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Inflation in the light of BICEP2 and PLANCK

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Abstract. The BICEP2/Keck+PLANCK joint analysis of the B-model polarization and polarization by foreground dust sets an upper bound on the tensor-to-scalar ratio of $r_{0.05} < 0.12$ at 95% CL. The popular Starorbinsky model Higgs-inflation or the conformally equivalent Higgs-inflation model allow low r values ($\sim 10^{-3}$). We survey the generalizations of the Starobinsky–Higgs models which would allow larger values ($r \sim 0.1$). The Starobinsky–Higgs inflation models require an exponential potential which can be naturally derived from SUGRA models. We show that a variation of the no-scale SUGRA model can give rise to the generalized Starobinsky models which give large r. We also examine non-standard boundary conditions which would allow a large deviation of the tensor spectral index from the slow roll values and propose that the presence of a thermal component in the tensor spectrum arises from Gibbons–Hawking temperature of the de-Sitter space.

Keywords. Inflation; supergravity; Gibbons–Hawking; BICEP2; PLANCK.

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1. Introduction

The detection of primordial gravitational waves of the kind predicted by inflation is of great interest as it would help in pinning down the particle physics model of inflation. BICEP2 Collaboration [1] announced the measurement of the B-mode polarization of the CMB with a tensor scalar ratio of $r = 0.16 \pm 0.07$ at 95% CL at scales $k \simeq 0.01$ Mpc^{-1}. This measurement was larger than the PLANCK-2013 [2] upper bound $r < 0.11$ obtained from the measurement of the temperature anisotropy at scale $k \simeq 0.002$ Mpc^{-1}. As the PLANCK-2013 measurements are at much larger angular scales compared to the BBICEP2, the two numbers can be reconciled in inflation models which allow either a large running of the scalar spectral index $\mathrm{d}n_s/\mathrm{dln}\, k \simeq -10^{-2}$ [2] or a large blue tilt of tensor spectrum $n_{\mathrm{T}} \sim 1$ [3–5]. The BICEP2 determination of r underestimated the contribution of the foreground dust polarization. After taking into account the measurement of dust polarization in the foreground by PLANCK-2014 [6], a joint analysis by BICEP2/Keck+PLANCK [7] now puts an upper bound of $r_{0.05} < 0.12$ at 95% CL. Finally, PLANCK-2015 [9] combined data from BICEP2/Keck+PLANCK and the temperature anisotropy and has lowered the bound to $r_{0.002} < 0.009$ which disfavours the $V(\phi) \propto \phi^2$ chaotic inflation models.

An independent analysis [8] of the BICEP2 and PLANCK-dust polarization data shows that the genus topology of the BICEP2 B-polarization map supports a primordial tensor wave origin of the BICEP2 signal and puts $r = 0.11 \pm 0.04$. Ongoing experiments like BICEP3 and Keck are expected to probe the values of the tensor ratio down to $r \sim 0.05$. The inflation models which predict r in the range of 0.05–0.1 are likely to be tested by the B-mode measurements. The models which have plateau potentials are favoured by the combination of the scalar spectral index $n_s = 0.968 \pm 0.006$ measured by PLANCK [9] and $r \sim 0.05$. The most prominent of the plateau potential models is the $R + (1/M^2)R^2$ Starobinsky model [10] where the longitudinal mode of graviton plays the role of inflation. It predicts $r \sim 0.003 - 0.005$ which means that if $r \sim 0.05$ is observed as indicated by BICEP2 the Starobinsky model would need to be modified. The 'Higgs-inflation' models [11] with the $R + \xi\phi^2 R$ curvature coupling of the inflation field leads to the same plateau potential as the Starobinsky model.

A natural framework for the higher-order gravity theories or the equivalent plateau potential theories is supergravity [12–20]. In §1, the Starobinsky model and its possible generalizations to models which yield higher values of the tensor ratio r are discussed. In §2, how the Starobinsky model as well as its variants can be derived from SUGRA models is described. There remains the possibility that the combination of temperature and polarization data at all angular scales demands a non-standard blue or red-tilted tensor spectra [3–5]. In §3, it is shown that taking a different assumption for the 'in' and 'out' vacuum states instead of the Bunch–Davies initial state can lead to large tilts on the tensor spectra. Observations of such large n_{T} would be a signature of the Hawking–Gibbons temperature [22] of the de-Sitter space at the time of inflation [23].

2. Starobinsky model and its generalization

The Starobinsky model consists of the quadratic curvature action

$$S = \frac{-M_p^2}{2} \int \mathrm{d}^4x \sqrt{-g} \left(R + \frac{1}{6M^2} R^2 \right). \tag{1}$$

This can be transformed to an equivalent scalar theory in the Einstein frame as follows. It is well known [24–26] that any $\mathcal{L} = (-M_p^2/2)\sqrt{-g} f(R)$ model can be transformed to a scalar theory by making the conformal transformation

$$g_{\mu\nu}(x) \longrightarrow \tilde{g}_{\mu\nu}(x) = \Omega(x) g_{\mu\nu}(x), \tag{2}$$

where the conformal factor $\Omega = f'(R) = \partial f/\partial R$ and tilde represents quantities in the Einstein frame. The Ricci scalar R in the two frames is related by

$$R = \Omega \left(\tilde{R} + 3\tilde{\Box} \ln \Omega - \frac{3}{2} \tilde{g}^{\mu\nu} \partial_\mu \ln \Omega \, \partial_\nu \ln \Omega \right). \tag{3}$$

Define the scalar as

$$\chi \equiv \sqrt{\frac{3}{2}} \ln \Omega M_p. \tag{4}$$

With these transformations, the $f(R)$ theory appears in the Einstein frame as the scalar theory with the action

$$S_{\mathrm{E}} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left[\frac{-M_p^2}{2} \tilde{R} + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + U(\chi) \right], \tag{5}$$

where the scalar potential in terms of χ is of the form

$$U(\chi) = \frac{(Rf' - f)M_p^2}{2f'^2}. \tag{6}$$

Using this procedure for the Starobinsky model (1), the equivalent scalar potential in the Einstein frame is of the form

$$U_S(\chi) = \frac{3}{4} M^2 M_p^2 \left(1 - \mathrm{e}^{-\sqrt{\frac{2}{3}}\chi/M_p}\right)^2. \tag{7}$$

The resultant potential is very flat (see the $\beta = 2$ curve of figure 1), which results in low values of the tensor to scalar ratio. Specifically, the Starobinsky model predicts $n_s \simeq 1 - 2/N$ and $r \simeq 12/N^2$, where N is the number of e-foldings. For $N = 50$–60, the Starobinsky model predicts $r = 0.003 - 0.005$.

The Higgs-inflation model [11]

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(-\frac{M_p^2}{2} \left(1 + \frac{\xi\phi^2}{M_p^2}\right) R + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{\lambda}{4}\phi^4 \right), \tag{8}$$

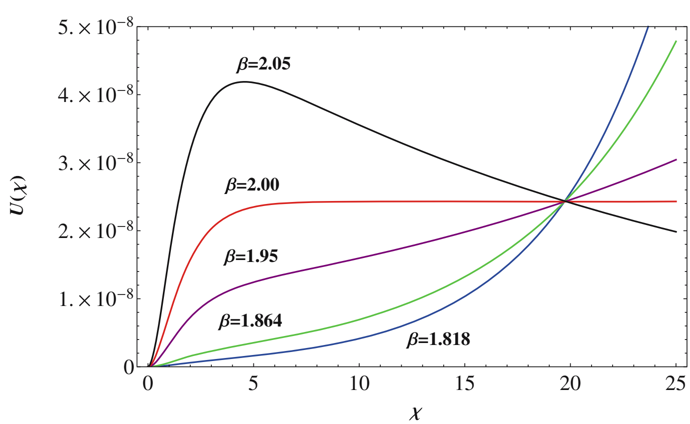

Figure 1. The nature of the potential (7) for different β values (with $M = 1.8 \times 10^{-4}$). The potential and the field values are in $M_p = 1$ units.

after a conformal transformation with the conformal factor $\Omega = (1 + \xi\phi^2/M_p^2)$ gives a potential in the Einstein frame

$$U(\chi) = \frac{\lambda^2 M_p^4}{\xi^2}\left(1 - \mathrm{e}^{-\sqrt{\frac{2}{3}}\chi/M_p}\right)^2, \tag{9}$$

which is identical in form to (6) and therefore has the same prediction for r as the Starobinsky model.

One generalization of the Starobinsky model which predicts a large value of r is the power-law curvature model [20],

$$S = \frac{-M_p^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}\right). \tag{10}$$

The action (10) after transformation to the Einstein frame gives a scalar potential of the form

$$U(\chi) = \frac{(\beta-1)}{2}\left(\frac{6M^2}{\beta^\beta}\right)^{1/(\beta-1)} \exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{2-\beta}{\beta-1}\right)\right] \times\left[1 - \exp\left(\frac{-2\chi}{\sqrt{6}}\right)\right]^{\beta/(\beta-1)}, \tag{11}$$

where $M_p = 1$. The potential (11) is plotted for different values of β in figure 1. We see that the potential becomes steep when β deviates from the Starobinsky value of $\beta = 2$. In ref. [20], we have shown that if $\beta \sim 1.81$, we can get tensor-to-scalar ratio r as large as 0.2 and satisfy all other CMB constraints.

3. Embedding R^β model in supergravity

The Starobinsky model can be derived from a SUGRA model either in the quadratic curvature or in the equivalent plateau potential scalar form. It was shown by Cecotti [12] that quadratic Ricci curvature terms can be derived in a supergravity theory by adding two chiral superfields in the minimal supergravity. A no-scale SUGRA model with a modulus field and the inflation field with a minimal Wess–Zumino superpotential give the same F-term potential in the Einstein frame as the Starobinsky model [13]. The range of tensor-to-scalar ratio r predicted by varying the parameters of this SUGRA model is in the range of 10^{-3}–10^{-2} [13]. The symmetry principle which can be invoked for the SUGRA generalization of the Starobinsky model is the spontaneous violation of superconformal symmetry [14]. The quadratic curvature can also arise from D-term in a minimal-SUGRA theory with the addition of a vector and chiral supermultiplets [15]. The Starobinsky model has been derived from the D-term potential of a SUGRA model [16–18]. Quartic powers of Ricci curvature in the bosonic Lagrangian can also be obtained in a SUGRA model from the D-term of higher order powers of the field strength superfield [18,19]. In this section, a SUGRA derivation of the power-law generalization [20] of the Starobinsky model which gives the potential (6) which yield larger values of r compared to the Starobinsky model has been outlined.

The F-term scalar potential in SUGRA depends on the combination [21] of the Kähler potential $K(\Phi_i)$ and the superpotential $W(\Phi_i)$ as

$$G \equiv K + \ln W + \ln W^*. \tag{12}$$

The potential in the Einstein frame is given by

$$V = \mathrm{e}^G \left[\frac{\partial G}{\partial \phi^i} K^i_{j*} \frac{\partial G}{\partial \phi^*_j} - 3 \right], \tag{13}$$

where K^i_{j*} is the inverse of the Kähler metric $K_i^{j*} \equiv K_i^{j*} \equiv \partial^2 K/\partial\phi^i \partial\phi^*_j$.

We choose the Kähler potential of the form

$$K = -3 \ln \left[T + T^* - \frac{(\phi + \phi^*)^n}{12} \right], \tag{14}$$

which can be motivated by a shift symmetry $T \to T + iC$, $\phi \to \phi + iC$ with C real, on the Kähler potential. Here T is a modulus field and ϕ is a matter field which plays the role of inflaton. The superpotential with a single chiral superfield Φ (whose scale component is ϕ) is chosen as

$$W(\Phi) = \frac{\mu}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3. \tag{15}$$

For this choice of Kähler potential (14), the potential for the scalar fields T and ϕ turns out to be

$$V = \frac{4(\phi + \phi^*)^{2-n}}{n(n-1)[T + T^* - \frac{(\phi+\phi^*)^n}{12}]^2} \left| \frac{\partial W}{\partial \phi} \right|^2 \tag{16}$$

and the kinetic term of the scalar is given by

$$K_i^{j*} \partial_\mu \phi^i \partial^\mu \phi^*_j = \frac{n(\phi + \phi^*)^{n-2}[(T + T^*)(n-1) + \frac{(\phi+\phi^*)^n}{12}]}{4[T + T^* - \frac{(\phi+\phi^*)^n}{12}]^2} \left| \partial_\mu \phi \right|^2. \tag{17}$$

Assuming that the T field gets a VEV $\langle T + T^* \rangle = 2\langle \mathrm{Re}\, T \rangle = c > 0$ and $\mathrm{Im}\langle T \rangle = \mathrm{Im}\langle \phi \rangle = 0$, the Einstein frame Lagrangian in terms of $\mathrm{Re}\langle \phi \rangle$ becomes

$$\mathcal{L}_\mathrm{E} = \frac{n(2\phi)^{n-2}[c(n-1) + \frac{(2\phi)^n}{12}]}{4[c - \frac{(2\phi)^n}{12}]^2} \left| \partial_\mu \phi \right|^2 - \frac{4(2\phi)^{2-n}}{n(n-1)[c - \frac{(2\phi)^n}{12}]^2} \left| \frac{\partial W}{\partial \phi} \right|^2. \tag{18}$$

To make the kinetic term canonical in $\mathcal{L}_\mathrm{E}$, we redefine the field ϕ to χ

$$\phi = \frac{1}{2} \left[\exp\left(\frac{2n\chi}{\sqrt{3n}} \right) + 6c(n+1) \right]^{1/n}. \tag{19}$$

The potential (16) in the Einstein frame with the assumption $\mu = \lambda/2$ reduces to the form

$$V = \frac{144\mu^2}{n(n-1)} \exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{3\sqrt{2}(2-n)}{\sqrt{n}}\right)\right] \times \left[1 - \exp\left(\frac{-2\chi}{\sqrt{3n}}\right) - \frac{9c(n^2-n-2)}{n}\exp\left(\frac{-2n\chi}{\sqrt{3n}}\right)\right]^2, \tag{20}$$

which is identical to the potential (6) derived from the R^β model. This toy model illustrates the form of the SUGRA embedding of a GUT Higgs which would lead to a viable inflation model.

4. Tilting tensor spectrum by Gibbons–Hawking radiation

In quantum field theory in curved space–times, it is well known that the spectrum of particles measured depends on the reference frame of the observer. In the generation of scalar and tensor spectra, it is assumed that the 'in' vacuum of the zero-point fluctuations which are amplified in the de-Sitter expansion is defined with respect to the conformal coordinates. The Bunch–Davies modes with respect to the conformal time

$$\phi_{in\,k}(\eta, \rho, \theta, \phi) = \frac{iH}{\sqrt{2k^3}} \mathrm{e}^{-ik\eta}(1 + ik\eta) j_l(k\eta) \frac{Y_{l,m}(\theta, \phi)}{\sqrt{4\pi}}, \tag{21}$$

can be related to the modes with respect to a different observer frame by a set of transformations defined by the complex valued α and β Bogoliubov coordinates. The two-point correlation with respect to a different 'out' observer will depend on the Bogoliubov coefficients

$$\langle 0_{\text{out}}|\phi_{\text{in}}(\mathbf{k}, \eta)\phi_{\text{in}}(\mathbf{k}', \eta)|0_{\text{out}}\rangle = \delta\left(k - k'\right) \frac{H^2}{k^3 M_p^2} \left[|\alpha_{\omega k}|^2 + |\beta_{\omega k}|^2 + 2\,\text{Re}\left(\alpha_{\omega k}\beta^\star_{\omega k}\right)\right]. \tag{22}$$

So, the tensor power spectrum will be observer-dependent and will depend upon the Bogoliubov coefficients [23]

$$P_{\text{T}} = \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{-2\epsilon} \left[|\alpha_{\omega k}|^2 + |\beta_{\omega k}|^2 + 2\,\text{Re}\left(\alpha_{\omega k}\beta^\star_{\omega k}\right)\right]. \tag{23}$$

There are two choices for the assumption of reference frame of the observer who measures inflationary perturbations [27–35]. There are, (a) the static observer following a geodesic trajectory in the de-Sitter space or (b) the asymptotic Minkowski observer in the post-inflation era. Both the static and the asymptotic Minkowski future observers measure a thermal spectrum of the Bunch–Davies perturbations

$$|\beta_{\omega k}|^2 = \frac{1}{\mathrm{e}^{\beta\omega} - 1}, \tag{24}$$

where again $\beta^{-1} = H/2\pi$ which is the Hawking–Gibbon temperature of the de-Sitter space. The two-point correlation depends upon the relative phase between α and β and

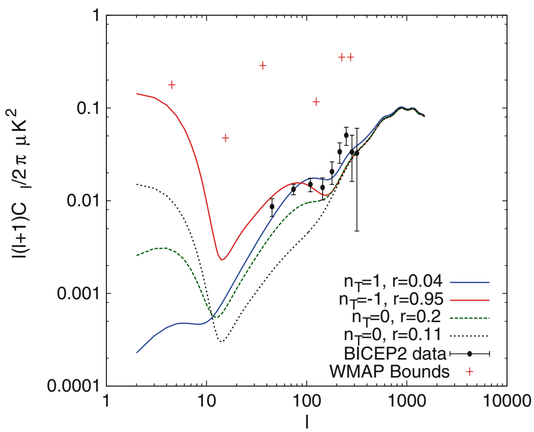

Figure 2. *B*-modes from modified as well as standard power spectrum with BICEP2 data and WMAP bounds.

the phase turns out to be different for the two cases. For the static observer, $\alpha\beta^* < 0$ and the tensor power spectrum turns out to be red-tilted with spectral index $n_{\rm T} \sim -1$,

$$
\begin{aligned}
P_{\rm T} &= \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}\left[\frac{(\mathrm{e}^{\frac{\pi k}{aH}}+1)^2}{(\mathrm{e}^{\frac{2\pi k}{aH}}-1)}\right] \\
&\simeq \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}\left[\frac{2}{\pi}\left(\frac{aH}{k}\right)\right], \qquad \text{for } k \ll aH. \qquad (25)
\end{aligned}
$$

On the other hand, for the post-inflation out observer $\alpha\beta^* < 0$ and the tensor power is blue-tilted with spectral index $n_{\rm T} \sim 1$,

$$
\begin{aligned}
P_{\rm T} &= \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}\left[\frac{(\mathrm{e}^{\frac{\pi k}{aH}}-1)^2}{(\mathrm{e}^{\frac{2\pi k}{aH}}-1)}\right] \\
&\simeq \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}\left[\frac{\pi}{2}\left(\frac{k}{aH}\right)\right], \qquad \text{for } k \ll aH. \qquad (26)
\end{aligned}
$$

In figure 2, the *B*-mode arising from these different power spectra are plotted with the BICEP2 data [23]. The blue-tilted $n_{\rm T} \sim 1$ spectrum reconciles the tension between the BICEP2 and the PLANCK-2013 bounds. If future measurement confirms this trend in the data, then this may indicate that the assumption of Bunch–Davies vacuum for calculating the two-point function may be too simplistic and there may be a signature of the Hawking–Gibbons temperature in the *B*-mode data.

5. Conclusions

Ongoing measurement of the *B*-mode polarization signal will be important for pinning down the particle physics model of inflation and may show the imprint of quantum gravity effects like the Hawking temperature of the de-Sitter space during inflation.

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