.8810$, which is consistent with condition (27) and at the boundary of the star $(\mathrm{d}p_\mathrm{r}/\mathrm{d}\rho)\,(r=R) < 1$ if $k < 22.7047$, which is also consistent with conditon (27). From eqs (24) and (26) we have

$$\frac{\mathrm{d}p_\mathrm{t}}{\mathrm{d}\rho} = \frac{-\left[X_2 + Y_2(r^2/R^2) + Z_2(r^4/R^4)\right]}{4\left(1+(r^2/R^2)\right)\left[(5+k)+(1-k)(r^2/R^2)\right]}. \tag{29}$$

Now, $(\mathrm{d}p_\mathrm{t}/\mathrm{d}\rho)\,(r=0) < 1$ if $k < 19.4283$, which is consistent with condition (27) and $(\mathrm{d}p_\mathrm{t}/\mathrm{d}\rho)\,(r=R) < 1$ if $k < 6.6371$, which is also consistent with condition (27). Hence for $0 \le k < 0.2446$ and $p_0 = [(k^2 - 22k + 8)/8]$, all the physical plausibility conditions are satisfied.

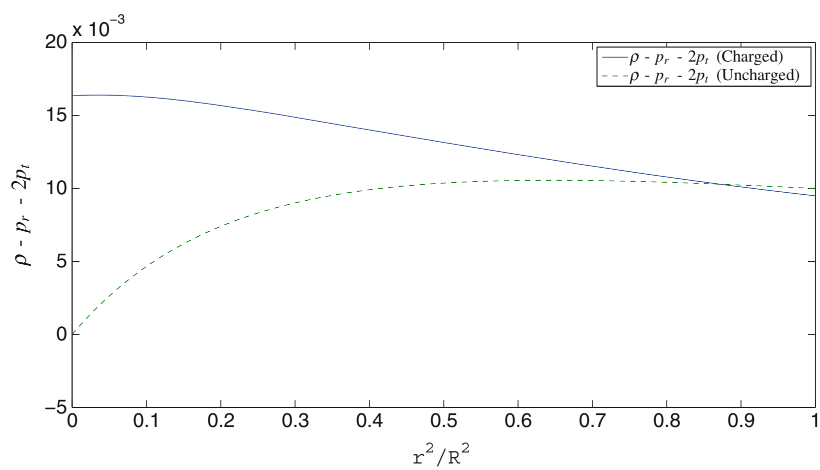

Figure 5. Variation of energy condition $(\rho - p_\mathrm{r} - 2p_\mathrm{t})$ (charged and uncharged) against r^2/R^2.

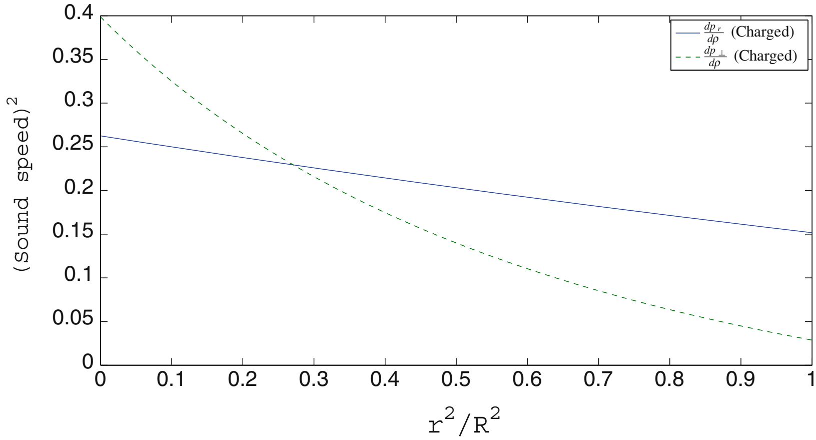

Figure 6. Variation of $\mathrm{d}p_{\mathrm{r}}/\mathrm{d}\rho$ and $\mathrm{d}p_{\mathrm{t}}/\mathrm{d}\rho$ (charged) against r^2/R^2.

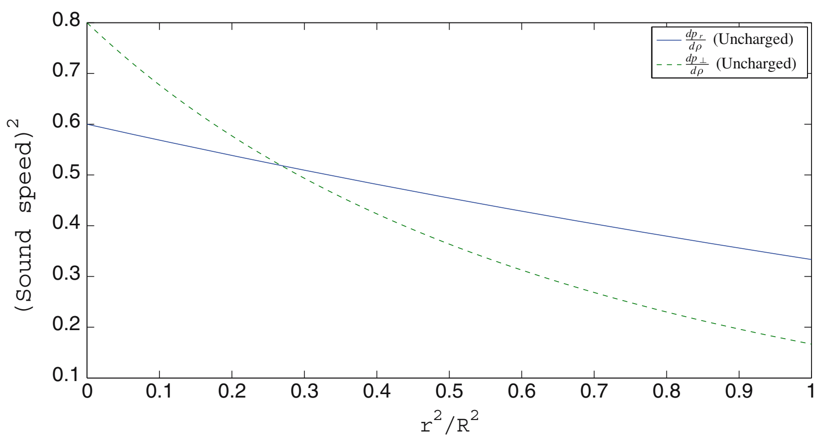

Figure 7. Variation of $\mathrm{d}p_{\mathrm{r}}/\mathrm{d}\rho$ and $\mathrm{d}p_{\mathrm{t}}/\mathrm{d}\rho$ (uncharged) against r^2/R^2.

4. Dicussion

Certain aspects of charged relativistic star on paraboloidal space-time are discussed. It is observed that all the physical plausibility conditions are satisfied for $0 \leq k \leq 0.2446$ and $p_0 = (k^2 - 22k + 8)/8$. The plots of ρ (charged, uncharged), p_{r} and p_{t} (charged), p_{r} and p_{t} (uncharged), anisotropy Δ (charged, uncharged), $\rho - p_{\mathrm{r}} - 2p_{\mathrm{t}}$ (charged, uncharged), $\mathrm{d}p_{\mathrm{r}}/\mathrm{d}\rho$ and $\mathrm{d}p_{\mathrm{t}}/\mathrm{d}\rho$ (charged), $\mathrm{d}p_{\mathrm{r}}/\mathrm{d}\rho$ and $\mathrm{d}p_{\mathrm{t}}/\mathrm{d}\rho$ (uncharged) over r^2/R^2 for $R = 10$, $k = 0.2$ and taking $G = c^2 = 1$ are shown in figures 1–7. It is observed that energy condition is satisfied throughout the star. When $k = 0$, the value of $p_0 = 1$ and the model reduces to the Sharma and Ratanpal [18] model. Hence, the model described here is the charged generalization of a particular case $p_0 = 1$ of uncharged Sharma and Ratanpal [18] model.

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References

[1] R Ruderman, *Annu. Rev. Astron. Astrophys.* **10**, 427 (1972)
[2] R L Bowers and E P T Liang, *Astrophys. J.* **188**, 657 (1974)
[3] L Herrera and N O Santos, *Phys. Rep.* **286**, 53 (1997)
[4] R Kippenhahn and A Weigert, *Stellar structure and evolution* (Springer Verlag, Berlin, Heidelberg, New York, 1990)
[5] A I Sokolov, *J. Exp. Theoret. Phys.* **52**, 575 (1980)
[6] M Cosenza, L Herrera, M Esculpi and L Witten, *J. Math. Phys.* **22**, 118 (1981)
[7] R Tikekar and V O Thomas, *Pramana – J. Phys.* **52**, 237 (1999)
[8] M K Mak and T Harko, *Proc. R. Soc. London A* **459**, 393 (2003)
[9] S Karmakar, S Mukherjee, R Sharma and S D Maharaj, *Pramana – J. Phys.* **68**, 881 (2007)
[10] P C Vaidya and R Tikekar, *J. Astrophys. Astron.* **3**, 325 (1982)
[11] B C Paul, P K Chattopadhyay, S Karmakar and R Tikekar, *Mod. Phys. Lett. A* **26**, 575 (2011)
[12] V O Thomas and B S Ratanpal, *Int. J. Mod. Phys. D* **16**, 1479 (2007)
[13] V O Thomas, B S Ratanpal and P C Vinodkumar, *Int. J. Mod. Phys. D* **14**, 85 (2005)
[14] R Tikekar and V O Thomas, *Pramana – J. Phys.* **64**, 5 (2005)
[15] L K Patel and S S Kopppar, *Ast. J. Phys.* **40**, 441 (1987)
[16] K Komathiraj and S D Maharaj, *Int. J. Mod. Phys. D* **16**, 1803 (2007)
[17] S Thirukkanesh and S D Maharaj, *Class. Quantum Grav.* **25**, 235001-1 (2008)
[18] R Sharma and B S Ratanpal, *Int. J. Mod. Phys. D* **22**, 1350074-1 (2013)
[19] M S R Delgaty and K Lake, *Comput. Phys. Commun.* **115**, 395 (1998)