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Dynamics of non-Markovianity in the presence of a driving field

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Abstract. We investigate a two-level system in a cavity QED by considering the effects of amplitude damping, phase damping and driving field. We have studied the non-Markovianity in resonance and non-resonance limits in the presence of these effects using Breuer–Laine–Piilo (BLP) non-Markovianity measure ($N_{\rm BLP}$). The evolution of the system is derived using the time convolutionless (TCL) master equation. In some conditions, it is shown that in the presence of a driving field, the $N_{\rm BLP}$ increases in the resonance and non-resonance limits. We have also found the exact solution of the master equation in order to investigate the effect of temperature- and environment-excited states. We have shown that the behaviour of non-Markovianity is very different from what one can see from the TCL approach. We have also presented some explanation about the behaviour of non-Markovianity in the exact solution using quantum discord (QD).

Keywords. Non-Markovianity; driving field; cavity QED; time convolutionless master equation; amplitude damping; phase damping; two-level atoms.

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1. Introduction

Quantum information science (QIS) has attracted great attention because of its theoretical aspects and also its applications such as quantum teleportation, quantum cryptography etc. [1,2]. The main problem in the realization of quantum computing or quantum information processing is decoherence [3] because of the inevitable coupling between quantum systems and their environment which causes information loss. The evolution of closed quantum systems is unitary but the real quantum systems are not closed or isolated and interact with their surrounding environments. Therefore these interactions can lead to dissipation of information [4–7] and contrary to the case of closed quantum systems, the dynamics of an open quantum system cannot be represented in terms of a unitary time evolution [8–10]. One can study the dynamics of the system by directly solving the master equation [8]. In this method we can find the evolution of the total system (principal system and its environment). The density matrix of the principal system can be studied by Markovian (memoryless) and non-Markovian approaches. In the Markovian processes, information flows from the system to the environment whereas in non-Markovian processes, it is possible that information flows back to the system. It may be noted that the dissipation of information can occurr in all the physical processes.

There are other methods also for studying the open quantum systems such as stochastic differential equation (SDE) and Langevin equation [11–13]. Breuer *et al* have introduced a criterion for the amount of non-Markovianity called measure of non-Markovianity (N_{BLP}) [14]. Recently, other criterions have been introduced by Bylicka *et al* [15] and Liu *et al* [16]. The latter provides a measure of non-unital non-Markovianity of quantum processes, which is a supplement to N_{BLP} measure. Haikka *et al* [17] studied the differences and analogies of the non-Markovianity measures. One can also show that the non-Makovianity can speed up the quantum evolution and therefore lead to a smaller quantum speed limit time [18]. We investigate the dynamics of a two-level system in a single-mode cavity with a driving field which has been proposed in [19–25]. It may be noted that this model is used in single atom detectors in strong coupling limit [26–29]. Also it is important to keep in mind that in experimental set-ups, atom remains inside the cavity only for a short time. So we study the quantum correlation between the system and its environment in a finite time. Werlang *et al* [30] have analysed various dissipative channels such as dephasing, depolarizing and generalized amplitude damping, assuming independent perturbation for two qubits under Markovian environment. Similar discussions can also be found in [31,32].

In this paper, we study the dynamics of non-Markovianity under system–environment coupling using the time convolutionless (TCL) master equation. We first write the Hamiltonian of the system and the environment and their interaction. We recall the N_{BLP} measure in §3 and derive the dynamics of this measure numerically, by considering different conditions in §4. In §5, the dynamics of the system is also investigated using the exact solution of master equation again and then we check the amount of non-Markovianity by the N_{BLP} measure. We derive the evolution of quantum discord between the system and the environment to explain some of the behaviours of the measure.

2. Model

Consider a two-level system coupled to an environment consisting of a radiation field. The total Hamiltonian reads:

$$H = H_0 + H_{\mathrm{I}}, \tag{1}$$

where H_0 is the self-Hamiltonian of the open system and the free Hamiltonian of the environment ($\hbar = 1$) [33–35]:

$$H_0 = \frac{\omega_0}{2}\sigma_z + \omega a a^\dagger + i\varepsilon_p(a - a^\dagger) \tag{2}$$

and H_{I} is the interaction terms of the system and the environment:

$$H_{\mathrm{I}} = g_1(\sigma_+ + \sigma_-)(a + a^\dagger) + g_2\sigma_+\sigma_-(a + a^\dagger). \tag{3}$$

The first term in (2) is related to the energy of a two-level system and ω_0 is the energy difference between two states $|0\rangle$ and $|1\rangle$. The second term is the cavity energy, ω is the single-mode cavity frequency and $a(a^\dagger)$ is the annihilation(creation) operator. The third term is the driving field with amplitude ε_p in dipole approximation. The parameter ε_p is related to the power output driving laser [21]. It creates and annihilates photons in cavity which is called pump Hamiltonian [19] and it is usually achieved using a laser (see figure 1). The interaction terms (3) contain the amplitude and phase damping. The amplitude-damping term is the dissipative interaction between the system and the environment that includes an exchange of energy [36–38]. The phase-damping term describes the loss of quantum coherence without loss of energy and leads to decoherence without relaxation [36,37]. $\sigma_z(|0\rangle\langle 0| - |1\rangle\langle 1|)$ is the Pauli matrix and $\sigma_- = |1\rangle\langle 0|(\sigma_+ = |0\rangle\langle 1|)$ is the atomic lowering (raising) operator. g_1 and g_2 denote the strengths of coupling constant for phase-damping and amplitude-damping effects, respectively.

In the weak coupling limit, the non-Markovian master equation in the interaction picture for reduced system with TCL approach can be written as [4]

$$\frac{\partial \rho_{\mathrm{S}}}{\partial t} = -\int_0^t \mathrm{d}\tau \mathrm{Tr}_{\mathrm{E}}[H_{\mathrm{I}}(t), [H_{\mathrm{I}}(t-\tau), \rho(t)]]. \tag{4}$$

Indices 'S' and 'E' refer to the system and the environment, respectively. The total Hamiltonian in the interaction picture can be written as

$$H_{\mathrm{I}}(t) = g_1(\sigma_+ \mathrm{e}^{i\omega_0 t} + \sigma_- \mathrm{e}^{i\omega_0 t})(a(t) + a^\dagger(t)) + g_2\sigma_+\sigma_-(a(t) + a^\dagger(t)). \tag{5}$$

Using the interaction picture, we get the following equation for $a(t)$:

$$\frac{\partial a(t)}{\partial t} = -i\omega a(t) - \varepsilon_p. \tag{6}$$

Assuming $a(t=0) = a$ we have

$$a(t) = A(t) + a\mathrm{e}^{-i\omega t}, \tag{7}$$

where

$$A(t) = \left(\frac{i}{\omega}\right)\varepsilon_p(1 - \mathrm{e}^{-i\omega t}). \tag{8}$$

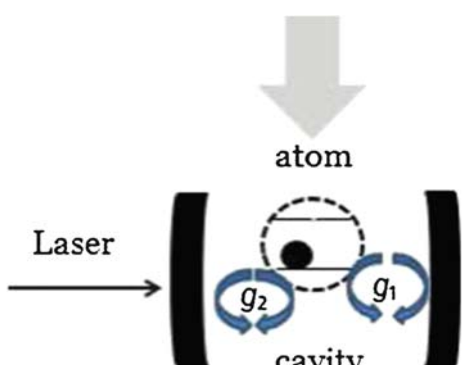

Figure 1. A two-level atom in a single mode cavity interacts with different couplings.

Substituting eq. (5) in eq. (4), by dropping the terms containing $e^{\pm i(\omega-\omega_0)}$ which is called rotating wave approximation (RWA), we get

$$\begin{aligned}\frac{\partial\rho(t)}{\partial t} &= -i[H_{\mathrm{LS}},\rho] + g_1^2\gamma_1(t)\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho\}\right).\\ &\quad + g_1^2[\gamma_2(t)(\sigma_-\rho\sigma_+ + \sigma_+\rho\sigma_- - \sigma_+\sigma_-\rho - \rho\sigma_-\sigma_+) + \mathrm{h.c.}]\\ &\quad + g_2^2\gamma_3(t)\left(\sigma_+\sigma_-\rho\sigma_+\sigma_- - \frac{1}{2}\{\sigma_+\sigma_-,\rho\}\right). \end{aligned} \tag{9}$$

H_{LS} is the Lamb shift Hamiltonian which describes a small shift in the energy of a two-level atom:

$$H_{\mathrm{LS}} = \left(\int_0^t (g_1^2\sin(\omega_0-\omega)\tau - g_2^2\sin\omega\tau)\mathrm{d}\tau\right)\sigma_+\sigma_-. \tag{10}$$

Also h.c. denotes the Hermitian conjugation. For convenience, we have also dropped the index S in the equations. It may be noted that eq. (9) has been written in zero temperature. Time-dependent rates $\gamma_i(t)$ are defined as

$$\begin{aligned}\gamma_1(t) &= \int_0^t 2\cos[(\omega_0-\omega)\tau]\mathrm{d}\tau,\\ \gamma_2(t) &= \int_0^t A(t)A^*(t-\tau)\mathrm{e}^{i\omega_0\tau}\mathrm{d}\tau,\\ \gamma_3(t) &= \int_0^t \left(2\cos\omega\tau + \left(\frac{\varepsilon_p}{\omega}\right)^2\sin\omega t\sin\omega(t-\tau)\right)\mathrm{d}\tau. \end{aligned} \tag{11}$$

Here $*$ indicates complex conjugation. The explicit solution of eq. (9) when $g_1 = 0$ (omitting the amplitude-damping effect) becomes

$$\begin{aligned}\rho_{00}(t) &= \rho_{00}(0),\\ \rho_{11}(t) &= 1-\rho_{00}(t),\\ \rho_{01}(t) &= \rho_{01}(0)\mathrm{e}^{-4g_2^2\int_0^t\gamma_2(t)\mathrm{d}t},\\ \rho_{10}(t) &= \rho_{10}(0)\mathrm{e}^{-4g_2^2\int_0^t\gamma_2(t)\mathrm{d}t}. \end{aligned} \tag{12}$$

Now we are ready to study the time evolution of the dynamical trace distance which will be calculated in the next section.

3. The measure of non-Markovianity

The non-Markovianity of the system can be studied by applying the N_{BLP} measure [14,18]. Trace distance (distinguishability) for the two states is defined as [1]

$$D(\rho_1(t),\rho_2(t)) = \frac{1}{2}\|\rho_1(t)-\rho_2(t)\|_1, \tag{13}$$

where $\|.\|_1$ denotes some appropriate trace norm. In the Markovian processes, the trace distance decreases between any two arbitrary states of the system. This means that the change of dynamical trace distance $\sigma(t)$ (which is called the information flow) is negative:

$$\sigma(\rho_{1,2};t) = \frac{\mathrm{d}}{\mathrm{d}t}D(\rho_1(t),\rho_2(t)) \le 0. \tag{14}$$

In the non-Markovian processes, $\sigma(t)$ is positive. Therefore, two states find more distinct shapes compared to their resemblance in previous times and therefore distinguishability between them increases [39].

The total amount of non-Markovianity can be quantified by

$$N_{\mathrm{BLP}} = \max_{\rho_{1,2}(0)} \frac{1}{2} \int [\sigma(\rho_{1,2}; t) + |\sigma(\rho_{1,2}; t)|]\mathrm{d}t. \tag{15}$$

Here maximization is performed on all the possible initial states $(\rho_1(0), \rho_2(0))$ which can be chosen for two states.

We consider the initial density matrix of the two-level system as

$$\rho(0) = \frac{1}{2}(I + \vec{b}(0) \cdot \vec{\sigma}), \tag{16}$$

where $\vec{b}(0)$ is the Bloch vector and σ_i with $i = x, y, z$ are the Pauli matrices. When $g_1 = 0$, eq. (14) becomes

$$\sigma(\rho_{1,2}; t) = \frac{(\Delta b_x(0)^2 + \Delta b_y(0)^2)\mathrm{e}^{F(t)}(4g^2\gamma_2(t))}{2\sqrt{(\Delta b_x(0)^2 + \Delta b_y(0)^2)\mathrm{e}^{F(t)} + \Delta b_z(0)^2}}. \tag{17}$$

After doing some calculations one can show that

$$F(t) = \frac{4g^2}{\omega^4}[\cos(\omega t + 1)(2\omega^2 - \varepsilon_p^2(\cos\omega t - 3)) - 4(\omega^2 + \varepsilon_p^2)]. \tag{18}$$

4. Numerical simulation

Our numerical simulations show that the maximum of the non-Markovianity measure occurs for the two initial states $(|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle))$ and $(|0\rangle, |1\rangle)$. Simulations have been performed by the numerical method used in [14]. The non-Markovianity N_{BLP} as a function of the detuning is plotted in ref. [38]. It is shown that the non-Markovianity induced by the phase-damping effect is larger than that induced by the amplitude effect. But in this paper we have investigated systems in which g_1 is a function of g_2. Figures 2 and 3 present the result of our calculations which have been done with independent values of coupling constants g_1 and g_2. In figure 2, the N_{BLP} measure is plotted as a function of ε_p in the resonance limit ($\omega_0 \approx \omega$) for different values of coupling constants. The N_{BLP} measure is zero when $g_1 > g_2$. This means that the environment is fully Markovian. The N_{BLP} measure is not zero when $g_1 < g_2$ but it remains almost constant as the driving field amplitude increases. A more interesting result is that the N_{BLP} increases as amplitude of the driving field increases when $g_1 = g_2$. It clearly indicates that the non-Markovian effects become brighter in systems with greater values of driving field when $g_1 = g_2$.

Figure 3 demonstrates N_{BLP} as a function of the driving field for different values of couplings in the non-resonance limit (e.g. $\omega_0 = 2\omega$). This figure shows that the measure increases as driving field amplitude increases for all the values of g_1 and g_2. Therefore, the non-Markovianity behaviour is brighter in systems at the non-resonance limit. It may be noted that for systems with $g_1 > g_2$, the measure becomes larger in comparison with N_{BLP} in $g_1 < g_2$. Our numerical calculations show that for $g_1 = 0$, the measure also increases

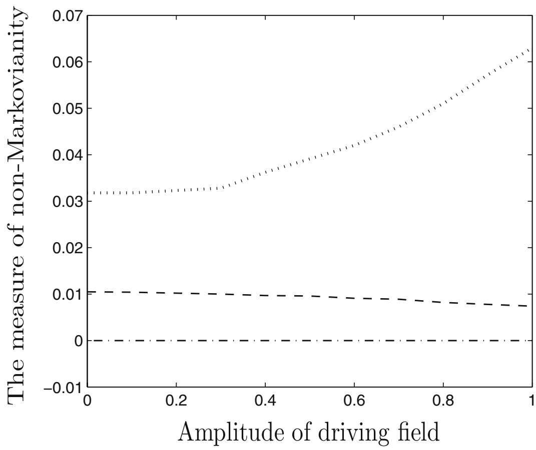

Figure 2. The non-Markovianity measure as a function of driving field amplitude in the resonance limit ($\omega_0 \approx \omega$). The diagram is plotted with $g_1 = g_2$ (dotted line), $g_1 > g_2$ (dash–dotted line) and $g_1 < g_2$ (dashed line).

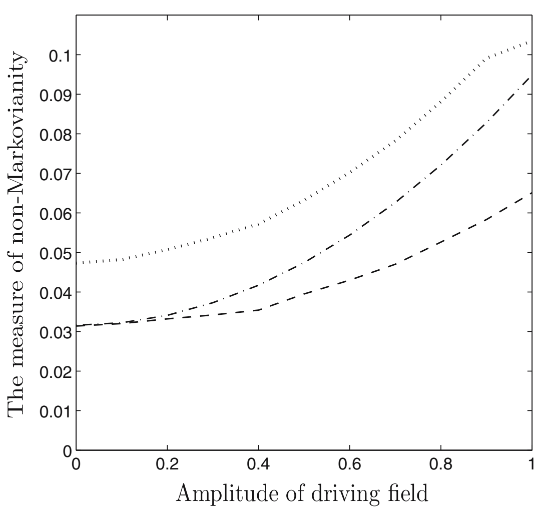

Figure 3. The non-Markovianity measure as a function of driving field amplitude in the non-resonance limit (e.g. $\omega_0 = 2\omega$) for $g_1 = g_2$ (dotted line), $g_1 > g_2$ (dash–dotted line) and $g_1 < g_2$ (dashed line).

with respect to the driving field amplitude. However, the increasing rate is smaller than what we can see in cases with $g_1 > g_2$.

5. Exact solution

It is clear that the evolution eq. (4) is an approximated model which is written using the TCL approach. The effects of temperature and the presence of the excited states of the environment is not included in the previous results. Therefore, we need an exact equation to study the evolution of the system by considering the above-mentioned effects. Thus, we have to find the N_{BLP} measure using the exact solution of the master equation [8,40]. The Schrödinger equation in the interaction picture is

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H_{\mathrm{I}}(t)|\Psi(t)\rangle. \tag{19}$$

We assume that the initial state of the total system is

$$|\Psi(0)\rangle = c_0|0\rangle_{\mathrm{S}}|0\rangle_{\mathrm{E}} + c_1|1\rangle_{\mathrm{S}}|0\rangle_{\mathrm{E}} + c_2|0\rangle_{\mathrm{S}}|n\rangle_{\mathrm{E}} + c_3|1\rangle_{\mathrm{S}}|n\rangle_{\mathrm{E}}, \tag{20}$$

where $|0\rangle_{\mathrm{S}} = \sigma_-|1\rangle_{\mathrm{S}}$ and $|1\rangle_{\mathrm{S}} = \sigma_+|0\rangle_{\mathrm{S}}$ indicate the ground and excited states of the system, respectively. The state $|0\rangle_{\mathrm{E}}$ denotes the vacuum state of the cavity and $|n\rangle_{\mathrm{E}}$ presents the state with one photon in the single-mode cavity (mode n). Also in relation (20) we have added a new term ($c_3(0)|1\rangle_{\mathrm{S}}|n\rangle_{\mathrm{E}}$) due to the phase-damping effect. To solve eq. (19), we substitute

$$|\Psi(t)\rangle = c_0(t)|0\rangle_{\mathrm{S}}|0\rangle_{\mathrm{E}} + c_1(t)|1\rangle_{\mathrm{S}}|0\rangle_{\mathrm{E}} + c_2(t)|0\rangle_{\mathrm{S}}|n\rangle_{\mathrm{E}} + c_3(t)|1\rangle_{\mathrm{S}}|n\rangle_{\mathrm{E}} \tag{21}$$

into the Schrödinger equation (19). The result is

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}c_0(t) &= -ig_1A^*(t)\mathrm{e}^{-i\omega_0 t}c_1(t),\\
\frac{\mathrm{d}}{\mathrm{d}t}c_1(t) &= -i\Big(-\frac{g_2\varepsilon_p}{\omega}\sin\omega t c_1(t) + g_2\mathrm{e}^{i\omega t}c_3(t)\\
&\qquad + g_1A(t)\mathrm{e}^{i\omega_0 t}c_0(t) + g_1\mathrm{e}^{i(\omega-\omega_0)t}c_2(t)\Big),\\
\frac{\mathrm{d}}{\mathrm{d}t}c_2(t) &= -i(g_1\mathrm{e}^{-i(\omega-\omega_0)t}c_1(t) + g_1A^*(t)\mathrm{e}^{-i\omega_0 t}c_3(t)),\\
\frac{\mathrm{d}}{\mathrm{d}t}c_3(t) &= -i\Big(-\frac{g_2\varepsilon_p}{\omega}\sin\omega t c_3(t) + g_2\mathrm{e}^{i\omega t}c_1(t) + g_1A(t)\mathrm{e}^{i\omega_0 t}c_2(t)\Big).
\end{aligned} \tag{22}$$

By the numerical solution of the above equations we can plot the N_{BLP} measure as a function of ε_p. Figures 4 and 5 demonstrate N_{BLP} with respect to the driving field amplitude in this situation. Comparing figures 4 and 2 clearly shows that the amount of non-Markovianity in this case is larger than the estimated non-Markovianity using the TCL approach. Also in the $g_1 > g_2$ case, information flows back to the system and the non-Markovianity effect appears again. The reason is that the environment is taken in its ground state in the TCL approach, while in the exact solution we can consider more complicated states for the environment by adding its excited states occurring easily in actual systems. It is an important result because it means that the non-Markovianity in real systems is noticeable and we cannot easily neglect its effects. Maximum non-Markovianty occurs for the initial states: $c_0(0) = 0, c_1(0) = 1, c_2(0) = 0, c_3(0) = 0$ and $c_0(0) = 0, c_1(0) = 0, c_2(0) = 1, c_3(0) = 0$ or $c_0(0) = \sqrt{\frac{1}{2}}, c_1(0) = \sqrt{\frac{1}{2}}, c_2(0) = 0, c_3(0) = 0$ and $c_0(0) = -\sqrt{\frac{1}{2}}, c_1(0) = \sqrt{\frac{1}{2}}, c_2(0) = 0, c_3(0) = 0$. It shows that the excited states of the environment have significant contributions to the amount of non-Markovianity as mentioned earlier. As presented in figure 4, the behaviour of N_{BLP} when $g_1 < g_2$ is different from the other situations. In this case N_{BLP} is almost constant, but

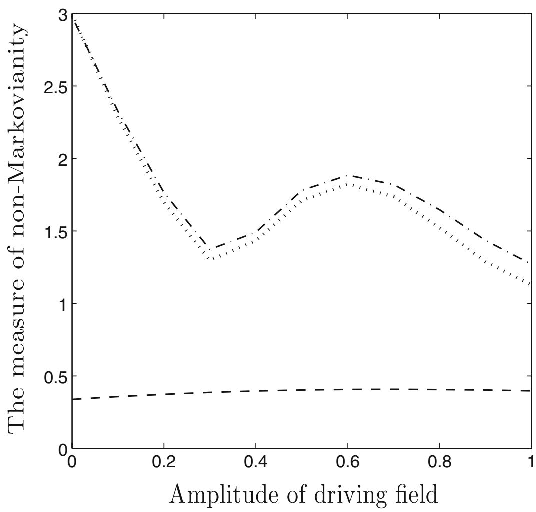

Figure 4. The non-Markovianity measure in the resonance limit ($\omega_0 \approx \omega$) for $g_1 = g_2$ (dotted line), $g_1 > g_2$ (dash–dotted line) and $g_1 < g_2$ (dashed line).

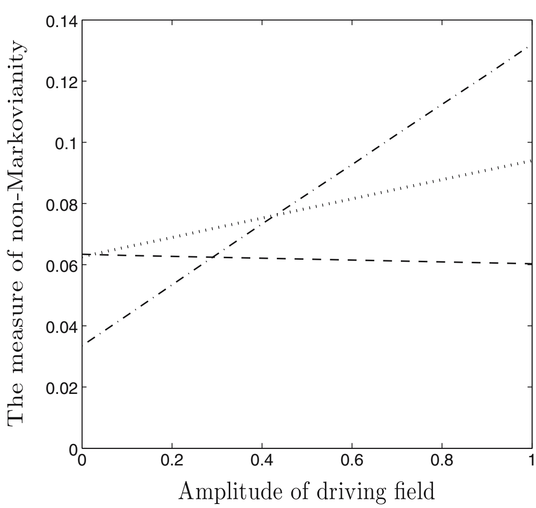

Figure 5. The non-Markovianity measure in the non-resonance limit (e.g. $\omega_0 = 2\omega$) for $g_1 = g_2$ (dotted line), $g_1 > g_2$(dash–dotted line) and $g_1 < g_2$ (dashed line).

when $g_1 = g_2$ and $g_1 > g_2$, they are oscillating and decreasing functions of the driving field amplitude, because the environment excited states are added to the initial states and the energy exchange occurred between the system and the environment. Rong-Chun Ge *et al* [41] have shown that the evolution of both classical and quantum correlations closely depend on the form of the initial states. We observe that in the non-resonance limit, the measure increases with the amplitude of the driving field but its amount remains lower than its value for the resonance limit when $g_1 \leq g_2$.

To understand the reason for these behaviours, we find the evolution of the quantum discord (QD) and the concurrence between the system and its environment. The concurrence is defined using an explicit formula for a mixed state of two qubits as:

$$C(\rho) = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{23}$$

where λ_i is the square root of the eigenvalues of $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in decreasing order (ρ^* denotes the complex conjugate of ρ) [42]. To find QD of the system we start with the density matrix of the total system in the basis $\{|1n\rangle, |10\rangle, |0n\rangle, |00\rangle\}$ and we do the projective measure on environment in the basis $\{\cos\theta|1\rangle_{\mathrm{E}} + \mathrm{e}^{i\phi}\sin\theta|0\rangle_{\mathrm{E}}, \mathrm{e}^{-i\phi}\sin\theta|1\rangle_{\mathrm{E}} -$

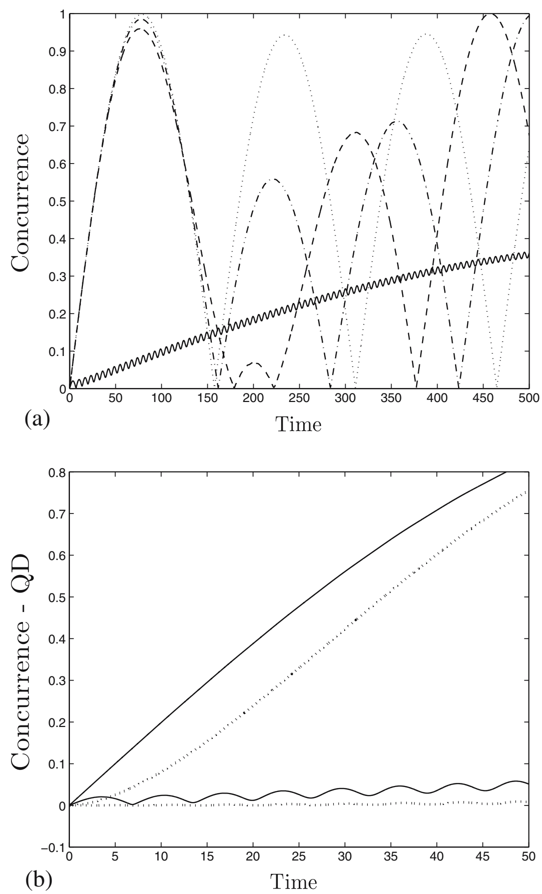

Figure 6. **(a)** The concurrence as a function of time for a two-level system and the single-mode cavity when $g_1 > g_2$ with $\varepsilon_p/\omega = 0.1$ (dotted line), $\varepsilon_p/\omega = 0.3$ (dash–dotted line) and $\varepsilon_p/\omega = 0.5$ (dashed line) and when $g_1 < g_2$ with $\varepsilon_p/\omega = 0.3$ (solid line) in the resonance limit. **(b)** Concurrence (solid line) and QD (dotted line) measure as functions of time for $g_1 > g_2$ (upper lines) and $g_1 < g_2$ (lower lines) in the resonance limit ($\varepsilon_p/\omega = 0.5$). The initial conditions are $c_0(0) = \sqrt{\frac{1}{2}}, c_1(0) = \sqrt{\frac{1}{2}}, c_2(0) = 0, c_3(0) = 0$ and $c_0(0) = 0, c_1(0) = 1, c_2(0) = 0, c_3(0) = 0$.

$\cos\theta|0\rangle_{\mathrm{E}}\}$. It may be noted that the QD measured the quantum correlations, including entanglement but is not limited to that, while the concurrence is mainly a measure for quantum entanglement. However, precise relation between concurrence and the QD is not known.

The QD and the concurrence have been calculated numerically and the results are demonstrated in figure 6. Our calculations show that the quantum correlations do not change for different values of ε_p in regions where $g_1 < g_2$. Therefore, we have plotted the concurrence just for $\varepsilon_p/\omega = 0.3$. Figure 6a shows the QD and concurrence for different values of ε_p in regions where $g_1 > g_2$. Figure 6b presents QD and concurrence as functions of time between times 0 and 50. The initial correlations between the system and the environment can fundamentally influence the dynamics of the system [43]. In the two limits considered, the initial correlations between the atom and the single-mode cavity are different. Therefore, two distinct behaviours are observed. It may be noted that the amount of non-Markovianity is related to the amount of quantum correlations (see figure 6). In non-resonance limit, the QD and the concurrence are negligible. Concurrence and QD are plotted in figure 6b as functions of time in the resonance limit. The concurrence is larger than quantum discord during the evolution which also has been reported before [44,45].

6. Conclusion

We considered two-level systems coupled to its environment and derived the evolution of density matrix of the system by TCL approach and also the exact solution of the master equation. We used the Breuer–Laine–Piilo (BLP) non-Markovianity measure (N_{BLP}) and obtained the dynamics of non-Markovianity in the resonance and non-resonance limits.

In the presence of the driving field, the non-Markovianity is an increasing function of amplitude of the driving field. We found that by decreasing the amplitude-damping effect, the measure of non-Markovianity decreases in the non-resonance limit. But in the resonance limit, the measure of non-Markovianity is increased by decreasing the amplitude-damping effect because of the energy exchange between the system and its environment. The results show that in both the limits, the changes of backflow of information from the environment to the system are negligible and it depends on the ratio ω/ω_0. To investigate the effects of the cavity excited states on non-Markovianity, the exact solution of the master equation was also considered. One can observe a significant amount of non-Markovianity in the exact solutions of the master equation. Such effects occurred due to the presence of excited state of the cavity which contributed in the initial states. It leads to flow back which extremely affects the non-Markovianity behaviours in the system. The N_{BLP} measure in the resonance limit finds greater values compared to other limits if it is calculated from the exact solution of the master equation. But in the TCL method, such a situation cannot be observed. We also studied the evolution of quantum discord and concurrence to find the reason of different behaviour of the measure. In the results of the exact solution one can observe three different types of evolution for quantum discord that depended on different values of coupling constants. Therefore, we can distinguish a regeneration of the non-Markovian effects in the system.

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