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Primordial gravitational waves, BICEP2 and beyond

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Abstract. Observations of the imprints of primordial gravitational waves on the anisotropies in the cosmic microwave background can provide us with unambiguous clues to the physics of the very early Universe. In this brief article, the implications of the detection of such signatures for the inflationary scenario has been discussed.

Keywords. Inflation; primordial gravitational waves.

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1. Introduction

Inflation is currently considered to be the most attractive scenario to describe the origin of perturbations in the early Universe [1,2]. It corresponds to a brief phase of accelerated expansion during the early stages of the radiation-dominated epoch. Inflation is typically achieved using scalar fields. It amplifies the tiny quantum fluctuations present in the scalar fields at the beginning of the epoch and converts them into classical perturbations which, in turn, leave their imprints as anisotropies in the cosmic microwave background (CMB). Over the last decade or so, the anisotropies in the CMB have been measured with ever increasing precision by missions such as WMAP [3–5], Planck [6–8] and BICEP2 [9,10]. These measurements of the CMB anisotropies as well as other cosmological data have led to rather strong constraints on the models of inflation [11–15].

Despite the strong constraints that have emerged, there exist a plethora of inflationary models that remain consistent with the data. The primordial tensor pertubations is expected to play an important role in this regard, as its detection can help in considerably lifting the degeneracy amongst the prevailing models. The announcement of the detection of the so-called B-mode polarization of the CMB and its interpretation as imprints of primordial tensor pertubations by BICEP2 had kindled exactly such a hope [9,10], before it was realized that the signals detected by BICEP2 can be completely attributed to foreground dust [16–19]. In this article, the importance of detection of the primordial tensor perturbations and the powerful role it can play in arriving at a unique model of inflation has been briefly outlined.

This article is organized as follows. In the following section, the classification of perturbations in the early Universe, into scalars, vectors and tensors is discussed. In §3 and 4, the generation of scalar and tensor pertubations during inflation and the corresponding imprints on angular power spectra of anisotropies in the CMB are described. Also, the effects due to weak gravitational lensing on the CMB is discussed. In §5, the implications of the detection of the primordial tensor modes for inflationary models are discussed. In §6, the current status of the original BICEP2 results and the constraints on the tensor-to-scalar ratio from Planck are outlined. The paper concludes in §7 with a very brief discussion on the forthcoming missions to detect the tensor-to-scalar ratio.

2. Classification of perturbations in the early Universe

The perturbations in the space–time (governed by the metric tensor) and matter (described by the stress–energy tensor) can be classified, for instance, based on their transformation properties under rotations on a constant time hypersurface in the Friedmann Universe [1,2]. The perturbations can, evidently, be classified as scalars, vectors and tensors. Scalar perturbations, such as the density and the pressure perturbations, are the dominant ones, lead to the primary imprints on the CMB and the large-scale structure. The tensor perturbations are essentially gravitational waves, which, as is well known, can be generated even in the absence of sources. It is difficult to sustain vector perturbations in an expanding Universe, and they decay rather rapidly. More importantly, they are simply not generated during an epoch such as inflation driven by scalar fields. As we shall discuss in some detail below, on large scales, it is the tensor perturbations that are essentially responsible for the B-mode polarization in the CMB. It is important to note that, at the linear order in the perturbations, the scalars and the tensors evolve independently, and leave their characteristic signatures on the CMB.

3. Generation of the perturbations during inflation

As we mentioned above, it is the quantum fluctuations associated with the scalar field driving inflation that are responsible for the generation of perturbations. The background evolution during inflation is determined by the inflationary model and the potential (and other functions in the case of non-canonical fields) that describes the scalar field. The background evolution in turn governs the behaviour of the scalar and the tensor perturbations. The perturbations are evolved from well-motivated, Minkowski-like, initial conditions (viz., those corresponding to the Bunch–Davies vacuum [20]), which are imposed when the modes are well inside the Hubble radius. In most single field models of inflation, the amplitudes of the scalar and the tensor perturbations freeze soon after they leave the Hubble radius. The perturbation spectra are evaluated at sufficiently late times when the amplitude of the modes have turned constant.

It is well known that many single-field models lead to inflation of the slow roll type. In the case of slow roll inflation driven by a canonical scalar field, it can be shown that the scalar and the tensor power spectra $\mathcal{P}_{\rm S}(k)$ and $\mathcal{P}_{\rm T}(k)$ can be expressed as follows [2]:

$$\mathcal{P}_{\rm S}(k) = \frac{H^2}{8\,\pi^2\, M_{\rm Pl}^2\,\epsilon_1}\left[1 - 2\,\epsilon_1\,(C+1) - \epsilon_2\,C - (2\,\epsilon_1 + \epsilon_2)\,\ln\left(\frac{k}{k_*}\right)\right], \quad (1)$$

$$\mathcal{P}_{\rm T}(k) = \frac{2H^2}{\pi^2\, M_{\rm Pl}^2}\left[1 - 2\,\epsilon_1\,(C+1) - 2\,\epsilon_1\,\ln\left(\frac{k}{k_*}\right)\right], \quad (2)$$

where H is the Hubble scale during inflation, $M_{\rm Pl} = (8\pi G)^{-1/2}$ denotes the reduced Planck mass and $C = \gamma_{\rm E} - 2 + \ln 2$, with $\gamma_{\rm E}$ being the Euler constant. The quantities ϵ_1 and ϵ_2 are the first two slow roll parameters, while k_* is the pivot scale at which the amplitude of the scalar power spectrum is often quoted. The scalar and the tensor spectral indices are defined as

$$n_{\rm S} = 1 + \frac{{\rm d}\ln \mathcal{P}_{\rm S}(k)}{{\rm d}\ln k}, \tag{3}$$

$$n_{\rm T} = \frac{{\rm d}\ln \mathcal{P}_{\rm T}(k)}{{\rm d}\ln k}. \tag{4}$$

During slow roll, it is clear from the above expressions for the power spectra that the scalar and the tensor spectral indices are given by $n_{\rm S} = 1 - 2\epsilon_1 + \epsilon_2$ and $n_{\rm T} = -2\epsilon_1$. As the parameters ϵ_1 and ϵ_2 are expected to be much smaller than unity, these results imply that slow roll inflation leads to nearly scale-invariant primordial spectra. Another important observable quantity is the tensor-to-scalar ratio r which is defined as

$$r(k) \equiv \frac{\mathcal{P}_{\rm T}(k)}{\mathcal{P}_{\rm S}(k)}. \tag{5}$$

It is useful to note from the above expressions for the power spectra that, if the weak scale dependence is ignored, one has $r \simeq 16\epsilon_1 = -8n_{\rm T}$, a relation that is often referred to as the consistency condition governing slow roll inflation in single field models [2].

4. Imprints of the primordial perturbations on the CMB

As we mentioned, the perturbations generated during inflation leave their imprints as anisotropies in the CMB. The CMB is a vestige of the radiation-dominated epoch, a period when the radiation was strongly coupled to matter. It streams to us virtually unimpeded from the scattering surface when it had last interacted with the matter, thereby carrying with it pristine information about these early epochs. The CMB is expected to be polarized due to the interactions with the electrons through Compton scattering (Thomson scattering, to be precise) prior to the epoch of decoupling. It is well known that Compton scattering produces polarization only when the incident field has a quadrupole moment [1]. But, the tight coupling between the electrons and the photons before decoupling leads to only a small quadrupole. This implies that the anisotropies in polarization in the CMB can be expected to be much smaller than the signal in the temperature.

Recall that, a propagating, plane and monochromatic electromagnetic wave, will in general, be elliptically polarized. Such a polarized wave is often described in terms of the so-called Stokes' parameters Q, U and V, with the intensity, say, I, of the radiation being given by $I^2 = Q^2 + U^2 + V^2$ (see, for instance, ref. [21]). The quantity V characterizes the circularity parameter that measures the ratio of the principal axes of the ellipse. The wave is said to have left- or right-handed polarization, if V is positive or negative, with a vanishing V corresponding to the linear polarization. The parameters Q or U determine the orientation of the ellipse.

Compton scattering leads to linear polarization. Apart from Compton scattering at the epoch of decoupling, the CMB photons are also polarized by weak gravitational lensing due to the intervening clustered matter, as they propagate towards us (however, gravitational lensing does not alter the linear nature of polarization). In the case of linear polarization, we require only the Stokes' parameters Q and U to characterize the electromagnetic wave. These two parameters can be thought of as the components of a symmetric and trace-free second rank tensor, and expressed in terms of two new quantities, say, E and B, as follows [22]:

$$\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix} \propto \left(\partial_i\, \partial_j - \frac{1}{2}\, \delta_{ij}\, \nabla^2\right) E + \epsilon_{k\,(i}\, \partial_{j)}\, \partial_k B.$$

Essentially, the two-dimensional vector describing the linearly polarized electromagnetic wave has been decomposed, using the conventional Helmholtz theorem, into a part involving the gradient of a scalar (viz. E) and a divergence-free part, involving the curl (viz. B). Clearly, while E is scalar, B is pseudoscalar.

The above decomposition of the CMB polarization in terms of the E and B modes proves to be convenient for studying the effects of the scalar and the tensor perturbations on the CMB anisotropies. It can be shown that, while the E modes are generated by both scalars and tensors, the B modes are produced only by the tensors [22]. Actually, as we mentioned, the polarization of the CMB is also affected due to gravitational lensing by intervening matter as it propagates towards us. Gravitational lensing causes shear and, due to this reason, it converts the E modes generated by the scalar perturbations into B modes. Because of the fact that gravitational lensing operates on small scales, it is expected to generate B modes only on the large multipole moments of the CMB corresponding to scales smaller than the horizon size at decoupling. In contrast, the amplitudes of the tensor perturbations remain constant once they leave the Hubble radius during inflation, and they decay in amplitude only after they re-enter the Hubble radius at late times. As a result, the strongest imprints of the primordial tensor perturbations on the CMB correspond to those modes which are outside the Hubble radius before decoupling. In figure 1, the theoretically computed CMB angular temperature, cross-correlation and the polarization power spectra generated due to the scalar and the tensor pertubations have been plotted as a function of the multipoles. The angular power spectrum in the B mode produced due to gravitational lensing has also been illustrated. The expected, complete B-mode power spectrum will be a sum of the contribution due to the primordial tensor perturbations and due to weak gravitational lensing. It is clear from the figure that, for a suitably large value of the tensor-to-scalar ratio r, the contribution due to the primordial tensor perturbations dominate at small multipoles. Therefore, the detection of B-mode polarization of the CMB on large scales is considered to be an unambiguous signature of the primordial tensor perturbations.

5. Observational constraints and implications for inflation

In order to arrive at constraints on the inflationary models, one often assumes the primordial spectra to be of the power law form, i.e., $\mathcal{P}_{\rm S}(k) = \mathcal{A}_{\rm S}\,(k/k_*)^{n_{\rm S}-1}$ and $\mathcal{P}_{\rm T}(k) = \mathcal{A}_{\rm T}\,(k/k_*)^{n_{\rm T}}$, with $\mathcal{A}_{\rm S}$, $\mathcal{A}_{\rm T}$, $n_{\rm S}$ and $n_{\rm T}$ being constants. It should be evident that, with suitable parametrization, these template spectra roughly correspond to the spectra (1) and (2) generated in slow roll inflation. Note that the above power spectra contain four parameters, viz., $\mathcal{A}_{\rm S}$, $\mathcal{A}_{\rm T}$, $n_{\rm S}$ and $n_{\rm T}$. Of these four parameters, as the amplitude of the CMB anisotropies is measured accurately, the scalar amplitude $\mathcal{A}_{\rm S}$ proves to be well constrained

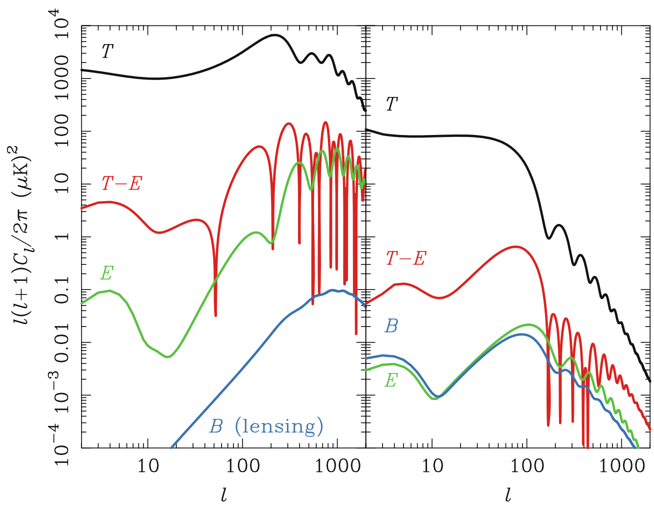

Figure 1. The different theoretically computed, CMB angular power and cross-correlation spectra – temperature T (in black), E (in green), B (in blue), and T–E (in red) – arising due to the scalars (on the left) and the tensors (on the right) corresponding to a tensor-to-scalar ratio of $r = 0.24$. The B-mode spectrum induced by weak gravitational lensing has also been shown (in blue) in the panel on the left. (This figure has been reproduced with permission from ref. [22].)

and is usually referred to as COBE normalization [23]. One further neglects the small tensor tilt and assumes that the tensor spectrum is strictly scale-invariant (which is nearly true in slow roll inflation). Of the two remaining parameters n_{S} and $\mathcal{A}_{\mathrm{T}}$, one considers the tensor-to-scalar ratio r evaluated at a given scale instead of the tensor amplitude $\mathcal{A}_{\mathrm{T}}$ and arrives at joint constraints on the parameters in the n_{S}–r plane. These constraints help us understand as to how the various inflationary models perform against the data.

The constraints from the Planck 2013 data had suggested that $n_{\mathrm{S}} \simeq 0.965$ [7,11]. Moreover, the data had indicated only an upper bound on the tensor-to-scalar ratio r, viz., that $r \lesssim 0.12$. In contrast, the BICEP2 data, from the observations of the B-mode polarization of the CMB, had indicated that $r \simeq 0.2$ (in this context, see figures 2 and 3). Further, from the data, it had been concluded that a vanishing r was ruled out at greater than 5-σ [9,10]. Such a claim of the detection of the primordial tensor modes by the BICEP2 team had tremendous implications for inflation.

In the case of canonical models, it can be shown that, during slow roll inflation, the tensor power spectrum can be written in terms of the potential $V(\phi)$ as [2]

$$\mathcal{P}_{\mathrm{T}}(k) \simeq \frac{2}{3\pi^2} \left(\frac{V}{M_{\mathrm{Pl}}^4}\right)_{k=aH}, \tag{6}$$

with the quantity on the right-hand side to be evaluated when the modes of interest leave the Hubble radius. Therefore, accurate knowledge of the scalar amplitude $\mathcal{A}_{\mathrm{S}}$ and the detection of the tensor-to-scalar ratio r imply that we can express the energy scale of inflation as [24]

$$V^{1/4} \simeq \left(\frac{3\,\pi^2\, r\, \mathcal{A}_{\mathrm{S}}}{2}\right)^{1/4} M_{\mathrm{Pl}} \simeq 3.2 \times 10^{16}\, r^{1/4}\,\mathrm{GeV}. \tag{7}$$

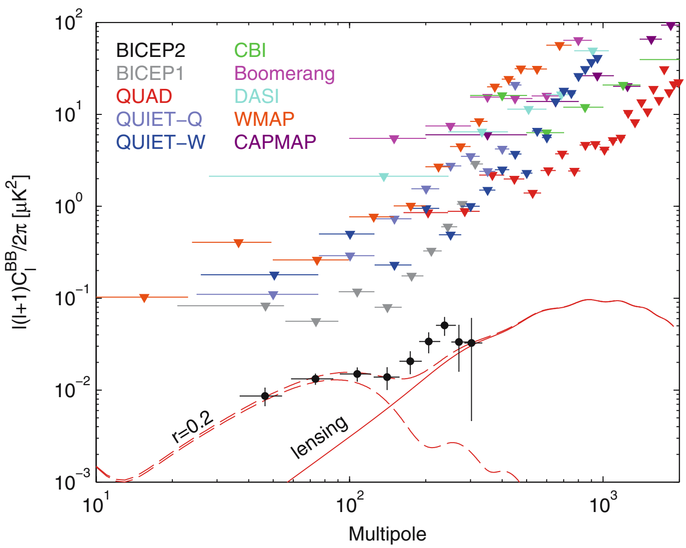

Figure 2. The observations of the B-mode angular power spectrum of the CMB by BICEP2 (the black dots with error bars). Also indicated in the figure are the upper limits from the various earlier experiments. The dashed and the solid red curves represent the contributions to B-mode angular power spectrum due to the primordial tensor perturbations (corresponding to $r = 0.2$) and weak gravitational lensing, respectively. (This figure has been reproduced from ref. [10].)

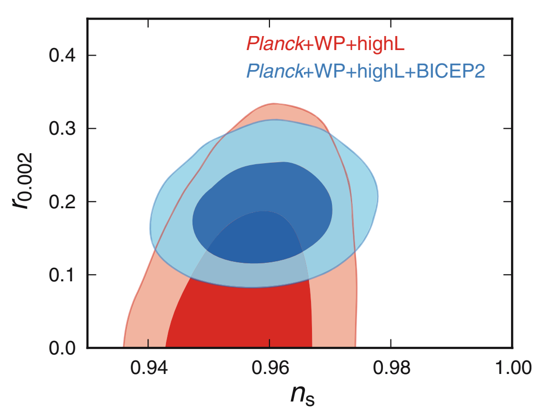

Figure 3. Constraints in the n_s–r plane from the BICEP2 and the Planck 2013 data. The lower bound on r (outlined by the blue regions) indicates a confirmed detection of the primordial tensor perturbations. It should be clarified that, actually, these constraints have been arrived at by also allowing for 'running' of the scalar power spectrum (i.e., a small variation of the spectral index n_s with wavenumber). (This figure has been reproduced from ref. [10].)

For instance, if $r \simeq 0.2$, $V^{1/4} \simeq 2.1 \times 10^{16}$ GeV. In other words, the detection of the primordial tensor modes by BICEP2 at the level of $r = 0.2$ had unambiguously suggested that inflation took place around the GUT scale. The announcement of the BICEP2 results had immediately provided hope that further observations may quickly point to a small class or even a unique model of inflation. However, as we shall discuss in the following section, certain doubts were cast about the BICEP2 results and conclusions within a couple of months of their announcement.

6. BICEP2 versus Planck

The Planck mission was to detect CMB anisotropies with exquisite precision and it was expected to be sensitive to a tensor-to-scalar ratio of $r \simeq 0.1$. Planck was a satellite-based mission and had carried out a survey of the entire sky. As we mentioned, the first year results from Planck had arrived at the upper bound on the tensor-to-scalar ratio of $r \lesssim 0.12$. The BICEP2 mission's primary goal was to measure the tensor-to-scalar ratio. It had focussed on a narrow region of the sky where the contamination due to possible foregrounds was supposed to be very small. As we discussed, initially, the BICEP2 team had announced the detection of primary tensor contribution corresponding a tensor-to-scalar ratio of $r \simeq 0.2$. Moreover, the BICEP2 team had concluded that the tensor-to-scalar ratio was non-zero at more than 5-σ. This was simply astounding, as it had suggested that we had detected direct signatures of physics operating at energy scales of $\mathcal{O}(10^{16})$ GeV. It was also highly promising as it strongly indicated that the primordial tensor modes can aid us in quickly arriving at a rather small class of viable models of inflation. It had been known that foreground galactic dust can create B modes and, hence, soon after the announcement from BICEP2, concerns were raised if the BICEP2 results can be attributed to the foreground dust [16,17]. In fact, preliminary efforts had pointed in such a direction [18]. A recent joint analysis by the Planck and the BICEP2 teams clearly attributes the signals detected by BICEP2 to dust [19]. Further, the Planck 2015 results point to only an upper bound of $r \lesssim 0.11$ [8,12]. In other words, the primordial tensor modes remain elusive and the search for them continues.

7. Beyond BICEP2

In this brief article, the tremendous implications that the detection of primordial gravitational waves could have on our understanding of the physics of the early Universe has been discussed. Not only will its detection immediately point to the energy scale of inflation, it can also help us quickly converge on a small class of viable models. For instance, the detection of imprints of the primordial tensor modes can help us examine if the tensor consistency condition holds true, with its violation possibly pointing to inflation driven by multiple scalar fields [25]. The BICEP2 results had flattered to deceive. But, the flurry of activity following the announcement of the original BICEP2 results has indicated the significant impact that the detection of the primordial tensor modes can have on our understanding of the physics of the early Universe. With many forthcoming missions such BICEP3 [26], EPIC [27], PRISM [28], LiteBIRD [29] and COrE [30] being planned to arrive at unprecedented constraints on r (roughly, these missions are expected to be sensitive to $r \simeq 10^{-3}$), the next decade seems to hold a lot of promise.

References

[1] E W Kolb and M S Turner, *The early Universe* (Addison-Wesley, Redwood City, California, 1990)
S Dodelson, *Modern cosmology* (Academic Press, San Diego, USA, 2003)
V F Mukhanov, *Physical foundations of cosmology* (Cambridge University Press, Cambridge, England, 2005)
S Weinberg, *Cosmology* (Oxford University Press, Oxford, England, 2008)
R Durrer, *The cosmic microwave background* (Cambridge University Press, Cambridge, England, 2008)
D H Lyth and A R Liddle, *The primordial density perturbation* (Cambridge University Press, Cambridge, England, 2009)
P Peter and J-P Uzan, *Primordial cosmology* (Oxford University Press, Oxford, England, 2009)
H Mo, F V d Bosch and S White, *Galaxy formation and evolution* (Cambridge University Press, Cambridge, England, 2010)
D Baumann and L McAllister, *Inflation and string theory*, arXiv:1404.2601 [hep-th]

[2] H Kodama and M Sasaki, *Prog. Theor. Phys. Suppl.* **78**, 1 (1984)
V F Mukhanov, H A Feldman and R H Brandenberger, *Phys. Rep.* **215**, 203 (1992)
J E Lidsey, A Liddle, E W Kolb, E J Copeland, T Barreiro and M Abney, *Rev. Mod. Phys.* **69**, 373 (1997)
A Riotto, arXiv:hep-ph/0210162
W H Kinney, astro-ph/0301448
J Martin, *Lect. Notes Phys.* **738**, 193 (2008)
J Martin, *Lect. Notes Phys.* **669**, 199 (2005)
J Martin, *Braz. J. Phys.* **34**, 1307 (2004)
B Bassett, S Tsujikawa and D Wands, *Rev. Mod. Phys.* **78**, 537 (2006)
W H Kinney, arXiv:0902.1529 [astro-ph.CO]
L Sriramkumar, *Curr. Sci.* **97**, 868 (2009), arXiv:0904.4584 [astro-ph.CO]
D Baumann, arXiv:0907.5424v1 [hep-th]
J Martin, arXiv:1502.05733 [astro-ph.CO]

[3] J Dunkley *et al*, *Astrophys. J. Suppl.* **180**, 306 (2009)
E Komatsu *et al*, *Astrophys. J. Suppl.* **180**, 330 (2009)

[4] D Larson *et al*, *Astrophys. J. Suppl.* **192**, 16 (2011)
E Komatsu *et al*, *Astrophys. J. Suppl.* **192**, 18 (2011)

[5] C L Bennett *et al*, *Astrophys. J. Suppl.* **208**, 20 (2013)
G Hinshaw *et al*, *Astrophys. J. Suppl.* **208**, 19 (2013)

[6] P A R Ade *et al*, arXiv:1303.5075 [astro-ph.CO]

[7] P A R Ade *et al*, arXiv:1303.5076 [astro-ph.CO]

[8] P A R Ade *et al*, arXiv:1502.01589 [astro-ph.CO]

[9] P A R Ade *et al*, arXiv:1403.4302 [astro-ph.CO]

[10] P A R Ade *et al*, *Phys. Rev. Lett.* **112**, 241101 (2014)

[11] P A R Ade *et al*, arXiv:1303.5082 [astro-ph.CO]

[12] P A R Ade *et al*, arXiv:1502.02114 [astro-ph.CO]

[13] J Martin, C Ringeval and V Vennin, arXiv:1303.3787 [astro-ph.CO]

[14] J Martin, C Ringeval, R Trotta and V Vennin, *J. Cosmol. Astropart. Phys.* **1403**, 039 (2014)

[15] J Martin, C Ringeval and V Vennin, arXiv:1407.4034 [astro-ph.CO]

[16] M J Mortonson and U Seljak, *J. Cosmol. Astropart. Phys.* **1410**, 035 (2014)

[17] R Flauger, J C Hill and D N Spergel, *J. Cosmol. Astropart. Phys.* **1408**, 039 (2014)

[18] R Adam *et al*, arXiv:1409.5738 [astro-ph.CO]

[19] P A R Ade *et al*, arXiv:1502.00612 [astro-ph.CO]
[20] T Bunch and P C W Davies, *Proc. R. Soc. London A* **360**, 117 (1978)
[21] G B Rybicki and A P Lightman, *Radiative processes in astrophysics* (Wiley-Interscience, New York, 1979)
[22] A Challinor, arXiv:1210.6008 [astro-ph.CO]
[23] E F Bunn, A R Liddle and M J White, *Phys. Rev. D* **54**, R5917 (1996)
[24] D H Lyth, *Phys. Rev. Lett.* **78**, 1861 (1997)
[25] L C Price, H V Peiris, J Frazer and R Easther, *Phys. Rev. Lett.* **114**, 031301 (2015)
[26] Z Ahmed *et al*, arXiv:1407.5928 [astro-ph.IM]
[27] J Bock *et al*, arXiv:0805.4207 [astro-ph]
[28] P Andre *et al*, *J. Cosmol. Astropart. Phys.* **1402**, 006 (2014)
[29] T Matsumura *et al*, arXiv:1311.2847 [astro-ph.IM]
[30] See, http://www.core-mission.org/