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Kalb–Ramond fields and the CMBR

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Abstract. Cosmological implications on the polarization of the cosmic microwave background radiation, of a Kalb–Ramond field interacting with gauge fields and gravity as dictated by quantum consistency of heterotic string theory are surveyed. A parity violating augmentation going beyond the dictates of string theory is shown to lead to possible appearance of a B mode generated in the cosmic microwave background (CMB) in the post-last scattering epoch. This generation of the B mode of CMB appears to be dramatic when the augmentation is embedded within a Randall–Sundrum braneworld scenario of the first kind.

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1. Introduction

A frequently asked question for string theorists is: do we live in a stringy world? The low-lying (massless) degrees of freedom of the heterotic string (and indeed of all closed strings) include the dilaton ϕ_{D} and the axion ϕ_{A} fields. If indeed the Universe is governed by the dynamics of string theory, degrees of freedom of the standard electroweak and strong theory must interact with these massless string fields to produce what must be important signatures of 'physics beyond the Standard Model'. It is therefore imperative that the dynamics of the Kalb–Ramond (KR) field, which is nothing but the Hodge-dual of the axion, be carefully analysed for such signatures involving probe fields of the Standard Model. Understandably, such signatures must be few and far between, or else we would have observed them already. The hope is that the imprints of these exotic massless string degrees of freedom are just around the corner of our discovery potential, and may become accessible any time in the near future.

Taking the Standard Model probes to be photon and graviton fields, quantum consistency of the heterotic string [1] leads to specific interactions with dilatons and axions. These interactions have been studied for their phenomenological implications at low energies in four dimensions [2,3]. What emerges is a phenomenon called 'cosmic' optical activity: the polarization plane of light from the most distant sources is seen to undergo a rotation, similar in some sense to Faraday rotation, but differing from it in one crucial aspect – the new optical activity is independent of wavelength of the emitted light. However, within a standard cosmological scenario, this interaction must have happened post-last scattering of the cosmic microwave radiation, and is therefore a tiny effect well within the observed bounds for typical coupling strengths which are invariably Planck-suppressed. Is there a situation though where this suppression may be offset by an unexpected enhancement? It turns out that if the heterotic string is compactified to five-dimensional anti-de Sitter space–time, then reduced to four dimensions through Randall–Sundrum warping of the first kind involving 'hidden' and 'visible' 3-branes, KR field couplings to Standard Model probes like photons and gravitons undergo 'antiwarping' [4–7]. If the KR field couplings are generalized to include parity violation [8], this antiwarping leads to an enhanced likelihood of conversion of the E-mode correlations of CMB to the elusive B-mode correlations. While the latest reported observation of the B-mode correlations [9] have been attributed to noise due to foreground dust [10], it is believed that the jury is till out on this issue. Under the circumstances, the generation of the B mode of CMB through parity-violating couplings of the KR field to photons still remains a viable option, albeit in the post-last scattering era.

This article is organized as follows: in §2, we briefly review the free KR fields and move on to their interactions with photons and gravitons as formulated within quantum-consistent heterotic string theory. In §3, we restrict the formulation to four dimensions and also to the $U(1)_{\rm em}$ sector of the augmentation of the KR field strength, for phenomenological purposes. The coupled KR–Maxwell field equations are solved, for small Planck-suppressed couplings, to leading order in this coupling, exhibiting optical activity involving achromatic rotation of the polarization plane, both in a Minkowski and a spatially flat Friedmann–Robertson–Walker background space–time. In §4, the stringy augmentation is generalized to include parity-violating terms which can be embedded within a supersymmetric scenario, even though they are not directly derivable from string theory. In §5, the KR field strength augmentation is embedded within a Randall–Sundrum scenario of the first kind [11], and the antiwarping of the parity-violating KR couplings leads to a huge enhancement in the likelihood of generation of the B-mode correlations from the E-mode polarization.

2. KR field interactions

KR fields are tensor fields characterized by a two-form potential B_{ab} with an Abelian vector-valued gauge transformation $B_{ab} \to B_{ab} + 2\partial_{[a}\xi_{b]}$ where ξ_a is the vectorial gauge parameter. The gauge-invariant field strength is defined as $H_{abc} \equiv \partial_{[a}B_{bc]}$ and its dynamics obeys the free-field action $S_{\rm KR} \sim \int H_{abc}H^{abc}$ and the consequent equation of motion $\partial_a H^{abc} = 0$. H_{abc} also satisfies its Bianchi identity $\partial_{[a}H_{bcd]} \equiv \partial_a^* H^{abc} = 0$. The equation of motion of H_{abc} implies that the 1-form field V_a Hodge-dual to it must be the gradient of a pseudoscalar – the axion field $\phi_{\rm A}$ which, by virtue of the Bianchi identity, obeys the free-field equation $\Box\phi_{\rm A} = 0$.

The effective action involving photons and the KR field, in the presence of gravity, has the form

$$S = \int \sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}H_{abc}H^{abc}\right]. \quad (1)$$

Note that there is no direct interaction of photons with the KR field. Such an interaction is introduced by embedding the action within the heterotic string theory: quantum consistency of this theory implied by the absence of gravitational and gauge anomalies, together with the requirement of $N = 1$ supersymmetry in four dimensions, warrants the augmentation of the KR field strength:

$$\begin{aligned} H_{abc} &\to \tilde{H}_{abc} \equiv H_{abc} + \frac{1}{3M_P}\left(\Omega^{(\mathrm{YM})}_{abc} + \Omega^{(\mathrm{GR})}_{abc}\right) \\ \Omega^{(\mathrm{YM})}_{abc} &\equiv A_{[a}\partial_b A_{c]} + A_{[a}A_b A_{c]} \text{ YM Chern–Simons} \\ \Omega^{(\mathrm{GR})}_{abc} &\equiv \omega_{[a}\partial_b \omega_{c]} + \omega_{[a}\omega_b\omega_{c]} \text{ GR Chern–Simons.} \end{aligned} \tag{2}$$

Compactification details determine the Yang–Mills gauge group, e.g., for compactifications on a Calabi–Yau 6-fold, the Yang–Mills gauge group in four dimensions is E_6.

3. Photon–KR interactions

Here, our focus is on the coupling of photons to KR fields induced by the augmentation (2). For this purpose, we restrict the Yang–Mills Chern–Simons 3-form to the $U(1)_{\mathrm{em}}$ Chern–Simons 3-form $\Omega^{(\mathrm{Max})}_{abc} = M_P^{-1} A_{[a}F_{bc]}$. This implies that for the augmented KR field strength $\tilde{H}_{abc}$ to be invariant under Maxwell $U(1)_{\mathrm{em}}$ gauge transformations $A_a \to A_a^{(\omega)} = A_a + \partial_a\,\omega$, the KR gauge potential must transform under $U(1)_{\mathrm{em}}$ as $B_{ab} \to B_{ab}^{(\omega)} = B_{ab} - \omega F_{ab}$. The effective interaction action is therefore given by

$$S_{\text{KR-Max}} = h\int \phi_{\mathrm{A}} F_{ab}\,{}^*F^{ab}, \quad h \sim M_P^{-1}\,, \tag{3}$$

with the consequent equations of motion and the Bianchi identities,

$$\begin{aligned} &\nabla_a H^{abc} = 0, && \text{(KR EoM)} \\ &\Box_g \phi_{\mathrm{A}} = h F_{ab}{}^* F^{ab} + O(h^2), && \text{(KR Bianchi)} \\ &\nabla_a F^{ab} = h\partial_a\phi_{\mathrm{A}} F^{ab} + O(h^2), && \\ &\nabla_a{}^* F^{ab} = 0. && \end{aligned} \tag{4}$$

In Minkowski space–time, assuming $\phi_{\mathrm{A}} = \phi_{\mathrm{A}}(t)$, $\dot{\phi}_{\mathrm{A}} = f_0$, the Maxwell equation above leads to $\Box\vec{B} = f_0 h\nabla\times\vec{B}$. This means that left-circular and right-circular polarizations of the electromagnetic wave must have different frequencies $\omega_\pm = ck(k \pm hf_0)$. This has been called 'cosmic optical effect (birefringence)', i.e., rotation of the polarization plane through the angle $\Delta\Theta_{\mathrm{cos}} \simeq 2hf_0\Delta t$. Now, observationally, such a birefringence may originate from multiple sources; for instance, intergalactic magnetic fields may lead to Faraday rotation of light from distant stars. However, the optical activity being discussed here has no dependence on the wavelength of the emitted light, so that even if a net birefringence is observed, the achromatic part can be easily separated from the frequency-dependent Faraday rotation.

In a spatially flat Friedmann–Robertson–Walker space–time, $f_0 = f_0(a(t))$ in conformal frame, so that $\Delta\Theta_{\mathrm{cos}} = hf_0(a)\Delta t(z)$ where z is the cosmological redshift, i.e., $\Delta t(z)$ may be called the 'lookback time'. However, because of the inherent suppression in the strength of the signal characterized by $hf_0(a)$ by $M^{P-1}_{\mathrm{Planck}}$, the observed angle of rotation is less than 10^{-9}. We should remark that this optical activity is different from that posited in [6] where an inherent anisotropy is attributed to space–time itself, while here space–time itself is isotropic, and the observed departure from isotropy in the CMB is attributed to its interaction with stringy degrees of freedom.

4. Parity-violating KR–photon interaction

We postulate a new augmented KR field strength beyond the requirements of heterotic string theory,

$$\tilde{H}_{abc} = H_{abc} + hA_{[a}[F_{bc]} + \zeta_- \, {}^*F_{bc]}], \quad \zeta \leqslant O(1). \tag{5}$$

This implies that the Hodge-dual $\phi_A \rightarrow \underbrace{\phi_A}_{P\text{-odd}} + \zeta_- \underbrace{\phi_S}_{P\text{-even}}$. One now has the modified KR-Maxwell action

$$S_{\text{KR-Max}} = \int \mathcal{L}, \; \mathcal{L} = \mathcal{L}_{P\text{-sym}} + \mathcal{L}_{P\text{-vio}}, \tag{6}$$

where

$$\mathcal{L}_{P\text{-sym}} = h[\phi_A F_{ab} \, {}^*F^{ab} + \zeta_- \phi_S F_{ab} F^{ab}],$$
$$\mathcal{L}_{P\text{-vio}} = h[\phi_A F_{ab} F^{ab} + \zeta_- \, \phi_S F_{ab} \, {}^*F^{ab}]. \tag{7}$$

This sort of interaction has the following consequences:

(1) The interaction $\phi_S F \cdot {}^*F$ term violates both P and T; it follows that the ‘arrow of time’ generates P-violation naturally.
(2) In the case of P-violation in weak interactions, C-invariance is important except for the small observed CP-violation. Here the P-violation is charge-blind.
(3) The proposed additional augmentation of the KR field strength can be easily embedded in $N = 1$ SUSY gauge theory: the gauge-kinetic function $\chi(\mathcal{S}) \rightarrow \chi(\mathcal{S}) + \zeta_- \, \tilde{\chi}(i\mathcal{S})$.
(4) For $\phi_S = \phi_S(t)$ in a cosmological context, the fine-structure constant becomes time-dep: $\alpha^{-1} \rightarrow \alpha^{-1} + h\zeta_- \phi_S(t)$. This means that α must have been small earlier.
(5) Such an interaction has the potential for the generation of P-violating CMB polarization anisotropy correlations from P-symmetric correlations [12]

The P-violating CMB polarization anisotropy correlations are generated as follows: recall that the temperature anisotropy is given by

$$\frac{\Delta T}{T}(\vec{n}) = \sum_{l,m} a^T_{lm} \, Y^T_{lm}(\vec{n}),$$

so that temperature correlations (covariances) can be expressed as

$$C^{TT}_l \equiv \langle a^T_{lm} a^T_{lm} \rangle_{\text{univ}}.$$

Similarly, the polarization anisotropy is expressed in terms of the pol tensor $\mathcal{P}_{\alpha\beta}$, $\alpha, \beta = 1, 2$, where

$$\mathcal{P}_{\alpha\beta} \equiv \frac{E_\alpha E^*_\beta}{|E_1|^2 + |E_2|^2} = \frac{1}{2} I_{\alpha\beta} + \xi_1(\sigma_1)_{\alpha\beta} + \xi_2(\sigma_2)_{\alpha\beta} + \xi_3(\sigma_3)_{\alpha\beta}. \tag{8}$$

The Stokes parameter $\xi_{1,3} \in [-1, 1]$ for linear polarization. The circular polarization parameter $\xi_2 = 0$ for Thomson scattering, so that $\mathcal{P}_{\alpha\beta} = \mathcal{P}_{\beta\alpha}$. For an observation direction characterized by the unit angle-vector $\vec{n}$,

$$\mathcal{P}_{\alpha\beta}(\vec{n}) = \mathcal{P}^E_{\alpha\beta}(\vec{n}) + \mathcal{P}^B_{\alpha\beta}(\vec{n}), \quad E \to \text{grad}, \quad B \to \text{curl}. \tag{9}$$

One now resorts to the multipole expansion of the polarization tensor, leading to

$$\mathcal{P}^{(E)}_{\alpha\beta} = \sum_{l,m} a^E_{lm} Y^E_{lm,\alpha\beta}(\vec{n}),$$
$$\mathcal{P}^{(B)}_{\alpha\beta} = \sum_{l,m} a^B_{lm} Y^B_{lm,\alpha\beta}(\vec{n}). \tag{10}$$

Observe that, under spatial parity, $P[Y^E_{lm,\alpha\beta}] = (-)^l = P[Y^T_{lm}]$, $P[Y^B_{lm,\alpha\beta}] = (-)^{l+1}$, so that one now has mixed-parity correlations $C^{TB}_l = 0 = C^{EB}_l$, if temperature or polarization distribution is P-sym which allows $C^{XX}_l \neq 0$, $X = T, E, B$. If the KR field dynamics is appropriate, cosmic birefringence together with P-violating interactions will certainly imply the generation of C^{TB}_l from C^{TE}_l, as demonstrated below.

Observe that the Y^E_{lm} and Y^B_{lm} tensors are orthogonal at every point in sky for the same l, m. Cosmic birefringence rotates the polarization axes equally everywhere, so that E, B polarization modes mix. Thus,

$$C^{TB}_l = C^{TE}_l \sin \Delta\Theta_{\cos},$$
$$C^{EB}_l = C^{EE}_l \sin \Delta\Theta_{\cos},$$
$$\Delta\Theta_{\cos} \sim h\dot{\phi}_{\mathrm{S}}\, \Delta t(z)\ . \tag{11}$$

However, $\dot{\phi}_{\mathrm{S}}$ as determined by the cosmological evolution of $\phi_{\mathrm{S}}(t)$ implies that $\Delta\Theta_{\cos}$ is Planck-suppressed in standard cosmology, and hence immeasurably small. Can it be still within measurable range?

5. Embedding parity-violating coupling of KR field within an RS1 scenario: Antiwarping

The Randall–Sundrum scenario [11] consists of an AdS_5 space–time compactified to $\mathcal{M}_4 \times S^1/Z_2$, where $\mathcal{M}_4$ is the standard four-dimensional (non-compact) space–time. In this scenario, the massless closed string modes characterized by the metric g_{ab} and the two-form Kalb–Ramond field B_{ab} propagate in $D = 5$ bulk space–time, and the standard strong-electroweak theory fields propagate on 'visible' 3-brane. Superheavy gauge fields like those appearing in grand unified theory are usually restricted to a 'hidden' 3-brane.

The five-dimensional compactified space–time is described by the warped metric

$$ds^2_{\mathrm{RS}} = \exp -2\sigma(y)\eta_{ab}\mathrm{d}x^a\mathrm{d}x^b - \mathrm{d}y^2, \tag{12}$$

where $\sigma(y) = |\kappa|y,\ y \in [0, \ell]$ and ℓ is the length of the compact dimension. In $D = 5$, the KR field strength $H_{ABC} = \epsilon_{ABCDE}\partial^D V^E,\ V^E = (V^a, \Phi),\ A, B, ... = 0, ..., 4$. The $D = 5$ KR Bianchi identity (after a standard heterotic string augmentation with Maxwell–Chern–Simons 3-form, can be expressed as

$$\Box_{\mathrm{RS}} V^M = M_5^{-3/2}\, {}^*F^{MNP} F_{NP}, \tag{13}$$

where M_5 is the five-dimensional Planck mass.

Restricting the five-dimensional electromagnetic vector potential to the non-compactified four dimensions, $A_{\mathrm{A}} = A_a(x) \Rightarrow F_{4a} = 0$; the 5-scalar field ϕ satisfies

$$\Box_{\mathrm{RS}}\Phi = M_5^{-3/2}\, {}^*F_{ab}F_{ab}\delta(y - \ell). \tag{14}$$

One now resorts to the eigenfunction expansion of the full 5-scalar ϕ in terms of the eigenfunctions $\chi_n(y)$ on the warped dimension,

$$\Phi(x, y) = \sum_n \Phi_n(x)\chi_n(y). \tag{15}$$

The zero mode fields in four-dimensional space–time satisfy the equation

$$\Box\Phi_0 = \frac{\exp\sigma(\ell)}{M_P}\, F \cdot {}^*F, \qquad M_P \simeq (M_5^3/\kappa)^{1/2}. \tag{16}$$

Similarly, the $D = 4$ Maxwell equation in Minkowski space–time is

$$\partial_a F^{ab} = \frac{\sqrt{6}\exp\sigma(\ell)}{M_P}\, {}^*F^{bc}\, \partial_c\Phi_0. \tag{17}$$

It is clear that this will now cause an optical activity, characterized by the antiwarped coupling constant $\exp\sigma(\ell)$. In fact, if the four-dimensional space–time is chosen to be a spatially flat Friedmann–Robertson–Walker space–time, the angle through which the polarization plane of the electromagnetic wave is rotated is given by

$$\Delta\Theta_{\cos} = |\omega_+ - \omega_-|\Delta t(z) = \frac{2\sqrt{6}\exp\sigma(\ell)}{M_P}\, \Delta t(z). \tag{18}$$

Now for the RS1 scenario to accommodate a light Higgs boson, we must choose $\exp\sigma(\ell) \sim O(10^{16})$! This seems to lead to a sizable increase in the signal of birefringent conversion of the TE correlations to the parity-violating TB correlations, leading to a B-mode polarization of the CMB in far excess of the observed bounds. Will this 'cosmic birefringence' be actually observable? Only the future can tell. If we decompose $\Phi = \Phi_- + \zeta_-\Phi_+\ ,\ P[\Phi_\pm] = \pm$, it is obvious that the generation of C_l^{TB} from C_l^{TE} via

$C_l^{TB} = C_l^{TE} \sin \Delta\Theta_{\text{cos}}$ will be a mechanism to reckon with, in so far as observation of the B mode is concerned. Calculation of the generated parity-violating correlations C_l^{TB} is a project to be undertaken in the near future.

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