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Is the size of θ_{13} related to the smallness of the solar mass splitting?

SOUMITA PRAMANICK and AMITAVA RAYCHAUDHURI*
Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700 009, India
*Corresponding author. E-mail: palitprof@gmail.com

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Abstract. θ_{13} is small compared to the other neutrino mixing angles. The solar mass splitting is about two orders smaller than the atmospheric splitting. We indicate how both could arise from a perturbation of a more symmetric structure. The perturbation also affects the solar mixing angle and can tweak alternate mixing patterns such as tribimaximal, bimaximal, or other variants to viability. For real perturbations only normal mass ordering with the lightest neutrino mass less than 10^{-2} eV can accomplish this goal. Both mass orderings can be accommodated by going over to complex perturbations if the lightest neutrino is heavier. The CP-phase in the lepton sector, fixed by θ_{13} and the lightest neutrino mass, distinguishes different options.

Keywords. Neutrino mixing; θ_{13}; solar splitting; perturbation.

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1. Introduction

The recent experimental observation [1] of non-zero θ_{13}

$$\sin^2 2\theta_{13} = 0.090^{+0.008}_{-0.009} \text{ (Daya Bay: 217 days, rate and spectrum)} \quad (1)$$

$$\sin^2 2\theta_{13} = 0.100 \pm 0.010 \text{ (stat)} \pm 0.015 \text{ (syst)}$$
$$\text{(RENO: 403 live days, rate only)} \quad (2)$$

has triggered much model building.

The flavour basis neutrino mass matrix is diagonalized by a unitary matrix U such that $U^T M_\nu U = \text{diag}(m_1, m_2, m_3)$. In the standard parametrization, the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) mixing matrix U is expressed as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (3)$$

As is evident, the entire PMNS matrix is completely determined by the oscillation observables which in turn dictates the lepton mixings. The association of the CP violating phase δ with s_{13} readily suggests the possibility of occurrence of CP violation now as θ_{13} is observed to be non-vanishing. From 1σ global fits the currently favoured values are [2]

$$\begin{aligned} \Delta m^2_{21} &= (7.50^{+0.18}_{-0.19}) \times 10^{-5}\,\text{eV}^2, \qquad \theta_{12} = (33.36^{+0.81}_{-0.78})^\circ, \\ |\Delta m^2_{31}| &= (2.473^{+0.070}_{-0.067}) \times 10^{-3}\,\text{eV}^2, \qquad \theta_{23} = (40.0^{+2.1}_{-1.5} \oplus 50.4 \pm 0.13)^\circ \\ &\qquad \theta_{13} = (8.66^{+0.44}_{-0.46})^\circ, \quad \delta = (300^{+66}_{-138})^\circ. \end{aligned} \tag{4}$$

The atmospheric mixing angle, θ_{23}, displays an intriguing deviation from exact maximal mixing ($\theta_{23} = \pi/4$). θ_{12} is also large but not maximal, while θ_{13} is the smallest of the three although it is close to its upper bound from the earlier data. In this sense, the latter is sometimes said to be large.

The mass spectrum harbours some fascinating unsettled issues. Although the magnitude of the solar and atmospheric neutrino mass splittings are now well measured, the absolute mass remains undetermined. Moreover, the sign of Δm^2_{31} remains unknown keeping the options open for both the normal ($m_1 < m_2 < m_3$) and the inverted ($m_3 < m_1 < m_2$) ordering depending upon whether this sign is positive or negative. Note that the solar splitting is about two orders of magnitude smaller than the atmospheric one. It is useful to define the ratio $R_{\text{mass}} \equiv |\Delta m^2_{21}/\Delta m^2_{31}| = (3.03 \pm 0.16) \times 10^{-2}$.

Therefore, the oscillation data evince two small quantities, namely θ_{13} and R_{mass}. This observation kindled the motivation of our present analysis of starting with both these small quantities vanishing and generating both by deploying a single perturbation, thereby relating them [3].

2. Perturbation theory

2.1 *The unperturbed picture*

To begin with, we assume that there is no solar splitting as well as $\theta_{13} = 0$ and produce both by a single perturbation. For three flavours of neutrinos in the absence of solar splitting the unperturbed mass matrix in the mass basis will appear as $M^0_{\text{mass}} = \text{diag}(m_1^{(0)}, m_1^{(0)}, m_3^{(0)})$. The masses $m_i^{(0)} (i = 1, 2, 3)$ are chosen real and positive by appropriately adjusting the Majorana phases. We define $m^\pm = (m_3^{(0)} \pm m_1^{(0)})$. m^- is positive (negative) for normal (inverted) mass ordering. The unperturbed mass matrix in the flavour basis is $M^{(0)}_{\text{flavour}} = U^0 \text{diag}(m_1^{(0)}, m_1^{(0)}, m_3^{(0)}) U^{0^T}$, where U^0 is the lowest-order leptonic mixing matrix. Note that all the mass matrices, both the unperturbed as well as the perturbation itself, are of Majorana nature and therefore symmetric. The columns of U^0 are the unperturbed flavour eigenstates. The charged lepton mass matrix is diagonal in the flavour basis under consideration. Popular lepton mixings like tribimaximal (TBM), bimaximal (BM) and the 'golden ratio' (GR) exhibit $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. They differ only in θ_{12}^0. After the discovery of non-zero θ_{13}, none of them is consistent with the data and thus have to be corrected. We opt for a general parametrization:

$$U^0 = \begin{pmatrix} \cos\theta_{12}^0 & \sin\theta_{12}^0 & 0 \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & \sqrt{\frac{1}{2}} \\ \frac{\sin\theta_{12}^0}{\sqrt{2}} & -\frac{\cos\theta_{12}^0}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \tag{5}$$

For $\sin\theta_{12}^0 = 0.577$, one gets the tribimaximal mixing, whereas $\sin\theta_{12}^0 = 0.707$ and $\sin\theta_{12}^0 = 0.526$ yield the BM and GR mixings respectively. At 1σ, $0.539 < \sin\theta_{12} < 0.561$. Therefore, none of the mixings above satisfy the 1σ limits of θ_{12}.

2.2 *The perturbation*

The symmetric perturbation mass matrix in the mass basis after the removal of an irrelevant constant part has the most general form:

$$M' = m^+ \begin{pmatrix} 0 & \gamma & \xi \\ \gamma & \alpha & \eta \\ \xi & \eta & \beta \end{pmatrix}. \tag{6}$$

The dimensionless entities $\alpha, \beta, \gamma, \xi, \eta$ should be small compared to unity for a valid perturbation theory. The perturbation – $\alpha, \beta, \gamma, \xi, \eta$ – can be real or complex.

A complex M' is not Hermitian, and therefore the combination $(M^0 + M')^\dagger (M^0 + M')$ has to be considered. $M^{0\dagger} M^0$ is the unperturbed term and $(M^{0\dagger} M' + M'^\dagger M^0) = M_{\rm pert}$ serves as perturbation to the lowest order. The unperturbed eigenvalues are $(m_i^{(0)})^2$ and the perturbation matrix is

$$M_{\rm pert} = m^+ \begin{pmatrix} 0 & 2m_1^{(0)}{\rm Re}(\gamma) & m^+\,{\rm Re}(\xi) - im^-\,{\rm Im}(\xi) \\ 2m_1^{(0)}{\rm Re}(\gamma) & 2m_1^{(0)}{\rm Re}(\alpha) & m^+\,{\rm Re}(\eta) - im^-\,{\rm Im}(\eta) \\ m^+{\rm Re}(\xi) + im^-\,{\rm Im}(\xi) & m^+{\rm Re}(\eta) + im^-\,{\rm Im}(\eta) & 2m_3^{(0)}{\rm Re}(\beta) \end{pmatrix}. \tag{7}$$

2.2.1 *The solar sector.*

Perturbation splits the degeneracy and determines the eigenstates which are rotated by an angle ζ with respect to the first two columns of U^0. The resultant solar mixing angle is now $\theta_{12} = \theta_{12}^0 + \zeta$. The 2×2 perturbation submatrix responsible for the entire solar story is

$$M'_{(2\times2)} = m^+\alpha \begin{pmatrix} 0 & r \\ r & 1 \end{pmatrix}, \quad \text{for real } M', \tag{8}$$

with $r = \gamma/\alpha$. For complex M', $r \equiv {\rm Re}(\gamma)/{\rm Re}(\alpha)$ and

$$(M_{\rm pert})_{(2\times2)} = 2m^+ m_1^{(0)}\,{\rm Re}(\alpha) \begin{pmatrix} 0 & r \\ r & 1 \end{pmatrix}, \quad \text{for complex } M'. \tag{9}$$

Table 1. The range of r in the perturbation (see eqs (8), (9)) for the TBM, BM and GR alternatives that produces a θ_{12} consistent with the global fits at 1σ.

Parameter	TBM		BM		GR	
	$r_{\min}$	$r_{\max}$	$r_{\min}$	$r_{\max}$	$r_{\min}$	$r_{\max}$
$r\ (\times 10^2)$	−4.59	−1.95	−23.1	−19.9	1.54	4.18

From simple calculation, one can easily show for real perturbation

$$m_{2,1} = m_1^{(0)} + m^+ \frac{\alpha}{2}\left[1 \pm \sqrt{1+4r^2}\right] \tag{10}$$

and for complex perturbation, one has

$$m_{2,1}^2 = (m_1^{(0)})^2 + 2m_1^{(0)}m^+ \frac{\text{Re}(\alpha)}{2}\left[1 \pm \sqrt{1+4r^2}\right]. \tag{11}$$

Up to small perturbative corrections m^+m^- gives the atmospheric mass splitting. Hence

$$R_{\text{mass}} = |(m_2^2 - m_1^2)/(m_3^2 - m_1^2)| = 2\frac{m_1^{(0)}}{|m^-|}\,\text{Re}(\alpha)\sqrt{1+4r^2}\ . \tag{12}$$

The angle ζ obtained from the above 2×2 submatrices – eqs (8), (9) – is $\zeta = \frac{1}{2}\tan^{-1}(2r)$. $r \neq 0$ is chosen so that the mass degeneracy is removed as well as the mixing angle is tuned within the allowed range. In table 1, we show the ranges of r for each of the three models.

2.2.2 *Generating* $\theta_{13} \neq 0$. With first-order corrections, the third wavefunction $|\psi_3\rangle$ is given by

$$|\psi_3\rangle = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + \bar{\xi}^* \begin{pmatrix} \cos\theta_{12}^0 \\ -\sin\theta_{12}^0/\sqrt{2} \\ \sin\theta_{12}^0/\sqrt{2} \end{pmatrix} + \bar{\eta}^* \begin{pmatrix} \sin\theta_{12}^0 \\ \cos\theta_{12}^0/\sqrt{2} \\ -\cos\theta_{12}^0/\sqrt{2} \end{pmatrix} \tag{13}$$

with

$$\bar{\xi} = \left(\frac{m^+}{m^-}\right)\text{Re}(\xi) + i\ \text{Im}(\xi),$$
$$\bar{\eta} = \left(\frac{m^+}{m^-}\right)\text{Re}(\eta) + i\ \text{Im}(\eta), \qquad \text{for complex } M'. \tag{14}$$

From the above the expressions for the real limit can be read. For simplicity, we assume $\theta_{23} = \pi/4$. Hence we get, $\left(\bar{\xi}/\bar{\eta}\right)^* = \tan\theta_{12}^0$. Now $\tan\theta_{12}^0$, being a real quantity, forces the phases of $\bar{\xi}$ and $\bar{\eta}$ to be exactly equal. Comparing $|\psi_3\rangle$ with eq. (3) one has

$$\sin\theta_{13}\,e^{-i\delta} = \left[\cos\theta_{12}^0\,\bar{\xi}^* + \sin\theta_{12}^0\,\bar{\eta}^*\right] = \frac{\bar{\xi}^*}{\cos\theta_{12}^0}\ , \tag{15}$$

θ_{13} and δ are now determined. It is evident that in the real case CP is conserved.

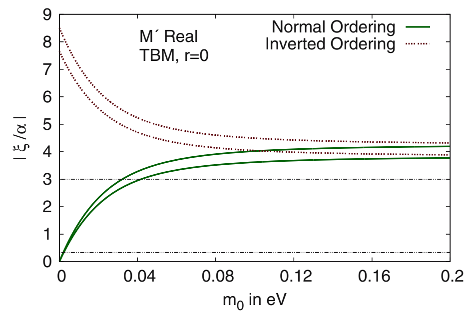

Figure 1. $|\xi/\alpha|$ is shown as a function of the lightest neutrino mass m_0 for both mass orderings when M' is real. The area between the two curves of the same type is allowed when θ_{13} is varied over its 1σ range. Also indicated are the values $\frac{1}{3}$ and 3 for $|\xi/\alpha|$ – black dot–dashed lines.

2.3 *Results*

Equation (12) gives an estimate of α, while ξ is obtained from eq. (15). This is utilized below to set bounds on the lightest neutrino mass, m_0.

2.3.1 *Real perturbation.* It is not unreasonable to demand that the elements of the perturbation matrix should be of the same order. In figure 1, $|\xi/\alpha|$ is shown for both mass orderings as a function of m_0 for the TBM case. We have also indicated where this ratio corresponds to the values 3 and 1/3 (dot–dashed black lines), two limits separated by an order of magnitude. For normal ordering, the ratio is within the above range only if 2.3 meV $\leq m_0 \leq$ 3.7 meV. If other experiments establish a larger value of m_0 then that could be an indication that M' must be complex. In the case of inverted ordering, α is more than an order of magnitude less than $|\xi|$ for almost the entire range of m_0. Thus, inverted ordering is a less favoured alternative if the perturbation is real but can be accommodated if it is complex.

2.3.2 *Complex perturbation.* We take a conservative standpoint such that the complex perturbation matrix elements differ only in the phase while each of them has the same magnitude, ϵ, i.e., $\alpha = \epsilon \exp(i\phi_\alpha)$, $\gamma = \epsilon \exp(i\phi_\gamma)$, $\xi = \epsilon \exp(i\phi_\xi)$. ϵ sets the scale of perturbation and is not entirely arbitrary. Equation (14) implies:

$$\left|\frac{m^+}{m^-}\right| \epsilon \;\geq\; |\bar{\xi}| \;\geq\; \epsilon. \tag{16}$$

These limits are presented respectively in figures 2a and 2b for the normal and inverted mass ordering. The upper and lower limits on ϵ are shown as the green dashed and blue solid curves. The two curves of each type show how the limit changes as θ_{13} is allowed to vary over its 1σ range. Tribimaximal mixing has been assumed for these plots.

In addition, eq. (12) also puts a bound indicated by the dotted maroon curves in figures 2a and 2b. It is seen that $\epsilon < 0.13$ is a safe choice.

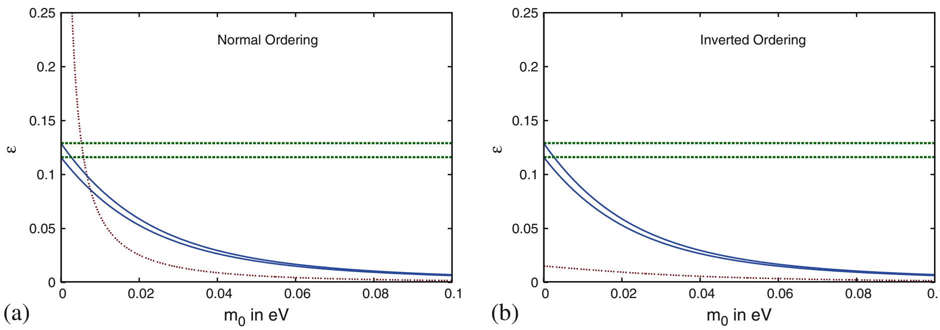

Figure 2. The limits on ϵ for (**a**) normal and (**b**) inverted mass orderings as a function of m_0. The upper (lower) limits from eq. (16) for TBM are the green dashed (blue solid) curves. The region between the curves of the same type corresponds to θ_{13} values in the 1σ range. The dotted maroon curves are the lower limits from solar splitting. Here $r = 0$ has been taken.

The phases ϕ_α, ϕ_γ and ϕ_ξ can be found from the solar splitting, θ_{12} and θ_{13}, respectively and δ can be predicted as shown in figure 3.

It is worthwhile to point out that the procedure for extracting δ using $|\bar{\xi}|$ leaves a two-fold uncertainty $\delta \leftrightarrow \pi + \delta$. Keeping this in mind we have shown δ in the first quadrant in figure 3 even though the 1σ range of the global fit – eq. (4) – would prefer the partner $\pi + \delta$ solution.

3. A mass model

In this segment, we focus on a definite model. As already shown, the two small quantities θ_{13} and $\Delta m^2_{\text{solar}}$ can originate from a single perturbation. We now intend to generate some other oscillation parameters perturbatively, causing them to get connected. Specifically, our present goal is to get non-zero θ_{12}, θ_{13}, $\Delta m^2_{\text{solar}}$ and also allow θ_{23} to deviate from $\pi/4$ starting from a scenario where both the solar mixing and splitting together with θ_{13} are zero and the atmospheric mixing is also maximal. Here a subdominant type-I see-saw contribution perturbs the dominant unperturbed mass matrix arising from a type-II see-saw.

3.1 *Origin of the unperturbed piece*

The unperturbed mass matrix remains the same as earlier. The mixing matrix now reduces to a simpler form obtained from eq. (5) setting $\theta^0_{12} = 0$.

We assume that the type-II see-saw together with a $\mu \leftrightarrow \tau$ symmetry produces the unperturbed piece. The $SU(2)_L \times U(1)_Y$ conserving Lagrangian is of the form

$$\mathcal{L}_{\text{type-II}} = \sum_{i,j} \frac{1}{2} h_{ij} (\nu^i_L)^T C^{-1} \nu^j_L \langle \Delta_L \rangle + \text{h.c.} \tag{17}$$

Here Δ_L is the usual scalar triplet whose VEV gives the Majorana mass and $\nu_L \equiv (\nu_e, \nu_\mu, \nu_\tau)^T_L$. $h_{ij} = h_{ji}$ owing to the symmetric nature of the Majorana mass matrix.

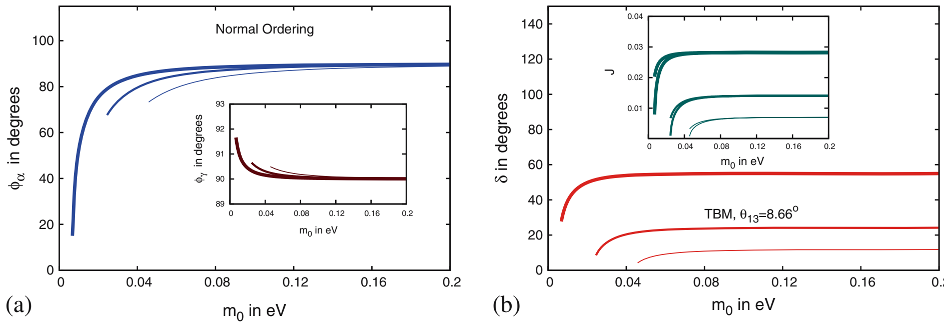

Figure 3. (**a**) ϕ_α for complex M' as a function of m_0 ((ϕ_γ) in the inset) for ϵ in decreasing order of thickness 0.1, 0.05, 0.025 for the TBM model. (**b**) δ as a function of m_0. Same values of ϵ are chosen in decreasing order of thickness and θ_{13} is taken at the best-fit value. In the inset is shown the Jarlskog parameter J for the chosen ϵ and the 1σ limits of θ_{13}. Both panels are for normal ordering. For inverted ordering $\delta \to (\pi - \delta)$ and J is unchanged.

The additional $\mu \leftrightarrow \tau$ symmetry causes $h_{22} = h_{33}$. We demand ν_e is unmixed and set $h_{12} = h_{13} = 0$. This can be achieved using a $\mathbb{Z}_2$ symmetry satisfying

$$\mathbb{Z}_2\colon \nu_{eL} \to \nu_{eL}; \quad (\nu_{\mu,\tau})_L \to -(\nu_{\mu,\tau})_L; \quad \Delta_L \to \Delta_L, \quad i, j = 1, 2, 3. \tag{18}$$

All these properties are obeyed by a general mass matrix in the flavour basis:

$$M^0_{\text{flavour}} = \begin{pmatrix} x & 0 & 0 \\ 0 & y & z \\ 0 & z & y \end{pmatrix}. \tag{19}$$

Its mass basis counterpart obtained with the help of U^0 leads to $M^0_{\text{mass}} = \text{diag}(x, y - z, y + z)$. Absence of solar splitting is guaranteed if $x = y - z$, i.e., $h_{22} - h_{23} = h_{11}$.

3.2 *Model for the perturbation*

The perturbation originates from a type-I see-saw. We choose the Dirac mass matrix to be proportional to the identity. The $SU(2)_L \times U(1)_Y$ preserving Lagrangian in this case is

$$\mathcal{L}_{\text{type-I}} = \sum_{i,j} \lambda_{ij} \bar{\nu}^i_L N^j_R \langle\Phi\rangle + \frac{1}{2} H_{ij} (N^i_R)^T C^{-1} (N^j_R) + \text{h.c}, \quad i, j = 1, 2, 3. \tag{20}$$

The VEV of the doublet scalar $\langle\Phi\rangle$ sets the Dirac mass scale (m_D). Choosing $\lambda_{ij} = \lambda_0 \delta_{ij}$, one gets $M_D = m_D \mathbb{I}$.

One of the singlet right-handed neutrinos is decoupled from the rest to yield the mass matrix of the desired form via the Majorana mass term given by the second term in the above Lagrangian. The Majorana nature requires $H_{ij} = H_{ji}$. The perturbation can be both real and complex depending upon H_{ij}. We discuss the real case first.

3.2.1 *Real perturbation.* Using type-I see-saw mechanism, we get the perturbation matrix in flavour basis as

$$M'^{\text{flavour}} = M_D^T M_R^{-1} M_D = \frac{m_D^2}{m_R}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{21}$$

choosing

$$M_R^{\text{flavour}} = m_R \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In mass basis:

$$M'^{\text{mass}} = U^{0T} M'^{\text{flavour}} U^0 = \frac{m_D^2}{\sqrt{2} m_R}\begin{pmatrix} 0 & 1 & 1 \\ 1 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 1 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \tag{22}$$

Using this perturbation matrix the corrected wavefunction $|\psi_3\rangle$ is

$$|\psi_3\rangle = \begin{pmatrix} \sigma \\ \frac{1}{\sqrt{2}}(1 - \frac{\sigma}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}}(1 + \frac{\sigma}{\sqrt{2}}) \end{pmatrix}, \quad \text{where} \quad \sigma \equiv \frac{m_D^2}{\sqrt{2} m_R m^-} = s_{13}\cos\delta. \tag{23}$$

As M' is real, CP-violation is absent. As m^- is positive (negative) for normal (inverted) ordering, δ is 0 (π) for normal (inverted) ordering. For the atmospheric mixing we have

$$\tan\theta_{23} = \frac{1 - \frac{\sigma}{\sqrt{2}}}{1 + \frac{\sigma}{\sqrt{2}}} = \tan(45^\circ - \varphi),$$

where

$$\varphi = \tan^{-1}\left(\frac{\sigma}{\sqrt{2}}\right) = \tan^{-1}\left(\frac{s_{13}\cos\delta}{\sqrt{2}}\right). \tag{24}$$

As $s_{13}\cos\delta$ is positive (negative) for normal (inverted) ordering, θ_{23} lies in the first and second octant for normal and inverted ordering respectively.

The 2×2 submatrix of M'^{mass} relevant for the solar sector is

$$M'^{\text{mass}}_{2\times2} = \frac{m_D^2}{\sqrt{2} m_R}\begin{pmatrix} 0 & 1 \\ 1 & \frac{1}{\sqrt{2}} \end{pmatrix} = \sqrt{2} m^- s_{13}\cos\delta \begin{pmatrix} 0 & 1 \\ 1 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{25}$$

This leads to

$$\Delta m^2_{\text{solar}} = 3\sqrt{2} s_{13}\cos\delta\, m^- m_1^{(0)}. \tag{26}$$

The solar mixing angle as already introduced in terms of ζ is given by

$$\theta_{12} = \zeta = \frac{1}{2}\tan^{-1}(2\sqrt{2}) = 35.26^\circ. \tag{27}$$

This is actually the θ_{12} of tribimaximal mixing. It can be inferred from eqs (4) and (27) that real M' is incapable of providing the observed value of θ_{12} within 1σ. But it is allowed at 3σ for which the region of lightest neutrino mass ranges from 2 to 3.25 meV for normal ordering. For inverted ordering, the solar splitting is too high to fit the data.

To accommodate θ_{12} within 1σ and produce CP violation, complex M' is imperative.

3.2.2 *Complex perturbation.* The perturbation is made complex by allowing phases in M_R, keeping $M_D \propto \mathbb{I}$ fixed.

$$M_R^{\text{flavour}} = m_R \begin{pmatrix} 0 & e^{-i\phi_1} & 0 \\ e^{-i\phi_1} & 0 & 0 \\ 0 & 0 & e^{-i\phi_3} \end{pmatrix}. \tag{28}$$

We shall restrict ourselves to the choice of $\phi_3 = 0$. Employing type-I see-saw:

$$M'^{\text{mass}} = U^{0T} M'^{\text{flavour}} U^0 = \sigma m^- \begin{pmatrix} 0 & e^{i\phi_1} & e^{i\phi_1} \\ e^{i\phi_1} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ e^{i\phi_1} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{29}$$

In mass basis,

$$M_{\text{pert}} = \frac{m_D^2}{\sqrt{2} m_R} \begin{pmatrix} 0 & 2m_1^{(0)} \cos\phi_1 & m^+ \cos\phi_1 - i\, m^- \sin\phi_1 \\ 2m_1^{(0)} \cos\phi_1 & \frac{2}{\sqrt{2}} m_1^{(0)} & -\frac{m^+}{\sqrt{2}} \\ m^+ \cos\phi_1 + i\, m^- \sin\phi_1 & -\frac{m^+}{\sqrt{2}} & \frac{2}{\sqrt{2}} m_3^{(0)} \end{pmatrix}. \tag{30}$$

In analogy to the real case, we write for the solar mixing angle

$$\theta_{12} = \zeta = \frac{1}{2} \tan^{-1}(2\sqrt{2}\cos\phi_1). \tag{31}$$

This is similar to eq. (27) apart from the factor of $\cos\phi_1$. One can utilize the 1σ values of θ_{12} as in eq. (4) to obtain a range for ϕ_1: $0.764 < \cos\phi_1 < 0.890$. Restricting ourselves to the lowest order in perturbation

$$\Delta m^2_{\text{solar}} = \sqrt{2}\sigma m_1^{(0)} m^- \sqrt{1 + 8\cos^2\phi_1}, \tag{32}$$

where σ is as defined in eq. (23). σ is positive and negative for normal and inverted ordering respectively. With the solar sector now resolved, we turn towards the other two mixing angles. As computed before,

$$|\psi_3\rangle = \begin{pmatrix} \sigma m^- z_1 \\ \frac{1}{\sqrt{2}}\left(1 - \frac{\sigma}{\sqrt{2}}\right) \\ \frac{1}{\sqrt{2}}\left(1 + \frac{\sigma}{\sqrt{2}}\right) \end{pmatrix}, \tag{33}$$

where

$$z_1 \equiv \frac{\cos\phi_1}{m^-} - i\,\frac{\sin\phi_1}{m^+}.$$

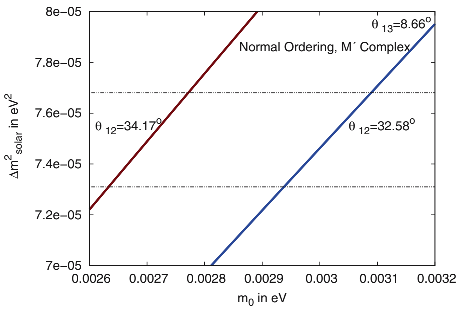

Figure 4. Solar splitting vs. m_0 for the central value of θ_{13} with θ_{23} in the first octant. The horizontal lines are the observed bounds. The blue (maroon) curve is for the lower (upper) bound of θ_{12}.

Thus, $s_{13}\mathrm{e}^{-i\delta} = \sigma m^- z_1$. Hence the identification of

$$s_{13} = \sigma |m^-| \sqrt{\frac{\cos^2\phi_1}{m^{-2}} + \frac{\sin^2\phi_1}{m^{+2}}} \quad \text{and} \quad \delta = \tan^{-1}\left(\tan\phi_1 \frac{m^-}{m^+}\right) \tag{34}$$

is obvious. The atmospheric mixing angle is given by $\tan\theta_{23} = \tan(45^\circ - \varphi')$ where $\varphi' = \sqrt{2} s_{13}(\cos\delta/\cos\phi_1)$. As the sign of $s_{13}\cos\delta$ is completely determined by sign of σ, one can clearly exclude the second (first) octant of θ_{23} for normal (inverted) ordering. One can always check the correspondence of this section with the real case by substituting $\phi_1 = 0$.

Our results for normal ordering are graphically represented in figure 4 for 1σ. Here solar splitting is expressed as a function of m_0. We have shown the results for central value of θ_{13} for θ_{23} in the first octant. The horizontal lines are the observed bounds of the solar splitting. The blue and maroon lines are for lower and upper bounds of θ_{12} respectively, that basically mark the range of m_0 in which our model holds. The CP-violating phase δ lies between 24° and 36°. The Jarlskog parameter ranges between 1.4×10^{-2} and 2×10^{-2}. In this case also our target cannot be achieved for inverted ordering.

4. Conclusions

We have proposed that the neutrino mass matrix has a structure in which θ_{13} and $\Delta m^2_{\text{solar}}$ are zero, $\theta_{23} = \pi/4$ and the atmospheric mass splitting, Δm^2_{atm}, is what is observed. The solar mixing angle θ_{12} can be chosen as zero or as dictated by popular mixing patterns such as tribimaximal mixing. This is a reasonably good reflection of the observed data though a few finer details are missing here. In addition, we speculate the presence of a smaller contribution, amenable to a perturbative treatment, that generates small parameters in the neutrino mixing sector, namely, θ_{13} and $\Delta m^2_{\text{solar}}$ and applies minor tweaks to θ_{12} and θ_{23}. CP-violation can also be incorporated. This leads to testable relationships between oscillation parameters. We also sketch a mass model based on the see-saw mechanism which embodies these features.

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