PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 86, No. 3
March 2016
pp. 515–526

Two-way quantum communication: Generalization of secure quantum information exchange to quantum network

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MS received 6 May 2014; revised 22 December 2014; accepted 12 January 2015
DOI: 10.1007/s12043-015-1034-4; ***e*Publication:** 15 August 2015

Abstract. The idea of secure quantum information exchange (SQIE) [*J. Phys. B: At. Mol. Opt. Phys.* **44**, 115504 (2011)] is introduced for the secure exchange of single qubit information states between two legitimate users, Alice and Bob. In the present paper, we extend this original SQIE protocol by presenting a scheme, which enables the secure exchange of n-single qubit information states among the n nodes of a quantum network, with the aid of a special kind of $4n$-qubit entangled state and the classical assistance of an extra participant Charlie. For experimental realization of our extended SQIE protocol, we suggest an efficient scheme for the generation of a special kind of $4n$-qubit entangled state using the interaction between highly detuned Λ-type three-level atoms and optical coherent field. Further, by discussing the various experimental parameters, we show that the special kind $4n$-qubit entangled state can be generated with the presently available technology.

Keywords. Quantum information; quantum teleportation; two-way quantum communication; coherent state; QED.

PACS Nos 03.67.Hk; 03.65.Ud; 03.67.−a

1. Introduction

In classical theory, the allowed states are simply classical bits ('0' and '1'), while in quantum theory, for a two-level system, the allowed states are $|0\rangle$ and $|1\rangle$, and also its linear superposition, i.e., $a|0\rangle + b|1\rangle$ with $|a|^2 + |b|^2 = 1$. Because of this linear superposition in quantum theory, for a bipartite system, we have the state $\frac{1}{\sqrt{2}}[|01\rangle + |10\rangle]$, which gives rise to an astonishing phenomenon called long-range EPR correlation [1], also termed as quantum entanglement [2].

A pure state of two or more quantum systems is said to be entangled [3], if it cannot be written as a product of the quantum states of the constituent systems. A mixed state is called entangled [4], if it cannot be written as a mixture of factorizable pure states, i.e., it cannot be written in the form,

$$\rho = \sum_j p_j \rho_j^{A_1} \otimes \rho_j^{A_2} \otimes \cdots \otimes \rho_j^{A_n}, \quad \text{with } p_j > 0 \text{ and } \sum_j p_j = 1.$$

Quantum entanglement makes possible many quantum information processing tasks, which are otherwise impossible in classical information theory. Quantum entanglement is widely used in quantum information processing tasks such as quantum teleportation [5], quantum cryptography [6], quantum superdense coding [7], quantum remote state preparation [8] etc.

Quantum teleportation (QT), first shown by Bennett *et al* [5], is a phenomenon in which an unknown quantum information state is destroyed at the sender's end (Alice) and a replica is created at the receiver's end (Bob) using long-range EPR correlation and transmitting classical information via classical channel from sender to receiver. After the introduction of the idea of QT, several theoretical studies on QT [9–12] have been done. Also several experiments have demonstrated QT with photonic-polarized states [13,14], atomic qubits [15,16] and quantum state of nucleus [17]. In some studies on QT, a third observer Charlie is introduced between two legitimate users, Alice and Bob. Introduction of a third observer increases the security of the QT protocol because now both Alice and Charlie have control on the teleportation process. This type of QT is called controlled QT. Many researchers proposed the controlled QT of single qubit information state using GHZ state or GHZ-class states [18,19] and W state [20]. However, in practical situations, there may be a need of sending a large amount of information encoded in multipartite states. For this reason, many researchers [21–23] proposed the teleportation of multipartite information states.

In all these studies on QT, user Alice sends information state and Bob gets an exact replica of the information state, i.e., this process has one-way quantum communication. If we require two-way quantum communication, then we have to switch two QT protocols in opposite directions between Alice and Bob. As completion of QT protocol requires sending of the classical information about Bell state measurement (BSM) from the sender to the receiver, which is required for performing suitable unitary transformation by the receiver. There may arise a situation when Bob gets classical information from Alice but he does not send his BSM result to Alice. Thus, this type of two-way quantum communication is insecure. Same problem will also arise when we switch two controlled QT processes in opposite directions.

To solve the above problems in the two-way quantum communication, Mishra *et al* [24] proposed a new idea called secure quantum information exchange (SQIE). The SQIE protocol enables the faithful exchange of two single qubit information states between Alice and Bob, with the aid of a special kind of six-qubit entangled (SSE) state and the classical assistance of Charlie, a third party. This protocol is secure in the sense that either both Alice and Bob get their required information states or in case of failure of this due to any reason, none of them gets any information state. Also Alice and Bob cannot reconstruct the required information states by communicating their BSM results classically to each other without the assistance of Charlie. However, classical communication between Alice and Bob is not allowed in the SQIE protocol. In the real world, there may be a need of secure exchange of a large amount of information encoded in multiqubit states. For this purpose, Maurya *et al* [25] have generalized their original SQIE protocol to secure exchange of information states of arbitrary number of qubits between Alice and Bob. Maurya *et al* [25] have also discussed about the effect on the security of SQIE protocol with respect to increase in the number of qubits going towards the controller Charlie. Xu *et al* [26] proposed cooperative two-way communication for the simultaneous exchange of two single qubit information states between Alice and Bob using Brown state as the quantum channel. The security of this protocol is the same as security of SQIE scheme [24]. But the success of this scheme is information state-dependent, which is one for a particular information state and for all other information states it is lesser. Success of our SQIE [24] scheme is independent of information state and is perfect for all information states.

To realize the quantum computer, there may be a large number of quantum processors (nodes) working apart. There may be a need of setting a quantum link among the nodes and also quantum communication among them. In order to set quantum communication between the nodes, in this paper, we extend our original SQIE protocol for the secure exchange of the quantum information states among a number of nodes working apart.

In this framework, we consider a quantum network composed of n nodes (observers), say, $\mathrm{Bob}^{(1)}$, $\mathrm{Bob}^{(2)}$, $\mathrm{Bob}^{(3)}, \ldots, \mathrm{Bob}^{(n)}$. Our aim is to exchange n single qubit information states among all observers in cyclic order, i.e., to teleport first information state from $\mathrm{Bob}^{(1)}$ to $\mathrm{Bob}^{(2)}$, simultaneously second information state from $\mathrm{Bob}^{(2)}$ to $\mathrm{Bob}^{(3)}$ and so on. $\mathrm{Bob}^{(n)}$ teleports the nth information state to $\mathrm{Bob}^{(1)}$. This exchange process must be done with the condition that either all observers get their required information states or in case of failure of this end result, nobody among them gets any information state. We present the extended SQIE protocol, which securely exchanges the n single qubit information states among the n observers with the aid of a special kind of $4n$-qubit entangled state and classical assistance of the extra participant Charlie. In order to experimentally realize our extended SQIE protocol, we present an efficient scheme for the generation of a special kind of $4n$-qubit entangled state using the interaction between Λ-type three-level atoms and optical coherent field. Further, we discuss the experimental feasibility of this scheme by considering some experimental aspects like cavity damping time, total flight time of atoms and conclude that a special kind of $4n$-qubit entangled state can be generated using the presently available technology.

2. Secure quantum information exchange among *n* nodes

Let us consider a quantum network composed of n observers, say, $\mathrm{Bob}^{(1)}$, $\mathrm{Bob}^{(2)}$, $\mathrm{Bob}^{(3)}, \ldots, \mathrm{Bob}^{(n)}$. Let $\mathrm{Bob}^{(1)}$ want to send single qubit information state $|\xi\rangle_{I_1} = [a_1|0\rangle + b_1|1\rangle]_{I_1}$ to $\mathrm{Bob}^{(2)}$, where $|0\rangle$ and $|1\rangle$ are two levels of a two-level system and a and b are unknown complex numbers satisfying the normalization condition $|a|^2 + |b|^2 = 1$. At the same time, $\mathrm{Bob}^{(2)}$ wants to send another single qubit information state $|\xi\rangle_{I_2} = [a_2|0\rangle + b_2|1\rangle]_{I_2}$ to $\mathrm{Bob}^{(3)}$, $\mathrm{Bob}^{(3)}$ wants to send single qubit information state $|\xi\rangle_{I_3} = [a_3|0\rangle + b_3|1\rangle]_{I_3}$ to $\mathrm{Bob}^{(4)}$ and so on. $\mathrm{Bob}^{(n)}$ wants to send single qubit information state $|\xi\rangle_{I_n} = [a_n|0\rangle + b_n|1\rangle]_{I_n}$ to $\mathrm{Bob}^{(1)}$. This information exchange process must be secure such that either all the observers get their required information states or if this end result is not obtained due to any reason, then nobody gets any information state.

To complete this task, we introduce an extra participant Charlie, who classically assists all the observers. We define a special kind of $4n$-qubit entangled state as

$$|\psi\rangle^{\mathrm{E}}_{A_1,B_1,A_2,B_2,A_3,B_3,\ldots,A_n,B_n,\{C\}} = \frac{1}{2^n}\left[\sum_{j_1,\,j_2,\ldots,\,j_n=1}^{4} |B\rangle^{(j_1)}_{A_1,B_2} \otimes |B\rangle^{(j_2)}_{A_2,B_3} \otimes |B\rangle^{(j_3)}_{A_3,B_4} \otimes \cdots \otimes |B\rangle^{(j_n)}_{A_n,B_1} \otimes \left(|\phi\rangle^{(j_1)}_{C_1,C_2} \otimes |\phi\rangle^{(j_2)}_{C_3,C_4} \otimes \cdots \otimes |\phi\rangle^{(j_n)}_{C_{2n-1},C_{2n}}\right)\right], \tag{1}$$

where each $|B\rangle^{(j_i)}$ is a standard bipartite Bell state given by

$$|B\rangle^{(1,\,2)} = \frac{1}{\sqrt{2}}[|00\rangle \pm |11\rangle], \quad |B\rangle^{(3,\,4)} = \frac{1}{\sqrt{2}}[|01\rangle \pm |10\rangle], \tag{2}$$

for $j_i = 1, 2, 3, 4$ respectively, modes $\{C\} \equiv (C_1, C_2, \ldots, C_{2n})$ and each $|\phi\rangle^{(j_i)}$ is the state in computational basis $\{|00\rangle,\ |11\rangle,\ |01\rangle,\ |10\rangle\}$ for $j_i = 1, 2, 3, 4$ respectively. A_i and B_i are modes with Bob$^{(i)}$ for $i = 1, 2, 3, \ldots, n$ respectively, while the modes $\{C\}$ are with Charlie. Superscript E refers to entangled state.

The state, given by eq. (1), can also be written as

$$|\psi\rangle^{\mathrm{E}}_{A_1,B_1,A_2,B_2,A_3,B_3,\ldots,A_n,B_n,\{C\}} = \frac{1}{2^n}\left[\sum_{j_1,\,j_2,\ldots,\,j_n=1}^{4} \sigma^{j_1}|B\rangle^{(1)}_{A_1,B_2} \otimes \sigma^{j_2}|B\rangle^{(1)}_{A_2,B_3} \otimes \sigma^{j_3}|B\rangle^{(1)}_{A_3,B_4} \otimes \cdots \otimes \sigma^{j_n}|B\rangle^{(1)}_{A_n,B_1} \otimes \left(|\phi\rangle^{(j_1)}_{C_1,C_2} \otimes |\phi\rangle^{(j_2)}_{C_3,C_4} \otimes \cdots \otimes |\phi\rangle^{(j_n)}_{C_{2n-1},C_{2n}}\right)\right], \tag{3}$$

where each σ^{j_i} is $I, \sigma_z, \sigma_x, \sigma_x\sigma_z$ for $j_i = 1, 2, 3, 4$ respectively. As all observers (n Bob and Charlie) have shared $4n$-qubit entangled state (3) and each Bob has single information state, we have a system consisting of $5n$ qubits and we write the composite state of this system as

$$|\psi\rangle_{I_1,\ldots,I_n,A_1,B_1,\ldots,A_n,B_n,\{C\}} = |\xi\rangle_{I_1} \otimes |\xi\rangle_{I_2} \otimes \cdots \otimes |\xi\rangle_{I_n} \otimes |\psi\rangle^{\mathrm{E}}_{A_1,B_1,A_2,B_2,A_3,B_3,\ldots,A_n,B_n,\{C\}}$$
$$= \frac{1}{2^n}\left[\sum_{j_1,\,j_2,\ldots,\,j_n=1}^{4} \sigma^{j_1}(|\xi\rangle_{I_1} \otimes |B\rangle^{(1)}_{A_1,B_2}) \otimes \sigma^{j_2}(|\xi\rangle_{I_2} \otimes |B\rangle^{(1)}_{A_2,B_3}) \otimes \sigma^{j_3}(|\xi\rangle_{I_3} \otimes |B\rangle^{(1)}_{A_3,B_4}) \otimes \cdots \otimes \sigma^{j_n}(|\xi\rangle_{I_n} \otimes |B\rangle^{(1)}_{A_n,B_1}) \otimes (|\phi\rangle^{(j_1)}_{C_1,C_2} \otimes |\phi\rangle^{(j_2)}_{C_3,C_4} \otimes \cdots \otimes |\phi\rangle^{(j_n)}_{C_{2n-1},C_{2n}})\right]. \tag{4}$$

The detailed scheme is shown in figure 1. Using the standard bipartite Bell states (2), the state (4) can also be written as

$$\begin{aligned}
&|\psi\rangle_{I_1,\ldots,I_n,A_1,B_1,\ldots,A_n,B_n,\{C\}} \\
&= \frac{1}{4^n}\left[\sum_{j_1,\,j_2,\ldots,\,j_n=1}^{4}\left\{\sum_{k_1,\,k_2,\ldots,\,k_n=1}^{4}(|B\rangle_{I_1,A_1}^{(k_1)}\otimes\sigma^{j_1}\sigma^{k_1}|\xi\rangle_{B_2})\right.\right. \\
&\quad\otimes(|B\rangle_{I_2,A_2}^{(k_2)}\otimes\sigma^{j_2}\sigma^{k_2}|\xi\rangle_{B_3}) \\
&\quad\left.\otimes(|B\rangle_{I_3,A_3}^{(k_3)}\otimes\sigma^{j_3}\sigma^{k_3}|\xi\rangle_{B_4})\otimes\cdots\otimes(|B\rangle_{I_n,A_n}^{(k_n)}\otimes\sigma^{j_n}\sigma^{k_n}|\xi\rangle_{B_1})\right\} \\
&\quad\left.\otimes(|\phi\rangle_{C_1,C_2}^{(j_1)}\otimes|\phi\rangle_{C_3,C_4}^{(j_2)}\otimes\cdots\otimes|\phi\rangle_{C_{2n-1},C_{2n}}^{(j_n)})\right], \qquad (5)
\end{aligned}$$

where each σ^{k_i} is $I, \sigma_z, \sigma_x, \sigma_x\sigma_z$ for k_i =1, 2, 3, 4 respectively.

Now each Bob$^{(i)}$ performs Bell state measurement (BSM) on his two qubits in modes I_i and A_i for $i = 1, 2, 3, \ldots, n$ respectively, while Charlie measures his qubits in the computational basis $\{|0\rangle, |1\rangle\}$. All Bobs convey their BSM results to Charlie through 2-bit classical channels. After getting the classical information from all Bobs and on the basis of Charlie's measurement results, Charlie decides about the 2-bit classical information to be conveyed to each Bob. On the basis of these classical information conveyed by Charlie, all Bobs perform the required unitary transformations on their particles in order

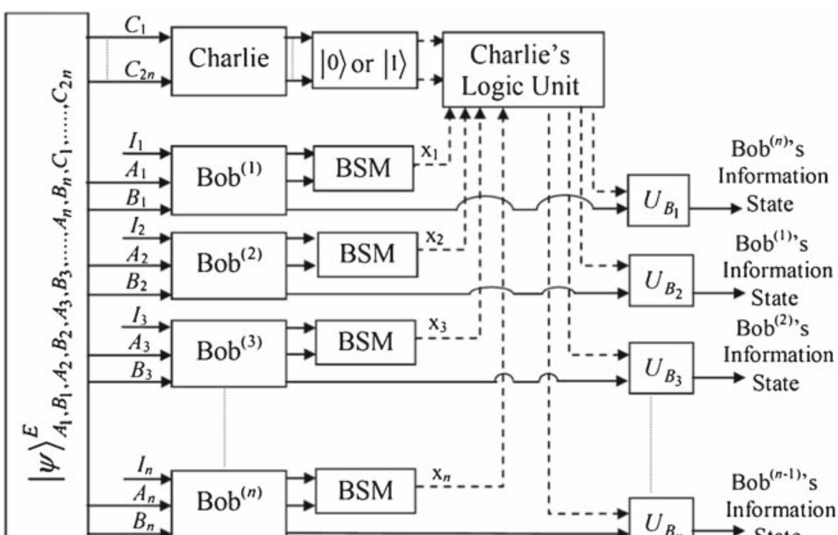

Figure 1. Scheme for extended SQIE protocol. Entangled modes $C_1, \ldots, C_{2n}$ are with Charlie, while information mode I_i and entangled modes A_i and B_i belong to Bob$^{(i)}$ for $i = 1, 2, 3, \ldots, n$. $x_1, x_2, x_3, \ldots, x_n$ are 2 c-bit classical information. U_{B_1}, $U_{B_2}, U_{B_3}, \ldots, U_{B_n}$ refer to the unitary operation to be performed by Bob$^{(1)}$, Bob$^{(2)}$, Bob$^{(3)}, \ldots,$ Bob$^{(n)}$ respectively for completing the extended SQIE protocol.

to generate exact replicas of the required quantum information states. If the result of Charlie's measurement is $j_1, j_2 \ldots j_n$ and the result of BSM of ith Bob is k_i, then from eq. (5), it is clear that $(i+1)$th Bob performs unitary transformation $(\sigma^{j_i}\sigma^{k_i})^\dagger$ on his qubit in mode B_{i+1} to generate the exact replica of the required information state.

Both the result of Charlie's measurement and result of BSM, are required to determine suitable unitary transformation, operation of which generates the required information state. Hence all Bobs cannot ignore Charlie by communicating classically to one another. However, in our scheme, classical communication among Bobs is not permitted. Due to any reason, if any one or more than one Bob do not send classical information to Charlie, then Charlie cancels the whole process without sending any information to all Bobs. Hence this SQIE process has the required security that is discussed in the beginning of this section.

3. Generation of entangled 4n-qubit state

In order to experimentally realize the extended SQIE scheme discussed in §2, we must have the ability to generate the entangled 4n-qubit state given by eq. (1), which is used as quantum channel. This state can also be written as

$$
\begin{aligned}
&|\psi\rangle^{\mathrm{E}}_{A_1,B_1,A_2,B_2,A_3,B_3,\ldots,A_n,B_n,\{C\}} \\
&\quad = \frac{1}{2^n}\left[\left(\sum_{j_1=1}^{4}|B\rangle^{(j_1)}_{A_1,B_2}\otimes|\phi\rangle^{(j_1)}_{C_1,C_2}\right)\otimes\left(\sum_{j_2=1}^{4}|B\rangle^{(j_2)}_{A_2,B_3}\otimes|\phi\rangle^{(j_2)}_{C_3,C_4}\right)\right. \\
&\qquad \left.\otimes\left(\sum_{j_3=1}^{4}|B\rangle^{(j_3)}_{A_3,B_4}\otimes|\phi\rangle^{(j_3)}_{C_5,C_6}\right)\otimes\cdots\otimes\left(\sum_{j_3=1}^{4}|B\rangle^{(j_n)}_{A_n,B_1}\otimes|\phi\rangle^{(j_n)}_{C_{2n-1},C_{2n}}\right)\right].
\end{aligned}
\tag{6}
$$

Hence it is enough to generate the state,

$$
\begin{aligned}
&\frac{1}{2}\left(\sum_{j_1=1}^{4}|B\rangle^{(j_1)}_{A_1,B_2}\otimes|\phi\rangle^{(j_1)}_{C_1,C_2}\right) \\
&\quad = \frac{1}{2}[|B\rangle^{(1)}\otimes|00\rangle+|B\rangle^{(2)}\otimes|11\rangle+|B\rangle^{(3)}\otimes|01\rangle \\
&\qquad +|B\rangle^{(4)}\otimes|10\rangle]_{A_1,B_2,C_1,C_2},
\end{aligned}
\tag{7}
$$

with n parallel set-ups.

In this section, we present an efficient scheme for the generation of the state (7). We consider the interaction of Λ-type three-level atom with optical coherent field. Level configuration of Λ-type three-level atom is shown in figure 2, where $|0\rangle$ and $|1\rangle$ are two degenerate ground levels and $|2\rangle$ is the excited level. Frequency of optical coherent field (ω_c) is largely detuned from atomic transition frequency ω_0, i.e., $\Delta=\omega_0-\omega_\mathrm{c}$ is large. In large detuning limit, the excited state $|2\rangle$ can be adiabatically eliminated during the interaction and effective Hamiltonian [27] can be expressed as

$$
H=-\lambda a^\dagger a[|0\rangle\langle 0|+|0\rangle\langle 1|+|1\rangle\langle 0|+|1\rangle\langle 1|],
\tag{8}
$$

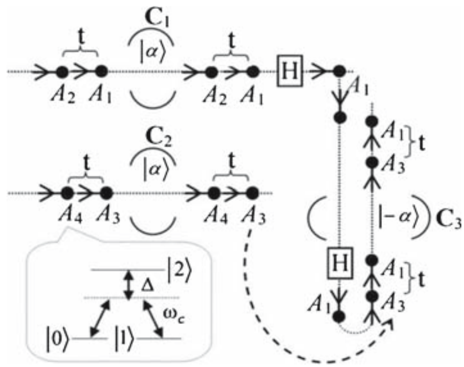

Figure 2. Scheme for the generation of state (7). Callout shows the level configuration of Λ-type three-level atom, where $|0\rangle$ and $|1\rangle$ are two degenerate ground levels and $|2\rangle$ is the excited level. $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$ refer to cavities initially prepared in optical coherent field $|\alpha\rangle_{\mathbf{C}_1}$, $|\alpha\rangle_{\mathbf{C}_2}$ and $|-\alpha\rangle_{\mathbf{C}_3}$ respectively. A_1, A_2, A_3 and A_4 denote the four atoms. The four atoms are initially in ground state $|0\,0\,0\,0\rangle_{A_1,A_2,A_3,A_4}$. $t = \pi/2\lambda$ is the interaction time of atom with cavity field. H refers to Hadamard operation.

where $\lambda = g^2/\Delta$. In [24], it is shown that for this system if interaction time satisfies $t = \pi/2\lambda$, then the state of the atom–cavity system evolves according to the following evolution:

$$|0,+\rangle \to |0,+\rangle; \quad |1,+\rangle \to |1,+\rangle;$$
$$|0,-\rangle \to -|1,-\rangle; \quad |1,-\rangle \to -|0,-\rangle \tag{9}$$

and

$$|A_+,\pm\alpha\rangle \to |A_+,\mp\alpha\rangle, \quad |A_-,\pm\alpha\rangle \to |A_-,\pm\alpha\rangle, \tag{10}$$

where $|\pm\rangle = [|\alpha\rangle \pm |-\alpha\rangle]$ are the unnormalized even and odd coherent states and $|A_\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$.

Four atoms in modes A_1, A_2, A_3 and A_4 are prepared initially in the states $|0\rangle_{A_1}$, $|0\rangle_{A_2}$, $|0\rangle_{A_3}$ and $|0\rangle_{A_4}$ respectively and two cavities $\mathbf{C}_1$ and $\mathbf{C}_2$ are prepared in optical coherent field state $|\alpha\rangle_{\mathbf{C}_1}$ and $|\alpha\rangle_{\mathbf{C}_2}$ respectively. The initial state of the atom–cavity system can be written as

$$|\psi(0)\rangle_{A_1,A_2,A_3,A_4,\mathbf{C}_1,\mathbf{C}_2} = |0000\rangle_{A_1,A_2,A_3,A_4} \otimes |\alpha,\alpha\rangle_{\mathbf{C}_1,\mathbf{C}_2}. \tag{11}$$

The detailed scheme for the generation of state (7) is shown in figure 2. First, let atom A_1 flies through the cavity $\mathbf{C}_1$ and at the same time, atom A_3 flies through the cavity $\mathbf{C}_2$. If we control the velocity of both atoms A_1 and A_3 such that each atom interacts with the optical coherent field for the time $t = \pi/2\lambda$, then the state of atom–cavity system will evolve according to the evolution given by eq. (9) and we get

$$|\psi(\pi/2\lambda)\rangle_{A_1,A_2,A_3,A_4,\mathbf{C}_1,\mathbf{C}_2} = \frac{1}{4}[(|0,+\rangle - |1,-\rangle)_{A_1,\mathbf{C}_1} \otimes |0\rangle_{A_2}]$$
$$\otimes [(|0,+\rangle - |1,-\rangle)_{A_3,\mathbf{C}_2} \otimes |0\rangle_{A_4}]. \tag{12}$$

Now atoms A_2 and A_4 pass through the cavities $\mathbf{C}_1$ and $\mathbf{C}_2$ respectively to interact with the optical coherent field for the interaction time $t = \pi/2\lambda$. The state of the system will evolve according to the evolution (9), giving,

$$
\begin{aligned}
&|\psi(\pi/\lambda)\rangle_{A_1,A_2,A_3,A_4,\mathbf{C}_1,\mathbf{C}_2} \\
&\quad = \frac{1}{4}[|00,+\rangle + |11,-\rangle]_{A_1,A_2,\mathbf{C}_1} \otimes [|00,+\rangle + |11,-\rangle]_{A_3,A_4,\mathbf{C}_2} \\
&\quad = \frac{1}{2}[|B\rangle^{(1)} \otimes |\alpha\rangle + |B\rangle^{(2)} \otimes |-\alpha\rangle]_{A_1,A_2,\mathbf{C}_1} \otimes [|B\rangle^{(1)} \otimes |\alpha\rangle \\
&\qquad + |B\rangle^{(2)} \otimes |-\alpha\rangle]_{A_3,A_4,\mathbf{C}_2},
\end{aligned} \tag{13}
$$

where $|B\rangle^{(1)}$ and $|B\rangle^{(2)}$ are the standard bipartite Bell states given by eq. (2).

As coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ are not orthogonal, $\langle\alpha|-\alpha\rangle = \mathrm{e}^{-2\alpha^2}$, but they become almost orthogonal for large mean photon number α^2. To distinguish between $|\alpha\rangle$ and $|-\alpha\rangle$, we now inject $|\alpha\rangle$ into each cavity, i.e., we make use of the displacement operator $D(\beta)|\alpha\rangle = |\alpha+\beta\rangle$, which results in $|\alpha\rangle \xrightarrow{D(\alpha)} |2\alpha\rangle$ and $|-\alpha\rangle \xrightarrow{D(\alpha)} |v\rangle$, where v is vacuum, and we get

$$
\begin{aligned}
&|\psi(\pi/\lambda)\rangle_{A_1,A_2,A_3,A_4,\mathbf{C}_1,\mathbf{C}_2} \\
&\quad = \frac{1}{2}[|B\rangle^{(1)}_{A_1,A_2} \otimes |B\rangle^{(1)}_{A_3,A_4} \otimes |2\alpha, 2\alpha\rangle_{\mathbf{C}_1,\mathbf{C}_2} \\
&\qquad + |B\rangle^{(1)}_{A_1,A_2} \otimes |B\rangle^{(2)}_{A_3,A_4} \otimes |2\alpha, v\rangle_{\mathbf{C}_1,\mathbf{C}_2} \\
&\qquad + |B\rangle^{(2)}_{A_1,A_2} \otimes |B\rangle^{(1)}_{A_3,A_4} \otimes |v, 2\alpha\rangle_{\mathbf{C}_1,\mathbf{C}_2} \\
&\qquad + |B\rangle^{(2)}_{A_1,A_2} \otimes |B\rangle^{(2)}_{A_3,A_4} \otimes |v, v\rangle_{\mathbf{C}_1,\mathbf{C}_2}].
\end{aligned} \tag{14}
$$

As for large $|\alpha|^2$, the state $|2\alpha\rangle$ has a very small probability of having no photons. Hence on performing photon counting measurement (PCM), state $|2\alpha\rangle$ gives nonzero count and state $|v\rangle$ gives zero count.

Now we perform PCM in both cavities $\mathbf{C}_1$ and $\mathbf{C}_2$. From eq. (14), it is clear that there are four possible PCM results and corresponding to these four results, there are four generated states. We tabulate the PCM results and the corresponding generated states in table 1.

Table 1. PCM results in both the cavities $\mathbf{C}_1$ and $\mathbf{C}_2$, and the generated states corresponding to these results.

Counts in cavity $\mathbf{C}_1$	Counts in cavity $\mathbf{C}_2$	Generated state
Nonzero	Nonzero	$\vert B\rangle^{(1)}_{A_1,A_2} \otimes \vert B\rangle^{(1)}_{A_3,A_4}$
Nonzero	Zero	$\vert B\rangle^{(1)}_{A_1,A_2} \otimes \vert B\rangle^{(2)}_{A_3,A_4}$
Zero	Nonzero	$\vert B\rangle^{(2)}_{A_1,A_2} \otimes \vert B\rangle^{(1)}_{A_3,A_4}$
Zero	Zero	$\vert B\rangle^{(2)}_{A_1,A_2} \otimes \vert B\rangle^{(2)}_{A_3,A_4}$

Let us consider that the generated state is the first state, i.e.,

$$\begin{aligned}|\psi\rangle_{A_1,A_2,A_3,A_4} &= |B\rangle^{(1)}_{A_1,A_2} \otimes |B\rangle^{(1)}_{A_3,A_4} \\ &= \frac{1}{2}[|00\rangle \otimes |00\rangle + |11\rangle \otimes |11\rangle + |01\rangle \otimes |01\rangle + |10\rangle \otimes |10\rangle]_{A_1,A_3,A_2,A_4}.\end{aligned} \tag{15}$$

Now Hadamard operation,

$$R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \text{with } |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

is applied on atoms A_1. Then state (15) becomes

$$\begin{aligned}|\psi\rangle_{A_1,A_3,A_2,A_4} = \frac{1}{2}[&|A_+0\rangle \otimes |00\rangle + |A_+1\rangle \otimes |01\rangle \\ &+ |A_-0\rangle \otimes |10\rangle + |A_-1\rangle \otimes |11\rangle]_{A_1,A_3,A_2,A_4}.\end{aligned} \tag{16}$$

We prepare a cavity $\mathbf{C}_3$ in optical coherent field state $|-\alpha\rangle_{\mathbf{C}_3}$. The complete state of the system is thus given as

$$\begin{aligned}|\psi\rangle_{A_1,A_3,A_2,A_4,\mathbf{C}_3} &= \frac{1}{2}[|A_+0\rangle \otimes |00\rangle + |A_+1\rangle \otimes |01\rangle + |A_-0\rangle \\ &\otimes|10\rangle + |A_-1\rangle \otimes |11\rangle]_{A_1,A_3,A_2,A_4} \otimes |-\alpha\rangle_{\mathbf{C}_3}.\end{aligned} \tag{17}$$

On sending atom A_1 through the cavity $\mathbf{C}_3$, for an interaction time $t = \pi/2\lambda$, the state of the system evolves according to evolutions in eq. (10) and we get

$$\begin{aligned}|\psi\rangle_{A_1,A_3,A_2,A_4,\mathbf{C}_3} &= \frac{1}{2}[|A_+0\rangle \otimes |00\rangle \otimes |\alpha\rangle + |A_+1\rangle \otimes |01\rangle \\ &\otimes|\alpha\rangle + |A_-0\rangle \otimes |10\rangle \otimes |-\alpha\rangle + |A_-1\rangle \otimes |11\rangle \\ &\otimes| - \alpha\rangle]_{A_1,A_3,A_2,A_4,\mathbf{C}_3}.\end{aligned} \tag{18}$$

Now Hadamard operation R is applied on atom A_1. Then state (18) becomes

$$\begin{aligned}|\psi\rangle_{A_1,A_3,A_2,A_4,\mathbf{C}_3} &= \frac{1}{2}[|00\rangle \otimes |00\rangle \otimes |\alpha\rangle + |01\rangle \otimes |01\rangle \otimes |\alpha\rangle \\ &+|10\rangle \otimes |10\rangle \otimes | - \alpha\rangle + |11\rangle \otimes |11\rangle \\ &\otimes| - \alpha\rangle]_{A_1,A_3,A_2,A_4,\mathbf{C}_3}.\end{aligned} \tag{19}$$

We let atom A_1 fly through the cavity $\mathbf{C}_3$ for time $t = \pi/2\lambda$ and then let atom A_3 fly through the cavity $\mathbf{C}_3$ for time $t = \pi/2\lambda$. Then, the states in modes A_1, A_3 and $\mathbf{C}_3$ evolve according to evolutions (9) and (10), giving,

$$|00,\alpha\rangle_{A_1,A_3,\mathbf{C}_3} \rightarrow \frac{1}{\sqrt{2}}[|B\rangle^{(1)}|\alpha\rangle + |B\rangle^{(2)}|-\alpha\rangle]_{A_1,A_3,\mathbf{C}_3},$$

$$|01,\alpha\rangle_{A_1,A_3,\mathbf{C}_3} \rightarrow \frac{1}{\sqrt{2}}[|B\rangle^{(3)}|\alpha\rangle + |B\rangle^{(4)}|-\alpha\rangle]_{A_1,A_3,\mathbf{C}_3},$$

$$|10,-\alpha\rangle_{A_1,A_3,\mathbf{C}_3} \rightarrow \frac{1}{\sqrt{2}}[|B\rangle^{(3)}|-\alpha\rangle - |B\rangle^{(4)}|\alpha\rangle]_{A_1,A_3,\mathbf{C}_3},$$

$$|11,-\alpha\rangle_{A_1,A_3,\mathbf{C}_3} \rightarrow \frac{1}{\sqrt{2}}[|B\rangle^{(1)}|-\alpha\rangle - |B\rangle^{(2)}|\alpha\rangle]_{A_1,A_3,\mathbf{C}_3}. \quad (20)$$

Using eq. (20) for modes A_1, A_3 and $\mathbf{C}_3$ in eq. (19), the final output state is written as,

$$|\psi\rangle_{A_1,A_3,A_2,A_4,\mathbf{C}_3} = \frac{1}{\sqrt{2}}[|\eta\rangle^{(1)}_{A_1,A_3,A_2,A_4}\otimes|\alpha\rangle_{\mathbf{C}_3}+|\eta\rangle^{(2)}_{A_1,A_3,A_2,A_4}\otimes|-\alpha\rangle_{\mathbf{C}_3}], \quad (21)$$

where

$$|\eta\rangle^{(1)}_{A_1,A_3,A_2,A_4} = \frac{1}{2}[|B\rangle^{(1)}\otimes|00\rangle - |B\rangle^{(2)}\otimes|11\rangle + |B\rangle^{(3)} \otimes|01\rangle - |B\rangle^{(4)}\otimes|10\rangle]_{A_1,A_3,A_2,A_4}, \quad (22)$$

$$|\eta\rangle^{(2)}_{A_1,A_3,A_2,A_4} = \frac{1}{2}[|B\rangle^{(1)}\otimes|00\rangle + |B\rangle^{(2)}\otimes|11\rangle + |B\rangle^{(3)} \otimes|01\rangle + |B\rangle^{(4)}\otimes|10\rangle]_{A_1,A_3,A_2,A_4}. \quad (23)$$

Now we inject $|\alpha\rangle$ into the cavity $\mathbf{C}_3$, then state (21) becomes

$$|\psi\rangle_{A_1,A_3,A_2,A_4,\mathbf{C}_3} = \frac{1}{\sqrt{2}}[|\eta\rangle^{(1)}_{A_1,A_3,A_2,A_4}\otimes|2\alpha\rangle_{\mathbf{C}_3}+|\eta\rangle^{(2)}_{A_1,A_3,A_2,A_4}\otimes|\nu\rangle_{\mathbf{C}_3}]. \quad (24)$$

On performing PCM in the cavity $\mathbf{C}_3$, from eq. (24), it is clear that the result nonzero count gives the generated state (22), while the result zero count gives the generated state (23). We see that state (23) is the required state and however, the state (22) can be converted into state (23) simply by applying local operation σ_z on atom A_2. Thus, for both PCM results, we obtain the required state, which we want to generate.

We now discuss about the experimental feasibility of our scheme, which generates the entangled $4n$-qubit state. The cavity [28], which can be used in the state generation process, is an open Fabry–Perot interferometer, made up of two carefully polished niobium mirrors facing each other (diameter of each mirror $D = 50$ mm). Optical field occupies only ∼10% size of the cavity, i.e., ∼5 mm. Mishra *et al* [24] have shown that the atom–cavity field interaction time $t \approx 10^{-4}$ s, cavity damping time $T_{\mathrm{D}} \approx 1$ s and velocity of atom should be 50 m/s. In the state generation process, we see that in all the atoms, atom A_1 travels the longest distance through the cavity and in the vacuum space. Hence if we show that the total flight time of atom A_1 is less than the cavity damping time T_{D}, then our scheme will be experimentally feasible.

Let us take $T(=10t = 10^{-3}$ s) as the time taken by the atom to cross the cavity, i.e., the atom travels 50 mm. If the oven (source of atoms) is placed 10 mm away from cavity $\mathbf{C}_1$, then atom A_1 will take time $T/5$ to reach the cavity $\mathbf{C}_1$ and further it will cross the cavity $\mathbf{C}_1$ in time T. If there is a separation of 10 mm between two cavities $\mathbf{C}_1$ and $\mathbf{C}_3$, then atom A_1 will take time $T/5$ to cross this separation (10 mm) and further time T to cross the cavity $\mathbf{C}_3$. After 10 mm separation from the cavity $\mathbf{C}_3$, atom A_1 gets reflected back to cavity $\mathbf{C}_3$, then atom A_1 will take time $2T/5$ to cross the separation (atom travels twice 10 mm separation, i.e., vacuum space) and takes time T to cross cavity $\mathbf{C}_3$. Thus total flight time of atom A_1 is $(4T/5) + 3T = 19T/5 \approx 4 \times 10^{-3}$ s, which is lesser than the cavity damping time $T_{\mathrm{D}} = 1$ s. Hence, our special kind of $4n$-qubit entangled state can be generated using the presently available technology.

4. Conclusions

In this paper, we have extended the original SQIE protocol, which enables the secure exchange of n single qubit information states among n nodes of a quantum network, with the aid of a special kind of $4n$-qubit entangled state and the classical assistance of an extra participant Charlie. For experimental realization of our extended SQIE protocol, we have suggested an efficient scheme for the generation of a special kind of $4n$-qubit entangled state using interaction between highly detuned Λ-type three-level atoms and optical coherent field. Further we have discussed the experimental feasibility of this scheme by considering some experimental parameters like cavity damping time and total flight time of atoms and concluded that the generation of a special kind of $4n$-qubit entangled state is within the reach of the presently available technology.

Acknowledgements

The authors are thankful to N Chandra and R Prakash for their interest in this work. The authors also would like to thank R S Singh, D K Singh, D K Mishra, R Kumar, P Kumar, Vikram Verma and Ajay K Yadav for helpful and stimulating discussions. One of the authors M K M acknowledges UGC for financial support under UGC–SRF fellowship scheme and A K M acknowledges CSIR for financial support under CSIR–SRF fellowship scheme.

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